**Project Summary**

*The goal of the project is to illustrate application of the material learned in Math 23C to the real-world financial data. When possible, we tried to make sure that the application is sound from the point of view of financial theory. However, the focus of the project was on the application of the tools rather than on making a breakthrough in the finance.*

**Data**

For the analysis, we downloaded data on VIX Index (Volatility Index of Chicago Board Option Exchange) and VIXY, an ETF that is supposedly linked to the performance of the index. The data represents daily prices (Open, High, Low, and Close) for both instruments for the period 2007-2020.

From the raw data we calculated daily and monthly returns, correlation of returns between VIX and VIXY, volatility of returns (standard deviation), and “beta” coefficient[[1]](#footnote-1). We also created two categorical variables to perform permutation tests, and to illustrate the use of boxplots.

**Research questions**

By applying the techniques we learned in class, we tried to answer the following questions

1. Does the tracking performance of VIXY differ for volatile and calm market regimes?[[2]](#footnote-2)
2. Are the returns of VIX Index normally distributed?
3. Can we model volatility of VIX index using some sort of well-defined distribution?
4. How we can apply Lagrange interpolation to forecast the trend of the index? Does it yield acceptable results?
5. How we can model VIX prices using the information from Lagrange interpolation and the distribution of volatility?
6. How we can apply Fourier analysis to the time series of VIX prices?

**Key findings**

VIXY tracking performance is generally subpar (low correlation with VIX Index returns, beta decay, and low beta (0.6)). The piece of good new is that the beta and the correlation increases in volatile market regimes.

The distribution of VIX returns is not well-behaved (large kurtosis and positive skewness). We tried several methods to fit the distribution (e.g. normal, skewed normal, Cauchy, re-centering distribution and fitting gamma and beta distributions, BoxCox transformation and fitting normal distribution). None of the attempts was acceptable after performing Chi-Squared test.

Application of Lagrange interpolation (extrapolation) to the moving average returns yield reasonable results in normal market conditions but tends to overshoot in the periods of exponential growth.

We can use historic distribution to model future index values, but the confidence interval is rather wide.

Fourier analysis allows modelling longer-term behavior of the index: expansion and contraction. However, the index returns are too noisy to allow modelling daily prices well.

1. Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), pp.425-442. [↑](#footnote-ref-1)
2. For motivation, see https://www.barrons.com/articles/no-your-etf-doesnt-track-the-vix-volatility-index-and-here-are-the-numbers-1403010972 [↑](#footnote-ref-2)