Engineering Mathematics 4 Previous Year Solved + Unsolved Question Paper from December 2014 to May 22

S.E. Comp / IT sem IV CBSGS M-2015 19/5/14

Sub:- AM IV

QP Code : **NP-19682**

(3 Hours) [Total Marks : 80

N. B. : (1) Question no. **one** is **compulsory**.
(2) Answer **any three** questions from **Q.2** to **Q.6**.

1. (a) If $A = \begin{bmatrix} x & 4x \\ 2 & y \end{bmatrix}$ has eigen values 5 and -1 then find values of x and y. 5

(b) Evaluate $\int_c (\bar{z} + 2z)\, dz$ along the circle c: $x^2 + y^2 = 1$. 5

(c) State true or false with justification: If the two lines of regression are $x + 3y - 5 = 0$ and $4x + 3y - 8 = 0$ then the correlation coefficient is $+0.5$. 5

(d) Find dual of following LP model 5

$$\max z = 2x_1 + 3x_2 + 5x_3$$

subject to

$$x_1 + x_2 - x_3 \geq -5$$
$$x_1 + x_2 + 4x_3 = 10$$
$$-6x_1 + 7x_2 - 9x_3 \leq 4$$

& $x_1, x_2 \geq 0$ and x_3 is unrestricted.

2. (a) Using Cauchy's integral formula, evaluate $\int_c \frac{(12z - 7)\, dz}{(z-1)^2 (2z+3)}$ where C: $|z + i| = \sqrt{3}$. 6

(b) Determine whether matrix A is derogatory $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. 6

(c) In a competitive examination, the top 15% of the students appeared will get grade 'A', while the bottom 20% will be declared fail. If the grades are normally distributed with mean % of marks 75 and S.D. 10, determine the lowest % of marks to receive grade A and the lowest % of marks that passes. 8

3. (a) The daily consumption of electric power (in millions of kwh) is r.v. X with PDF $f(x) = k\, xe^{-x/3}$, $x > o$. Find k and the probability that on a given day the electricity consumption is more than expected electricity consumption. 6

[TURN OVER

(b) Using Simplex method, solve the following LPP 6

$$\max\ z = 15x_1 + 6x_2 + 9x_3 + 2x_4$$
$$\text{s.t. } 2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
$$\&\ \ x_1, x_2, x_3, x_4 \ge 0$$

(c) Obtain ALL Taylor's and Laurent's series expansions of function $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ about $z = 0$. 8

4. (a) Find the moment generating function of Poisson distribution and hence find mean and variance. 6

(b) Obtain the equation of the line of regression of **cost on age** from the following table giving the age of a car of certain make and the annual maintenance cost. Also find maintenance cost if age of the car is 9 years. 6

Age of car (in years) : x	2	4	6	8
Maintenance cost : y (in thousands)	5	7	8.5	11

(c) Show that the matrix A is diagonalizable, find its diagonal form and transforming matrix, if $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. 8

5. (a) A sample of 8 students of 16 years each shown up a mean systolic blood pressure of 118.4 mm of Hg with S.D. of 12.17 mm. While a sample of 10 students of 17 years each showed the mean systolic BP of 121.0 mm with S.D. of 12.88 mm during in investigation. The investigator feels that the systolic BP is related to age. Do you think that the data provides enough reasons to support investigator's feeling at 5% LoS? Assume the distribution of systolic BP to be normal. 6

(b) Using Cauchy's residue theorem, show that $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta = \frac{\pi}{6}$. 6

[TURN OVER

(c) Using dual simplex method, solve **8**

$$\max z = -2x_1 - x_3$$
$$\text{s.t. } x_1 + x_2 - x_3 \geq 5$$
$$x_1 - 2x_2 + 4x_3 \geq 8$$
$$\& \quad x_1, x_2, x_3 \geq 0$$

6. (a) A total of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them, 1872 were men and the rest were women. A total of 2257 individuals were in favour of the proposal and 917 were opposed to it. A total of 243 men were undecided and 442 women were opposed to the proposal. Do you justify on the hypothesis that there is no association between sex and attitude, at 5% LoS. **6**

(b) Using Kuhn – Tucker's method solve **6**

Maximize $Z = 2x_1^2 + 12x_1x_2 - 7x_2^2$

Subject to the constraints $2x_1 + 5x_2 \leq 98$ and $x_1, x_2 \geq 0$

(c) (i) Average mark scored by 32 boys is 72 with standard deviation of 8 while that for 36 girls is 70 with standard deviation of 6. Test at 1% LoS whether the boys perform better than the girls. **4**

(ii) If the first four moments of a distribution about the value 4 of the random variable are –1.5, 17, –30 and 108 then find first four raw moments. **4**

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S·E. Comps & IT   21/11/14
AM-IV (K.T.)
(CBGS).

QP Code : 12413

(3 Hours)   [Total Marks : 80

N.B. (1) Question No. 1 is **compulsory**.
(2) Answer any **three** questions from Question Nos. 2 to 6.

1. (a) Evaluate $\int_C (z - z^2)$ where C is the upper half of the circle $|z| = 1$. What is the value of the integral for the lower half of the same circle ? 5

(b) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$. Find the eigen values of $A^3 + 5A + 8I$. 5

(c) The regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$. Find (1) mean of x and y and (2) coefficient of correlation between x and y. 5

(d) A machine is claimed to produce nails of mean length 5 cm. and standard deviation of 0·45 cm. A random sample of 100 nails gave 5·1 cm. as average length. Does the performance of the machine justify the claim ? Mention the level of significance you apply. 5

2. (a) Show that the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory. 6

(b) Evaluate $\int \frac{z+3}{z^2 + 2z + 5} dz$, where C is the circle (i) $|z| = 1$. (ii) $|z + 1 - i| = 2$. 6

(c) The mean inside diameter of a sample of 200 washers produced by a machine is 0·502 cm and the standard deviation is 0·005 cm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0·496 to 0·508 cm, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed. 8

3. (a) A continuous random variable X has the following probability law $f(x) = kx^2 e^{-x}$, $x \geq 0$. Find k, mean and variance. 6

(b) Solve the following LPP by Simplex method :— 6

Max $z = x_1 + 4x_2$
Subject to $2x_1 + x_2 \leq 3$
$3x_1 + 5x_2 \leq 9$
$x_1 + 3x_2 \leq 5$
$x_1, x_2 \geq 0$

(c) Find Laurent's series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ when 8

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$.

[TURN OVER

4. (a) The means of two random samples of size 9 and 7 are 196·42 and 198·82 respectively. The sums of the squares of the deviation from the means are 26·94 and 18·73 respectively. Can the samples be considered to have been drawn from the same population ? 6

(b) Calculate the correlation coefficient from the following data : 6

X :	23	27	28	29	30	31	33	35	36	39
Y :	18	22	23	24	25	26	28	29	30	32

(c) Show that the following matrix is Diagonalizable. Find the transforming matrix and the Diagonal matrix. 8

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

5. (a) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6

(b) Evaluate the following integral by contour integration 6

$$\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}.$$

(c) Use Kuhn Tucker method to solve the NLPP :-- 8

$$\text{Max } Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$$

$$\begin{aligned} \text{St} \quad x_1 + x_2 &\le 2 \\ 2x_1 + 3x_2 &\le 12 \\ x_1, x_2 &\ge 0. \end{aligned}$$

6. (a) For special security in a certain protected area, it was decided to put three lighting bulbs on each pole. If each bulb has a probability p of burning out in the first 100 hours of service, calculate the probability that at least one of them is still good after 100 hours. 6
If $p = 0{\cdot}3$, how many bulbs would be needed on each pole to ensure 99% safety that atleast one is good after 100 hours ?

(b) Use Duality to solve the following LPP : 6

$$\text{Max } Z = 2x_1 + x_2$$

$$\begin{aligned} \text{Subject to} \quad 2x_1 - x_2 &\le 2 \\ x_1 + x_2 &\le 4 \\ x_1 &\le 3 \\ x_1, x_2 &\ge 0 \end{aligned}$$

(c) The number of car accidents in a metropolitan city was found to be 20, 17,12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use χ^2 test to check whether these frequencies are in agreement with the belief that occurrence of accidents was the same during 10 months period. Test at 5% level of Significance. 8

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S.E. (Comp & IT) - IV (CBSGS)

A.M. IV

15/5/2015

Q.P. Code : 3541

(3 Hours) [Total Marks : 80

N.B.: (1) Question **No.1** is **compulsory**.
(2) Attempt any **three** questions from Question No. **2** to **6**.
(3) Use of stastical Tables permitted.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Show that $\int_C \log z \, dz = 2\pi i$, where C is the unit circle in the z - plane. 5

(b) If $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ then find the eigen values of $4A^{-1} + 3A + 2I$. 5

(c) It is given that the means of x and y are 5 and 10. If the line of regression of y on x is parallel to the line $20y = 9x + 40$, estimate the value of y for $x = 30$. 5

(d) Find the dual of the following L.P.P. 5

Maximise $Z = 2x_1 - x_2 + 3x_3$

Subject to
$x_1 - 2x_2 + x_3 \geq 4$
$2x_1 + x_3 \leq 10$
$x_1 + x_2 + 3x_3 = 20$
$x_1, x_3 \leq 0$, x_2 unrestricted.

2. (a) Evalute $\int_C \frac{z+2}{z^3 - 2z^2} dz$, where C is the circle $|Z - 2 - i| = 2$ 6

(b) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. 6

(c) In a distribution exactly normal 7% of items are under 35 and 89% of the items are under 63. Find the probability that an item selected at random lies between 45 & 56. 8

3. (a) A continuous random variable has probability density function $f(x) = 6(x - x)^2$, $0 \leq x \leq i$. Find (i) mean (ii) variance. 6

(b) Solve the following L.P.P. by simplex method 6

Maximise $Z = 4x_1 + 3x_2 + 6x_3$

Subject to
$2x_1 + 3x_2 + 2x_3 \leq 440$
$4x_1 + 3x_3 \leq 470$
$2x_1 + 5x_2 \leq 430$
$x_1, x_2, x_3 \leq 0$

3. (c) Find all possible Laurent's expansions of the function 8

$$f(z)=\frac{7z-2}{z(z-2)(z+1)} \quad \text{about } z=-1$$

4. (a) Find the moment generating function of Binomial distribution & hence find mean and variance. 6

(b) Calculate the correlation coefficient from the following data : 6

x :	100	200	300	400	500
y :	30	40	50	60	70

(c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ 8

is diagonalisable. Find the transforming matrix and the diagonal matrix.

5. (a) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69,69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. 6

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^3}$, $a > 0$ using contour integration. 6

(c) Use Kuhn - Tucker conditions to solve the following N.L.P.P. 8

Maximise $Z = 8x_1 + 10x_2 - x_1^2 - x_2^2$

subject to $3x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

6. (a) A die was thrown 132 times and the following frequencies were observed. 6

No. obtained :	1	2	3	4	5	6	Total
Frequency :	15	20	25	15	29	28	132

(b) Using duality solve the following L. P. P. 6

Maximise $Z = 5x_1 - 2x_2 + 3x_3$

Subject to

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 3$$
$$x_1 + 3x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

(c) (i) A random sample of 50 items gives the mean 6.2 and standard deviation 10.24, can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? 4

(ii) Find the M.G.F. of the following distribution. 4

X :	– 2	3	1
P (X = x)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Hence find first four central moments.

S.E.-Sem IV (Comp & I.T.), 23/11/15
AM-IV

Q.P. Code : 5316

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. one is compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Use of stastical Tables permitted.

(4) Figures to the right indicate full marks

1. (a) Evaluate the line integral $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x$ 5

(b) State Cayley-Hamilton theorem & verify the same for $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ 5

(c) The probability density function of a random variable x is

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	2k	0.3	K

Find i) k ii) mean iii) variance 5

(d) Find all the basic solutions to the following problem

Maximize $z = x_1 + 3x_2 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

and $x_1, x_2, x_3 \geq 0$ 5

2. (a) Find the Eigen values and the Eigen vectors of the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ 6

(b) Evaluate $\oint_c \frac{dz}{z^3(z+4)}$ where c is the circle $|z| = 2$ 6

(c) If the heights of 500 students is normally distributed with mean 68 inches and standard deviation of 4 inches, estimate the number of students having heights i) less than 62 inches, ii) between 65 and 71 inches. 8

[TURN OVER

MD-Con. 8175-15.

3. (a) Calculate the coefficient of correlation from the following data

x	30	33	25	10	33	75	40	85	90	95	65	55
y	68	65	80	85	70	30	55	18	15	10	35	45

6

(b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 100 such samples, how many would you expect to contain 3 defectives i) using the Binomial distribution, ii) Poisson distribution. 6

(c) Show that the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. 8

4. (a) Fit a Poisson distribution to the following data

x	0	1	2	3	4	5	6	7	8
f	56	156	132	92	37	22	4	0	1

6

(b) Solve the following LPP using Simplex method

Maximize $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$ 6

(c) Expand $f(z) = \dfrac{2}{(z-2)(z-1)}$ in the regions

$i)\ |z| < 1,\ ii)\ 1 < |z| < 2,\ iii)\ |z| > 2$ 8

5. (a) Evaluate using Cauchy's Residue theorem $\oint_C \dfrac{1-2z}{z(z-1)(z-2)} dz$ where c is $|z| = 1.5$ 6

[TURN OVER

(b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6

(c) Solve the following LPP using the Dual Simplex method

Minimize $z = 2x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + 3x_2 + 5x_3 \geq 2$

$3x_1 + x_2 + 7x_3 \leq 3$

$x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$. 8

6. (a) Solve the following NLPP using Kuhn-Tucker conditions

Maximize $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to $2x_1 + x_2 \leq 5$; *and* $x_1, x_2 \geq 0$ 6

(b) In an experiment on immunization of cattle from Tuberculosis the following results were obtained

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	155	912
Total	1024	182	1206

Use χ^2 Test to determine the efficacy of vaccine in preventing tuberculosis. 6

(c) i) The regression lines of a sample are $x + 6y = 6$ *and* $3x + 2y = 10$ find a) sample means $\bar{x}$ *and* $\bar{y}$ b) coefficient of correlation between x *and* y 4

ii) If two independent random samples of sizes 15 & 8 have respectively the means and population standard deviations as

$\bar{x}_1 = 980, \bar{x}_2 = 1012 : \sigma_1 = 75, \sigma_2 = 80$

Test the hypothesis that $\mu_1 = \mu_2$ at 5% level of significance. 4

MD-Con. 8175-15.

Sem-IV (CBGS)
Computer/Applied Math-IV
& IT

Q. P. Code : 541302

(3 Hours)

[Total Marks: 80

N.B. : (1) Question No. one is compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Use of stastical Tables permitted.

(4) Figures to the right indicate full marks

1. (a) Find the Eigen values of $A^2 + 2I$, where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 0 \\ 3 & 5 & 3 \end{bmatrix}$ and I is the Identity matrix of order 3. 5

(b) Evaluate the line integral $\int_0^{1+i} (x^2 + iy)dz$ along the path $y = x$ 5

(c) If x is a continuous random variable with the probability density function given by

$$f(x) = \begin{cases} k(x - x^3) & 0 \le x \le 1 \\ 0 & otherwise \end{cases},$$

Find i) k ii) the mean of the distribution. 5

(d) Compute Spearman's rank correlation coefficient from the following data

X	18	20	34	52	12
Y	39	23	35	18	46

5

2. (a) Is the following matrix Derogatory? Justify.

$$\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$ 6

(b) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 2$ 6

(c) The marks of 1000 students in an Examination are found to be normally distributed with mean 70 and standard deviation 5, estimate the number of students whose marks will be i) between 60 and 75 ii) more than 75. 8

[Turn over

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3. (a) Solve the following non-linear programming problem using Kuhn-Tucker conditions

Maximize $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to $2x_1 + x_2 \leq 5$; *and* $x_1, x_2 \geq 0$ 6

(b) Fit a Binomial distribution to the following data

x	0	1	2	3	4	5	6
F	5	18	28	12	7	6	4

6

(c) Is the following matrix diagonalizable? If yes, find the transforming matrix and the diagonal matrix.

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

8

4. (a) Solve the following LPP using Simplex method

Maximize $z = 4x_1 + x_2 + 3x_3 + 5x_4$

Subject to $-4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$ 6

(b) If a random variable X follows the Poisson distribution such that $P(X = 1) = 2P(X = 2)$, find the mean, the variance of the distribution and $P(X = 3)$ 6

(c) Expand $f(z) = \frac{1}{z(z-2)(z+1)}$ in the regions

$i)\ |z| < 1, ii)\ 1 < |z| < 2, iii)\ |z| > 2$ 8

[Turn over

5. (a) Evaluate using Cauchy's Residue theorem $\oint_c \frac{2z-1}{z(2z+1)(z+2)} dz$ where c is

$|z| = 1.$ 6

(b) A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:

$5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6$

Can it be concluded that the stimulus will increase the blood pressure (at 5% level of significance.)? 6

(c) Solve the following LPP using the Dual Simplex method

Maximise $z = -3x_1 - 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$ 8

6.(a) Find the equations of lines of regression for the following data

x	5	6	7	8	9	10	11
y	11	14	14	15	12	17	16

6

(c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ using contour integration. 6

(b) In an experiments on pea breeding, the following frequencies of seeds were obtained

Round and Yellow	Wrinkled and yellow	Round and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9: 3: 3: 1.

Examine the correspondence between theory and experiment using Chi-square Test 8

IV / COMP/I.T/CBGS/App. maths IV / 24th NOV'16

KARJAT, RAIGAD * K. G. C. E. * EXAM

QP Code : 541304

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. **one** is **compulsory.**
(2) Answer **any three** questions from **Q.2 to Q.6**
(3) Use of stastical Tables permitted.
(4) Figures to the **right** indicate **full** marks
(5) Assume suitable data wherever applicable.

1. (a) Find the Eigenvalues and eigenvectors of the matrix. 5

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(b) Evaluate the line integral $\int_0^{1+i} (x^2 + iy)dz$ along the path $y = x$ 5

(c) Find k and then E (x) for the p.d.f. 5

$$f(x) = \begin{cases} k(x-x^2) & ,0 \leq x \leq 1, k > 0 \\ 0 & , \text{otherwise} \end{cases}$$

(d) Calculate Karl pearson's coefficient of correlation from the following data. 5

x	100	200	300	400	500
y	30	40	50	60	70

2. (a) Show that the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is non- derogatory. 6

(b) Evaluate $\int \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z-1|=3$ 6

(c) If x is a normal variate with mean 10 and standard deviation 4 find 8
(i) $P(|x-14|<1)$ (ii) $P(5 \leq x \leq 18)$ (ii) $P(x \leq 12)$

3. (a) Find the relative maximum or minimum (if any) of the function 6
$Z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100$
(b) If x is Binomial distributed with E (x) =2 and V (x) = 4/3, find the probability distribution of x. 6

(c) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A^{50}. 8

4. (a) Solve the following L.P.P. by simplex method 6

Minimize $z = 3x_1 + 2x_2$

Subject to
$3x_1 + 2x_2 \leq 18$
$0 \leq x_1 \leq 4$
$0 \leq x_2 \leq 6$
$x_1, x_2 \geq 0.$

(b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6

(c) Find Laurent's series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ 8

When (i) $|z| < 1$, (ii) $1<|z|<2$ (iii) $|z|>2$

5. (a) Evaluate $\int_c \frac{z^2}{(z-1)^2 (z+1)} dz$ where C is $|z| = 2$ using residue theorem 6

(b) The regression lines of a sample are $x+6y=6$ and $3x+2y=10$ Find 6
(i) Sample means $\bar{x}$ and $\bar{y}$
(ii) Correlation coefficient between x and y. Also estimate y When $x = 12$

(c) A die was thrown 132 times and the following frequencies were observed 8

No.obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Using χ^2-test examine the hypothesis that the die is unbiased.

6\. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2+x+2}{x^4+10x^2+9}\,dx$ using contour integration. 6

(b) If a random variable x follows Poisson distribution such that $P(x=1)=2P(x=2)$ Find the mean and the variance of the distribution. Also find $P(x=3)$. 6

(c) Use Penalty method to solve the following L.P.P. 8

Minimize $z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \geq 5$

$x_1 + 2x_2 \geq 6$

$x_1, x_2 \geq 0$.

(3 Hours) **[Total Marks :80**

N.B. : (1) Questions no. **1** is compulsory.
(2) Attempt any **three** questions from Q. 2 to Q. 6
(3) Use of statistical table permitted.
(4) Figures to the **right** indicate **full** marks.

1. (a) Evaluate $\int_C (z-z^2)\,dz$, where C is the upper half of the circle $|z|=1$. **5**

(b) If $A=\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$, then find the eigen values of $6A^{-1}+A^3+2I$ **5**

(c) State whether the following statement is true or false with reasoning : " The regression coefficients between 2x and 2y are the same as those between x and y." **5**

(d) Construct the dual of the following L.P.P. **5**

Maximise $Z=3x_1+17x_2+9x_3$

Subject to $x_1-x_2+x_3 \geq 3$

$-3x_1+2x_3 \leq 1$

$2x_1+x_2-5x_3=1$

$x_1, x_2, x_3 \geq 0$

2. (a) Evaluate , $\int_c \frac{e^{2z}}{(z+1)^4}\,dz$, where C is the circle $|Z-1|=3$ **6**

(b) Show that the matrix $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is derogatory. **6**

(c) A manufacturer knows from his experience that the resistance of resistors he produces is normal with $\mu=100$ ohms and standard deviation $\sigma=2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms? **8**

3. (a) A discrete random variable has the probability distribution given below: **6**

x	-2	-1	0	1	2	3
p(x)	0.2	k	0.1	2k	0.1	2k

Find k, the mean and variance

[TURN OVER]

(b) Solve the following L.P.P. by simplex method **6**

Maximise $Z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

(c) Expand $f(z) = \dfrac{z^2 - 1}{z^2 + 5z + 6}$ around $z = 0$, indicating region of convergence. **8**

4. (a) Find the first two moments about the origin of Poisson distribution and hence find mean and variance. **6**

(b) Calculate R and r from the following data : **6**

x	12	17	22	27	32
y	113	119	117	115	121

(R - the rank correlation coefficient, r - correlation coefficient)

(c) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. **8**

Find the transforming matrix and the diagonal matrix.

5. (a) A tyre company claims that the lives of tyres have mean 42,000 kms with S.D of 4000 kms. A change in the production process is believed to result in better product. A test sample of 81 new tyres has a mean life of 42,500 kms. Test at 5% level of significance that the new product is significantly better than the old one. **6**

(b) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{5 + 3\sin\theta}$ using Cauchy's residue theorem. **6**

(c) Using the Kuhn-Tucker conditions solve the following N.L.P.P. **8**

Minimise $Z = 7x_1^2 + 5x_2^2 - 6x_1$

Subject to $x_1 + 2x_2 \leq 10$

$x_1 + 3x_2 \leq 9$

$x_1, x_2 \geq 0$

[TURN OVER]

6. (a) 300 digits were chosen at random from a table of random numbers. The frequency of digits was as follows. **6**

Digit	0	1	2	3	4	5	6	7	8	9	Total
Frequency	28	29	33	31	26	35	32	30	31	25	300

Using χ^2-test examine the hypothesis that the digits were distributed in equal numbers in the table.

(b) Use the dual simple method to solve the following L.P.P. **6**

Minimise $Z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1, x_2 \geq 0$

(c) (i) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. **4**

(ii) A random variable X has the following probability distribution **4**

x	0	1	2	3
p(x)	$1/6$	$1/3$	$1/3$	$1/6$

Find M.G.F about the origin and hence first four raw moments.

MATHEMATICS SOLUTION

(CBCGS MAY – 2018 SEM - 4)

BRANCH – COMPUTER ENGINEERING

Q1] A) Find all the basics solutions to the following problem: (5)

Maximise : $\mathbf{z = x_1 + 3x_2 + 3x_3}$

Subject to : $\mathbf{x_1 + 2x_2 + 3x_3 = 4}$

$\mathbf{2x_1 + 3x_2 + 5x_3 = 7}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION :-

Maximise : $z = x_1 + 3x_2 + 3x_3$

Subject to : $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

$x_1, x_2, x_3 \geq 0$

No of basic solution	Non basic variable	Basic variable	Equation and values of basic variables	Is solution feasible?	Is solution degenerate	Value of Z	Is solution optimal
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 7$ $x_1 = 2, x_2 = 1$	Yes	No	5	Yes
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$ $x_1 = 1, x_3 = 1$	Yes	No	4	No
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$ $x_2 = -1, x_3 = 2$	no	no	-	-

---

Q1] B) Evaluate $\int_0^{1+2i} z^2 dz$, along the curve $2x^2 = y$ **(5)**

SOLUTION :-

$y = 2x^2$

$dy = 4xdx$

$z = x + iy$

$dz = dx + idy = dx + i4xdx \;=\; (1 + 4xi)dx$ …………… (1)

$\int_0^{1+2i} (x + iy)^2 dz \;=\; \int_0^{1+2i} x^2 - y^2 + 2ixydz = \int_0^1 (x^2 - 4x^4 + 2ix.2x^2)(1 + i4x)dx$

$\int_0^1 (x^2 - 4x^4 + 4ix^3 + i4x^3 - 16x^5 i - 16x^4)dx \;=\; \int_0^1 (x^2 - 20x^4 + 8ix^3 - 16x^5 i)dx$

$= \left[\frac{x^3}{3} - \frac{20x^5}{5} + 2i - \frac{16x^6 i}{6}\right]_0^1 \;=\; \frac{1}{3} - 4 + 2i - \frac{8}{3}$

ans $: - \; -\frac{11}{3} - \frac{2i}{3}$

Q1] C) A random sample of size 16 from a normal population showed a mean of 103.75cm and sum of squares of derivations from the mean 843.75 cm^2. Can we say that the population has a mean of 108.75? **(5)**

SOLUTION :-

First we calculate sample standard deviation

$s^2 = \frac{\sum(xi - \bar{x})^2}{n} = \frac{843.75}{16} = 52.73$

(i) The null hypothesis $H_0 : \mu = 108.75$
Alternate hypothesis $H_a : \mu \neq 108.75$

(ii) $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{103.75 - 108.75}{\sqrt{52.73}/\sqrt{15}} = -2.67$
$|t| = 2.67$

(iii) Level of significance ; $\alpha = 0.05$

(iv) t_α for 5% level of significance and degree of freedom
$v = 16 - 1 = 15$ is 2.131

(v) $2.67 > 2.131$

i.e $t_{calc} > t_{obs}$

we cannot say that mean population is 108.75

Q1] D) If $A = \begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$; Find : sinA **(5)**

$A = \begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$; TF : sinA

The characteristics equation is;

$$\begin{bmatrix} \frac{\pi}{2} - \lambda & \pi \\ 0 & \frac{3\pi}{2} - \lambda \end{bmatrix} = 0$$

$$\left(\frac{\pi}{2} - \lambda\right)\left(\frac{3\pi}{2} - \lambda\right) = 0$$

Let $\varphi(A) = \sin A = \alpha_1 A + \alpha_0 I$

Λ satisfies the above equation we have

$$\sin\lambda = \alpha_1\lambda + \alpha_0$$

Putting $\lambda = \frac{\pi}{2}$ and $\frac{3\pi}{2}$

$$\sin\left(\frac{\pi}{2}\right) = \alpha_1\left(\frac{\pi}{2}\right) + \alpha_0$$

$1 = \alpha_1\left(\frac{\pi}{2}\right) + \alpha_0$ ………………………. (1)

$$\sin\left(\frac{3\pi}{2}\right) = \alpha_1\left(\frac{3\pi}{2}\right) + \alpha_0$$

$-1 = \alpha_1\left(\frac{3\pi}{2}\right) + \alpha_0$ ………………………. (2)

Equation (1) – (2);

$$2 = \alpha_1\left(\frac{\pi}{2} - \frac{3\pi}{2}\right)$$

$2 = \alpha_1(-\pi) \qquad = \qquad \alpha_1 = -\frac{2}{\pi}$

And $\alpha_0 = 2$

$$\sin A = -\frac{2}{\pi}\begin{bmatrix} \frac{\pi}{2} & \pi \\ 0 & \frac{3\pi}{2} \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mathbf{\sin A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}}$$

Q2] A) Evaluate $\int_c^0 \frac{dz}{z^3(z+4)}$, where c is the circle $|z| = 2$ (6)

SOLUTION:-

Poles are $z = 0$ and $z = -4$

$z = 0$ lies inside the circle

residue at $z = o \; = \frac{1}{2!}\lim_{z\to 0}\left\{\frac{d^2}{dz^2}\left[z^3 . \frac{1}{z^3(z+4)}\right]\right\} = \frac{1}{2}\lim_{z\to 0}\left\{\frac{d^2}{dz^2}\left[\frac{1}{(z+4)}\right]\right\}$

$= \frac{1}{2}\lim_{z\to 0}\left\{\frac{d}{dz}\left[-\frac{1}{(z+4)^2}\right]\right\} = \frac{1}{2}\lim_{z\to 0}\left\{\left[\frac{2(z+4)}{(z+4)^2}\right]\right\} = \frac{1}{64}$

Residue at $z = -4 = 0$ as it lies outside the circle.

$$\boldsymbol{\varphi f(z)dz = 2\pi i\left(\frac{1}{64}\right) = \frac{\pi i}{32}}$$

Q2] B) Memory capacity of 9 students was tested before and after a course of mediation for a month. State whether the course was effective or not from the data below. (6)

Before	**10**	**15**	**9**	**3**	**7**	**12**	**16**	**17**	**4**
After	**12**	**17**	**8**	**5**	**6**	**11**	**18**	**20**	**3**

SOLUTION :-

Before	10	15	9	3	7	12	16	17	4
After	12	17	8	5	6	11	18	20	3

Calculating difference between after – before,

X	2	2	-1	2	-1	-1	2	3	-1
$di = (X_i - 2)$	0	0	-3	0	-3	-3	0	1	-3
$di^2 = (X_i - 2)^2$	0	0	9	0	9	9	0	1	9

$$\bar{X} = a + \frac{\sum di}{n} = 2 + \frac{-11}{9} = 0.7778$$

$$\sum(X_i - \bar{X})^2 = \sum di^2 - \frac{(\sum di)^2}{n} = 37 - \frac{(-11)^2}{9} = 23.5556$$

$$S^2 = \frac{\sum(X_i - \bar{X})^2}{n} = \frac{23.5556}{9} = 2.6173$$

Null hypothesis ; $\mu = 0$

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n-1}} = \frac{0.7778 - 0}{\sqrt{2.6173}\,.\sqrt{8}}$$

t at $\alpha = 0.05$ for $v = 9 - 1 = 8$ is 2.306

$t_{cal} < t_{critical}$

Hypothesis is accepted

The students are not benefitted.

Q2] C) Solve the following LPP using simple method. (8)

Maximise ; $\mathbf{z = 6x_1 - 2x_2 + 3x_3}$

Subject to $\mathbf{2x_1 - x_2 + 2x_3 \leq 2}$

$\mathbf{x_1 + 4x_3 \leq 4}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION:-

Maximise ; $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

Expressing in standard form;

$$z - 6x_1 + 2x_2 - 3x_3 + 0s_1 + 0s_2 = 0$$

$$2x_1 - x_2 + 2x_3 + s_1 + 0s_2 = 2$$

$$x_1 + 0x_2 + 4x_3 + 0s_1 + s_2 = 4$$

Iteration no	Basic variable	Coefficients of					RHS soln	ration
		x_1	x_2	x_3	s_1	s_2		
0	Z	-6	2	-3	0	0	0	
s_1 leaves	s_1	2	-1	2	1	0	2	1
x_1 enters	s_2	1	0	4	0	1	4	4
1	z	0	-1	3	3	0	6	
s_2 leaves	x_1	1	-1/2	1	1/2	0	1	-2
x_2 enters	s_2	0	1/2	3	-1/2	1	3	6
2	Z	0	0	9	2	2	12	
	x_1	1	0	4	0	1	4	
	x_2	0	1	6	-1	2	6	

$\mathbf{x_1 = 4;\ x_2 = 6;\ x_3 = 0\ \text{and}\ z_{max} = 12}$

Q3] A) Find the Eigen values and Eigen vectors of the following matrix, (6)

$$\mathbf{A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}}$$

SOLUTION:-

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

Characteristics equation is given as follows:

$$\lambda^3 - 4\lambda^2 + (-1 - 6 + 6)\lambda + 4 = 0$$

$$\lambda^3 - 4\lambda^2 - \lambda + 4 = 0$$

$\lambda^2(\lambda - 4) - 1(\lambda - 4) = 0$

$(\lambda - 4)(\lambda^2 - 1) = 0$

$\lambda = 4$ and $\lambda = \pm 1$

For $\lambda = 4$

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & -2 \\ -1 & -4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 6x_2 + 6x_3 = 0$

$1x_1 - 1x_2 + 2x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{bmatrix} 0 & 6 \\ 1 & 2 \end{bmatrix}} = \frac{x^3}{\begin{bmatrix} 0 & 6 \\ 1 & -1 \end{bmatrix}} = t$$

$$\frac{x_1}{18} = \frac{x_2}{6} = \frac{x_3}{6} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3t \\ t \\ t \end{bmatrix}$$

For $\lambda = 1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 + x_3 = 0$

$-x_1 - 4x_2 - 4x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 2 & 1 \\ -4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ -1 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix}} = t$$

$$\frac{x_1}{-4} = \frac{x_2}{3} = \frac{x_3}{-2} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4t \\ -3t \\ -2t \end{bmatrix}$$

For $\lambda = -1$

$$\begin{bmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 + 6x_2 + 6x_3 = 0$

$-x_1 - 4x_2 - 2x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 6 \\ -1 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 6 \\ -1 & -4 \end{vmatrix}} = t$$

$$\frac{x_1}{12} = \frac{x_2}{4} = \frac{x_3}{-14} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6t \\ 2t \\ -7t \end{bmatrix}$$

Eigen values = 4,1,-1

Eigen vectors = [3t t t],[-4t 3t -2t], [6t 2t -7t]

Q3] B) For a normal distribution 30% items are below 45% and 8% are above 64. Find the mean and variance of the normal distribution. (6)

SOLUTION:-

30% items below 45%

50-30 = 20% items are between 45 and m

8%are above 64

50-8= 42 % items are between m and 64

From table we find that

Area 0.2 corresponds to z = 0.525

Area 0.42 corresponds to z = 1.40

0.2 area is to the left of m

Hence z = -0.525

$\frac{45-m}{\sigma} = -0.525; \frac{64-m}{\sigma} = 1.4$

$45 - m = -0.525\sigma$

And $64 - m = 1.4\sigma$

We get, $\sigma = 9.87$

M = 50.1818

Mean = 50.1818; var =$\sigma^2 = 97.4169$

Q3] C) Obtain Laurent's series for $f(z) = \frac{1}{z(z+2)(z+1)}$ about $z = -2$ **(8)**

SOLUTION:-

$f(z) = \frac{1}{z(z+2)(z+1)}$ about z = -2

Applying partial fraction,

$$\frac{1}{z(z+2)(z+1)} = \frac{A}{z} + \frac{B}{z+2} + \frac{C}{z+1}$$

$$1 = A(z+2)(z+1) + B(z)(z+1) + C(z)(z+2)$$

Put z = -2

$$1 = B(-2)(-1) = B = \frac{1}{2}$$

Put z = 0

$$1 = 2A - 1\,, \quad A = \frac{1}{2}$$

Put z=-1

$$1 = -c, \quad c = 1$$

$$\frac{1}{z(z+2)(z+1)} = \frac{1}{2z} + \frac{1}{2(z+2)} - \frac{1}{z+1} = \frac{1}{2[(z+2)-2]} + \frac{1}{2(z+2)} - \frac{1}{[(z+2)-1]}$$

Let z+2 = u, we get,

$$= \frac{1}{2[u-2]} + \frac{1}{2u} - \frac{1}{u-1} = -\frac{1}{4\left(1-\frac{u}{2}\right)} + \frac{1}{2u} + \frac{1}{1-u}$$

$$\frac{1}{z(z+2)(z+1)} = \frac{1}{2u} - \frac{1}{4}\left[1 + \frac{u}{2} + \left(\frac{u}{2}\right)^2 + \cdots..\right] + [1 + u + u^2 \ldots..]$$

Q4] A) An ambulance services claims that it takes on an average 8.9 min to reach the destination in emergency calls. To check this the Licensing Agency has then timed on 50 emergency calls, getting a mean 0f 9.3 min with a S.D. 1.6 min. is the claim acceptable at 5% LOS? **(6)**

SOLUTION:-

$\overline{X} = 9.3$

$\mu = 8.9$

SD = 1.6 min

n = 50

we have $z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{9.3-8.9}{50/\sqrt{1.6}} = \frac{0.4\sqrt{1.6}}{50} = 0.0101$

$\alpha = 0.05$

Z at 5% level of significance = 1.96

0.0101 < 1.96

Thus the hypothesis is acceptable.

Q4] B) Using the Residue theorem Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$ (6)

SOLUTION:-

$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$

Put z = $e^{i\theta}$, $dz = ie^{i\theta}d\theta$

$dz = iz d\theta \quad d\theta = \frac{dz}{iz}$

And $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} = \frac{z+\left(\frac{1}{z}\right)}{2}$

$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \int_c^1 \frac{z^2}{5+4\left(\frac{z+\frac{1}{z}}{2}\right)} \frac{dz}{iz}$

$\int_0^{2\pi} \frac{e^{2i\theta}}{5+4\cos\theta} d\theta = \int_c^1 \frac{z^2}{i(2z^2+5z+2)} dz$

$c : |z| = 1$

Now the poles are given by $2z^2 + 5z + 2 = 0$

$(2z + 1)(z + 2) = 0$

$z = -\frac{1}{2} \quad \text{and} \quad -2$

for $z = -\frac{1}{2}$

Residue = $\lim_{z \to -1/2} \left(z + \frac{1}{2}\right) \frac{z^2}{2\left[z+\frac{1}{2}\right][z+2]i} = \frac{(-\frac{1}{2})^2}{2\left[-\left(\frac{1}{2}\right)+2\right]i} = \frac{1}{12i}$

Z = -2 lies outside the circle;

Residue = 0

$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta = 2\pi i \left(\frac{1}{12i}\right) = \frac{\pi}{6}$$

$$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\, d\theta = \text{real part of } \int_0^{2\pi} \frac{e^{2i\theta}}{5+4\cos\theta} d\theta = \frac{\pi}{6}$$

Q4] C) (i) If 10% of the rivets produced by a machine are defective, find the probability that out of 5 randomly chosen rivets at the most two will be defective.

(ii) If x denotes the outcome when a fair die is tossed, find M.G.F. of x and hence, find the mean and variance of x. (8)

SOLUTION:-

(1)

$p = 0.1$

$q = 1 - p = 0.9$

$n = 5$

P at most will be defective:

P(0 defective)+ P(1 defective) + P(2 defective)

$5C_0(0.1)^2(0.9)^5 + 5C_1(0.1)^1(0.9)^4 + 5C_2(0.1)^2(0.9)^3 = 0.59049 + 0.328 + 0.0792 = 0.99$

(2) Moment generating function,

$M_0(t) = E\left(e^{txi}\right) = \sum P_i e^{txi} = \frac{1}{6}e^t + \frac{1}{6}e^{2t} + \cdots \ldots \ldots \ldots . . \frac{1}{6}e^{6t}$

$= \frac{1}{6}(e^t + e^{2t} + \cdots \ldots \ldots . . e^{6t})$

$\mu_1' = \text{mean} = \frac{d}{dt[M_{0(t)}]} = \frac{1}{6}[e^t + 2e^{2t} + \cdots \ldots \ldots . . 6e^{6t}]$

$\text{at } t = 0; \text{mean} = \frac{1}{6}[1 + 2 + 3 \ldots \ldots . . +6] = \frac{21}{6} = \frac{7}{2}$

$\text{variance} = \mu_2' = \frac{d^2}{dt^2}\left[M_{0(t)}\right] = \frac{1}{6}[e^t + 4e^{2t} + \cdots \ldots \ldots . . 36e^{6t}]$

$= \frac{1}{6}[1 + 4 + 9 \ldots \ldots . +36] = \frac{91}{6}$

$\mu_2 = \mu_2' - {\mu_1'}^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$

$\mathbf{mean} = \frac{7}{2}$ **and variance** $= \frac{35}{12}$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory

$$\mathbf{A} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ **(6)**

SOLUTION:-

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Characteristics equation :

$\lambda^3 - 12\lambda^2 + (8 + 14 + 14)\lambda - 32 = 0$

Λ = 8,2,2

Let us assume $(x - 8)(x - 2) = x^2 - 10x + 16$ annihilates A

Now $A^2 - 10A + 16I$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - 10 \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + 16 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 0 & 0 \\ 0 & -16 & 0 \\ 0 & 0 & -16 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\mathbf{x^2 - 10x + 16}$ **annihilates A thus f(x) is the monic polynomial of lowest degree**

$\mathbf{minimal\ polynomial = x^2 - 10x + 16}$

Q5] B)Justify if there is any relationship between sex and color for the following data.

(6)

colour	**male**	**female**	**Total**
Red	**10**	**40**	**50**
White	**70**	**30**	**100**
Green	**30**	**20**	**50**
total	**110**	**90**	**200**

SOLUTION :-

colour	male	female	Total
Red	10	40	50
White	70	30	100
Green	30	20	50
total	110	90	200

Null hypothesis, H_0, there is no relationship

Alternative hypothesis , H_a there is an relationship

Calculation of test statistics;

Red and male : $\frac{110\times 50}{200} = 27.5$

White and male : $\frac{110\times 100}{200} = 55$

Green and male : $\frac{110\times 50}{200} = 27.5$

Red and female : $\frac{90\times 50}{200} = 22.5$

White and female : $\frac{90\times 100}{200} = 45$

Green and female : $\frac{90\times 50}{200} = 22.5$

O	E	$(\mathrm{O}-\mathrm{E})^2$	$(\mathrm{O}-\mathrm{E})^2/\mathrm{E}$
27.5	10	306.25	30.625
55	70	225	3.2143

27.5	30	6.25	0.2083
22.5	40	306.25	7.6562
45	30	225	7.5
22.5	20	6.25	0.3125 X = 49.5288

$\alpha = 0.05$

Degree of freedom = $(r-1)(c-1) = (3-1)(2-1) = 2$

Critical value = 5.991

$X^2_{calc} \; > \; X^2_{table}$

thus null hypothesis is rejected

There is relationship between colour gender.

Q5] C) Use the dual simplex method to solve the following L.P.P. (8)

Minimise

$\mathbf{z = 2x_1 + x_2}$

Subject to $\mathbf{3x_1 + x_2 \geq 3}$

$\mathbf{4x_1 + 3x_2 \geq \; 6}$

$\mathbf{x_1 + 2x_2 \leq 3}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

Minimise

$z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq \; 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq - \ 6$

$x_1 + 2x_2 \leq 3$

Introducing stack variables

$Z' - 2x_1 - x_2 - 0s_1 - 0s_2 - 0s_3$

$-3x_1 - x_2 + s_1 - 0s_2 - 0s_3 = \ -6$

$x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 3$

Iterations no	Basics variables	Coefficients of					RHS Solution
		x_1	x_2	s_1	s_2	s_3	
0	Z'	-2	-1	0	0	0	0
s_2 leaves	s_1	-3	-1	1	0	0	-3
x_2 enters	s_2	-4	-3	0	1	0	-6
	s_3	1	2	0	0	1	3
Ratio:		-1/2	-1/3				
1	Z'	2	2	0	-1	0	6
	s_1	-5/3	0	0	1/3	0	-1
	x_2	4/3	1	0	1/3	0	2
	s_3	-5/3	0	0	-2/3	1	-1
Ratio:		-6/5			3/2	0	
2		11/3	2	0	-1/3	0	7
s_3 leaves	s_1	0	0	0	1	-1	0
x_1 enters	x_2	0	1	0	3	-4/3	6/5
	x_1	1	0	0	2/5	-3/5	3/5

$x_1 = \frac{3}{5}; \ \ x_2 = \frac{6}{5};$

$Z_{min} = 2\left(\frac{3}{5}\right) + \frac{6}{5} = \frac{12}{5}$

$\mathbf{Z_{min}} = \frac{\mathbf{12}}{\mathbf{5}}$

Q6] A)Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1} where **(6)**

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-2} \\ \mathbf{-1} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{1} \end{bmatrix}$$

SOLUTION :-

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Characteristics equation,

$$\lambda^3 - 5\lambda^2 + (3+1+5)\lambda - 1 = 0$$

$$\lambda^3 - 5\lambda^2 + (9)\lambda - 1 = 0$$

Cayley Hamilton theorem states that;

$$A^3 - 5A^2 + 9A - I = 0 \qquad \text{.....................} (1)$$

$$A^2 = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}^2 = \begin{bmatrix} -1 & 12 & -4 \\ -4 & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}^3 = \begin{bmatrix} -13 & 42 & -2 \\ -11 & 9 & 10 \\ 10 & -22 & -3 \end{bmatrix}$$

Substituting the values in equation (1)

We get, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Multiply the above equation by A^{-1}

$$A^{-1} = A^2 - 5A + 9I$$

$$A^{-1} = \begin{bmatrix} -1 & 12 & -4 \\ -4 & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\mathbf{A^{-1}} = \begin{bmatrix} \mathbf{3} & \mathbf{2} & \mathbf{6} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{2} & \mathbf{5} \end{bmatrix}$$

Q6] B) The probability Distribution of a random variable X is given by (6)

X	**-2**	**-1**	**0**	**1**	**2**	**3**
P(X=x)	**0.1**	**k**	**0.2**	**2k**	**0.3**	**k**

Find k, mean and variance.

SOLUTION:-

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

$\sum P_i = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$0.6 + 4k = 1$

$4k = 0.4$

$k = 0.1$

X	-2	-1	0	1	2	3
P(X=x)	0.1	0.1	0.2	0.2	0.3	0.1

$\text{Mean} = E(X) = \sum P_i x_i = -0.2 - 0.1 + 0.2 + 0.6 + 0.3 = 0.8$

$E(X^2) = \sum P_i {x_i}^2 = 0.4 + 0.1 + 0.2 + 1.2 + 0.9 = 2.8$

$\text{Variance} = E(X^2) - [E(X)]^2 = 2.8 - 0.64 = 2.16$

$\mathbf{E(X^2) = 2.8}$

Variance = 2.16

Q6] C)Using Kuhn- Tucker conditions solve the following NLPP (8)

Maximise : $\mathbf{Z = 2x_1 - 7x_2 + 12x_1x_2}$

Subject to : $\mathbf{2x_1 + 5x_2 \leq 98}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION:-

Maximise : $Z = 2x_1 - 7x_2 + 12x_1x_2$

Subject to : $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

We rewrite the given problem as:

$f(x) = 2x_1^2 - 7x_2^2 + 12x_1x_2$

$h(x_1x_2) = 2x_1 + 5x_2 - 98$

Kuhn tucker conditions are:

$\frac{\partial f}{\partial x_1} - \frac{\lambda\,\partial h}{\partial x_1} = 0;\ \frac{\partial f}{\partial x_2} - \frac{\lambda\,\partial h}{\partial x_2} = 0$

$\lambda h(x_1, x_2) = 0;\ \ h(x_1, x_2) \leq 0,\ \lambda \geq 0$

We get,

$4x_1 + 12x_2 - \lambda(2) = 0$ ………………… (1)

$12x_1 - 14x_2 - \lambda(5) = 0$ ……………………(2)

$\lambda(2x_1 + 5x_2 - 98) = 0$ ………………… (3)

$2x_1 + 5x_2 - 98 \leq 0$ ……………… (4)

$\lambda \geq 0$ ………… (5)

From (3) we get either $\lambda = 0$ or $(2x_1 + 5x_2 - 98) = 0$

Case 1: $\lambda = 0$ and $(2x_1 + 5x_2 - 98) \neq 0$

From 1 and 2,

$4x_1 + 12x_2 = 0$

$12x_1 - 14x_2 = 0$

On solving simultaneously we get $x_1 = x_2 = 0$

Case 2:

$\lambda \neq 0$ and $2x_1 + 5x_2 - 98 = 0$

$4x_1 + 12x_2 - \lambda(2) = 0$

$12x_1 - 14x_2 - \lambda(5) = 0$

$\lambda = \frac{12x_1 - 14x_2}{5}$

Equation I : $4x_1 + 12x_2 - 2\left[\frac{12x_1 - 14x_2}{5}\right] = 0$

$20x_1 + 60x_2 - 24x_1 + 28x_2 = 0$

$-4x_1 + 88x_2 = 0$ (divide through by 4)

$-x_1 + 22x_2 = 0$

Put $x_1 = 22x_2$ in 4

$2(22x_2) + 5x_2 = 98$

$44x_2 + 5x_2 = 98$

$49x_2 = 98$

$x_2 = 2$

$2x_1 + 10 = 98$

$2x_1 = 88$

$x_1 = 44$

These values satisfy all conditions,

$Z_{max} = 2(1936) - 7(4) + 12(44)(2) = 4900$

$\mathbf{Z_{max} = 4900}$

QP CODE: 40558

MATHEMATICS SOLUTION

(CBCGS SEM – 4 NOV 2018)

BRANCH – COMPUTER ENGINEERING

Q1] A) Find all the basic solutions to the following problem: **(5)**

Maximise : $\mathbf{z = x_1 + x_2 + 3x_3}$

Subject to : $\mathbf{x_1 + 2x_2 + 3x_3 = 9}$

$\mathbf{3x_1 + 2x_2 + 2x_3 = 15}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION:-

Maximise : $z = x_1 + x_2 + 3x_3$

Subject to : $x_1 + 2x_2 + 3x_3 = 9$

$3x_1 + 2x_2 + 2x_3 = 15$

$x_1, x_2, x_3 \geq 0$

No of basic solution	Non basic variables	Basic variable	Equation and the value of basic variables	Is the solution feasible	Is the solution degenerate	Value of Z
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 9$ $3x_1 + 2x_2 = 15$ $x_1 = 3$ $x_2 = 3$	yes	no	6
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 9$ $3x_1 + 2x_3 = 15$ $x_1 = 3.86$ $x_3 = 1.71$	yes	no	8.99
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 9$ $2x_2 + 2x_3 = 15$ $x_2 = 13.5$ $x_3 = -6$	no	no	-

---

Q1] B) Evaluate $\oint zdz$ from $z = 0$ to $z = 1 + i$ along the curve $z = t^2 + it$ **(5)**

SOLUTION :-

When $z = 0$, $t = 0$

When $z = 1 + i$; $t = 1$

$z = t^2 + it$

$dz = (2t + i)dt$

$\int_0^{1+i} zdz = \int_0^1 (t^2 + it)(2t + i)dt = \int_0^1 (2t^3 + it^2 + 2it^2 - t)dt = \int_0^1 2t^3 - t + 3it^2)dt$

$= \left[\frac{t^4}{2} - \frac{t^2}{2} + it^3\right]_0^1 = i$

$\oint zdz = i \quad$ **where $z = t^2 + it$ along $z = 0$ to $i + 1$**

Q1] C) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160cm. Can it be reasonably regarded that in the population, the mean height is 165cm, and the standard deviation is 10cm? **(5)**

SOLUTION :-

Null hypothesis : $\mu = 160$

Alternate hypothesis : $\mu \neq 160$

$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{165 - 160}{10/\sqrt{100}} = 5$

$|z| = 5$

At 5% level of significance, z is 1.96.

$z_{cal} > z_{critical}$

Null hypothesis is rejected.

Therefore, No it wouldn't be reasonable to suppose the assumption.

Q1] D) The sum of the Eigen values of a 3 X 3 matrix is 6 and the product of the Eigen value id also 6. If one of the Eigen value is one, find the other two Eigen values. (5)

SOLUTION:-

Let the Eigen values be λ_1, λ_2 and λ_3

$\lambda_1 + \lambda_2 + \lambda_3 = 6$

And $\lambda_1 . \lambda_2 . \lambda_3 = 6$

Given : one Eigen value = 1

Let $\lambda_1 = 1$

$1 + \lambda_2 + \lambda_3 = 6$

$\lambda_2 + \lambda_3 = 5$

$\lambda_3 = (5 - \lambda_2)$

And $(\lambda_2 - \lambda_3) = 6$

$\lambda_2(5 - \lambda_2) = 6$

$5\lambda_2 - {\lambda_2}^2 = 6$

${\lambda_2}^2 - 5\lambda_2 + 6 = 0$

${\lambda_2}^2 - 3\lambda_2 - 2\lambda_2 + 6 = 0$

$\lambda_2(\lambda_2 - 3) - 2(\lambda_2 - 3) = 0$

$\lambda_2 = 2 \quad \text{or} \quad \lambda_2 = 3$

If $\lambda_2 = 2$ and $\lambda_3 = 3$ and if $\lambda_2 = 3$ and $\lambda_3 = 2$

The Eigen values are 1,2,3.

Q2] A) Evaluate $\oint \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where c is the circle |z| = 1 for n = 1, n = 3 (6)

SOLUTION:-

$\oint \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where c is a is a circle $|z| = 1$

For n = 1 we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z_0 - \frac{\pi}{6} = 0$

$Z_0 = \frac{\pi}{6}$ pt lies inside the circle ; |z| = 1

$\int \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)$ { By Cauchy's Integra formula}

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz = 2\pi i \sin^2\left(\frac{\pi}{6}\right) = 2\pi i\left(\frac{1}{2}\right)^6 = \frac{2\pi i}{64} = \frac{\pi i}{32}$$

For n =3; we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z = \frac{\pi}{6}$ lies inside the circle , $|Z| = 1$

Order of pole = 3

$\int \frac{f(z)}{(Z-Z_0)^n} = \frac{2\pi i}{(n-1)!} f^{n-1}(z_0)$

$f(z) = \sin^6 z$

$f'(z) = 6\sin^5 z.\cos z$

$f''(z) = 6[5\sin^4 z\cos^2 z - \sin^5 z.\sin z] = 6[5\sin^4 z\cos^2 z - \sin^6 z]$

$f''(z) = 6\left[5\sin^4\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{6}\right) - \sin^6\left(\frac{\pi}{6}\right)\right] = \frac{21}{16}$

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz = \frac{2\pi i}{2!} \cdot \frac{21}{16} = \frac{21\pi i}{16}$$

Q2] B) The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy? Use Chi-square test at 5% Level of significance (6)

	Smokers	Non smokers
Literates	**83**	**57**
Illiterates	**45**	**68**

Solution:-

	Smokers	Non smokers	Total
Literates	83	57	140
Illiterates	45	68	113
Total	128	125	253

Expected frequency (literate and smokers) = $\frac{140\times128}{253} = 70.83$

Expected frequency (literate and non - smokers) = $\frac{140\times125}{253} = 69.17$

Expected frequency (illiterate and smokers) = $\frac{113\times128}{253} = 57.17$

Expected frequency (illiterate and non - smokers) = $\frac{113\times125}{253} = 55.83$

O	E	$(0-\mathrm{E})^2$	$\frac{(\mathrm{O}-\mathrm{E})^2}{\mathrm{E}}$
83	70.83	148.11	2.0911
57	69.17	148.11	2.1412
45	57.17	148.11	2.5907
68	55.83	148.11	2.6529
			$\mathrm{X}^2 = 9.4759$

Level of significance = (0.05)

Degree of freedom = (r-1)(c-1) = (2-1)(2-1) = 1

Critical value at 1 degree of freedom at 5% level of significance is 3.84.

$\mathbf{X^2}_{\mathbf{table}} < \mathbf{X^2}_{\mathbf{calc}}$

There is no association.

Q2] C) Solve the following LPP using Simple Method **(8)**

Maximise $\mathbf{z = 3x_1 + 5x_2}$

Subject to $\mathbf{3x_1 + 2x_2 \leq 18, \quad x_1 \leq 4, \; x_2 \geq 6, \quad x_1, x_2 \geq 0}$

SOLUTION :-

$z = 3x_1 + 5x_2$

$3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \geq 6$

We first express the given problem in standard form;

$z - 3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

$3x_1 + 2x_2 + 1s_1 + 0s_2 + 0s_3 = 0$

$x_1 + 0s_1 + 0s_2 + 0s_3 = 0$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Simple table

Iteration number	Basic variables	Coefficient of					RHS soln	ratio
		x_1	x_2	s_1	s_2	s_3		
0	Z	-3	-5	0	0	0	0	
s_3 leaves	s_1	3	2	1	0	0	18	9
x_2 enters	s_2	1	0	0	1	0	4	-
	s_3	0	1	0	0	1	6	6
1	Z	-3	0	0	0	5	30	
s_1 leaves	s_1	3	0	1	0	-2	6	2
x_1 enters	s_2	1	0	0	1	0	4	4
	x_2	0	1	0	0	1	6	-
2	Z	0	0	1	0	3	36	
	x_1	1	0	1/3	0	-2/3	2	
	s_2	0	0	-1/3	1	2/3	2	
	x_2	0	1	0	0	1	6	

$x_1 = 2; \quad x_2 = 6;$

$$\mathbf{Z_{max} = 3(x_1) + 5(x_1) = 3(2) + 5(6) = 6 + 30 = 36}$$

Q3] A) Find the Eigen values and Eigen vectors of the following matrix. (6)

$$\mathbf{A} = \begin{bmatrix} \mathbf{4} & \mathbf{6} & \mathbf{6} \\ \mathbf{1} & \mathbf{3} & \mathbf{2} \\ \mathbf{-1} & \mathbf{-4} & \mathbf{-3} \end{bmatrix}$$

SOLUTION :-

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

The characteristics equation is given by

$$\lambda^3 - 4\lambda^2 + (-1 - 6 + 6)\lambda - 4 = 0$$

$$\lambda^3 - 4\lambda^2 - \lambda - 4 = 0$$

$$\lambda^2(\lambda - 4) - 1(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda^2 - 1) = 0$$

$$\lambda = 4 \quad \text{and} \quad \lambda = +1, -1$$

For $\lambda = 4$

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & 2 \\ -1 & -4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 + 6x_2 + 6x_3 = 0$$

$$1x_1 - 1x_2 + 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix}} = t$$

$$\frac{x_1}{18} = \frac{x_2}{6} = \frac{x_3}{6} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3t \\ t \\ t \end{bmatrix}$$

For $\lambda = 1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

$-x_1 - 4x_2 - 4x_3 = 0$

$\frac{x_1}{\begin{vmatrix} 2 & 1 \\ -4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 5 \\ -1 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix}} = t$

$\frac{x_1}{-4} = \frac{x_2}{3} = \frac{x_3}{-2} = t$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4t \\ 3t \\ -2t \end{bmatrix}$

For $\lambda = -1$

$\begin{bmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$5x_1 + 6x_2 + 6x_3 = 0$

$-x_1 - 4x_2 - 2x_3 = 0$

$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 6 \\ -1 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 6 \\ -1 & -4 \end{vmatrix}} = t$

$\frac{x_1}{12} = \frac{x_2}{4} = \frac{x_3}{-14} = t$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6t \\ 2t \\ -7t \end{bmatrix}$

Eigen values = 4,1,-1

Eigen vector = [3t t t], [-4t 3t -2t], [6t 2t -7t]

Q3] B) The income of a group of 10,000 persons were found to be normally distributed with mean of Rs 750 and standard deviation of Rs 50. What is the lowest income of richest 250? (6)

SOLUTION :-

Standard normal variate; $Z = \frac{X-m}{\sigma} = \frac{X-750}{50}$

If we have to consider the richest 250 persons, then probability that a person selected at random will be one of them is $\frac{250}{10000} = 0.025$

Area from $(z = 0 \text{ to } z = \text{this value}) = 0.5 - 0.025 = 0.475$

From the table, we find that the area from $z = 0$ to $z = 1.96$ is 0.475

The required $z = 1.96$

When $z = 1.96$, $1.96 = \frac{X-750}{50}$

$X - 750 = 1.96 \times 50 = 848$

Lowest income of richest 250 persons= Rs 848.

Q3] C) Expand $\frac{z^2-1}{z^2+5z+6}$ around z = 0 **(8)**

SOLUTION :-

Degree of numerator = degree of denominator

Dividing numerator by denominator

$f(z) = 1 - \frac{8}{z+3} + \frac{3}{z+2}$

case (i); when |z| < 2

$f(z) = 1 - \frac{8}{3\left(1+\frac{z}{3}\right)} + \frac{3}{2\left(1+\frac{z}{2}\right)} = 1 - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} + \frac{3}{2}\left(1+\frac{z}{2}\right)^{-1} = 1 - \frac{8}{3}\left(1 - \frac{z}{3} + \frac{z^2}{9} + \cdots\right) + \frac{3}{2}\left(1 - \frac{z}{2} + \frac{z^2}{4} + \cdots\right)$

case (ii); when 2 < |z| < 3

$f(z) = 1 - \frac{8}{3\left(1+\frac{z}{3}\right)} + \frac{2}{z\left(1+\frac{z}{2}\right)} = 1 - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} + \frac{2}{z}\left(1+\frac{z}{2}\right)^{-1}$

$= 1 - \frac{8}{3}\left(1 - \frac{z}{3} + \frac{z^2}{9} + \cdots\right) + \frac{2}{z}\left(1 - \frac{2}{z} + \frac{4}{z^2} + \cdots\right)$

Case (iii), when |z| > 3

$f(z) = 1 - \frac{8}{z\left(1+\frac{3}{z}\right)} + \frac{2}{z\left(1+\frac{2}{z}\right)} = 1 - \frac{8}{z}\left(1+\frac{3}{z}\right)^{-1} + \frac{2}{z}\left(1+\frac{2}{z}\right)^{-1}$

$$\mathbf{f(z) = 1 - \frac{8}{z}\left(1 - \frac{3}{z} + \frac{9}{z^2} + \cdots\right) + \frac{2}{z}\left(1 - \frac{2}{z} + \frac{4}{z^2} + \cdots\right)}$$

Q4] A)The mean breaking strength of cables supplied by a manufacturer is 1800 with S.D. 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cable has increased. In order to test the claim a sample of 50 cables are tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS. (6)

SOLUTION :-

Null hypothesis : $\mu = 1800$

alternate hypothesis : $\mu \neq 1800$

$|Z| = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850-180}{\frac{100}{\sqrt{50}}} = 3.53$

The value of Z at 1% level of significance = 2.576

$Z_{calc} > \ Z_{critical}$

Null hypothesis is rejected

Therefore the claim is not supported.

Q4] B) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ (6)

SOLUTION :-

$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$

Let $z = e^{i\theta} d\theta$

$dz = ie^{i\theta} d\theta$

$d\theta = \frac{dz}{ie^{i\theta}}$

$d\theta = \frac{dz}{iz}$ where C is unit circle $|z| = 1$

$\int_0^{2\pi} \frac{1}{5-3\left(\frac{z^2+1}{2z}\right)} \left(\frac{dz}{iz}\right) = \int_0^{2\pi} \frac{-2dz}{(10z-3z^2-3)i} = \int_0^{2\pi} \frac{-2dz}{(-10z+ \quad ^2+3)i} = \int \frac{-2}{(3z-1)(z-3)i}$

Z-3 lies outside the circle.

Residue of f(z) at $z = \frac{1}{3} = \lim_{z\to\frac{1}{3}}\left(z-\frac{1}{3}\right)\frac{-2}{i(3z-1)(z-3)} = \lim_{z\to\frac{1}{3}}\left(\frac{3z-1}{3}\right)\frac{-2}{i(3z-1)(z-3)} =$
$\frac{1}{3i}\left(-\frac{2}{\frac{1}{3}-3}\right) = \frac{1}{4i}$

$\int_0^{2\pi}\frac{d\theta}{5-3\cos\theta} = 2\pi i\left(\frac{1}{4i}\right) = \frac{\pi}{2}$

$$\boldsymbol{\int_0^{2\pi}\frac{d\theta}{5-3\cos\theta} = \text{real part of } \left(\int_0^{2\pi}\frac{d\theta}{5-3\cos\theta}\right) = \frac{\pi}{2}}$$

Q4] C) (1) Out of 1000 families with 4 children each, how many would you expect to have (a) at least one boy (b) at most 2 girls.

(2)Find the Moment Generating Function of POISSON Distribution and hence find its mean. (8)

SOLUTION :-

(1)

BBBB	BBBG	BBGB	BGBB
BBGG	BGGB	BGBG	BGGG
GGGG	GGGB	GGBG	GBGG
GGBB	GBBG	GBGB	GBBB

a) P(at last one boy) $= \frac{15}{16}$

Families having at least one boy = $N \times P = 1000 \times \frac{15}{16} = 37.5 = 938$

b) P(at most 2 girls) = P(2 girls) + P(3 girls) +P(4 girls) = $\frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16}$

Families having at most 2 girls = $N \times P = 1000 \times \frac{11}{16} = 687.5 = 688$

938 families have at least 1 boy
688 families have at most 2 girls.

(1) Moment generating function about origin:
$M_0(t) = E(e^{tx}) = \sum p_i e^{txi} = \sum nC_x p^x q^{n-x}.e^{tx} = \sum nC_x q^{n-x}(pe^t)^x$
$M_0(t) = (q+pe^t)^n$
Differentiating $M_0(t)$ and putting $t = 0$ to find mean.
$\frac{d}{dt}[M_0(t)] = n(q+pe^t)^n\, pe^t = np[e^t(q+pe^t)^{n-1}]$

$$\frac{d}{dt}[M_0(t)] = np(q+p)^{n-1} = np \qquad \dots\dots(p+q=1)$$

$$\mathbf{mean = np}$$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory:

$$\mathbf{A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}}$$ **(6)**

SOLUTION :-

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

Characteristics equation,

$\lambda^3 - 3\lambda^2 + (3+0-0)\lambda - 1 = 0$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$ .... (1)

$(\lambda - 1)^3 = 0$

λ = 1,1,1

let us find minimal polynomial;

$(x-1)(x-1) = 0$

Assuming; $x^2 - 2x + 1 = 0$ annihilates A

$A^2 - 2A + I = 0$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -3 & 3 \\ 3 & -8 & 6 \end{bmatrix}$$

$$2A = 2\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & -6 & 6 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 - 2A + I \neq 0$

It is not a minimal polynomial.

Minimal polynomial = $(x-1)(x-1)(x-1)$

As degree of freedom of minimal polynomial is equal to order

The matrix is non derogatory

Q5] B) The means of two random samples of sizes 9 and 7 are 196 and 199 respectively. The sum of the squares of the deviations from the mean is 27 and 19 respectively. Can the samples be regarded to have been drawn from the same normal population? (6)

SOLUTION :-

$n_1 = 9 \qquad n_2 = 7 \qquad \overline{X_1} = 196 \qquad X_2 = 199$

Null hypothesis $H_0: \mu_1 = \mu_2$

alternative hypothesis : $\mu_1 \neq \mu_2$

$$S_p = \sqrt{\frac{\sum(X_i - \overline{X})^2 + \sum(Y_i - \overline{Y})^2}{n_1 + n_2 - 2}} = \sqrt{\frac{27+19}{14}} = 1.8126$$

Standard error of the difference between the means

$$SE = sp \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.8261\sqrt{\frac{1}{9} + \frac{1}{7}} = 0.9135$$

$$t = \frac{\overline{X_1} - \overline{X_2}}{SE} = \frac{196-199}{0.9135} = -3.2841$$

$|t| = 3.2841$

Table value of t at $\alpha = 0.05$ for

$v = 9 + 7 - 2 = 14$ degree of freedom is 2.145

Decision : since $t_{calc} > t_{table}$

Null hypothesis is rejected

The sample cannot be considered to have been drawn from same population.

Q5] C) Use the dual simplex method to solve the following L.P.P (8)

Minimize : $\mathbf{z = 6x_1 + 3x_2 + 4x_3}$

Subject to : $\mathbf{x_1 + 6x_2 + x_3 = 10}$

$$\mathbf{2x_1 + 3x_2 + x_3 = 15}$$

$$\mathbf{x_1, x_2, x_3 \geq 0}$$

SOLUTION :-

Minimize : $z = 6x_1 + 3x_2 + 4x_3$

Subject to : $x_1 + 6x_2 + x_3 = 10$

$$2x_1 + 3x_2 + x_3 = 15$$

$$x_1, x_2, x_3 \geq 0$$

We first express the given problem using $\leq$ in the given constraints.

$x_1 + 6x_2 + x_3 \leq 10$ and $x_1 + 6x_2 + x_3 \geq 10$

$2x_1 + 3x_2 + x_3 \leq 15$ and $2x_1 + 3x_2 + x_3 \geq 15$

Multiply these equations by -1

$x_1 + 6x_2 + x_3 \leq 10$ and $-x_1 - 6x_2 - x_3 \leq -10$

$2x_1 + 3x_2 + x_3 \leq 15$ and $-2x_1 - 3x_2 - x_3 \leq -15$

Introducing stack variables,

Minimise : $z = 6x_1 + 3x_2 + 4x_3 - 0s_1 - 0s_2 - 0s_3 - 0s_4$

i.e. $z - 6x_1 - 3x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

subject to : $x_1 + 6x_2 + x_3 + s_1 + 0s_2 + 0s_3 + 0s_4 = 10$

$$-x_1 - 6x_2 - x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = -10$$

$$2x_1 + 3x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 15$$

$$-2x_1 - 3x_2 - x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = -15$$

Iteration no	Basic variable	Coefficients of							RHS solution
		x_1	x_2	x_3	s_1	s_2	s_3	s_4	
0	Z	-6	-3	-4	0	0	0	0	0
s_4 leaves	s_1	1	6	1	1	0	0	0	10
x_2 enters	s_2	-1	-6	-1	0	1	0	0	-10
	s_3	2	3	1	0	0	1	0	15
	s_4	-2	-3	-1	0	0	0	1	-15
Ratio		3	1	4					
1	Z	-4	0	-3	0	0	0	-1	15
s_4 leaves	s_1	-3	0	-1	1	0	0	0	-20
x_1 enters	s_2	3	0	0	0	1	0	-2	20
	s_3	0	0	0	0	0	1	1	0
	x_2	2/3	1	1/3	0	0	0	-1/3	5
Ratio		4/3		3					
2	Z	0	0	-5/3	-4/3	0	0	-11/3	125/3
	x_1	1	0	1/3	-1/3	0	0	-2/3	20/3
	s_2	0	0	0	0	1	0	0	0
	s_3	0	0	0	0	0	1	1	0
	x_2	0	0	1/2	1/2	0	0	1/9	5/9

$x_1 = \frac{20}{3}; \quad x_2 = \frac{5}{9}; \quad x_3 = 0$

$$\mathbf{Z = 6\left(\frac{20}{3}\right) + 3\left(\frac{5}{9}\right) + 0 \;=\; \frac{125}{3}}$$

Q6] A) Show that the matrix A satisfies Cayley- Hamilton theorem and hence find $\mathbf{A^{-1}}$ **(6)**

Where $\mathbf{A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}}$

SOLUTION :-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

Characteristics equation,

$$\lambda^3 - (1 - 1 - 1)\lambda^2 + (-3 - 10 - 5)\lambda - 40 = 0$$

$$\lambda^3 - \lambda^2 - 18\lambda - 40 = 0 \qquad \text{............ (1)}$$

By Cayley Hamilton theorem;

Matrix A satisfies characteristics equation

$$A^3 + A^2 - 18A + 40I = 0$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 44 & 33 & 46 \\ 24 & 13 & 74 \\ 52 & 14 & 8 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 14 & 3 & 8 \\ 12 & 9 & -2 \\ 2 & 4 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 44 & 33 & 46 \\ 24 & 13 & 74 \\ 52 & 14 & 8 \end{bmatrix} + \begin{bmatrix} 14 & 3 & 8 \\ 12 & 9 & -2 \\ 2 & 4 & 14 \end{bmatrix} - \begin{bmatrix} 18 & 36 & 54 \\ 36 & -18 & 72 \\ 52 & 14 & 8 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus the theorem is verified.

Multiplying (i) by A^{-1}

$$40A^{-1} = A^2 + A - 18I$$

$$A^{-1} = \frac{1}{40}(A^2 + A - 18I)$$

$$A^{-1} = \frac{1}{40}\begin{bmatrix} 14 & 3 & 8 \\ 12 & 9 & -2 \\ 2 & 4 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$\mathbf{A^{-1} = \frac{1}{40}\begin{bmatrix} -3 & 5 & 11 \\ 14 & -10 & 2 \\ 5 & 5 & -5 \end{bmatrix}}$$

Q6] B) The probability Distribution of a random variable X is given by (6)

X	**-2**	**-1**	**0**	**1**	**2**	**3**
P(X=x)	**0.1**	**k**	**0.2**	**2k**	**0.3**	**k**

Find k, mean and variance.

SOLUTION :-

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

$\sum P_i = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$0.6 + 4k = 1$

$4k = 0.4$

$k = 0.1$

X	-2	-1	0	1	2	3
P(X=x)	0.1	0.1	0.2	0.2	0.3	0.1

$\text{mean} = E(X) = \sum p_i x_i = -0.2 - 0.1 + 0.2 + 0.6 + 0.3 = 0.8$

$E(X^2) = \sum p_i {x_i}^2 = (0.1 \times 4) + 0.1 + 0 + 0.2 + (4 \times 0.3) + (9 \times 0.1) = 0.4 + 0.1 + 0.2 + 1.2 + 0.9 = 2.8$

$\text{variance} = E(X^2) - [E(X)]^2 = 2.8 - 0.8^2 = 2.8 - 0.64 = 2.16$

Variance = 2.16

Mean = 0.8

Q6] C) Using Kuhn-Tucker conditions, solve the following NLPP (8)

Maximise $\mathbf{Z = {x_1}^2 + {x_2}^2}$

Subject to ; $\mathbf{x_1 + x_2 - 4 \leq 0}$

$\mathbf{2x_{1+}x_2 - 5 \leq 0}$

$\mathbf{x_1, x_1 \geq 0}$

SOLUTION :-

$Z = {x_1}^2 + {x_2}^2$

Subject to ; $x_1 + x_2 - 4 \leq 0$

$2x_{1+}x_2 - 5 \leq 0$

$x_1, x_1 \geq 0$

We write the problem as

$f(x_1, x_2) = {x_1}^2 + {x_2}^2$

$h_1(x_1, x_2) = \ x_1 + x_2 - 4$

$h_2(x_1, x_2) = \ 2x_1 + x_2 - 5$

The kuhn jucker conditions for maxima are:

$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_1} = 0$

$2x_1 - \lambda_1 - 2\lambda_2 = 0$ …………………… (1)

Also, $\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h_1}{\partial x_2} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$

$2x_2 - \lambda_1(1) - \ \lambda_2(1) = 0$ …………………… (2)

$\lambda_1(x_1 + x_2 - 4) = 0$ …………………… (3)

$\lambda_2(2x_1 + x_2 - 5) = 0$ …………………… (4)

$x_1 + x_2 - 4 \leq 0$ …………………… (5)

$2x_1 + x_2 - 5 \leq 0$ …………………… (6)

$x_1, x_2 > 0$ ………………… (7)

$\lambda_1, \lambda_2 \geq 0$ ……………………(8)

Case 1: $\lambda_1 = \ \lambda_2 = 0$

From 1 and 2 we get,

$2x_1 = 0$ and $2x_2 = 0$

$x_1 = \ x_2 = 0$

This is a trivial solution

Case 2: $\lambda_1 = 0; \quad \lambda_2 \neq 0$

To find x_1 and x_2, we get

$2x_1 = 2\lambda_2$ and $2x_2 = \lambda_2$

From (4) we get, $2x_1 + 2x_2 = 5$

$2\lambda_2 - \frac{\lambda_2}{2} = 5$, $\frac{5\lambda_2}{2} = 5$

$\lambda_2 = 2$

$x_1 = 2; \quad x_2 = 1$

But these values do not satisfy 5 and 6

Thus reject the pair

Case 3: $\lambda_1 \neq 0; \ \lambda_2 = 0$

From (1) and (2) we get $2x_1 = \lambda_1; \quad 2x_2 = \lambda_2$

$x_1 = x_2$

From (3) we get, $x_1 + x_2 = 4$

Put $x_1 = x_2$

$2x_2 = 4 \qquad x_2 = 2 \qquad x_1 = 2$

These values satisfy the equation,

$z_{max} = {x_1}^2 + {x_2}^2 = 2^2 + 2^2 = 8$

Case 4: $\lambda_1 \neq 0, \lambda_2 \neq 0$

From 3 and 4, $x_1 + x_2 = 4$ and $2x_1 + x_2 = 5$

On solving these simultaneously we get,

$x_1 = 1$ and $x_2 = 3$

Putting in equation (1) and (2)

$\lambda_1 + 2\lambda_2 = 2$ and $\lambda_1 + \lambda_2 = 6$

Solving these, $\lambda_2 = -4$ and $\lambda_1 = 10$

$\lambda_1 > 0$ and $\lambda_2 < 0$ we reject the pair

Required solution = $\mathbf{x_1 = x_2 = 2 \quad and \quad z_{max} = 8}$

Q.P Code: 67635

MATHEMATICS SOLUTION

SEM 4 (CBCGS – MAY 2019)

BRANCH – COMPUTER ENGINEERING

Q1] A) Find the basic, feasible and degenerate solutions for the following equations: (5)

$$\mathbf{2x_1 + 6x_2 + 2x_3 + x_4 = 3; \quad 6x_1 + 4x_2 + 4x_3 + 6x_4 = 2}$$

SOLUTION :-

$2x_1 + 6x_2 + 2x_3 + x_4 = 3$

$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$

Since there are four variables and 2 constraints there are $4C_2 = 6$ basic solutions

No of basic solution	No of basic variables	Basic variables	Equation and the values of basic variables	Is the solution feasible	degenerate
1	$x_3 = 0$ $x_4 = 0$	x_1, x_2	$2x_1 + 6x_2 = 3$ $6x_1 + 4x_2 = 2$ $x_1 = 0, x_2 = \frac{1}{2}$	Yes	Yes
2	$x_2 = 0$ $x_4 = 0$	x_1, x_3	$2x_1 + 3x_3 = 3$ $6x_1 + 4x_3 = 2$ $x_1 = -2, x_3 = \frac{7}{2}$	No	No
3	$x_1 = 0$ $x_4 = 0$	x_2, x_3	$6x_2 + 2x_3 = 3$ $4x_2 + 4x_3 = 2$ $x_2 = \frac{1}{2}, x_3 = 0$	Yes	Yes
4	$x_1 = 0$ $x_3 = 0$	x_2, x_4	$6x_2 + x_4 = 3$ $4x_2 + 6x_4 = 2$ $x_2 = \frac{1}{2}, x_4 = 0$	Yes	Yes
5	$x_2 = 0$ $x_3 = 0$	x_1, x_4	$2x_1 + x_4 = 3$ $6x_1 + 4x_4 = 2$ $x_1 = \frac{8}{3}, x_2 = \frac{-7}{3}$	No	No
6	$x_1 = 0$ $x_2 = 0$	x_3, x_4	$2x_3 + x_4 = 3$ $4x_3 + 6x_4 = 2$ $x_3 = 2, x_4 = -1$	No	No

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Q1] B)Integrate the function $f(z) = x^2 + ixy$ from A = (1,1) to B(2,4) along the curve x = t and $y = t^2$ **(5)**

SOLUTION:-

$f(z) = x^2 + ixy$ from A = (1,1) to B(2,4) along the wave x = t and y = t^2

Putting $x = t$ and $y = t^2$ in $f(z)$

We get $f(z) = t^2 + it^3$

$d(z) = dx + idy = dt + 2itdt = dt(1 + 2it)$

$\int_A^B f(z)dz = \int_1^2 (t^2 + it^3)(1 + 2it)dt = \int_1^2 t^2 + 2it^3 + it^3 - 2t^4\, dt = \int_1^2 (t^2 - 2t^4 + 3it^3)dt$

$= \left[\frac{t^3}{3} - \frac{2t^5}{5} + \frac{3it^4}{4}\right]_1^2 = \frac{8}{3} - \frac{2\times32}{5} + \frac{(3i)(16)}{4} - \left[\frac{1}{3} - \frac{2}{5} + \frac{3i}{4}\right] = \frac{8}{3} - \frac{64}{5} + 12i - \frac{1}{3} + \frac{2}{5} - \frac{3i}{4}$

$= \frac{7}{3} - \frac{62}{5} + \frac{45i}{4} = -\frac{151}{15} + \frac{i45}{4}$

Answer:- $-\frac{151}{15} + \frac{i45}{4}$

Q1] C) A machine is set to produce metal plates of thickness 1.5cms with S.D. of 0.2cms. a sample of 100 plates produced by the machine gave an average thickness of 1.52 cms. Is the machine fulfilling the purpose? Test at 1% level of significance. **(5)**

SOLUTION :-

We have $\overline{X} = 1.52$; $\mu = 1.5$

$\sigma = 0.2$

sample size(n) = 100

$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{1.52 - 1.5}{0.2/\sqrt{100}} = \frac{0.2}{2.0} = 0.1$

$Z = 0.1$

But at $\alpha = 1\%$ we have $Z = 2.576$

$Z_{calc} < Z_{obs}$

Yes the machine is fulfilling its purpose.

Q1] D) The sum of the Eigen values of a 3 x 3 matrix is 6 and the product of the Eigen values is also 6. If one of the Eigen value is one, find the other two Eigen values. (5)

SOLUTION :-

Let the eigen values be λ_1, λ_2 and λ_3

$\lambda_1 + \lambda_2 + \lambda_3 = 6$

And $\lambda_1 . \lambda_2 . \lambda_3 = 6$

Given : one eigen value = 1

Let $\lambda_1 = 1$

$1+\lambda_2 + \lambda_3 = 6$

$\lambda_2 + \lambda_3 = 5$

$\lambda_3 = (5 - \lambda_2)$

And $(\lambda_2 - \lambda_3) = 6$

$\lambda_2(5 - \lambda_2) = 6$

$5\lambda_2 - {\lambda_2}^2 = 6$

${\lambda_2}^2 - 5\lambda_2 + 6 = 0$

${\lambda_2}^2 - 3\lambda_2 - 2\lambda_2 + 6 = 0$

$\lambda_2(\lambda_2 - 3) - 2(\lambda_2 - 3) = 0$

$\lambda_2 = 2 \quad \text{or} \quad \lambda_2 = 3$

If $\lambda_2 = 2$ and $\lambda_3 = 3$ and if $\lambda_2 = 3$ and $\lambda_3 = 2$

The eigen values are 1,2,3.

Q2] A) Evaluate $\oint \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where c is the circle |z| = 1 for n =1, n=3 **(6)**

SOLUTION:-

$\oint \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where c is a is a circle $|z| = 1$

For n = 1 we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z_0 - \frac{\pi}{6} = 0$

$Z_0 = \frac{\pi}{6}$ pt lies inside the circle ; |z| = 1

$\int \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)$ { By Cauchy's Integra formula}

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz = 2\pi i \sin^2\left(\frac{\pi}{6}\right) = 2\pi i\left(\frac{1}{2}\right)^6 = \frac{2\pi i}{64} = \frac{\pi i}{32}$$

For n =3; we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z = \frac{\pi}{6}$ lies inside the circle , $|Z| = 1$

Order of pole = 3

$\int \frac{f(z)}{(Z-Z_0)^n} = \frac{2\pi i}{(n-1)!} f^{n-1}(z_0)$

$f(z) = \sin^6 z$

$f'(z) = 6\sin^5 z.\cos z$

$f''(z) = 6[5\sin^4 z\cos^2 z - \sin^5 z.\sin z] = 6[5\sin^4 z\cos^2 z - \sin^6 z]$

$f''(z) = 6\left[5\sin^4\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{6}\right) - \sin^6\left(\frac{\pi}{6}\right)\right] = \frac{21}{16}$

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz = \frac{2\pi i}{2!} \cdot \frac{21}{16} = \frac{21\pi i}{16}$$

Q2] B) Solve the following LPP using Simplex method **(6)**

Maximize $\mathbf{z = 3x_1 + 5x_2}$;

Subject to $\mathbf{3x_1 + 2x_2 \leq 18}$

$\mathbf{x_1 \leq 4}$;

$\mathbf{x_2 \geq 6}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

$z = 3x_1 + 5x_2$; $\quad 3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$; $\quad x_2 \geq 6$

We first express the problem in standard form;

$z - 3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

$3x_1 + 2x_2 + 1s_1 + 0s_2 + 0s_3 = 0$

$x_1 + 0s_1 + s_2 + 0s_3 = 0$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Simple table

Iteration no	Basic variables	Coefficient of					RHS soln	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	Z	-3	-5	0	0	0	0	
s_3 leaves	s_1	3	2	1	0	0	18	9
x_2 enters	s_2	1	0	0	1	0	4	-
	s_3	0	1	0	0	1	6	6
1	Z	-3	0	0	0	5	30	
s_1 leaves	s_1	3	0	1	0	-2	6	2
x_1 enters	s_2	1	0	0	1	0	4	4
	x_2	0	1	0	0	1	6	-
2	Z	0	0	1	0	3	36	
	x_1	1	0	1/3	0	-2/3	2	
	s_2	0	0	-1/3	1	2/3	2	

	x_2	0	1	0	0	1	6	

$x_1 = 2;\ \ x_2 = 6$

$$\mathbf{Z_{max} = 3(x_1) + 5(x_1) = 3(2) + 5(6) = 6 + 30 = 36}$$

Q2] C) The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy? Use chi-square test at 5% Level of significance. (8)

	Smokers	**Non smokers**
Literates	**40**	**35**
Illiterates	**35**	**85**

SOLUTION :-

	Smokers	Non smokers	Total
Literates	40	35	75
Illiterates	35	85	120
Total	75	120	195

Expected frequency (literate and smokers) = $\frac{75\times75}{195} = 28.846$

Expected frequency (literate and non - smokers) = $\frac{75\times120}{195} = 46.15$

Expected frequency (illiterate and smokers) = $\frac{120\times75}{195} = 46.15$

Expected frequency (illiterate and non - smokers) = $\frac{120\times120}{195} = 73.85$

Null hypothesis, H_0 : no association

Alternate hypothesis , H_a : there is association

O	E	$(0-E)^2$	$\frac{(O-E)^2}{E}$
40	28.85	124.32	4.3093
35	46.15	124.32	2.6938
35	46.15	124.32	2.6938
85	73.85	124.32	1.6834
			$X^2 = 11.3803$

Level of significance = (0.05)

Degree of freedom = (r-1)(c-1) = (2-1)(2-1) = 1

Critical value at 1 df for 5% level of significance is 3.84.

$\mathbf{X^2}_{\mathbf{table}} < \mathbf{X^2}_{\mathbf{calc}}$

There is no association.

Q3] A) Find the Eigen values and Eigen vectors of the following matrix. (6)

$$\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{10} & \mathbf{5} \\ \mathbf{-2} & \mathbf{-3} & \mathbf{-4} \\ \mathbf{3} & \mathbf{5} & \mathbf{7} \end{bmatrix}$$

SOLUTION :-

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

The characteristics equation is given by

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

$\lambda = 3,2,2$

For $\lambda = 3$

$[A - 3I]X = 0$

$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 10x_2 + 5x_3 = 0$

$-2x_1 - 6x_2 - 4x_3 = 0$

$3x_1 + 5x_2 + 4x_3 = 0$

Consider last two equations,

$$\frac{x_1}{\begin{vmatrix} -6 & -4 \\ 5 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & -4 \\ 3 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & -6 \\ 3 & 5 \end{vmatrix}} = t$$

$$\frac{x_1}{-4} = -\frac{x_2}{4} = \frac{x_3}{8} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

For $\lambda = 2$

$[A - 2I]X = 0$

$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 10 & 5 \\ 0 & 15 & 16 \\ 0 & -25 & -25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank of matrix = 3 and number of variables = 3

$x_1 + 10x_2 + 5x_3 = 0$

$-2x_1 - 5x_2 - 4x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 10 & 5 \\ -5 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 5 \\ -2 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 10 \\ -2 & -5 \end{vmatrix}} = t$$

$$\frac{x_1}{-15} = -\frac{x_2}{6} = \frac{x_3}{15} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -15 \\ -6 \\ 15 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -5 \end{bmatrix}$$

Eigen values = 3,2

Eigen vector = [1 1 -2], [5 -2 -5]

Q3] B) The incomes of a group of 10,000 person's were found to be normally distributed with mean of Rs 750 and standard deviation of Rs. 50 . what is the lowest income of richest 250? (6)

SOLUTION :-

Standard normal variate; $Z = \frac{(X-m)}{\sigma} = \frac{X-750}{50}$

If we have to consider the richest 250 persons, then probability that a person selected at random will be one of them is $\frac{250}{10000} = 0.025$

Area from (z=0 to z = this value) = 0.5-0.025 = 0.475

From the table, we find that the area from z = 0 to z = 1.96 is 0.476 the required z = 1.96;

$1.96 = \frac{X-750}{50}$

$X - 750 = 1.96 \times 50 = 845$

Lowest income of richest 250 persons = Rs 848

Q3] C) Obtain Taylor's and Laurent's expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ indicating region of convergence (8)

SOLUTION:-

$f(z) = \frac{z-1}{z^2-2z-3} = \frac{z-1}{(z+1)(z-3)}$

Applying partial fractions;

$\frac{z-1}{z^2-2z-3} = \frac{A}{z+1} + \frac{B}{z-3}$

$(z-1) = A(z-3) + B(z+1)$

Put z = 3

3-1 = B(4)

2 = 4B

B = $\frac{1}{2}$

Put z = -1

-2 = A(-4)

A = $\frac{1}{2}$

$f(z) = \frac{1}{2(z+1)} + \frac{1}{2(z-3)} = \frac{1}{2}[\frac{1}{z+1} - \frac{1}{z-3}]$

(1) for $|z| < 1$

$f(z) = \frac{1}{2}\left[(1+z)^{-1} - \frac{1}{3}\left(\frac{1}{1-\frac{z}{3}}\right)\right]$

$$= \frac{1}{2}\left[\left(1 - z + \frac{z^2}{2} + \cdots\right) - \frac{1}{3}\left(1 - \frac{z}{3}\right)^{-1}\right] = \frac{1}{2}\left[\left(1 - z + \frac{z^2}{2} + \cdots\right) - \frac{1}{3}\left(1 + \frac{z}{3} + \frac{z^2}{9} + \cdots\right)\right]$$

$$= \frac{1}{2}\left[\frac{2}{3} - \frac{10z}{9} + \frac{29z^2}{27} - \cdots\right]$$

(2) For 1 < |z| < 3

$$f(z) = \frac{1}{2}\left[\frac{1}{z}(1+z)^{-1} - \frac{1}{3}\left(\frac{1}{1-\frac{z}{3}}\right)^{-1}\right] = \frac{1}{2}\left[\frac{1}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2} \ldots\right) - \frac{1}{3}\left(1 - \frac{z}{3} + \frac{z^2}{9} + \cdots\right)\right]$$

(3) For |z| > 3

$$f(z) = \frac{1}{2}\left[\frac{1}{z+1} + \frac{1}{z-3}\right] = \frac{1}{2}\left[\frac{1}{z}\left(1 + \frac{1}{z}\right)^{-1} + \frac{1}{z}\left(1 - \frac{3}{z}\right)^{-1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \cdots\right] + \frac{1}{z}\left(1 + \frac{3}{z} + \frac{z^2}{9} + \cdots.\right)^{-1}\right]$$

$$= \frac{1}{z}\left[1 + \frac{1}{z} + \frac{5}{z^2} + \cdots\right] \quad = \frac{1}{z} + \frac{1}{z^2} + \frac{5}{z^3} + \cdots$$

(1) Gives taylor series (2) and (3) gives Laurent series.

Q4] A) A man buys 100 electric bulbs of each of two well-known makes taken at random from stock for testing purpose. He finds that 'make A' has a mean life of 1300 hrs with a S.D. of 82 hours and 'make B' has a mean life of 1248 hours with S.D. of 93 hours. Discuss the significance of these results. (6)

SOLUTION :-

Null hypothesis : $\mu_1 = \mu_2$, alternate hypothesis : $\mu_1 \neq \mu_2$

a) We have $n_1 = n_2 = 100$

$\sigma_1 = 82$

$\sigma_2 = 93$

$$SE = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} = \sqrt{\frac{82^2+93^2}{100}} = 12.3988$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{SE} = \frac{1300-1248}{12.3988} = 4.1939$$

Level of significance = 5%

Critical value = 1.96

$Z_{calc} > Z_{obs}$

There is significant difference.

Q4] B) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ (6)

SOLUTION :-

$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$

Let $z = e^{i\theta} d\theta$

$dz = ie^{i\theta} d\theta$

$d\theta = \frac{dz}{ie^{i\theta}}$

$d\theta = \frac{dz}{iz}$ where C is unit circle $|z| = 1$

$$\int_0^{2\pi} \frac{1}{5-3\left(\frac{z^2+1}{2z}\right)} \left(\frac{dz}{iz}\right) = \int_0^{2\pi} \frac{-2dz}{(10z-3z^2-3)i} = \int_0^{2\pi} \frac{-2dz}{(-10z+3z^2+3)i} = \int \frac{-2dz}{(3z-1)(z-3)i}$$

Z-3 lies outside the circle.

Residue of f(z) at $z = \frac{1}{3} = \lim_{z\to\frac{1}{3}} \left(z - \frac{1}{3}\right) \frac{-2}{i(3z-1)(z-3)} = \lim_{z\to\frac{1}{3}} \left(\frac{3z-1}{3}\right) \frac{-2}{i(3z-1)(z-3)} = \frac{1}{3i}\left(-\frac{2}{\frac{1}{3}-3}\right) = \frac{1}{4i}$

$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta} = 2\pi i \left(\frac{1}{4i}\right) = \frac{\pi}{2}$

$$\boldsymbol{\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta} = \textbf{real part of} \left(\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}\right) = \frac{\pi}{2}}$$

Q4] C) (1) Out of 1000 families with 4 children each, how many would you expect to have (a) at least one boy (b) at most 2 girls.

(2)Find the Moment Generating Function of Binomial Distribution and hence find its mean. (8)

SOLUTION :-

(1)

BBBB	BBBG	BBGB	BGBB
BBGG	BGGB	BGBG	BGGG
GGGG	GGGB	GGBG	GBGG
GGBB	GBBG	GBGB	GBBB

a) P(at last one boy) $= \frac{15}{16}$

Families having at least one boy = $N \times P = 1000 \times \frac{15}{16} = 37.5 = 938$

b) P(at most 2 girls) = P(2 girls) + P(3 girls) +P(4 girls) = $\frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16}$

Families having at most 2 girls = $N \times P = 1000 \times \frac{11}{16} = 687.5 = 688$

938 families have at least 1 boy

688 families have at most 2 girls.

(1) Moment generating function about origin:

$M_0(t) = E(e^{tx}) = \sum p_i e^{txi} = \sum nC_x p^x q^{n-x}. e^{tx} = \sum nC_x q^{n-x}(pe^t)^x$

$M_0(t) = (q + pe^t)^n$

Differentiating $M_0(t)$ and putting $t = 0$ to find mean.

$\frac{d}{dt}[M_0(t)] = n(q + pe^t)^n\, pe^t = np[e^t(q + pe^t)^{n-1}]$

$\frac{d}{dt}[M_0(t)] = np(q + p)^{n-1} = np \qquad \ldots\ldots(p + q = 1)$

$\mathbf{mean = np}$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{-3} & \mathbf{3} \end{bmatrix}$$ **(6)**

SOLUTION :-

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

Characteristics equation :

$\lambda^3 - 3\lambda + (3 - 0 - 0)\lambda - 1 = 0$

$\lambda^3 - 3\lambda + 3\lambda - 1 = 0$

$(\lambda - 1\,)^3 = 0$

$\lambda = 1,1,1$

Let us find minimal polynomial.

$(x - 1)(x - 1) = 0$

Assuming ; $x^2 - 2x + 1 = 0 \quad$ annihilates A

$A^2 - 2A + I$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -3 & 3 \\ 3 & -8 & 6 \end{bmatrix}$$

$$2A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & -6 & 6 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 - 2A + I \neq 0$

It is not a minimal polynomial

Minimal polynomial = (x-1)(x-1)(x1)

As a degree of minimal polynomial is equal to order the matrix is non derogatory

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Q5] B) The means of two random samples of sizes 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the derivatives from the means are 26.94 and 18.73 respectively. Can the samples be regarded to have been drawn from the same normal population? (6)

SOLUTION:-

Null hypothesis $H_0: \mu_1 = \mu_2$

alternative hypothesis : $\mu_1 \neq \mu_2$

(ii) calculation of test statistic

$$S_p = \sqrt{\frac{\sum(X_i-\bar{X})^2+\sum(Y_i-\bar{Y})^2}{n_1+n_2-2}} = \sqrt{\frac{26.94+18.73}{9+7-2}} = \sqrt{\frac{14.67}{14}} = 1.87$$

Standard error of the difference between the means

$$SE = sp \times \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 1.81\sqrt{\frac{1}{9}+\frac{1}{7}} = 0.91$$

$$t = \frac{\overline{X_1}-\overline{X_2}}{SE} = \frac{196.42-198.82}{0.91} = -2.64$$

|t| = 2.64

Level of significance = 5%

Critical value; $v = 9 + 7 - 2 = 14$ degrees of freedom is 2.145

Decision; $t_{calc} > t_{table}$

Thus; null hypothesis is rejected alternative hypothesis is accepted

The sample cannot be considered to have been drawn from same population.

5] C) Use the dual simplex method to solve the following L.P.P. (6)

Minimize : $\mathbf{z = x_1 + x_2}$

Subject to : $\mathbf{2x_1 + x_2 \geq 2}$

$\mathbf{-x_1 - x_2 \geq 1}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

Minimize : $z = x_1 + x_2$

Subject to : $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

We first express the given problem using $\leq$ in the given constraints.

Minimize; $z = x_1 + x_2$

$-2x_1 - x_2 \leq -2$

$x_1 + x_2 \le 1$

Introducing stack variables s_1, s_2, we have

$z = x_1 + x_2 - 0s_1 - 0s_2$

$z - x_1 - x_2 - 0s_1 - 0s_2 = 0$

Subject ; $-2x_1 - x_2 + s_1 - 0s_2 = -2$

$x_1 + x_2 + 0s_1 + s_2 = -1$

Simple table

Iteration number	Basic variables	Coefficient of				RHS soln
		x_1	x_2	s_1	s_2	
0	Z	-1	-1	0	0	0
s_2 leaves	s_1	-2	-1	1	0	-2
x_1 enters	s_2	1	1	0	1	-1
Ratio		½	1			
1	Z	0	-½	-½	0	1
	x_1	1	½	-½	0	1
	s_2	0	½	½	1	-2
Ratio		-	-1	-1	-	

Since s_2 row is –ve, s_2 leaves; but since all ratios are –ve the LPP has no feasible solution.

Q6] A) Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1} (6)

Where $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

SOLUTION :-

$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

Characteristics equation,

$\lambda^3 - 6\lambda^2 + (4 + 3 + 4)\lambda - 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ .... (1)

λ = 1,3,2

By cayley Hamilton theorem;

Matrix A satisfies equation (1)

$A^3 - 6A^2 + 11A - 6I = 0$

$$A^3 = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 0 & -13 \\ 0 & 8 & 0 \\ -13 & 0 & 14 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 5 \end{bmatrix}$$

$A^3 - 6A^2 + 11A - 6I = 0$

$$= \begin{bmatrix} 14 & 0 & -13 \\ 0 & 8 & 0 \\ -13 & 0 & 14 \end{bmatrix} - \begin{bmatrix} 30 & 0 & -24 \\ 0 & 24 & 0 \\ -24 & 0 & 30 \end{bmatrix} + \begin{bmatrix} 22 & 0 & -11 \\ 0 & 22 & 0 \\ -11 & 0 & 22 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Cayley Hamilton theorem is satisfied.

Multiplying (i) by A^{-1}

$A^3 - 6A^2 + 11A - 6A^{-1} = 0$

$$6A^{-1} = \begin{bmatrix} 5 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 12 & 0 & -6 \\ 0 & 12 & 0 \\ -6 & 0 & 12 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\mathbf{A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}}$$

Q6] B) A random variable X has the probability distribution (6)

$\mathbf{P(X = x) = \frac{1}{8} . 3C_x; \quad x = 0, 1, 2, 3}$ **. find the mean and variance.**

SOLUTION :-

$P(X = x) = \frac{1}{8} . 3C_x; \quad x = 0,1,2,3$

X	0	1	2	3
P(X= x)	1/8	3/8	3/8	1/8

Mean = $\sum p_i x_i \;=\; 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{1}{8} = \frac{10}{8}$

$E(X^2) = \sum p_i x_i^2 = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 4 \times \frac{3}{8} + 9 \times \frac{3}{8} = 0 + \frac{3}{8} + \frac{12}{8} + \frac{27}{8} = \frac{42}{8}$

$\text{variance} = E(X^2) - [E(X)]^2 = \frac{42}{8} - \left[\frac{10}{8}\right]^2 \;= 3.6875$

Mean = 1.25; $\mathbf{variance = 3.6875}$

Q6] C) Using Kuhn-Tucker conditions, solve the following NLPP (8)

Maximize : $\mathbf{Z = 10x_1 + 10x_2 - x_1{}^2 - x_2{}^2}$

Subject to ; $\mathbf{x_1 + x_2 \leq 8}$

$\mathbf{-x_{1+}x_2 \leq 5}$

$\mathbf{x_1, x_1 \geq 0}$

SOLUTION :-

Maximize : $Z = 10x_1 + 10x_2 - x_1{}^2 - x_2{}^2$

Subject to ; $x_1 + x_2 \leq 8$

$-x_{1+}x_2 \leq 5$

$x_1, x_1 \geq 0$

We write the problem as

$f(x_1, x_2) = 10x_1 + 10x_2 - x_1{}^2 - x_2{}^2$

$h_1(x_1, x_2) = \; x_1 + x_2 - 8$

$h_2(x_1, x_2) = \; -x_1 + x_2 - 5$

The kuhn jucker conditions are:

$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$

$10 - 2x_1 - \lambda_1(1) - \; \lambda_2(1) = 0$

$10 - 2x_2 - \lambda_1 - \lambda_2 = 0$ .... (1)

Also, $\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h_1}{\partial x_2} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$

$10 - 2x_2 - \lambda_1(1) - \lambda_2(1) = 0$ .... (2)

$\lambda_1(-x_1 + x_2 - 5) = 0$ .... (3)

$\lambda_2(-x_1 + x_2 - 5) = 0$ .... (4)

$x_1 + x_2 - 8 \leq 0$ .... (5)

$-x_1 + x_2 - 5 \leq 0$ .... (6)

$x_1, x_2 > 0$ .... (7)

For minima; $\lambda_1, \lambda_2 \geq 0$

Case 1: $\lambda_1 = \lambda_2 = 0$

From 1 and 2 we get,

$10 - 2x_1 = 0$ and $10 - 2x_2 = 0$

$x_1 = x_2 = 5$

These values do not satisfy all the equation

Case 2: $\lambda_1 = 0$; $\lambda_2 \neq 0$

To find x_1 and x_2, we eliminate λ_2 from

$10 - 2x_1 + \lambda_2 = 0$ and $10 - 2x_2 - \lambda_2 = 0$

Adding these equation,

We get $20 - 2x_1 - 2x_2 = 0$

$x_1 + x_2 = 10$

$\lambda_2 \neq 0$ we get from (4); $-x_1 + x_2 = 5$

Adding the two we get $2x_2 = 15$

$x_2 = 7.5$

$x_1 = 10 - 7.5 = 2.5$

But equation 5 is not satisfied reject this pair.

Case 3: $\lambda_1 \neq 0; \ \ \lambda_2 = 0$

Eliminating λ_1 from,

$16 - 2x_1 - \lambda_1 = 0$ and $10 - 2x_2 - \lambda_1 = 0$

By subtraction;

$-2x_1 + 2x_2 = 0$

$x_1 = x_2$

$\lambda_1 \neq 0$ from $3, x_1 + x_2 = 8$

$2x_1 = 8$

$x_1 = 4$ and $x_2 = 4$

From 10-2$x_1 - \lambda_1 = 0$

We get, $\lambda_1 = 10 - 2x_1 = 2$

These values satisfy the condition,

$z_{max} = 10(4) + 10(4) - 16 - 16 = 48$

Case 4: $\lambda_1 \neq 0, \lambda_2 \neq 0$

From 3 and 4,

$x_1 + x_2 = 8$

And -$x_1 + x_2 = 5$

Adding the two , we get $2x_2 = 13$

$x_2 = 6.5$

$x_1 = 8 - 6.5 = 1.5$

For these values of x_1 and x_2

$-\lambda_1 + \lambda_2 = -10 + 2x_1 = -7$

$-\lambda_1 - \lambda_2 = -10 + 2x_2 = 3$

Adding the two we get, $-2\lambda_1 = -4$

And $\lambda_2 = -\lambda_1 - 3 = -5$

We reject this pair, $\mathbf{z_{max} = 48}$

Q.P. CODE: 79693

MATHEMATICS SOLUTION

(DEC-2019 SEM-4 COMPS)

Q1] A) Find all the basics solutions to the following problem: (5)

Maximise : $\mathbf{z = x_1 + 3x_2 + 3x_3}$

Subject to : $\mathbf{x_1 + 2x_2 + 3x_3 = 4}$

$\mathbf{2x_1 + 3x_2 + 5x_3 = 7}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION :-

Maximise : $z = x_1 + 3x_2 + 3x_3$

Subject to : $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

$x_1, x_2, x_3 \geq 0$

No of basic solution	Non basic variable	Basic variable	Equation and values of basic variables	Is solution feasible?	Is solution degenerate	Value of Z	Is solution optimal
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 7$ $x_1 = 2, x_2 = 1$	Yes	No	5	Yes
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$ $x_1 = 1, x_3 = 1$	Yes	No	4	No
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$ $x_2 = -1, x_3 = 2$	no	no	-	-

---

Q1] B) Evaluate $\int_c^1 (z - z^2)dz$, where c is upper half of the circle |z| = 1 **(5)**

SOLUTION:-

Evaluate $\int_c^1 (z - z^2)dz \qquad |z| = 1$

$z = e^{i\theta} \qquad \therefore dz = e^{i\theta}d\theta$ and θ varies from 0 to π

$$\int_c^1 (z - z^2)dz = \int_0^\pi (e^{i\theta} - e^{2i\theta})e^{i\theta}.id\theta = i\int_0^\pi (e^{2i\theta} - e^{3i\theta})d\theta = i\left[\frac{e}{2i} - \frac{e}{3i}\right]_0^\pi$$

$$= \left[\frac{e^{2i}}{2} - \frac{e^{3i\pi}}{2} - \frac{1}{2} + \frac{1}{3}\right] = \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{2} + \frac{1}{3}\right] = \frac{2}{3}$$

The value of the integral for the lower half of the same circle in the same positive direction i.e when θ varies from π to 2π.

$$\int_c^1 (z - z^2)dz = i\left[\frac{e^{2i\theta}}{2i} - \frac{e^{3i\theta}}{3i}\right]_\pi^{2\pi} = i\left[\frac{e^{4i\pi}}{2i} - \frac{e^{6i\pi}}{3i} - \frac{e^{2i\pi}}{2i} + \frac{e^{3i\pi}}{3i}\right]$$

$$= \left[\frac{\cos4\pi + i\sin4\pi}{2} - \frac{\cos6\pi + i\sin6\pi}{3} - \frac{\cos2\pi + i\sin2\pi}{2} + \frac{\cos3\pi + i\sin3\pi}{2}\right] = \left[\frac{1}{2} - \frac{1}{3} - \frac{1}{2} - \frac{1}{3}\right] = -\frac{2}{3}$$

Q1] C) Ten individuals are chosen at random from a population and heights are found to be 63,63,64,65,66,69,69,70,70,71 inches. Discuss the suggestion that the height of universe is 65 inches. **(5)**

SOLUTION:-

N = 10(< 30, so it is small sample)

Null hypothesis(H_0): $\mu = 65$

Alternate hypothesis(H_a): $\mu \neq 65$

LOS = 5%

Degree of freedom = n-1=10-1 =9

Critical value(t_x) = 2.2622

Values (x_i)	$D_i = x_i - 67$	${D_i}^2$
63	-4	16
63	-4	16
64	-3	9
65	-2	4
66	-1	1

69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
Total	0	88

$\bar{d} = \frac{\sum d_i}{n} = \frac{0}{10} = 0$

$\bar{x} = a + \bar{d} = 67 + 0 = 67$

Since sample is small,

$s = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{88}{10} - \left(\frac{0}{10}\right)^2} = 2.9965$

S.E = $\frac{s}{\sqrt{n-1}} = \frac{2.9965}{\sqrt{9}} = 0.9888$

Test statistics

$t_{cal} = \frac{\bar{x}-\mu}{S.E} = \frac{67-65}{0.988} = 2.0227$

Decision

Since $|t_{cal}| < t_x$, H_0 is accepted.

The man height of the universe is 65 inches

Q1] D) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ then find A^{100} **(5)**

SOLUTION:-

$A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$

The characteristics equation is

$\begin{bmatrix} 2-\lambda & 3 \\ -3 & -4-\lambda \end{bmatrix} = 0$

$(2-\lambda)(-4-\lambda)-(-3\times 3)=0$

$-8-2\lambda+4\lambda+\lambda^2+9=0$

$(\lambda+1)^2=0$

$\lambda=\ -1$

Since matrix is of order 2 we consider

$\varphi(A)=\ A^{100}=\ \alpha_1 A+\alpha_0 I$

λ satisfies this equation $\qquad \lambda^{100}=\ \alpha_1\lambda+\alpha_0 I$

Putting $\lambda=\ -1$

$(-1)^{100}=\ \alpha_1(-1)+\alpha_0 I$

$1=\ -\alpha_1+\alpha_0 I$

$100\lambda^{99}=\alpha_1+0$

$\alpha_1\ =\ -100$

$1=\ -\alpha+\alpha_0$

$\alpha_0\ =\ -99$

$A^{100}=-100A-99I$

$A^{100}=\ -100\begin{bmatrix}2 & 3\\ -3 & -4\end{bmatrix}-99\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-200-99 & -300-0\\ 300-0 & 400-99\end{bmatrix}=\begin{bmatrix}-299 & -300\\ 300 & 301\end{bmatrix}$

Q2] A) Evaluate $\int_c^1 \frac{z+2}{(z-3)(z-4)}$ dz, where c is the circle |z| = 1 **(6)**

SOLUTION:-

$\int_c^1 \frac{z+2}{(z-3)(z-4)}dz \qquad\qquad |z|=1$

$|z|=1$ is a circle with center at the origin and radius 1 hence both points z = 3 and z = 4 lie outside the circle C and f(z) is analytic in C

By Cauchy's theorem $\int_c^1 \frac{z+2}{(z-3)(z-4)}dz=0$

Q2] B) An I.Q test was administered to 5 persons and after they were trained. The results are given below. (6)

Test whether there is change in I.Q after the training program use 1% LOS

	I	**II**	**III**	**IV**	**V**
I.Q before training	**110**	**120**	**123**	**132**	**125**
I.Q after training	**120**	**118**	**125**	**136**	**121**

SOLUTION:-

	I	II	III	IV	V
I.Q before training	110	120	123	132	125
I.Q after training	120	118	125	136	121

deviation in each case is 10 -2 2 4 -4
sum of d=10
d^2=100+4+4+16+16=140

$\mathrm{d} = \text{sum of} \frac{\mathrm{d}}{\mathrm{n}} = \frac{10}{2} = 5 \quad \text{and } \mathrm{s} = \sqrt{\frac{\mathrm{d}^2 - \mathrm{nd}}{\mathrm{n}-1}} = \sqrt{\frac{140- \ (4)}{5-1}} = 5.673$

Hence there is no change of IQ after training since given
t0.01(4) = 4.6

Q2] C) Solve the following LPP using Simplex Method (8)

Maximize $z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \le 10$

$2x_1 + 5x_2 \le 20$

$2x_1 + 3x_2 \le 18$

$x_1, \ x_2 \ge 0$

Solution:-

We first express the given problem in standard form.

Maximize $z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

i.e. $z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Subject to $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 10$

$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 20$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 18$

We put this information in tabular form as follows.

Iteration no	Basic variable	Coefficients of					RHS soln	ration
		x_1	x_2	s_1	s_2	s_3		
0	Z	-4	-10	0s	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	8/5	0	1	-1/5	0	6	15/4
x_1 enters	x_2	2/5	1	0	1/5	0	4	10
	s_3	4/5	0	0	-3/5	1	6	15/2
2	Z	0	0	0	-1/5	0	40	
	x_1	1	0	5/8	-1/8	0	15/4	
	x_2	0	1	-1/4	1/5	0	5/2	
	s_3	0	0	-1/2	-1/2	1	3	

$x_1 = \frac{15}{4};\ x_2 = \frac{5}{2}$ and $z_{max} = 40$

This is an alternative solution. But this does not improve the above optimal solution.

Thus we have two solutions, $x_1 = 0;\ x_2 = 4$ and $z_{max} = 40$

And $x_1 = \frac{15}{4};\ x_2 = \frac{5}{2}$ and $z_{max} = 40$

If there are two solutions to a problem then there are infinite number solutions.

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $X_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$, $X_2 = \begin{bmatrix} \frac{15}{4} \\ \frac{5}{2} \end{bmatrix}$, $then\ X = \lambda\ \ X_1 + (1-\lambda)X_2\ \ for\ 0 \leq\ \lambda \leq 1$

i.e. $X = \begin{bmatrix} \frac{15}{4}(1-\lambda) \\ 4 + \frac{5}{2}(1-\lambda) \end{bmatrix}$

Gives infinite number of feasible solutions, all giving $z_{max}\ = 40$

Thus we get two points A(0, 4) and G(15/4, 5/2) giving the same maximum value of z(=40).

Q3] A) Find the Eigen values and Eigen vectors of the following matrix. (6)

A = $\begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{bmatrix}$

SOLUTION:-

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

The characteristics equation is

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} = 0$$

After simplification we get,

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$(\lambda - 1)(\lambda - 1)(\lambda + 5) = 0$

$\lambda = 1,1,5$

Hence 1,1,5 are the Eigen values.

(1) For $\lambda = 1\ [A - \lambda_1 I]X = 0$ gives

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_2 - R_1,\ \ R_3 - R_1$

$$\begin{bmatrix}1 & 2 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

We see that the rank of the matrix is 1 and number of variables is 3. Hence there are 3-1 = 2 linearly independent solutions i.e there are two parameters we shall denote these parameters by s and t.

Putting $x_2 = -5,\ \ x_3 = -t,$ we get $x_1 = -2x_2 - x_3 = 2s + t$

$$X = \begin{bmatrix}2s+t\\-s+0\\0-t\end{bmatrix} = s\begin{bmatrix}2\\-1\\0\end{bmatrix} + t\begin{bmatrix}1\\0\\-1\end{bmatrix}$$

$$x_1 = \begin{bmatrix}2\\-1\\0\end{bmatrix},\ x_2 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

The vectors $\begin{bmatrix}2\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}$ are linearly independent.

Hence corresponding to λ=1 the Eigen vectors are $\begin{bmatrix}2\\-1\\0\end{bmatrix}$ and $\begin{bmatrix}1\\0\\-1\end{bmatrix}$

(1) For $\lambda = 5,\quad [A - \lambda_1 I]X = 0$ gives

$$\begin{bmatrix}-3 & 2 & 1\\1 & -2 & 1\\1 & 2 & -3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \text{ by } R_{13}\begin{bmatrix}1 & 2 & -3\\1 & -2 & 1\\-3 & 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By $R_2 - R_1,\ \ R_3 + 3R_1$

$$\begin{bmatrix}1 & 2 & -3\\0 & -4 & 4\\0 & 8 & -8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By $R_3 + 2R_2$

$$\begin{bmatrix}1 & 2 & -3\\0 & -4 & 4\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$x_1 + 2x_2 - 3x_3 = 0$ and $-4x_2 + 4x_3 = 0$

Putting $x_3 = t$ we get $x_2 = t$ and $x_1 = -2x_2 + 3x_3 = -2t + 3t = t$

$X = \begin{bmatrix}t\\t\\t\end{bmatrix} = t\begin{bmatrix}1\\1\\1\end{bmatrix}$ hence corresponding to λ = 5, the Eigen vector is $\begin{bmatrix}1\\1\\1\end{bmatrix}$

Q3] B) If the height of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights between 65 and 71 inches. (6)

SOLUTION:-

Evaluating no of students having height between 65 and 71 inches

When X = 65

$z = \frac{65-68}{4} = -\frac{3}{4} = -0.75$

When $x = 71$

$z = \frac{71-6}{4} = \frac{3}{4} = 0.75$

$P(-0.75 < Z < 0.75) = Area\ from\ (Z = 0\ to\ Z = -0.75) + Area\ from\ (Z = 0\ to\ 0.75)$

$\frac{1}{\sqrt{2\pi}}\int_{-0.75}^{0} C^{-\frac{1}{2}z^2dz} + \frac{1}{\sqrt{2\pi}}\int_{0}^{0.75} C^{-\frac{1}{2}z^2dz} = 0.2734 + 0.2734 = 0.5468$

Probability of students having height between 65 to 71 inches is 0.5468

No of students = $N \times P = 500 \times 0.5468 = 273$

No of students having height between 65 to 71 inches is 273

Q3] C) Obtain Taylors and Laurents expansion of $f(z) = \frac{z^2-1}{z^2+5z+6}$ around z = 0 (8)

SOLUTION:-

Since the degree of the numerator is equal to the degree of the denominator we first divide the numerator by the denominator.

$f(z) = \frac{z^2-1}{z^2+5z+6} = 1 - \frac{5z+7}{z^2+5z+6}$

Let $\frac{-5z-7}{z^2+5z+6} = \frac{a}{z+3} + \frac{b}{z+2}$ $\quad -5z - 7 = a(z+2) + b(z+3)$

When z = -2 b = 3 when z = -3 and a = -8

$$f(z) = \frac{z^2-1}{z^2+5z+6} \;\; = 1 - \frac{8}{z+3} + \frac{3}{z+2} \qquad \text{.................... (1)}$$

Case (1): when |z| < 2 we write

$$f(z) = 1 - \frac{8}{3\left[1+\left(\frac{z}{3}\right)\right]} + \frac{3}{2\left[1+\left(\frac{z}{2}\right)\right]}$$

When |z| < 2, clearly |z| < 3

$$f(z) = 1 - \frac{8}{3}\left[1+\left(\frac{z}{3}\right)\right]^{-1} + \frac{3}{2}\left[1+\left(\frac{z}{2}\right)\right]^{-1} = 1 - \frac{8}{3}\left[1-\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)^2 - \;\ldots\ldots.\right] + \frac{3}{2}\left[1-\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)^2 - \;\ldots\ldots\right]$$

Case (2): when 2 < |z| < 3 we write

$$f(z) = 1 - \frac{8}{3\left[1+\left(\frac{z}{3}\right)\right]} + \frac{3}{z\left[1+\left(\frac{z}{2}\right)\right]} \;=\; 1 - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} + \frac{3}{z}\left(1+\frac{2}{z}\right)^{-1}$$

$$f(z) = 1 - \frac{8}{3}\left[1-\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)^2 - \;\ldots..\right] + \frac{3}{z}\left[1-\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)^2 - \;\ldots\ldots..\right]$$

Case (3): when |z| > 3 we write

$$f(z) = 1 - \frac{8}{z\left[1+\left(\frac{3}{z}\right)\right]} + \frac{3}{z\left[1+\left(\frac{2}{z}\right)\right]}$$

When |z| > 3 clearly |z| > 2

$$f(z) = 1 - \frac{8}{z}\left(1+\frac{3}{z}\right)^{-1} + \frac{3}{2}\left(1+\frac{2}{z}\right)^{-1} = 1 - \frac{8}{z}\left[1-\left(\frac{3}{z}\right)+\left(\frac{3}{z}\right)^2-\left(\frac{3}{z}\right)^3+\cdots\right] + \frac{3}{z}\left[1-\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)^2-\left(\frac{2}{z}\right)^3+\cdots\right]$$

Q4] A) A machine is claimed to produce nails of mean length 5cms and standard deviation of 0.45cm. A random sample of 100 nails gave 5.1 as their average length. Does the performance of the machine justify the claim? Mention the level of significance you apply. (6)

SOLUTION:-

$\overline{X} = 5\text{cm}$

SD = 0.45cm

$\mu = 5.1\text{cm}$

n = 100

we have,

$z = \left|\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\right| = \left|\frac{5-5.1}{0.45/\sqrt{100}}\right| = 2.22$

Level of significance $\alpha = 0.05$

Critical value : the value of Z_α at 5% level of significance is 1.96

The performance of the machine increases.

Q4] B) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ (6)

SOLUTION:-

Let $e^{i\theta} = z$ $\quad e^{i\theta}.\, i d\theta = dz;$

$d\theta = \frac{dz}{iz}$ and $\sin\theta = \frac{z^2-1}{2iz}$

$I = \int_c^{iz} \frac{1}{5+3\left(\frac{z^2-1}{2iz}\right)}\left(\frac{dz}{iz}\right) = \int_c^1 \frac{2}{3z^2+10iz} dz = \int_c^1 \frac{2}{(3z+i)(z+3i)} dz$

Where c is the circle |z|=1

Now the poles of f(z) are given by $(3z+i)(z+3i) = 0$, $z = -\left(\frac{i}{3}\right)$ and $z = -3i$ are simple poles. But $z = -\left(\frac{i}{3}\right)$ lies inside and z = -3i lies outside the circle |z| = 1

Resides $\left(\text{at } z = -\frac{i}{3}\right) = \lim_{z\to -\frac{i}{3}}\left[z+\frac{i}{3}\right].\frac{2}{(3z+i)(z+3i)} = \lim_{z\to -\frac{i}{3}} \frac{2}{3(z+3i)} = \frac{1}{4i}$

$I = 2\pi i\left(\frac{1}{4i}\right) = \frac{\pi}{2}$

Q4] C) (1) In a certain manufacturing process 5% of the tools produced turnout to be defective. Find the probability that in a sample of 40 tools at most 2 will be defective.

(2)A random variable x has the probability distribution $\mathbf{P(X = x), = \frac{1}{8} \,.3C_x, x = 0, 1, 2, 3.}$ $\mathbf{find\ the\ moment\ generating\ function\ of\ x}$ (8)

SOLUTION:-

(1) n = 40 p = 0.05 $f(x) = \frac{e^{-2}.2^x}{x}$

n = np = $40 \times 0.05 = 2$ use poisson p(at most 2) = $p(x \leq 2) = \frac{e^{-2}.2^0}{0} + \frac{e^{-2}.2^1}{1!} + \frac{e^{-2}.2^2}{2!} = 0.675$

(2) $P(X = x), = \frac{1}{8} \,.3C_x, x = 0,1,2,3$

$P(X = 0) = \frac{1}{8}.3C_0 = \frac{1}{8}$

$P(X = 1) = \frac{1}{8}.3C_1 = \frac{3}{8}$

$P(X = 2) = \frac{1}{8}.3C_2 = \frac{3}{8}$

$P(X = 3) = \frac{1}{8}.3C_3 = \frac{1}{8}$

Moment generating function, $M_0(t) = E(e^{txi}) = \sum p_i\, e^{txi}$

From above values

$M_0(t) = \frac{1}{8}e^{t(0)} + \frac{3}{8}e^{t(1)} + \frac{3}{8}e^{t(2)} + \frac{1}{8}e^{t(3)} = \frac{1}{8}\left[1 + 3e^{t(0)} + 3e^{t(2)} + e^{t(3)}\right] = \frac{1}{8}(1 + e^t)^3$

$M_0(t) = \frac{1}{8}\frac{1}{8}(1 + e^t)^3$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory

$$\mathbf{A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}}$$ **(6)**

SOLUTION:-

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Characteristics equation :

$\lambda^3 - 12\lambda^2 + (8 + 14 + 14)\lambda - 32 = 0$

Λ = 8,2,2

Let us assume $(x - 8)(x - 2) = x^2 - 10x + 16$ annihilates A

Now $A^2 - 10A + 16I$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - 10 \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + 16 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 0 & 0 \\ 0 & -16 & 0 \\ 0 & 0 & -16 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x^2 - 10x + 16$ annihilates A thus f(x) is the monic polynomial of lowest degree

$\text{minimal polynomial} = x^2 - 10x + 16$

Q5] B) In an industry 200 workers employed for a specific job were classified according to their performance and training received to test independence of training received and performance. The data are summarized as follows: (6)

Performance	Good	Not good	Total
Trained	**100**	**50**	**150**
Untrained	**20**	**30**	**50**
Total	**120**	**60**	**200**

Use χ^2 − test for independence at 5% level of significance and write your conclusion.

SOLUTION:-

Null Hypothesis, H_o there is no independence

Alternative Hypothesis, H_a there is an independence

Calculate of test statistics;

Trained and good = $\frac{120\times150}{200} = 90$

Untrained and good = $\frac{120\times50}{200} = 30$

Trained and not good = $\frac{80\times150}{200} = 60$

untrained and not good = $\frac{80\times50}{200} = 20$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
90	100	100	1
30	20	100	5
60	50	100	2
20	30	100	3.33 X = 11.33

$\alpha = 0.05$

Degree of freedom = $(r-1)(c-1) \;=\; (2-1)(2-1) \;=\; 1$

Critical value = 3.841

$\chi^2\text{cal} \;>\; \chi^2\text{table}$

Thus Null hypothesis is rejected

There is an independence relationship.

Q5] C) Use the dual simplex method to solve the following L.P.P (8)

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$\mathbf{4x_1 + 3x_2 \geq 6}$

$\mathbf{x_1 + 2x_2 \leq 3}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

-$3x_1 - x_2 \leq -3$

-$4x_1 - 3x_2 \leq - 6$

$x_1 + 2x_2 \leq 3$

Introducing stack variables

$Z' - 2x_1 - x_2 - 0s_1 - 0s_2 - 0s_3$

$-3x_1 - x_2 + s_1 - 0s_2 - 0s_3 = -6$

$x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 3$

Iterations no	Basics variables	Coefficients of					RHS Solution
		x_1	x_2	s_1	s_2	s_3	
0	Z'	-2	-1	0	0	0	0
s_2 leaves	s_1	-3	-1	1	0	0	-3
x_2 enters	s_2	-4	-3	0	1	0	-6
	s_3	1	2	0	0	1	3
Ratio:		-1/2	-1/3				
1	Z'	2	2	0	-1	0	6
	s_1	-5/3	0	0	1/3	0	-1
	x_2	4/3	1	0	1/3	0	2
	s_3	-5/3	0	0	-2/3	1	-1
Ratio:		-6/5			3/2	0	
2		11/3	2	0	-1/3	0	7
s_3 leaves	s_1	0	0	0	1	-1	0
x_1 enters	x_2	0	1	0	3	-4/3	6/5
	x_1	1	0	0	2/5	-3/5	3/5

$x_1 = \frac{3}{5}$; $x_2 = \frac{6}{5}$;

$Z_{\min} = 2\left(\frac{3}{5}\right) + \frac{6}{5} = \frac{12}{5}$

$Z_{\min} = \frac{12}{5}$

Q6] A) Show that the matrix A satisfies Cayley- Hamilton theorem and hence find A^{-1} (6)

Where A = $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

We know that,

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} 1-\lambda & 3 & 7 \\ 4 & 2-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{bmatrix} \right| = 0$

$(1-\lambda)[(2-\lambda)(1-\lambda) - 6] - 3[4(1-\lambda) - 7] + 7[8 - (2-\lambda)] = 0$

$(1-\lambda)[2 - 2\lambda - \lambda + \lambda^2 - 6] - 3[4 - 4\lambda - 7] + 7[\lambda + 6] = 0$

$\lambda^2 - 3\lambda - 4 + \lambda^3 + 3\lambda^2 + 4\lambda + 12\lambda + 9 + 7\lambda + 42 = 0$

$\lambda^3 + 4\lambda^2 + 10\lambda + 47 = 0$

The characteristics equation of A is

$P(\lambda) = \lambda^3 + 4\lambda^2 + 10\lambda + 47 = 0$

By cayley-Hamilton theorem, A is satisfies its characteristics equation

So that replace λ with A

$P(A) = A^3 + 4A^2 + 10A + 47 = 0$ ………………… (1)

Since |A| = 35 $\neq 0$ by A^{-1} is exists.

Multiply equation (1) by A^{-1} we get,

$A^{-1}(A^3 + 4A^2 + 10A + 47) = 0$

$$A^2 = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 11 & 14 \end{bmatrix}$$

$A^2 + 4A + 10I + 47A^{-1} = 0$

$A^2 + 4A + 10I = -47A^{-1}$

$A^{-1} = \frac{1}{-47}[A^2 + 4A + 10I]$

$$A^{-1} = \frac{1}{-47}\left[\begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 11 & 14 \end{bmatrix} + \begin{bmatrix} 4 & 12 & 28 \\ 16 & 8 & 12 \\ 4 & 8 & 4 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}\right] = -\frac{1}{47}\begin{bmatrix} 34 & 35 & 51 \\ 31 & 40 & 49 \\ 14 & 19 & 28 \end{bmatrix}$$

Q6] B) A discrete random variable has the probability density function given below (6)

$X = x_i$	**-2**	**-1**	**0**	**1**	**2**	**3**
$P(x_i)$	**0.2**	**k**	**0.1**	**2k**	**0.1**	**2k**

Find K, Mean, Variance.

SOLUTION:-

$X = x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.2	k	0.1	2k	0.1	2k

$\sum P(x_i) = 1$

$0.2 + k + 0.1 + 2k + 0.1 + 2k = 1$

$0.4 + 5k = 1$

$5k = 1 - 0.4 = 0.6$

$k = \frac{0.6}{5} = 0.12$

$X = x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.2	0.12	0.1	0.24	0.1	0.24

Mean = $\sum x_i P(x_i) = (-2 \times 0.2) + (-1 \times 0.12) + (0 \times 0.1) + (1 \times 0.24) + (2 \times 0.1) + (3 \times 0.24)$

Mean = $-0.4 - 0.12 + 0 + 0.24 + 0.2 + 0.72 = 0.64$

Mean = E(x) = 0.64

$$E(X^2) = [4 \times 0.2] + [1 \times 0.12] + [0] + [1 \times 0.24] + [4 \times 0.1] + [3 \times 0.24]$$

$E(X^2) = 2.28$

Variance = $E(X^2) - [E(X)]^2 = 2.28 - 0.64^2 = 2.28 - 0.4096 = 1.8704$

Q6] C)Using Kuhn- Tucker conditions solve the following NLPP (8)

Maximize : $\mathbf{Z = 2x_1 - 7x_2 + 12x_1x_2}$

Subject to : $\mathbf{2x_1 + 5x_2 \leq 98}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION:-

Maximize : $Z = 2x_1 - 7x_2 + 12x_1x_2$

Subject to : $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

We rewrite the given problem as:

f(x) = $2x_1^2 - 7x_2^2 + 12x_1x_2$

$h(x_1x_2) = 2x_1 + 5x_2 - 98$

Kuhn tucker conditions are:

$\frac{\partial f}{\partial x_1} - \frac{\lambda \partial h}{\partial x_1} = 0; \frac{\partial f}{\partial x_2} - \frac{\lambda \partial h}{\partial x_2} = 0$

$\lambda h(x_1, x_2) = 0;\ h(x_1, x_2) \leq 0,\ \lambda \geq 0$

We get,

$4x_1 + 12x_2 - \lambda(2) = 0$ ………………… (1)

$12x_1 - 14x_2 - \lambda(5) = 0$ …………………..(2)

$\lambda(2x_1 + 5x_2 - 98) = 0$ .... (3)

$2x_1 + 5x_2 - 98 \leq 0$ .... (4)

$\lambda \geq 0$ .... (5)

From (3) we get either λ = 0 or $(2x_1 + 5x_2 - 98) = 0$

Case 1: λ = 0 and $(2x_1 + 5x_2 - 98) \neq 0$

From 1 and 2,

$4x_1 + 12x_2 = 0$

$12x_1 - 14x_2 = 0$

On solving simultaneously we get $x_1 = x_2 = 0$

Case 2:

$\lambda \neq 0$ and $2x_1 + 5x_2 - 98 = 0$

$4x_1 + 12x_2 - \lambda(2) = 0$

$12x_1 - 14x_2 - \lambda(5) = 0$

$\lambda = \frac{12\ \ _1 - 14x_2}{5}$

Equation I : $4x_1 + 12x_2 - 2\left[\frac{12x_1 - 14x_2}{5}\right] = 0$

$20x_1 + 60x_2 - 24x_1 + 28x_2 = 0$

$-4x_1 + 88x_2 = 0$ (divide through by 4)

$-x_1 + 22x_2 = 0$

Put $x_1 = 22x_2$ in 4

$2(22x_2) + 5x_2 = 98$

$44x_2 + 5x_2 = 98$

$49x_2 = 98$

$x_2 = 2$

$2x_1 + 10 = 98$

$2x_1 = 88$

$x_1 = 44$

These values satisfy all conditions,

$Z_{max} = 2(1936) - 7(4) + 12(44)(2) = 4900$

$Z_{max} = 4900$

S.E Comp Sem-IV EM-IV
R-19

Q.C. 92377

University of Mumbai

Examination First Half 2022 under cluster __ (Lead College: __________)

Examinations Commencing from 16 MAY 2022 to 30 MAY 2022

Program: __BE COMPUTER ENGINEERING __________

Curriculum Scheme: Rev2019 (C scheme)

17 MAY 2022

Examination: SE Semester : IV

Course Code: _CSC 401____ and Course Name: ___Engineering Mathematics_IV

Time: 2 hour 30 minutes Max. Marks: 80

==

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	If X is a Poisson variate and P(X=1)=P(X=2) ,then $E(X^2)$ is
Option A:	1
Option B:	5
Option C:	8
Option D:	6
2.	If $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ Eigen value of Adj. A are
Option A:	5,6,2
Option B:	2,3,6
Option C:	5,3,6
Option D:	1,3,6
3.	If $f(z) = \frac{3z^2+z}{z^2-1}$, then residue of f(z) at z= -1 is
Option A:	1
Option B:	-1
Option C:	2
Option D:	-2
4.	The value of $\int_C \frac{\cos \pi z}{z^2-1} dz$ where C is the circle $\lvert z\rvert = 1/2$
Option A:	πi
Option B:	$2\pi i$
Option C:	0
Option D:	$-\pi i$
5.	According to Time shifting property of z-transform, if X(z) is the z-transform of x(n) then what is the z-transform of x(n-k)?
Option A:	$z^{-k}X(z)$
Option B:	$z^{k}X(z)$
Option C:	X(z+k)
Option D:	X(z-k)
6.	The value of $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ is
Option A:	$\frac{a^{n+1}-b^{n+1}}{a+b}$

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Option B:	$\dfrac{a^{n+1}+b^{n+1}}{a-b}$
Option C:	$\dfrac{a^{n+1}-b^{n+1}}{a-b}$
Option D:	$\dfrac{a^{n+1}+b^{n+1}}{a+b}$
7.	If a random variable X follows Poisson distribution such that P(X=0)=6P(X=3), find the mean and variance of the distribution.
Option A:	$mean = 1, variance = 1$
Option B:	$mean = 1, variance = -1$
Option C:	$mean = 1, variance = 2$
Option D:	$mean = 1, variance = -2$
8.	In normal distribution
Option A:	Mean = Median = Mode
Option B:	Mean < Median < Mode
Option C:	Mean> Median > Mode
Option D:	Mean ≠ Median ≠ Mode
9.	If the primal LPP has an unbounded solution then the dual has
Option A:	Unbounded solution
Option B:	Bounded solution
Option C:	Feasible solution
Option D:	Infeasible solution
10.	The value of Lagrange's multiplier λ for the following NLPP is Optimize $z = 6x_1^2 + 5x_2^2$ Subject to $x_1 + 5x_2 = 7$ $x_1, x_2 \geq 0$
Option A:	$\lambda = 31/84$
Option B:	$\lambda = 84/31$
Option C:	$\lambda = 13/74$
Option D:	$\lambda = 31/64$

Q2	**Solve any Four out of Six** **5 marks each**
A	Given $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, find the eigenvalues of A. Also find eigenvalues of $4A^{-1}$ and eigenvector of $A^2 - 4I$.
B	Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path (i) $x^2 = y$ (ii) y = x
C	Find $Z\{2^k \cos(3k + 2)\}, k \geq 0$.
D	The following table gives the number of accidents in a city during a week. Find whether the accidents are uniformly distributed over a week Day | Sun | Mon | Tue | Wed | Thu | Fri | Sat | Total

	No. of accidents	13	15	9	11	12	10	14	84

E	Solve by Simplex Method *Maximise* $z = 7x_1 + 5x_2$ Subject to $-x_1 - 2x_2 \geq -6$ $4x_1 + 3x_2 \leq 12$ $x_1, x_2 \geq 0$
F	Solve the following NLPP *Maximise* $z = -2x_1^2 - x_2^2 + 10x_1 + 4x_2$ Subject to $2x_1 + x_2 \leq 5$ $x_1, x_2 \geq 0$

Q3	Solve any Four out of Six **5 marks each**
A	Find the Eigen values and Eigen Vectors of the following matrix. $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.
B	Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $\lvert z \rvert = 3$
C	Obtain inverse z-transform $\frac{z+2}{z^2-2z-3}$, $1 < \lvert z \rvert < 3$
D	The height of six randomly chosen sailors are in inches: 63,65,68,69,71,72. The height of 10 randomly chosen soldiers are: 61,62,65,66,69,69,70,71,72 and 73. discuss in lie that these data throw on the suggestion that the soldiers on an average are taller than the sailors
E	Solve by the dual Simplex Method *Minimise* $z = 6x_1 + 3x_2 + 4x_3$ Subject to $x_1 + 6x_2 + x_3 = 10$ $2x_1 + 3x_2 + x_3 = 15$ $x_1, x_2 \geq 0$
F	Find the relative maximum or minimum of the function $z = x_1^2 + x_2^2 + x_3^2 - 8x_1 - 10x_2 - 12x_3 + 100$

Q4	Solve any Four out of Six **5 marks each**
A	Show that the following matrix is diagonalizable. Also find the diagonal form and a diagonalizing matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$
B	Evaluate $\int_C \frac{4z^2+1}{(2z-3)(z+1)^2} dz,\ C: \lvert z \rvert = 4$ using Cauchy's residue theorem.
C	Find the inverse z-transforms of $F(z) = \frac{z}{(z-1)(z-2)}$; $\lvert z \rvert > 2$

D	If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches, estimate the number of students having heights (i) greater than 72 inches (ii) less than 62 inches (iii) between 65 and 71 inches.
E	Using Simplex method Maximize $z = 10x_1 + 6x_2 + 5x_3$ Subject to $2x_1 + 2x_2 + 6x_3 \le 300$ $10x_1 + 4x_2 + 5x_3 \le 600$ $x_1 + x_2 + x_3 \le 100$ $x_1, x_2, x_3 \ge 0$
F	Using Lagrange's multiplier optimize $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$ subject to $x_1 + 2x_2 = 2$ $x_1, x_2 \ge 0$