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Matrices (18M)

1) Eigen Values & Eigen vectors.
2) Diagonal Matrix 100%
3) Cayley-Hamilton Theorem.
4) Function of Square Matrix (5M) keliye aa sakta!

Type1:- Eigen Values & Eigen vectors

Eg:-1 Find Eigen Values & Eigen Vector of

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}_{3\times 3}$$

Sol:-

A) Eigen Values

i) $(A-\lambda I)X = 0$ → Unknown Matrix, → Null Matrix.

A - Lambda I into Capital(X)

$$\begin{bmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As it is 3x3 Matrix

∴ There are 3 unknowns & 3 '0's.

→ Shortcut to write this matrix is:-

Subtract (λ) from every diagonal element.

NOTE:- Sirf highlighted wale diagonal mai se hi Lambda (λ) subtract karna hai.

ii)

$|A-\lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -8 & -2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$$

Determinant of $|A-\lambda I|$

- Hamare pass (2x2) matrix solve karne keliye ke determinant ko solve karne ka method hai.

- ~~But~~ (3x3) ko bhi traditional way mai solve karte thai.
Nowadays no need to calculate traditionally.

- Scientific calcis have the feature where in the determinant is calculated.

- But see the above matrix, usme sirf number nahi hai, it has lambda too which cannot be calculated on calci.

- So, for that we have the formula:-

Formula:-

$\lambda^3 -$ (sum of diagonal elements)$\lambda^2 +$ (sum of minors of diagonal elements)$\lambda - |A| = 0$.

* Logic to remember formula:-
- Degree kam hote ja raha hai
- Sign is alternative
- Change of one word i.e. minors

$\lambda^3 -$ (sum of Diagonal elements) $\lambda^2 +$
(sum of minors of diagonal elements) $\lambda - |A| = 0$.

$\lambda^3 - (6)\lambda^2 + (11)\lambda - 6 = 0$. ← characteristic Eqn.

$\boxed{\lambda = 3, 2, 1}$ ← Eigen values / Characteristic roots

- Sum of Diagonal elements :-

(Q) mai jaane ka then Direct Diagonal elements ko add karke laane ka ans & that becomes the value of sum of diagonal elements.

- Sum of minors of Diagonal Elements :-

1st minor :-

- Hide 1st row
- Hide 1st col
- Write the remaining :- $\begin{vmatrix} -3 & -2 \\ -4 & 1 \end{vmatrix} = (-3 \times 1) - (-2 \times -4)$
$= -3 - 8 = \boxed{-11}$

2nd minor :- $\begin{vmatrix} 8 & -2 \\ 3 & 1 \end{vmatrix} = (8 \times 1) - (-2 \times 3)$
$= 8 + 6 = \boxed{14}$

3rd minor :- $\begin{vmatrix} 8 & -8 \\ 4 & -3 \end{vmatrix} = (8 \times -3) - (-8 \times 4)$
$= -24 + 32 = \boxed{8}$

$-11 + 14 + 8 = \boxed{11}$ → sum of minors of diagonal elements.

NOTE:- These eqⁿs are not solvable on calci, as RHS mai 0 hai.

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B) Eigen Vector (X)

$\lambda = 1$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$7x_1 - 8x_2 - 2x_3 = 0$

$4x_1 - 4x_2 - 2x_3 = 0$

Cramer's Rule :- It requires only 2 DIFFERENT equation

$$\frac{+x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x_1}{\cancel{24}\,8} = \frac{\cancel{+}x_2}{\cancel{+}\cancel{22}\,6} = \frac{x_3}{4}$$

Divide by 2:- $\frac{x_1}{4} = \frac{x_2}{3} = \frac{x_3}{2}$

$$X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

1st & 3rd nahi liya bcz dono same hai

If we divide 1st eqn by (2) we get it same as that of (3)

$\lambda = 2$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$6x_1 - 8x_2 - 2x_3 = 0$$

$$4x_1 - 5x_2 - 2x_3 = 0$$

$$\frac{+x_1}{\begin{vmatrix} -8 & -2 \\ -5 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 6 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix}}$$

$$\frac{x_1}{16-10} = \frac{\cancel{-}x_2}{\cancel{-}4} = \frac{x_3}{2}$$

$$\frac{x_1}{6} = \frac{x_2}{4} = \frac{x_3}{2}$$

Divide by 2 :- $\frac{x_1}{3} = \frac{x_2}{2} = \frac{x_3}{1}$

$$X_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 - 8x_2 - 2x_3 = 0$

$4x_1 - 6x_2 - 2x_3 = 0.$

$$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -6 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -8 \\ 4 & -6 \end{vmatrix}}$$

$$\frac{x_1}{4} = \frac{x_2}{2} = \frac{x_3}{2}$$

Divide by 2:- $\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{1}$

$\therefore \; X_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Eg:-2 Find Eigen Values and Eigen vectors of $A^3 + I$

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}_{3\times3}$$

Sol:-

A) Eigen Values

i) $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$.

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0.$$

$\lambda^3 -$ (sum of diagonal elements) $\lambda^2 +$ (sum of minors of diagonal elements) $\lambda - |A| = 0$.

$$= \lambda^3 - (7)\lambda^2 + (11)\lambda - 5 = 0.$$

$\boxed{\lambda = 5, 1, 1}$

Calci par solve karte time (2) values hi aaye. But we need (3). So, repeat the last one i.e 1 → that is 3rd value.

B) Eigen Vector (X)

$\lambda = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3x_1 + 2x_2 + 1x_3 = 0$

$x_1 - 2x_2 + x_3 = 0.$

$$\therefore \frac{x_1}{\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -3 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{+x_3}{\begin{vmatrix} -3 & 2 \\ 1 & -2 \end{vmatrix}}$$

$$\therefore \frac{x_1}{2-(-2)} = \frac{-x_2}{-3-1} = \frac{x_3}{6-2}$$

$$\therefore \frac{x_1}{4} = \frac{\cancel{-}x_2}{\cancel{-}4} = \frac{x_3}{4}$$

Divide by 4 :- $\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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$\lambda = 1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Yaha Cramer's Rule apply nahi hoga bcz Cramers bolta hai ki usko 2 different eq^n chaiye. But yaha pe ~~dono~~ sab same hai which means 2 different equation milna possible nahi hai.
- Hence we use Row Transformation.

$\boxed{R_2' = R_2 - R_1 \quad R_3' = R_3 - R_1}$ R_2' & R_3' → New Row Values

$$\begin{array}{cc} NZ & R_1 \\ Z & R_2 \\ Z & R_3 \end{array} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

r = Rank = No. of non-zero rows

$\boxed{r = 1}$

n = Number of unknowns

$\boxed{n = 3}$

$x_1 + 2x_2 + x_3 = 0$ — Valid Equation (1)

Assume:-

Assume, $x_1 = s$, $x_2 = t$

$\boxed{\text{No. of assumptions :- } n-r}$

Ham kisi ko bhi assume kar sakte hai
Guidance is to assume the values as s & t.
NOTE:- Don't assume zero

$\therefore$ Eqn (1) becomes

$s + 2t + x_3 = 0$

$\therefore \quad x_3 = -s - 2t$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s \\ t \\ -s-2t \end{bmatrix} = \begin{bmatrix} s+0t \\ 0s+t \\ -s-2t \end{bmatrix}$$

$\uparrow$ Start mai 's' and 't' assume kiya tha toh dono assumptions saath mai dikhne chaiye

$\uparrow$ yaha jaha 's' or 't' nahi thai, vaha hamne add kar diye

$$= \begin{bmatrix} s \\ 0s \\ -s \end{bmatrix} + \begin{bmatrix} 0t \\ t \\ -2t \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} s + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} t$$ ... 's' & 't' ko common liya.

$$\therefore X2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore X3 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

Eg:-3 Find Eigen Values of B where

$B = A^2 + 2A + I - 6A^{-1}$

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$ Triangular Matrix

Sol?:-

A) Eigen values

i) $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 1-\lambda & 2 & -3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$.

$$\begin{vmatrix} 1-\lambda & 2 & -3 \\ 0 & 3-\lambda & 2 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0.$$

iii) $\lambda^3 - (\text{sum of DE})\lambda^2 + (\text{Sum of minors of D.E})\lambda - |A| = 0$.

$\therefore$ $\lambda^3 - (2)\lambda^2 + (-5)\lambda + 6 = 0$

$\boxed{\lambda = -2, 1, 3}$ ← Eigen Values

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Triangular Matrix:-

Jab Diagonal elements ke niche YA upar sab '0' hai ton vo matrix ke Eigen values = Diagonal elements.

(Q) mai jo Matrix diya hai, usme mai bhi diagonal ke neeche wale sab '0' hai.

So, Eigen values that we got are equal to Diagonal elements.

Eigen values of B :-

HINT:- Convert the asked (Q) to Lambda form.

Why?

Bcz jo hamko old values mili hai they are in the form of Lambda.

$$B = A^2 + 2A + I - 6A^{-1}$$

Ousre jagah pe 'I' becomes 1. Matrix mai I is Identity Matrix.

$$= \lambda^2 + 2\lambda + 1 - 6\lambda^{-1}$$

$$= \lambda^2 + 2\lambda + 1 - \frac{6}{\lambda}$$

$\lambda = -2$ | $\lambda = 1$ | $\lambda = 3$

$\cancel{4} - \cancel{4} + 1 + 3 = \boxed{4}$

$1 + 2 + 1 - 1$ $\boxed{3}$

$9 + 6 + 1 - 3$

Eg:-4 Find Eigen values of $A^2 - 2A + 3I$ & adj A
adjoint

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

Triangular Matrix.

Sol^n:- A) Eigen values

i) $(A - \lambda I)X = 0$.

$$\therefore \begin{bmatrix} 2-\lambda & 3 \\ 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$.

$$\begin{vmatrix} 2-\lambda & 3 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

iii)

$$\lambda^2 - (\text{sum of D.E})\lambda + |A| = 0$$

$$\lambda^2 - (3)\lambda + 2 = 0.$$

$$\therefore \boxed{\lambda = 2, 1}$$

→ As Triangular Matrix
∴ Eigen values = Diagonal elements

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B) Eigen values of $A^2 - 2A + 3I$ are:- 2, 3.

$\downarrow$

$\lambda^2 - 2\lambda + 3I(1)$ {I becomes 1}.

For $\lambda = 1$

$1 - 2 + 3 = \boxed{2}$.

For $\lambda = 2$

$4 - 4 + 3 = \boxed{3}$.

C) Eigen values of Adjoint of A are:- 2, 1

$$\boxed{\text{Formula:- } \frac{|A|}{\lambda}}$$

For $\lambda = 1$

$\therefore \frac{|A|}{\lambda} = \frac{2}{1} = 2$

For $\lambda = 2$

$\therefore \frac{|A|}{\lambda} = \frac{2}{2} = 1$

Type 2:- Diagonal Matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \rightarrow \text{Fully Loaded}$$

$$\Downarrow$$

$$\begin{bmatrix} p & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & r \end{bmatrix}$$

Ek fully loaded matrix ko ek particular matrix mai convert karna.

Particular Format:-

1) All diagonal elements exists
2) Diagonal ke upar sab zero.
3) Diagonal ke neeche sab zero.

Particular Format mai convert karne ke liye:-

Formula:- $D = M^{-1} A M$

- $A \rightarrow$ (Q) mai diya hai.
- $M = [X_1 \; X_2 \; X_3]$

  $\downarrow$
- Eigenvectors ko ek matrix mai patak do.
- M^{-1} = Inverse of $(M) \rightarrow$ calculator se aayega

M

Modal M | Diagonalising M | Transforming M

NOTE:-
All fully loaded matrix cannot be converted to Diagonal matrix.

RULES:-

1) If Eigen values are DIFFERENT then matrix is "100% Diagonalisable.

2) If Eigen values are REPEATED then check:-
   1) AM → Algebraic Multiplicity
   2) GM → Geometric Multiplicity.

How to calculate AM?

→ 'λ' kitni baar repeat hua hai.
Here we mean the values of λ.

How to calculate GM?

Formula:- $n - r$

n → no. of unknowns
r → Rank → no. of non-zero rows.

2.1

If AM = GM

Matrix is Diagonalisable

Else

Matrix is non Diagonalisable.

Eg:-1 Show that matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

is diagonalisable.

Also find M & D.

Sol:-

A) Eigen Values

i) $(A-\lambda I)X = 0.$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A-\lambda I| = 0.$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0.$$

$$\lambda^3 - (\text{sum of D.E})\lambda^2 + (\text{sum of minors of D.E})\lambda - |A| = 0$$

$$\therefore \lambda^3 - (18)\lambda^2 + (45)\lambda - 0 = 0.$$

$$\boxed{\lambda = 0, 3, 15}$$

$\because$ Eigen values are Different

$\therefore$ Matrix is Diagonalisable.

B) Eigen Vector

$\lambda = 0$:

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$8x_1 - 6x_2 + 2x_3 = 0$$
$$2x_1 - 4x_2 + 3x_3 = 0$$

$$\frac{+x_1}{\begin{vmatrix} -6 & 2 \\ -4 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 8 & -6 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-18-(-8)} = \frac{-x_2}{24-4} = \frac{x_3}{-32-(-12)}$$

$$\frac{x_1}{-\cancel{18+8}\,10} = \frac{-x_2}{20} = \frac{x_3}{-20}$$

Divide by (-10):- $\dfrac{x_1}{1} = \dfrac{x_2}{2} = \dfrac{x_3}{2}$

$$\therefore X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -7 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 - 6x_2 + 2x_3 = 0$$
$$2x_1 - 4x_2 + 3x_3 = 0.$$

$$\frac{+x_1}{\begin{vmatrix} -6 & 2 \\ -4 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 2 \\ 2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & -6 \\ 2 & -4 \end{vmatrix}}$$

$$\frac{x_1}{-18-(-8)} = \frac{-x_2}{15-4} = \frac{x_3}{-20+12}$$

$$\frac{x_1}{-10} = \frac{-x_2}{11} = \frac{x_3}{-8}$$

$$\therefore X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$\lambda = 15$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 - 8x_2 - 4x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} -6 & 2 \\ -8 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -7 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -7 & -6 \\ -6 & -8 \end{vmatrix}}$$

$$\frac{x_1}{24+16} = \frac{-x_2}{28+12} = \frac{x_3}{56-36}$$

$$\frac{x_1}{40} = \frac{-x_2}{40} = \frac{x_3}{20}$$

Divide by 20:- $\frac{x_1}{2} = \frac{-x_2}{2} = \frac{x_3}{1}$

$\therefore \; X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

c) Diagonal Matrix

$$D = M^{-1} A M$$

$$M = [X_1 \quad X_2 \quad X_3]$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \frac{1}{9} & \frac{2}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{1}{9} & \frac{-2}{9} \\ \frac{2}{9} & \frac{-2}{9} & \frac{1}{9} \end{bmatrix}$$

$$D = \begin{bmatrix} 1/9 & 2/9 & 2/9 \\ 2/9 & 1/9 & -2/9 \\ 2/9 & -2/9 & 1/9 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

Eigen values

Eg:-2 Show that matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

is diagonalisable
Also find A & D.

Sol:-

A) Eigen Values (i) $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

ii) $|A - \lambda I| = 0$.

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0.$$

$\lambda^3 - (\text{sum of DE})\lambda^2 + (\text{sum of minors of DE})\lambda - |A| = 0.$

$\lambda^3 - (12)\lambda^2 + (36)\lambda - 32 = 0.$

$\boxed{\lambda = 8, 2, 2}$

B) Eigen Vector (X)

$\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 - 5x_2 - x_3 = 0$

$2x_1 - x_2 - 5x_3 = 0.$

$$\frac{x_1}{\begin{vmatrix} -5 & -1 \\ -1 & -5 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & -1 \\ 2 & -5 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & -5 \\ 2 & -1 \end{vmatrix}}$$

$$\frac{x_1}{25-1} = \frac{-x_2}{10+2} = \frac{x_3}{2+10}.$$

$$\frac{x_1}{24} = \frac{-x_2}{12} = \frac{x_3}{12}$$

Divide by 12:- $\frac{x_1}{2} = \frac{-x_2}{1} = \frac{x_3}{1}$

$$\therefore X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\lambda = 2$ {Repeat}

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 = \frac{R_1}{2}$, $R_2 = \frac{R_2}{-1}$

$$\begin{bmatrix} 2 & -1 & 1 \\ 2 & -1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 - R_1$; $R_3 = R_3 - R_1$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$r = 1$, $n = 3$.

$2x_1 - x_2 + x_3 = 0$.

Assume:- $x_1 = s$ & $x_2 = t$

$\therefore$ $2s - t + x_3 = 0$

$\therefore$ $x_3 = -2s + t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} s \\ t \\ -2s+t \end{bmatrix} = \begin{bmatrix} s+0t \\ 0s+t \\ -2s+t \end{bmatrix}$$

$$= \begin{bmatrix} s \\ 0s \\ -2s \end{bmatrix} + \begin{bmatrix} 0t \\ t \\ t \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} s + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} t$$

$$\therefore X_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\therefore X_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

c) Diagonal Matrix

$$D = M^{-1} A M$$

$$M = \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$M^{-1} = \cancel{\begin{bmatrix} 1 & 1/2 & -1/2 \\ -1 & -1 & 1 \\ 1 & 3/2 & -1/2 \end{bmatrix}} \quad \begin{bmatrix} 1/3 & -1/6 & 1/6 \\ 1/3 & 1/3 & -1/3 \\ 1/3 & 5/6 & 1/6 \end{bmatrix}$$

$$D = \begin{bmatrix} 1/3 & -1/6 & 1/6 \\ 1/3 & 1/3 & -1/3 \\ 1/3 & 5/6 & 1/6 \end{bmatrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\therefore D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$