

Media over QUIC relay topology optimization

1 MoQ relay topology optimization for a single track

1.1 Motivation

- Media over QUIC uses a full-mesh of relays to transmit user tracks from a single publisher to multiple subscribers.
- This works well on small networks and provides small-enough delays, but generates large traffic that might be expensive in large deployments.

1.2 High-level idea

- Given a MoQ relay network, link usage costs and delays, a single media track, set of streams between the publisher and its subscribers, a delay budget for each stream, assign a relay topology to this track so that
 1. stream delays are below their delay budget, and
 2. the link usage cost is minimal in the relay topology.

1.3 Problem formulation

- Instance:
 - relay network: $G(V, E)$, with $v \in V$ being the relays and $e \in E$ are the links between the relays
 - link costs: $c : E \rightarrow \mathbb{N}$
 - link delays: $d : E \rightarrow \mathbb{N}$
 - a publisher: $P \in V$
 - set of subscribers: $\forall s \in S : s \in V$ (note: these are relays that are subscribing to the track)
 - stream delay budget: $D_s \in \mathbb{N}$
 - streams/flows $f \in F$:
 - * each with path from publisher to subscriber across G , and
 - * each with a vector R_f with a size of relays in the stream path, which encodes roles of the participating relays as:
$$\begin{cases} -1, & \text{if node is a } \textit{publisher}, \\ +1, & \text{if node is a } \textit{subscriber}, \\ 0, & \text{otherwise.} \end{cases}$$
- Solution: an assignment of track topology: $G_t(V_t, E_t) : V_t \subset V, E_t \subset E$
- Measure:
 - the sum of link costs along the track topology:

$$\sum_{e \in E_t} c(e)$$

1.4 LP without the delay constraints

- new variables:
 - $c_{ij} : (i, j) \in E$, link cost of (i, j)
 - $x_{ij}^f : (i, j) \in E, f \in F$, transmission bitrate of stream f on link (i, j)
 - $y_{ij} : (i, j) \in E$, link usage of the track on (i, j)

- the LP:

$$\begin{aligned}
\min \quad & \sum_{(i,j) \in E} c_{ij} y_{ij} \\
& y_{ij} \geq x_{ij}^f & \forall (i,j) \in E, \forall f \in F \\
& \sum_{j:(j,i) \in E} x_{ji}^f - \sum_{j:(i,j) \in E} x_{ij}^f = R_i^f & \forall i \in V, \forall f \in F \\
& x_{ij}^f \geq 0 & \forall (i,j) \in E, \forall f \in F \\
& y_{ij} \geq 0 & \forall (i,j) \in E
\end{aligned}$$

1.5 LP with delay constraints

- new variables:
 - $c_{ij} : (i,j) \in E$, link cost of (i,j)
 - $x_{ij}^f : (i,j) \in E, f \in F$, transmission bitrate of stream f on link (i,j)
 - $y_{ij} : (i,j) \in E$, link usage of track on (i,j)
 - $z_{ij}^f \in 0, 1$: link is in the path of stream f
- the LP:

$$\begin{aligned}
\min \quad & \sum_{(i,j) \in E} c_{ij} y_{ij} \\
& y_{ij} \geq x_{ij}^f & \forall f \in F \\
& \sum_{j:(j,i) \in E} x_{ji}^f - \sum_{j:(i,j) \in E} x_{ij}^f = R_i^f & \forall i \in V, \forall f \in F \\
& x_{ij}^f \geq 0 & \forall (i,j) \in E, \forall f \in F \\
& y_{ij} \geq 0 & \forall (i,j) \in E \\
& z_{ij}^f * M \geq x_{ij}^f & \forall (i,j) \in E, \forall f \in F \\
& \sum_{i,j \in E} z_{ij}^f * d_{ij} \leq D^f & \forall f \in F
\end{aligned}$$

1.6 Notes

- Video-on-Demand and similar latency-insensitive use-cases might work without delay budget
- similar topics:
 - multicast routing literature
 - [minimum Steiner tree](#)

2 MoQ relay topology optimization with multiple tracks

2.1 High-level idea

- Given a MoQ relay network, link usage costs and delays, a set of media tracks, each with its set of streams between a publisher and its subscribers, a delay budget for each stream, assign a relay topology to each track so that
 1. all stream delays are below their delay budget, and
 2. the link usage cost is minimal along each track relay topology.

2.2 Problem formulation

- Instance:
 - relay network: $G(V, E)$, with $v \in V$ being the relays and $e \in E$ are the links between the relays
 - link costs: $c : E \rightarrow \mathbb{N}$
 - link delays: $d : E \rightarrow \mathbb{N}$
 - set of tracks: $t \in T$
 - track publishers: $P^t \in V, \forall t \in T$
 - set of subscribers: $\forall s \in S^t : s \in V$ (note: these are relays that are subscribing to a track)
 - stream delay budgets : $D_s^t \in \mathbb{N} : \forall t \in T$
 - track streams/flows $f \in F^t, \forall t \in T$:
 - * each with path from publisher to subscriber across G , and
 - * each with a vector R_f with a size of relays in the stream path, which encodes roles of the participating relays as:

$$\begin{cases} -1, & \text{if node is a publisher,} \\ +1, & \text{if node is a subscriber,} \\ 0, & \text{otherwise.} \end{cases}$$
- Solution: an assignment of each track topology G_t to each $t \in T$ track: $G_t(V_t, E_t) : V_t \subset V, E_t \subset E, \forall t \in T$
- Measure:
 - the sum of link costs along each track topology:

$$\sum_{t \in T} \sum_{e \in E_t} c(e) \quad (1)$$

2.3 LP without the delay constraints

- new variables:
 - $c_{ij} : (i, j) \in E$, link cost of (i, j)
 - $x_{ij}^f : (i, j) \in E, f \in F^t, t \in T$, transmission bitrate of stream f of track t on link (i, j)
 - $y_{ij}^t : (i, j) \in E, t \in T$, link usage of track t on (i, j)
- the LP:

$$\begin{aligned} \min \quad & \sum_{t \in T} \sum_{(i,j) \in E} c_{ij} y_{ij}^t \\ & y_{ij}^t \geq x_{ij}^f \quad \forall (i, j) \in E, \forall f \in F^t : \forall t \in T \\ & \sum_{j:(j,i) \in E} x_{ji}^f - \sum_{j:(i,j) \in E} x_{ij}^f = R_i^f \quad \forall i \in V, \forall f \in F^t : \forall t \in T \\ & x_{ij}^f \geq 0 \quad \forall (i, j) \in E, \forall f \in F^t : \forall t \in T \\ & y_{ij}^t \geq 0 \quad \forall (i, j) \in E, \forall t \in T \end{aligned}$$

2.4 LP with delay constraints

- new variables:
 - $c_{ij} : (i, j) \in E$, link cost of (i, j)
 - $x_{ij}^f : (i, j) \in E, f \in F^t, t \in T$, transmission bitrate of stream f of track t on link (i, j)
 - $y_{ij}^t : (i, j) \in E, t \in T$, link usage of track t on (i, j)
 - $z_{ij}^f \in 0, 1$: link is in the path of stream f
- the LP:

min

$$\sum_{t \in T} \sum_{(i,j) \in E} c_{ij} y_{ij}^t$$

$$\begin{aligned} y_{ij}^t &\geq x_{ij}^f \\ \sum_{j:j,i \in E} x_{ji}^f - \sum_{j:i,j \in E} x_{ij}^f &= R_i^f \\ x_{ij}^f &\geq 0 \\ y_{ij}^t &\geq 0 \\ z_{ij}^f * M &\geq x_{ij}^f \\ \sum_{i,j \in E} z_{ij}^f * d_{ij} &\leq D^f \end{aligned}$$

$$\forall f \in F^t : \forall t \in T$$

$$\forall i \in V, \forall f \in F : \forall t \in T$$

$$\forall (i,j) \in E, \forall f \in F^t : \forall t \in T$$

$$\forall (i,j) \in E, \forall t \in T$$

$$\forall (i,j) \in E, \forall f \in F^t : \forall t \in T$$

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