2. POLYNOMIALS

1. Constant : A number having a fixed numerical value is called a constant.

2. Variable : A number which can take various numerical values is known as variable.

   Ex : $x, y, z, a, b, p, q \ldots$

3. Algebraic Expression : An algebraic Expression is an Expression made up of variables and constants along with mathematical operators.

   Ex : $8x+7$, $-5x^2-13$, $5x^3+\frac{8}{x^2}+y$, etc.

4. Polynomial : A polynomial is an algebraic expression in which the exponent on any variable is a whole number.

   Ex : $2x+5$, $3x^2+5x+6$, $-5y \ldots$

Polynomials	Not Polynomials
$2x$	$4x^{1/2}$
$\frac{1}{3}x-4$	$3x^2+4x^{-1}+5$
x^2-2x-1	$4+\frac{1}{x}$

5. Degree of a polynomial : The hig power of x in a polynom $P(x)$ is called the degr of the polynomial P(

6. Linear Polynomial : A polynomia degree 1 is called a lin polynomial.

Ex : $3x+5$, $7x-8$, $-9x$...

7. Quadratic polynomial : A polyno of degree 2 is called a Quadratic polynomial.

Ex: x^2-5x+6, $2x^2-5$, $7x$

The general form a Quadratic p mial in variable x is ax^2+ c $(a,b,c \in R, a \neq 0)$.

8. Cubic polynomial :

A polynomial of degree 3 is called a cubic Polynomial.

Ex:- $5x^3 - 4x^2 + x - 1$, $2x^3 - 3x + 5$, $-3x^3 - 10$, ...

The general form a cubic polynomial in variable x is $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$.

9. The general form of n^{th} degree polynomial in one variable x:

$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots +$ $a_{n-1}x + a_n$ is a polynomial of n^{th} degree, where $a_0, a_1, a_2 \ldots\ldots, a_{n-1}, a_n$ are real coefficients and $a_0 \neq 0$.

10. Value of a polynomial at a given point:

The value of $P(x)$ at $x = k$ is $p(k)$

11. Zero of a polynomial:

A real number k is said to be a zero of a polynomial $P(x)$, if $p(k) = 0$

12. The zero of the linear polynomial $ax + b$ is $\frac{-b}{a} = \frac{-(\text{Constant term})}{\text{coefficient of } x}$

13. The graph of $y = ax + b$ is a straight line which intersects the x-axis at Exactly one point $\left(\frac{-b}{a}, 0\right)$

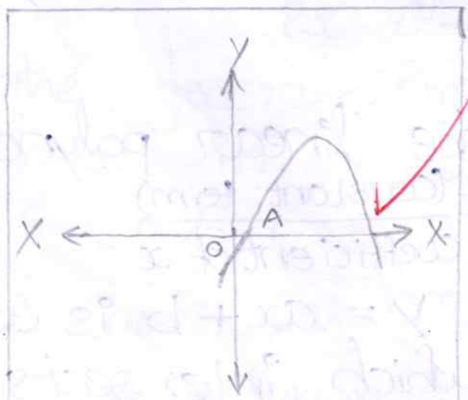
14. The x-coordinate of the point wh the graph of $y = ax+b$ intersects t x-axis is the zero of the polynom $ax+b$

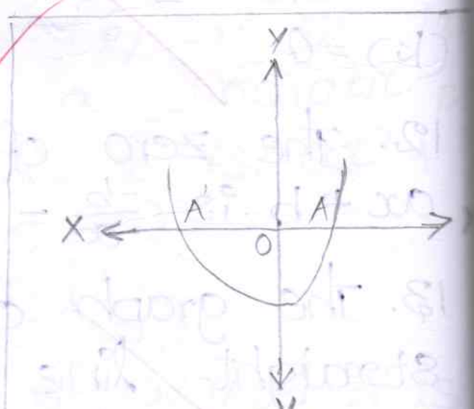
2.1 Geometrical Meaning Of The Zeroes Of A Polynomial.

A real number k is ~~said~~ ~~to be~~ ~~zero~~ of polynomial $P(x)$ if $p(k)=0$

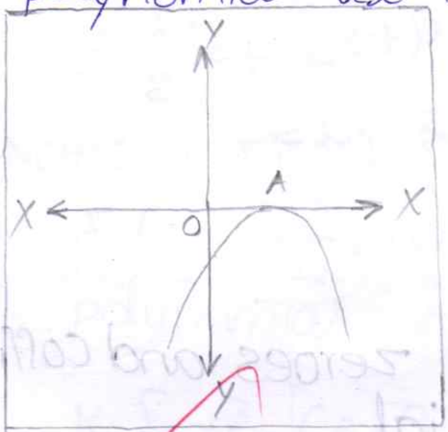
1. the graph of $y = ax^2+bx+c$ $(a \neq 0$ Either opens upwards like $\vee$ (if $a>$ or opens downwards like $\cap$ (if a the shape of these curves are called parabolas.

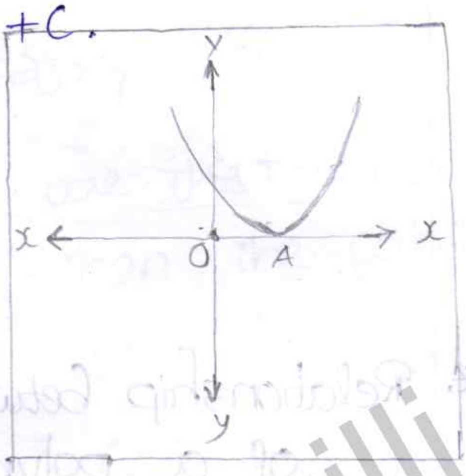
2. the graph of $y = ax^2+bx+c$ (intersects x-axis at two points (and $(\beta, 0)$ then α, β are the zer of the polynomial ax^2+bx+c



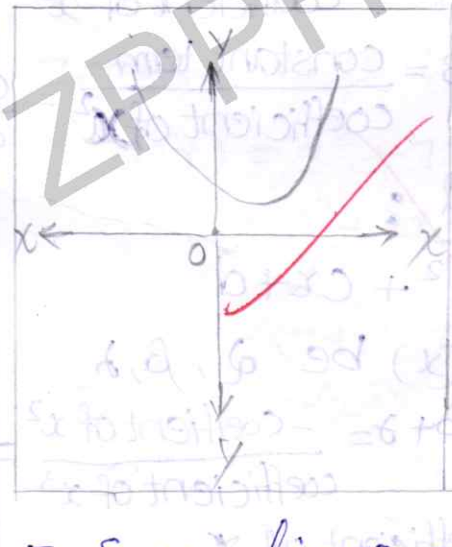


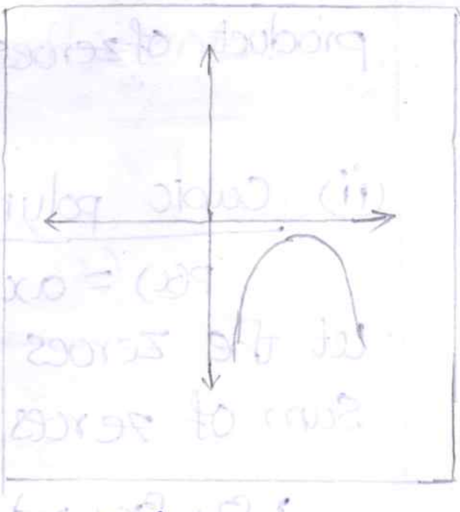
3. The graph of $y = ax^2 + bx + c$ $(a \neq 0)$ touches X-axis one point at $(\alpha, 0)$ then 'α' is only one zero of the polynomial $ax^2 + bx + c$.





4. The graph of $y = ax^2 + bx + c$ $(a \neq 0)$ does not intersects x-axis then the polynomial $ax^2 + bx + c$ has no real zeroes.





5. Every linear polynomial have at most one zero.

6. Every Quadratic polynomial have at most two zeroes.

7. Every Cubic polynomial have at most three zeroes

ax^2+bx+c

α, β | a b c

$\alpha+\beta$

$3u-12$

$3(u-4)=0$

$u-4=\frac{0}{3}=0$

$u=4$

α so is 4

$a=0$ $b=3$ $c=-$

*. Relationship between zeroes and coffic[ients] of a polynomial.

(i) Quadratic polynomial:

$P(x)=ax^2+bx+c$ (General form)

Let α and β are zeroes of $P(x)$.

Sum of zeroes $=\alpha+\beta=\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}=$

product of zeroes $=\alpha\beta=\frac{\text{constant term}}{\text{coefficient of } x^2}=\frac{c}{a}$

(ii) Cubic polynomial:

$P(x)=ax^3+bx^2+cx+d$

Let the zeroes of $P(x)$ be α, β, γ

Sum of zeroes $=\alpha+\beta+\gamma=\frac{-\text{coefficient of } x^2}{\text{coefficient of } x^3}=$

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{\text{coefficient of } x}{\text{coefficient of } x^3}=\frac{c}{a}$

Product of zeroes $=\alpha\beta\gamma=\frac{-\text{constant term}}{\text{coefficient of } x^3}$

Note 1 :- if α and β the zeroes of a polynomial ~~if α~~ than the quadratic polynomial is

$$k\{x^2-(\alpha+\beta)x+\alpha\beta\}$$

Note 2:- if α, β, γ are the zeroes of cubic polynomial than the cubic polynomial is

$$k\{x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma\}$$

<u>Example 2</u>: find the zeroes of the quadratic polynomial $x^2+7x+10$, and verify the relationship between the zeroes and the coefficients.

Ans:- $P(x) = x^2+7x+10$ $\quad 1\times10=10$, $2\times5=10$

$= x^2+2x+5x+10$

$= x(x+2)+5(x+2)$

$=(x+2)(x+5)$

$P(x)=0 \Rightarrow (x+2)(x+5)=0$

$\therefore x+2=0$ or $x+5=0$

$x=-2$ or $x=-5$

$\therefore$ zeroes of

-b/a

-b/a

-d/a

Example 3 : find the zeroes of th polynomial x^2-3 and verify the relationship between the zeroe and the coefficients.

Sol:- $P(x) = x^2-3 = x^2-(\sqrt{3})^2 = (x+\sqrt{3})(x-$

$P(x) = 0 \Rightarrow (x+\sqrt{3})(x-\sqrt{3}) = 0 \quad \because a^2-b^2=(a-$

$\therefore x+\sqrt{3}=0$ or $x-\sqrt{3}=0$

$\therefore x=-\sqrt{3}$ or $x=\sqrt{3}$

$\alpha = -\sqrt{3} \quad \beta = +\sqrt{3}$

comparing $P(x)$ with ax^2+bx+c, $a=1$, b
$c=$

$\alpha+\beta = -\sqrt{3}+\sqrt{3} = 0 = \frac{-0}{1} = \frac{-b}{a}$

$\alpha\beta = (-\sqrt{3})(\sqrt{3}) = -(\sqrt{3})^2 = -3 = \frac{-3}{1} = \frac{c}{a}$

Example 4 : find a Quadratic polyno the sum and product of whos zeroes are -3 and 2, respecti

Ans :- $\alpha = -3$, $\beta = 2$

Required polynomial is ~~$k\{x^2-(\alpha+$~~

$k\{x^2-(\alpha+\beta)x+\alpha\beta\}$

$\Rightarrow k\{x^2-(-3+2)x+(-3)\times 2\}$

$\Rightarrow k\{x^2+x-6\}$

If $k=1$, then required polynomial becomes x^2+x-6

Sum of the Zeroes $= \left(\frac{3}{2}\right) + \left(\frac{-1}{3}\right) = \frac{9-2}{6} = \frac{7}{6}$

$= \frac{-(-7)}{6} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} = \frac{-b}{a}$

product of the zeroes $= \left(\frac{3}{2}\right) \times \left(\frac{-1}{3}\right) = \frac{-3}{6}$

$\therefore \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

(v) $t^2 - 15$

sol:- $P(x) = t^2 - 15$

$= t^{2^2} - (\sqrt{15})^2 = (t+\sqrt{15})(t-\sqrt{15})$

To find zeroes let $P(t) = 0$

$t + \sqrt{15} = 0$ or $(t - \sqrt{15}) = 0$

$t = \sqrt{15}$ or $t = \sqrt{15}$

The Zeroes of the polynomial $p(t)$

$= t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$

sum of the Zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1}$

$= \frac{-(\text{coefficient of } t)}{\text{coefficient of } t^2} = \frac{-b}{a}$

product of the zeroes $= \sqrt{15} \times (-\sqrt{15}) = -15$

$\frac{-15}{1} = \frac{\text{constant term}}{\text{Coefficient of } t^2} = \frac{c}{a}$

(iv) $4u^2+8u$

Sol:- $P(u) = 4u^2+8u$

$= 4u(u+2)$

$P(u) = 0 \Rightarrow 4u(u+2) = 0$

$\Rightarrow 4u = 0$ or $u+2 = 0$

$\Rightarrow u = \frac{0}{4} = 0$ or $u = -2$

$\therefore \alpha = 0, \beta = -2$.

Comparing $P(x)$ with ax^2+bx+c,

$a = 4, b = 8, c = 0$

$\alpha+\beta = 0-2 = -2 = \frac{-8}{4} = \frac{-b}{a}$

$\alpha\beta = 0(-2) = 0 = \frac{0}{4} = \frac{c}{a}$

2. Find a Quadratic polynomial with the given numbers as sum and product of its zeroes respectively.

Sol:- (i) $\frac{1}{4}, -1$ $\alpha+\beta = \frac{1}{4}$ $\alpha\beta = -1$

Required polynomial is

$k\{x^2-(\alpha+\beta)x+\alpha\beta\}$

$\Rightarrow k\{x^2-\frac{1}{4}x+(-1)\}$

$\Rightarrow k\{x^2-\frac{x}{4}-1\}$

If $k=4$, required polynomial becom[es]

$4x^2-x-4$

(ii) $\sqrt{2}, \frac{1}{3}$ $\alpha+\beta=\sqrt{2}$, $\alpha\beta=\frac{1}{3}$

Sol :- Required polynomial is

$k\{x^2-(\alpha+\beta)x+\alpha\beta\}$

$\Rightarrow k\{x^2-\sqrt{2}x+\frac{1}{3}\}$

If $k=3$ the required polynomial becomes $3x^2-3\sqrt{2}x+1$

(iii) $0, \sqrt{5}$

Sol :- Sum of the zeroes $= \alpha+\beta=0$

product of zeroes $=\alpha\beta=\sqrt{5}$

The Quadratic polynomial is

$k\{x^2-(\alpha+\beta)x+\alpha\beta\}$

$\Rightarrow k\{x^2-(0)x+\sqrt{5}\}$

$\Rightarrow k\{x^2+\sqrt{5}\}$

Quadratic polynomial $= x^2+\sqrt{5}$ (when $k=1$)

(iv) 1, 1

Sol :- $\alpha+\beta=1$, $\alpha\beta=1$

Quadratic polynomial $= k\{x^2-(\alpha+\beta)x+\alpha\beta\}$

$\Rightarrow k\{x^2-(1)x+1\}$

$= k\{x^2-x+1\}$

Required one Quadratic polynomial $=$

$[x^2-x+1]$ (when $k=1$)