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Deconvolution of Galaxy Images

Abstract

Image denoising and deblurring is an issue that has been tackled for long time. Deconvolution is a problem of reversing a convolution. Deconvolution methods help in denoising and deblurring images to produce clearer ones. We will use Low-Rank method and compare it to the sparse method.

Problem

In astronomical image processing, removing the noise introduced by the Point Spread Function (PSF) is a crucial problem. Due to the noise, the deconvolution becomes a hard task that needs regularization. Taking into consideration the assumption that galaxies belong to a low-rank dimensional space, this work will use the low-rank matrix approximation as a regularization to remove the PSF noise.

This problem can be viewed as a linear inverse problem: $\mathbf{y} = \mathbf{Hx} + \mathbf{n}$; where $\underline{\mathbf{y}}$ is the noisy image, $\underline{\mathbf{H}}$ is the convolution with the PSF, $\underline{\mathbf{x}}$ is the image to be recovered and $\underline{\mathbf{n}}$ is the noise content. This, also, can be written as $\mathbf{Y} = \hat{\mathbf{H}}(\mathbf{X}) + \mathbf{N}$; where $\underline{\mathbf{Y}}$ is a matrix whose rows are the noisy images, $\hat{\mathbf{H}}(\mathbf{X})$ is a matrix of convolution of the images with their corresponding PSF, $\underline{\mathbf{X}}$ is a matrix whose rows are the images to be recovered and $\underline{\mathbf{N}}$ is a matrix whose rows are the noise contents.

For Low-Rank method:

Objective: $\min_X \frac{1}{2}\left\|Y - \hat{\mathrm{H}}(X)\right\|_2^2 + \lambda\|X\|_*$ (1)

This is a Linear Least Squares problem which aims to minimize the residual Y – Ĥ(X), where:

$\lambda = \alpha\sigma_{est}\sqrt{\max\left(n+1, rank(X)\right)}\,\rho(\hat{\mathrm{H}})$ is a regularization control parameter.

α is a multiplayer

$\sigma_{est} = 1.4826 \times\ MAD(Y)$, where MAD for median absolute deviation

$\rho(\hat{\mathrm{H}})$ is the spectral radius of Ĥ

$\|X\|_* = \sum_{k=1}^{rank(X)} \sigma_k(X)$, the nuclear norm, is a regularization parameter, since the problem itself is ill-posed; $\sigma_k(X)$ being the kth largest singular value

Constraints: X ≥ 0; i.e.: all pixels in the recovered images are positive.

Input: Noisy galaxy images with the corresponding PSFs (41 × 41 pixels).

Output: De-noised images

Dataset: [1], 10000 galaxy images, down sampled to 96x96 pixels with the galaxy centered then cropped to 41x41 pixels, 597 PSFs of 41x41 pixels.

Url: http://www.cosmostat.org/software/sf_deconvolve

Method:

To solve this problem, the primal-dual splitting technique described in Condat [2] algorithm 3.1, which solves $\min_X F(X) + G(X) + H(L(X))$, where F is the differentiable convex function (1) with gradient ∇F, G and H are functions with proximity operators and L is a linear operator. The algorithm produces primal (X) variable, once the algorithm converges it is considered the solution of the problem, and dual (Y) variables

Algorithm 1 Choose the proximal parameters $\tau > 0$, $\varsigma > 0$, the positive relaxation parameter, ξ, and the initial estimate $(\mathbf{X}_0, \mathbf{Y}_0)$. Then iterate, for every $k \geq 0$

1: $\tilde{\mathbf{X}}_{k+1} = \mathrm{prox}_{\tau G}(\mathbf{X}_k - \tau\nabla F(\mathbf{X}_k) - \tau\mathcal{L}^*(\mathbf{Y}_k))$

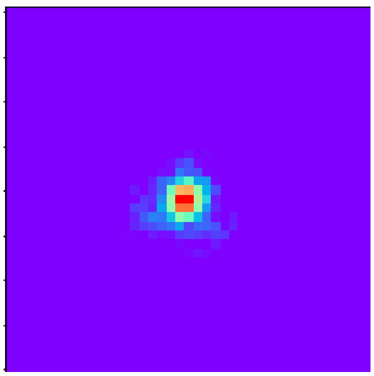
2: $\tilde{\mathbf{Y}}_{k+1} = \mathbf{Y}_k + \varsigma\mathcal{L}(2\tilde{\mathbf{X}}_{k+1} - \mathbf{X}_k) - \varsigma\mathrm{prox}_{K/\varsigma}\left(\frac{\mathbf{Y}_k}{\varsigma} + \mathcal{L}(2\tilde{\mathbf{X}}_{k+1} - \mathbf{X}_k)\right)$

3: $(\mathbf{X}_{k+1}, \mathbf{Y}_{k+1}) := \xi(\tilde{\mathbf{X}}_{k+1}, \tilde{\mathbf{Y}}_{k+1}) + (1-\xi)(\mathbf{Y}_k, \mathbf{Y}_k)$

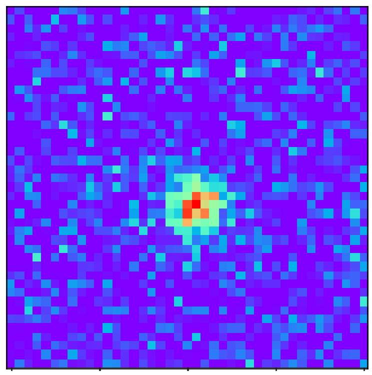
Data Preparation:

We sampled 500 galaxy images from the mentioned dataset, normalized the sum of pixels to add up to 1, convolved each one with a random PSF and finally added Gaussian noise.

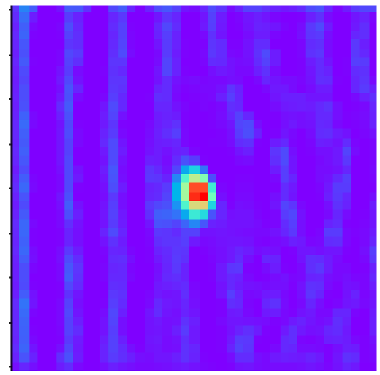
Initial Results (Low-Rank):



Original



Noised



Denoised

Sparsity method implementation

In this problem we will be calculating **L1** norm. We will be setting the smallest coefficients to zero and reducing the amplitudes of the largest coefficients which contain the most useful information.

Minimization with sparse regularization is implemented by solving the following problem:

$$\min_{x} \quad \frac{1}{2}\left\|Y - \hat{\mathrm{H}}(X)\right\|_2^2 + \left\|W^{(k)} \odot \Phi(X)\right\|_1$$

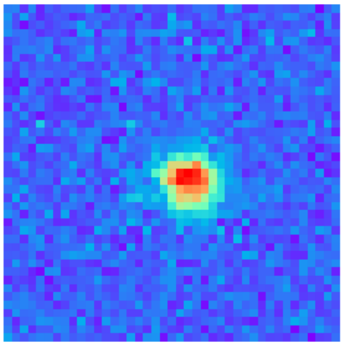
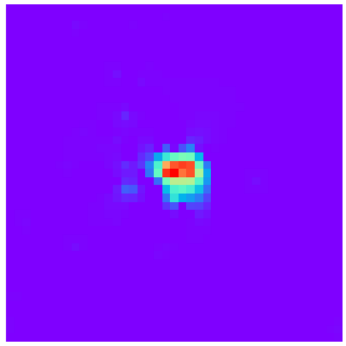
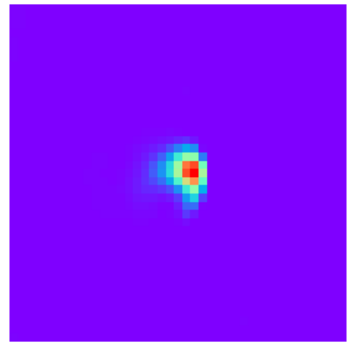
Subject to: X > 0; All the pixels in the recovered image should be positive

Φ: matrix that transforms the Galaxy images into a domain which is more sparse due to the fact that the Galaxy images are not sparse in the pixel domain(starlet transform).

$\boldsymbol{W^{(k)}}$: weighting matrix, where **K** is the reweighting index.

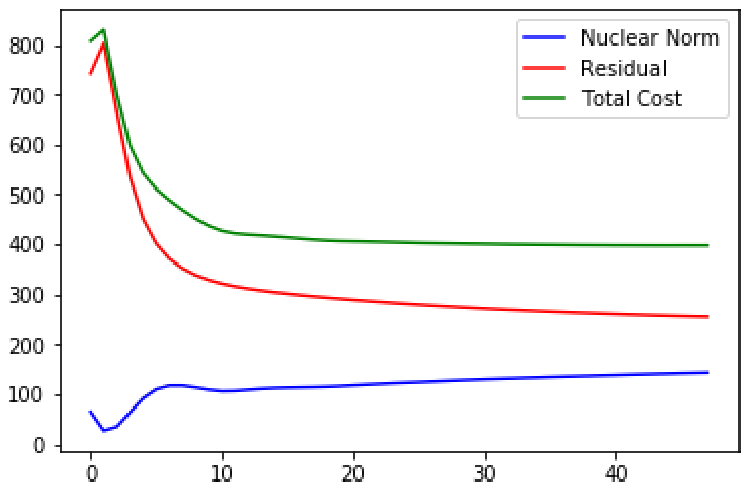
$\odot$: Entry-Wise Product.

Final Results:

Noised image	Sparse	Low-Rank
		

As seen above, Low-Rank can produce better result than sparse method.

Cost Graphs:



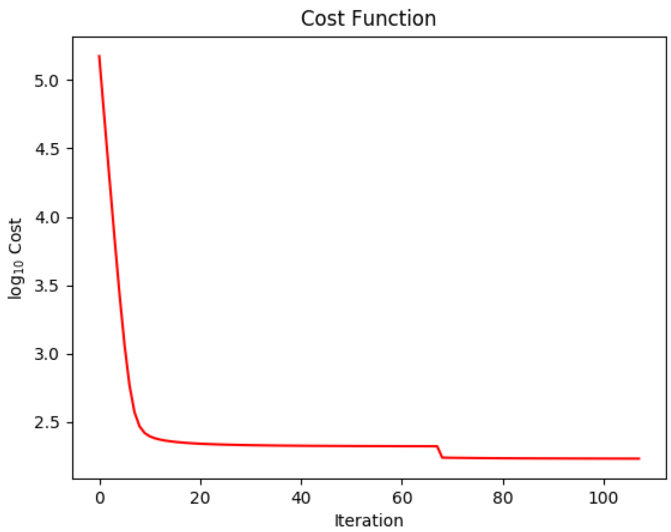
Low-Rank

The residual ($\frac{1}{2}\left\|Y - \hat{\mathrm{H}}(X)\right\|_2^2$) decreases with more iterations

The nuclear norm $\|X\|_*$ increases with more iterations

The total cost $\frac{1}{2}\left\|Y - \hat{\mathrm{H}}(X)\right\|_2^2 + \lambda\|X\|_*$ decreases with more iterations

The algorithm converges when the change in total cost becomes less than 0.0001



Sparse

The total cost $\frac{1}{2}\left\|Y - \hat{\mathrm{H}}(X)\right\|_2^2 + \left\|W^{(k)} \odot \Phi(X)\right\|_1$ decreases with more iterations

The sudden drop at the 65$^{\text{th}}$ iteration is due to reweighting (occurred only once because the number of reweightings is set to 1)

<u>Conclusion:</u>

The results show that as the number of galaxy images increases, Low-Rank performance improves, because with sufficiently large number of images, the rank of the matrix can be significantly reduced. For future work, we will try to improve the current method to be able to perform blind deconvolution (with unknown PSF).

<u>References:</u>

1. Farrens, S., FM Ngolè Mboula, and J-L. Starck. "Space variant deconvolution of galaxy survey images." (2017).
2. Condat, Laurent. "A primal–dual splitting method for convex optimization involving Lipschitzian, proximable and linear composite terms." Journal of Optimization Theory and Applications 158.2 (2013): 460-479.