

The union types

A compound object may come from different types. For example, a component of a [world wide web home page](#) may contain normal text, pictures, audio data, and so on, each belonging to a different category. The [union types](#) allow us to define such compound objects.

The outline of this part:

- [Union type declaration](#)
- [Is function](#)
- [A specification with union types](#)

Union type declaration

Let T_1, T_2, \dots, T_n be n types. Then, a union type T constituted from these types is declared in the format:

$$T = T_1 \mid T_2 \mid \dots \mid T_n$$

A value of T can come from one of the types T_1, T_2, \dots, T_n .

It is important to keep T_1, T_2, \dots, T_n **disjoint** so that any value of type T can be precisely determined to belong to which constituent type.

Example:

Color = {<Red>, <Blue>, <Yellow>}

Key = char

Digits = set of nat

the union type Hybrid can then be declared as:

Hybrid = Color | Key | Digits

the following values belong to the type Hybrid:

<Red>

<Blue>

'b'

'5'

{10, 20, 100}

No operators can be built on a union type except the equality (=) and inequality (<>).

For example,

<Red> = <Blue> <=> false

<Red> <> {3, 5, 8} <=> true

'b' = 'b' <=> true

is function

When writing specifications there may be a situation that requires a precise type of a given value. Such a type can be determined by applying a built-in function known as **is** function:

is_T(x)

This function is a predicate that yields **true** when the type of value **x** is **T** (any type is possible); otherwise, it yields **false**.

Examples:

is_Color(<Red>) <=> true

is_Hybrid(<Red>) <=> true

Specification with a union type

Suppose we want to write a program that scans a specification in SOFL and records different kinds of **tokens** in different tables. We first declare **Token** as a union type:

Token = EnglishLetter | Identifier | SpecialCharacter

where **EnglishLetter**, **Identifier**, and **SpecialCharacter** are supposed to have been declared before.

We then build a process `Record-Token` to record different tokens obtained by scanning the current text in different tables.

```
process Record-Token(token: Token)
  ext wr english_char_table: seq of EnglishLetter
    wr identifier_table: seq of Identifier
    wr special_char_table: seq of SpecialCharacter
  post (is_EnglishLetter(token) =>
    english_char_table = conc(~english_char_table, [token])) and
    (is_Identifier(token) =>
    identifier_table = conc(~identifier_table, [token])) and
    (is_SpecialCharacter(token) =>
    special_char_table = conc(~special_char_table, [token]))
  comment
    The token is recorded in the corresponding table.
end_process
```


Class exercise 9

1. Define a union type **School** with the constituent types **ElementarySchool**, **JuniorHighSchool**, **HighSchool**, and **University**, assuming that all the constituent types are given types.
2. Let **s1** and **s2** be two variables of the type **set of Hybrid**. Let **s1** = {<Red>, 3, 'b'} and **s2** = {<Blue>, 'a', 'b', 9}. Evaluate the expressions:
 - a. **card(s1) = card(s2) <=> ?**
 - b. **union(s1, s2) = ?**
 - c. **inter(s1, s2) = ?**
 - d. **diff(s1, s2) = ?**