

M4 All Required Formulas

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A) Complex Integration:

$$z = x + iy, \quad z = re^{i\theta}$$

$$\bar{z} = x - iy = re^{-i\theta}$$

$$|z| = \sqrt{x^2 + y^2} = |x + iy| = r$$

- Steps for Complex Integration :
- 1) Equation
 - 2) Derivative
 - 3) $f(z)$
 - 4) dz
 - 5) Integration.

Euler's Formula :

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

B) Gauchy's Theorem:

1) Non Repeated Pole :

$$\int_C \frac{f(z)}{(z - z_0)} dz = 2\pi i f(z_0)$$

\downarrow pole

2) Repeated Pole :

$$\int_C \frac{f(z)}{(z - z_0)^n} dz = \frac{2\pi i}{(n-1)!} f^{(n-1)}(z_0)$$

\downarrow pole

Circle

Standard Circle
↓

only $|z|$

General Circle
↓

$|z - \text{something}|$

$$\frac{d}{dx} f(x)^n = n f(x)^{n-1} \frac{d}{dx} f(x)$$

c) Residue :

i) Non Repeated Pole :

$$\text{Residue at } z = z_0 = \lim_{z \rightarrow z_0} (z - z_0) \times F(z)$$

Transfer multiply by

ii) Repeated Pole :

$$\text{Residue at } z = z_0 = \lim_{z \rightarrow z_0} \frac{d^{n-1}}{dz^{n-1}} (z - z_0)^n \times F(z)$$

$(n-1)!$

iii) Cauchy's Residue Theorem :

$$I = 2\pi i [\text{Sum of Residues}]$$

D) Taylor's and Laurent's Series:

i) Standard Expansions:

$$1) \sin x = + \frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$2) \cos x = + \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$3) \sinh x = + \frac{x^1}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$4) \cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$5) e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!}$$

$$6) e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$$

ii) Binomial Expansion:

$$1) (1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

$$2) (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \dots$$

$$3) (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!} x^2 \dots$$

$$4) (1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!} x^2 \dots$$

E) Poisson Distribution :

$$1) P(X=x) = \frac{e^{-m} m^x}{x!}$$

$$2) \text{Mean } \quad ? \quad m \text{ OR } (np)$$

Variance

Final Answer

Find Probability

Find Number

$$= p(x)$$

$$= N \times p(x)$$

where $N = \text{large sample.}$

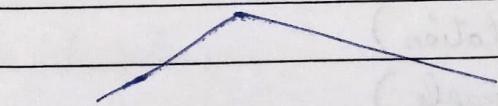
F) Normal Distribution :

$\mu = \text{Mean} = \text{Expectation}$

$\delta = \text{standard Deviation.}$

- If variance is given then $\delta = \sqrt{\text{variance}}$.

$z \rightarrow \text{Normal Variable}$



$$z = x - \mu$$

$$\delta$$

$$z = \frac{x - \mu}{\delta}$$

$$\delta / \sqrt{n}$$

↑ use this formula when
"Central Limit Theorem"

is given.

6) Sampling Theory:

i) Large Sample Test ($n > 30$)

1) Interval Estimation

Single Sample

$$\bar{x} \pm z_{\alpha} \hat{\delta_x}$$

Two Samples

$$|\bar{x}_1 - \bar{x}_2| \pm z_{\alpha} \cdot S.E.$$

(Standard Error)

$$\hat{\delta_x} = \frac{\delta}{\sqrt{n}}$$

$$S.E. = \sqrt{\frac{\delta_1^2}{n_1} + \frac{\delta_2^2}{n_2}}$$

2) Testing of Claims

Single Sample

$$z = \frac{\bar{x} - u}{\delta / \sqrt{n}}$$

Two Sample

$$z = \frac{\bar{x}_1 - \bar{x}_2}{S.E.}$$

- u = Mean (Population)

- \bar{x} = Mean (Sample)

- δ = Standard Deviation = $\sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

L: Flexibility | Tolerance | Level of Significance.

$$\alpha = 1 \% \quad = \quad 2.58$$

$$\alpha = 5 \% \quad = \quad 1.96$$

$$\alpha = 10 \% \quad = \quad 1.64$$

ii) \Rightarrow Small Sample Test ($n < 30$)

1) Interval Estimation

\downarrow
Single Sample

$$\bar{x} \pm t \cdot \hat{\sigma}_x$$

\downarrow
Two Samples

$$|\bar{x}_1 - \bar{x}_2| \pm t \cdot S.E.$$

$$\hat{\sigma}_x = \frac{s}{\sqrt{n-1}}$$

$$S.E. = sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$sp = \sqrt{\frac{n_1 \beta_1^2 + n_2 \beta_2^2}{n_1 + n_2 - 2}}$$

(standard proportion)

2) Testing of Claims

\downarrow
Single Sample

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}}$$

\downarrow
Two Samples

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S.E.}$$

3) Raw Data :

$$i) \bar{X} = A + \frac{\sum d}{n} \quad \left| \begin{array}{l} A = \text{Assumed Mean} \\ d = x - A \end{array} \right.$$

$$ii) \sum (x - \bar{x})^2 = \sum d^2 - \frac{(\sum d)^2}{n} \quad \left[\begin{array}{l} \text{Sum of squares of} \\ \text{deviations from mean.} \end{array} \right]$$

$$iii) \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

4) Degree of Freedom
(choice)

Single Sample

$$df = n - 1$$

Two Samples

$$df = n_1 + n_2 - 2$$

iii) χ^2 - Test (Chi-square test)

$$\text{Formula: } \chi^2 = \sum \frac{(O - E)^2}{E}$$

where O: Observed Frequency.

E: Expected Frequency.

Expected Frequency calculation for

Raw Data

$$E = \frac{\sum f}{n}$$

Tabular Data

$$E = R_T \times C_T$$

where R_T = Row Total, C_T = Column Total,
 F_T = Final Total.

$$\frac{df}{T} = \frac{\text{Raw Data}}{\text{Tabular Data}}$$

$$df = (r-1)(c-1)$$

r = Row

c = Column.

H) Z - Transform :

Definition : If $f(k)$ be any sequence then,

$$z \left\{ f(k) \right\} = \sum_{k=-\infty}^{\infty} f(k) z^{-k} = F(z)$$

$$\bullet 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$

$$= (1-x)^{-1}$$

$$\bullet 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \dots = e^{\text{second term}} = e^x$$

$$\bullet \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad \bullet \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\bullet \sinh \theta = \frac{e^\theta - e^{-\theta}}{2} \quad \bullet \cosh \theta = \frac{e^\theta + e^{-\theta}}{2}$$

• Standard Functions :

$$1) z \left\{ \sin \alpha k \right\} = \frac{z \cdot \sin \alpha}{z^2 - 2z \cdot \cos \alpha + 1}$$

$$2) z \left\{ \cos \alpha k \right\} = \frac{z^2 - z \cdot \cos \alpha}{z^2 - 2z \cdot \cos \alpha + 1}$$

$$3) z \left\{ \sinh \alpha k \right\} = \frac{z \cdot \sinh \alpha}{z^2 - 2z \cdot \cosh \alpha + 1}$$

$$4) z \left\{ \cosh \alpha k \right\} = \frac{z^2 - z \cdot \cosh \alpha}{z^2 - 2z \cdot \cosh \alpha + 1}$$

$$5) z \left\{ v(k) \right\} = \frac{z}{z-1}$$

• Compound Angles :

$$1) \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$2) \sinh(A+B) = \sinh A \cdot \cosh B + \cosh A \cdot \sinh B$$

$$3) \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$4) \cosh(A+B) = \cosh A \cdot \cosh B + \sinh A \cdot \sinh B.$$

• Properties :

1) Change of Scale Property :

$$\text{If } z \left\{ f(k) \right\} = F(z)$$

$$\text{then } z \left\{ c^k f(k) \right\} = F(z/c)$$

2) Convolution :

If $f(k)$ and $g(k)$ be the sequence -

$$z \left\{ f(k) * g(k) \right\} = F(z) * G(z)$$

3) Shifting Property :

$$\text{If } z \left\{ f(k) \right\} = F(z)$$

$$\text{then } z \left\{ f(k+n) \right\} = z^n z \left\{ f(k) \right\}$$

$$z \left\{ f(k-n) \right\} = z^{-n} z \left\{ f(k) \right\}$$

4) Multiplication by k :

$$\text{If } z \left\{ f(k) \right\} = F(z)$$

$$\text{then } z \left\{ k f(k) \right\} = \left(-z \frac{d}{dz} \right) F(z)$$

$$z \left\{ k^2 f(k) \right\} = \left(-z \frac{d}{dz} \right) \left(-z \frac{d}{dz} \right) F(z)$$

I) Inverse Z- Transform :

$$z^{-1} \left\{ F(z) \right\} = f(k)$$

= coeff of z^{-k}

J) Matrices :

- Characteristic Equation :

If 3×3 matrix,

$$\lambda^3 - (\text{sum of diagonal elements})\lambda^2 + (\text{sum of minors of diagonal elements})\lambda - |A| = 0$$

- Diagonal Matrix :

$$D = M^{-1} A M$$

M = Model Matrix | Transforming Matrix | Diagonalizing matrix

$M = [x_1 \ x_2 \ x_3]$ where x_1, x_2, x_3 are the Eigen vectors.

- Matrix is diagonalizable if -

1) All eigen values are different.

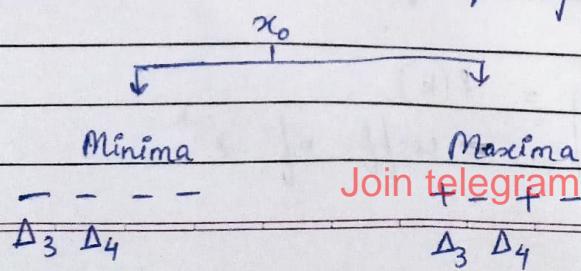
2) If eigen values are repeated, then check for A_m and G_m . If $A_m = G_m$ then matrix is diagonalizable.

A_m = Algebraic Multiplicity.

G_m = Geometric Multiplicity.

K) NLPP

- 1) Optimization with One Equality :



Determinant

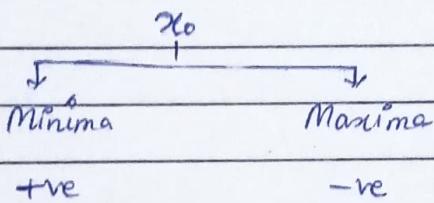
$$\Delta_{n+1} = \Delta_{3+1} = \Delta_4 = x_1$$

↑
(no of variable)

x_1	$\frac{\partial h}{\partial x_1}$	$\frac{\partial h}{\partial x_2}$	$\frac{\partial h}{\partial x_3}$
0	$\frac{\partial^2 L}{\partial x_1^2}$	$\frac{\partial^2 L}{\partial x_1 \partial x_2}$	$\frac{\partial^2 L}{\partial x_1 \partial x_3}$
$\frac{\partial h}{\partial x_1}$	$\frac{\partial^2 L}{\partial x_1^2}$	$\frac{\partial^2 L}{\partial x_1 \partial x_2}$	$\frac{\partial^2 L}{\partial x_1 \partial x_3}$
x_2	$\frac{\partial h}{\partial x_2}$	$\frac{\partial^2 L}{\partial x_2^2}$	$\frac{\partial^2 L}{\partial x_2 \partial x_3}$
x_3	$\frac{\partial h}{\partial x_3}$	$\frac{\partial^2 L}{\partial x_3^2}$	$\frac{\partial^2 L}{\partial x_3 \partial x_2}$

4x4

2) Optimization with Two Equality :



Bordered Hessian Matrix :

$$H_B^B = \begin{bmatrix} 0 & P \\ P' & g \end{bmatrix}$$

Null Matrix

$$\rightarrow D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

	x_1	x_2	x_3
$p = h_1$	$\frac{\partial h_1}{\partial x_1}$	$\frac{\partial h_1}{\partial x_2}$	$\frac{\partial h_1}{\partial x_3}$
	$\frac{\partial h_1}{\partial x_1}$	$\frac{\partial h_1}{\partial x_2}$	$\frac{\partial h_1}{\partial x_3}$
h_2	$\frac{\partial h_2}{\partial x_1}$	$\frac{\partial h_2}{\partial x_2}$	$\frac{\partial h_2}{\partial x_3}$
	$\frac{\partial h_2}{\partial x_1}$	$\frac{\partial h_2}{\partial x_2}$	$\frac{\partial h_2}{\partial x_3}$

	x_1	x_2	x_3
x_1	$\frac{\partial^2 L}{\partial x_1^2}$	$\frac{\partial^2 L}{\partial x_1 \partial x_2}$	$\frac{\partial^2 L}{\partial x_1 \partial x_3}$
x_2	$\frac{\partial^2 L}{\partial x_2 \partial x_1}$	$\frac{\partial^2 L}{\partial x_2^2}$	$\frac{\partial^2 L}{\partial x_2 \partial x_3}$
x_3	$\frac{\partial^2 L}{\partial x_3 \partial x_1}$	$\frac{\partial^2 L}{\partial x_3 \partial x_2}$	$\frac{\partial^2 L}{\partial x_3^2}$

3) Kuhn - Tucker Conditions

$$1) \frac{\partial L}{\partial x_1} = 0$$

$$2) \frac{\partial L}{\partial x_2} = 0$$

$$3) h(x_1, x_2) \leq 0 \leftarrow \text{Judge}$$

$$4) \lambda h(x_1, x_2) = 0 \leftarrow \text{Decision maker}$$

$$5) x_1, x_2 \geq 0$$