Question 1 [6 points]

HealthLink Supplies, a well-known distributor of medical equipment, currently operates a central warehouse in Ottawa, Ontario stocked with 2,900,000 units of medical supplies. In an effort to optimize their nationwide distribution network, they are considering the feasibility of establishing regional warehouses. This decision-making process involves selecting suitable locations from a pool of 27 potential sites, identified as $i = 1, 2, ..., 27$. For each of these potential sites, there exists a cost associated with establishing a warehouse, denoted as f_i. In addition, there is a another cost, represented as c_i, incurred for transporting each unit from the central Ottawa warehouse to site i. HealthLink Supplies' primary goal is to strategically position these regional warehouses to ensure timely and efficient deliveries to healthcare facilities across Canada. To achieve this, the plan for establishing the regional warehouses must meet the following conditions:

- No location can store more than 375,000 or less than 175,000 units (if chosen).
- At least four locations must be chosen amongst sites 6-16.
- No more than 6 locations must be chosen amongst the even numbered sites.
- If location 1 or 2 is chosen then the sites 5, 6, and 7 cannot be chosen.
- If any location from 19-22 is chosen, locations 24, 26, and 27 cannot be chosen.
- If any location from 1-5 is chosen, at least one odd site from 21-27 must be chosen.
- The number of locations chosen in 1-14 must equal the number of locations chosen in 15-27.
- The sum of all units at sites 1-9 and 19-27 must be equal.

Develop a mixed-integer linear programming (MILP) model with the objective of minimizing the total cost (see file *costs.csv*) associated with the diversification of the supply chain for HealthLink Supplies.

(a) What type of costs does the objective function contain?

(b) Write down the constraints associated with linking the integer and binary decision variables.

(c) Write down the constraint associated with: *If location 1 is chosen then site 5 cannot be.*

(d) Write down the constraint associated with: *If any location from 19-22 is chosen, 27 cannot be.*

(e) How many decision variables are in the MILP formulation?

(f) After solving the MILP model, what is the optimal cost?

(g) In the optimal solution, how many warehouses will be selected?

(h) Considering the significant expense associated with opening a warehouse, when would it make sense for HealthLink to open a facility but not utilize its full capacity?

(i) In reality, what does it indicate when the per unit cost of transporting equipment from the central Ottawa warehouse to each location varies as a function of the site number?

(j) The model attempts to minimize the cost of opening warehouses and transporting equipment from Ottawa to each of the chosen sites. Comment on whether you think this is an appropriate objective if the company's main concern is to geographically diversify their inventory.

Question 2 [6 points]

Baffin Bay company operates a network of n retail stores, identified as $i = 1, ..., n$. As the summer selling season approaches, each store must determine the optimal order quantity y_i. Demand at each retailer is independent and represented by the random variable D_i. The product's selling price is a fixed $49.99 and the unit ordering cost is $24.44, with these values being consistent across all stores. Additionally, there is no salvage value for unsold inventory at the end of the summer.

Retailers may not be entirely independent. In the summer selling season, if retailer j experiences a stockout, retailer i (where $i \neq j$) can provide surplus inventory to fulfill this demand, incurring an additional cost of c_{ij} to ship the product from one store to another. To understand the dynamics of this application, consider a problem with two time periods. In the first period, each retailer places an order with Baffin Bay's supplier. In the second period, retailers receive their order and commence fulfilling customer demands. Demand fulfillment at retailer i can be achieved either from the stock they ordered, denoted as y_i, or by receiving products transshipped from another retailer j, where x_{jik} is the amount sent from retailer j to retailer i in scenario k. Note that in scenario k, retailer i can also send some of their own inventory to help satisfy the demand of retailer j, i.e., x_{ijk}. Regardless, the retailers cannot place any additional orders with the supplier during this period.

(a) When determining the order quantity for each retailer, it's important not to order too much (unsold inventory cost at the end of period two) and not to order too little (opportunity cost of excess inventory left over at the end of period two). Write down these per unit costs.

(b) Suppose that no transshipment in the second stage is allowed. Using information in the accompanying file (the demand distribution at each retail store), create a stochastic program using a Monte Carlo simulation of 50 trials with 100 scenarios per trial, to determine the optimal order quantity for each store. What is the average cost associated with the optimal plan? *Hint: the objective function should minimize the sum of the two costs from part (a).*

(c) Now assume that transshipment between retail stores is allowed in the second stage. Using information in the accompanying files - the demand distribution at each retail store (*distributions.csv*) and the cost associated with sending (rows) a single unit of inventory from one retail store to (columns) another retail store (*cost_matrix.csv*) - create a stochastic program using a Monte Carlo simulation of 50 trials with 100 scenarios per trial, to determine the optimal order quantity for each store. What is the average cost associated with the optimal plan?

(d) Why does the model that allows transshipment lead to a reduction in expected costs?

(e) What is the expected value of perfect information (EVPI) with transshipment? *Hint: you do not need to formulate an optimization problem or solve any computational model to answer this.*

(f) Using a sample size of 500 scenarios and a single trial, compute the value of the stochastic solution (VSS) with transshipment. That is, find the EEV and compute the VSS?

(g) What managerial intuition does the EVPI and VSS provide in this context?