1. $Y \sim N(a, b^2)$, $X|Y \sim N(Y, 6^2)$

忘记机写答案。

$$f(y) = \frac{1}{\sqrt{2\pi b^2}} \cdot e^{-\frac{(y-a)^2}{2b^2}}, \quad f(x|y) = \frac{1}{\sqrt{2\pi 6^2}} \cdot e^{-\frac{(x-y)^2}{26^2}}$$

$$f(y|x) = \frac{f(y) \cdot f(x|y)}{\int f(y) f(x|y)\, dy}$$

$$f(y) \cdot f(x|y) \propto e^{-\frac{(y-a)^2}{2b^2}} \cdot e^{-\frac{(x-y)^2}{26^2}}$$

$$\propto e^{-\frac{b^2x + 6^2a}{2b^26^2}\left(y - \frac{忘了}{b^2+6^2}\right)^2}$$

正态、 $f(y|x) \sim N\left(\frac{忘了}{b^2+6^2}, \frac{b^2x+6^2a}{b^2 6^2}\right)$

2. P_0: $\pi(P_0) f(X|P_0) = \frac{1}{3} \prod_{i=1}^{n} C_N^{X_i} P_0^{X_i} (1-P_0)^{N-X_i}$

P_1: $\pi(P_1) f(X|P_1) = \frac{2}{3} \prod_{i=1}^{n} C_N^{X_i} P_1^{X_i} (1-P_1)^{N-X_i}$

$P_0 + P_1$

$$\hat{P} = E(P) = \frac{P_0 \cdot \frac{1}{3} \prod_{i=1}^{n} C_N^{X_i} P_0^{X_i} (1-P_0)^{N-X_i} + P_1 \cdot \frac{2}{3} C_N^{X_i} P_1^{X_i} (1-P_1)^{N-X_i}}{\frac{1}{3} \prod_{i=1}^{n} C_N^{X_i} P_0^{X_i} (1-P_0)^{N-X_i} + \frac{2}{3} \prod_{i=1}^{n} C_N^{X_i} P_1^{X_i} (1-P_1)^{N-X_i}}$$

3. $2n\lambda_1\bar{x} \sim \chi^2(2n)$

$2m\lambda_2\bar{y} \sim \chi^2(2m)$

$$\frac{2n\lambda_1\bar{x}/2n}{2m\lambda_2\bar{y}/2m} \sim F(2n, 2m)$$

$H_0\ \frac{\frac{1}{\lambda_2}}{\frac{1}{\lambda_1}} \geq 1 \qquad H_1\ \frac{\frac{1}{\lambda_2}}{\frac{1}{\lambda_1}} < 1$

$$P\left\{\frac{\bar{y}}{\bar{x}} < k\right\} = \alpha.$$

$$\frac{\bar{x}}{\bar{y}} > \frac{1}{k}$$

$$\frac{1}{k} = F_\alpha(2n, 2m)$$

4. ① $\frac{3}{2}\bar{x}$ ② $E(\ )^2 \Rightarrow D(\ ) + [E(\ )]^2.$

5. ① 忘了

② $(\theta+1)\log\frac{1}{x} \sim \Gamma(1)$

$(\theta+1)\log\frac{1}{x} = Y.$

$Y \sim \Gamma(1)$

$2n\bar{Y} \sim \chi^2(2n)$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} (\theta+1)\log\frac{1}{X}.$$

和①证相等

6. 卡分检验

7. 方差检验

8. ① $\bar{Y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{X} = [1, \bar{X}]\begin{bmatrix}\hat{\beta}_0\\ \hat{\beta}_1\end{bmatrix}$

$$\hat{\beta}_0 = [1, 0]\begin{bmatrix}\hat{\beta}_0\\ \hat{\beta}_1\end{bmatrix}$$

A

$$\begin{bmatrix}1, & \bar{X}\\ 1, & 0\end{bmatrix}\begin{bmatrix}\hat{\beta}_0\\ \hat{\beta}_1\end{bmatrix} \sim N\left[A\begin{bmatrix}\beta_0\\ \beta_1\end{bmatrix},\ A\sigma^2 S^{-1}A^T\right]$$

$(\bar{Y}, \hat{\beta}_0)$ 是二维正态.

μ_1, μ_2 好求

$\sigma_0^2, \sigma_1^2, \rho$ 通过 $A\sigma^2 S^{-1}A^T$ 求.

② $\dfrac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}}\sqrt{L_{xx}} \sim t(n-2)$