High Performance Computing (Oct 8)

Question

Suppose a scalar processor and a vector processor. The scalar processor is 32 times faster than the vector processor when executing a program with only scalar instructions. But, the vector processor has vector instructions, each of which can replace 256 scalar instructions, so execution of a single vector instruction is 256 times faster than that of 256 scalar instructions on the vector processor. This is not a very realistic assumption but just a simplified performance model of vector processor. The vectorization ratio based on the vector instruction fraction is 95%. Then what is the ratio of vector processor's execution time over the scalar one's? How does the ratio change for the vectorization ratio of 99%?

Answer

Let T_s be the execution time of the scalar processor. Then, if the program is not vectorized at all, it is obvious that the execution time of the vector processor is given by $T_v = 32T_s$. If 95% of the instruction is vectorized, that portion is executed 256 times faster on the vector processor. Hence, the execution time is calculated by

$$T_v = 32 \times 0.05T_s + 32 \times \frac{0.95T_s}{256} = (1.6 + 0.11875)T_s = 1.71875T_s.$$

The ratio of vector processor's execution time over the scalar one's is 1.71875. In other words, the vector processor works about 1.7 times slower than the scalar processor. If the vectorization ratio is 99%, the ratio is 0.199 calculated by

$$T_v = 32 \times 0.01T_s + 32 \times \frac{0.99T_s}{256} = (0.32 + 0.12375)T_s = 0.44375T_s.$$

The ratio drastically decreases to 0.44375, meaning that the vector processor works about 2.25 times faster than the scalar processor.

This example clearly shows that only a very small increase of 4% in vectorization ratio leads to significant performance improvement.