1. Real Numbers

1. Natural Numbers (N) : Counting numbers; $\{1, 2, 3, 4 \ldots\}$
2. whole Numbers (W): Natural Numbers along with zero ; $\{0, 1, 2, 3, 4 \ldots\}$
3. Integers (Z) : whole and their negatives; $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$
4. Rational Numbers (Q) : Numbers that can be Expressed as quotient or fraction p/q where p and q are integers and $q \neq 0$.
5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization q is of the form $2^n \times 5^m$ where n and m are non-negative integers. then x has a decimal Expansion which terminates.

   Ex : (i) $\frac{16}{125} = \frac{16}{5^3} = 0.120 \rightarrow$ Terminating decimal.

   (ii) $\frac{7}{8} = \frac{7}{2^3} = 0.875 \rightarrow$ Terminating decimal.

   (iii) $\frac{7}{20} = \frac{7}{2^2 \times 5} = 0.35 \rightarrow$ Terminating decimal.

6. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not the form $2^n \times 5^m$, where

n and m are non-negative integers. Then, x has a decimal Expansion which is non-terminating repeating (recurring).

Ex: (i) $\frac{11}{12} = \frac{11}{2^2 \times 3} = 0.91666\ldots = 0.91\overline{6}$ → Non-terminating repeating decimal.

(ii) $\frac{5}{14} = \frac{5}{2 \times 7} = 0.357142857142857\ldots = 0.3\overline{571428}$ → Nonterminating decim

(iii) $\frac{11}{30} = \frac{11}{2 \times 3 \times 5} = 0.3666 = 0.3\overline{6}$ — " "

7. If the prime factorisation of q is of the form $2^m . 5^n$ then the decimal Expansion of the rational number p/q, (p and q are Co-prime) will terminate after m decimalplace if m>n, after n decimalplace if n>m

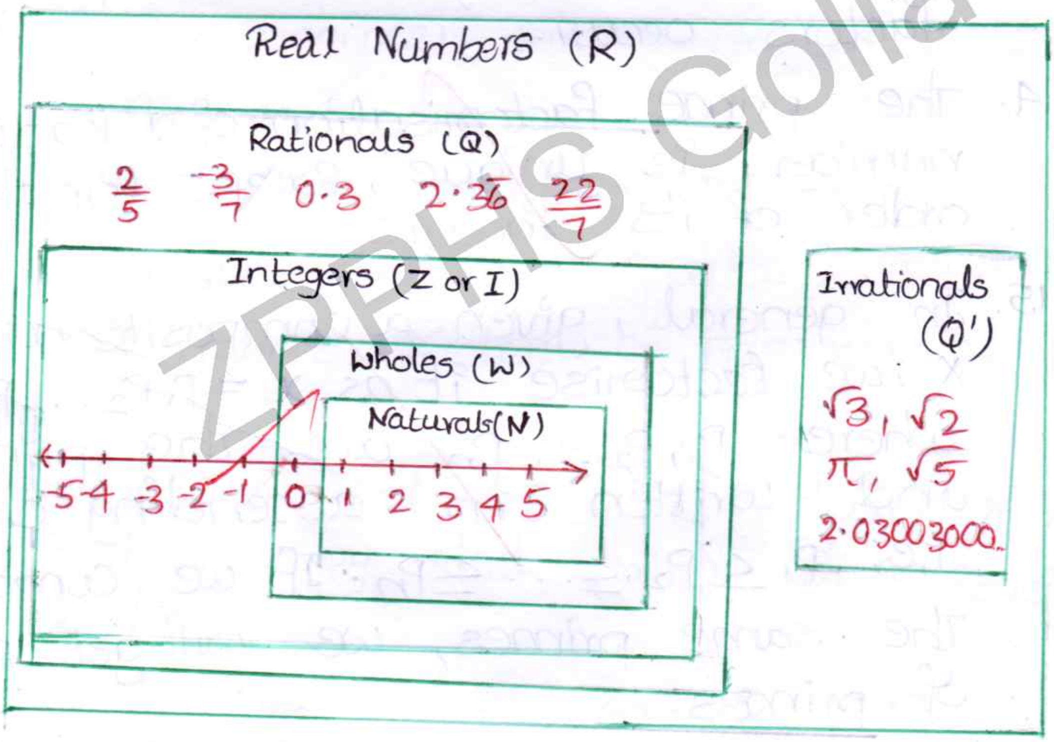
Ex: (i) $\frac{33}{2^2 \times 5}$ will terminate after two decimal places.

(ii) $\frac{14587}{1250} = \frac{14587}{2 \times 5^4}$ will terminate after 4 decimal places.

8. <u>Irrational Numbers</u>: Numbers that cannot be Expressed as

fractions, such as the square root of 2 ($\sqrt{2}$) or π (pi).

10. Real Numbers (R): Real Numbers are a set of numbers that include all rational and irrational numbers. they can be positive, negative, or zero. Examples of real numbers include integers, fractions, decimals and square roots of non-perfect squares (irrational numbers).



11. Euclid's Division Lemma: This is a fundamental concept in number theory. It states that for any such that: $a = bq + r$, where $0 \leq r < b$.

12. Euclidean Algorithm: this is a metho
for finding the greatest common
divisor (GCD) of two numbers. It
involves repeatedly applying Euclid
Division Lemma.

13. theorem 1.1 (Fundamental theorem of
Arithmetic): Every Composite numb
can be Expressed (factorised) as
product of primes, and this
factorisation is unique, apart fr
the order in which the prime
factors occur.

14. the prime factorisation of a natura
number is unique, Except for the
order of its factors.

15. In general, given a composite numb
x, we factorise it as $x = P_1 P_2 \ldots P_n$,
where $P_1, P_2 \ldots P_n$ are the prime
and written in ascending orde
i,e, $P_1 \leq P_2 \leq \ldots \leq P_n$. If we combin
the same primes, we will get pow
of primes.

16. the prime factorisation of a compo
number contains 5 and 2 it Ends
with 0.

Example 1: consider the numbers 4n, where n is a natural number. check whether there is any value of n for which 4^n Ends with the digit zero.

Sol:- $4n = (2^2)^n = 2^{2n}$

5 is not in prime factorisation of 4^n so, there is no natural numbers n for which 4^n Ends with the digit zero.

HCF (Highest Common factor):
product of the smallest power of Each common prime factor of the numbers.

LCM (lowest common multiple):
Product of the greatest power of Each prime factor of the numbers.

Example 2: find the LCM and HCF of 6 and 20 by the prime factorisation method.

Sol:- $6 = 2^1 \times 3^1$ and $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$.

HCF $(6, 20) = 2^1 = 2$

LCM $(6, 20) = 2^2 \times 3^1 \times 5^1 = 60$

HCF and LCM Relationship:
for any two positive integers a and b

$$HCF(a,b) \times LCM(a,b) = a \times b$$

$$\therefore HCF(a,b) = \frac{a \times b}{LCM(a,b)} \text{ and } LCM(a,b) = \frac{a \times b}{HCF(a,b)}$$

Example 3: Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Sol :- $96 = 2^5 \times 3, 404 = 2^2 \times 101$

$HCF(96, 404) = 2^2 = 4$

$LCM(96, 404) = \frac{96 \times 404}{HCF(96, 404)} = \frac{96 \times 404}{4}$

$= 96 \times 101 = 9696.$

Example 4: Find the HCF and LCM of 6, 72 and 120 using the prime factorisation method.

Sol :- $6 = 2 \times 3, 72 = 2^3 \times 3^2, 120 = 2^3 \times 3 \times 5$

$HCF(6, 72, 120) = 2^1 \times 3^1 = 2 \times 3 = 6$

$LCM(6, 72, 120) = 2^3 \times 3^1 \times 5^1 = 8 \times 9 \times 5 = 36$

Note: The product of the three numb is not Equal to the product of the HCF and LCM.

EXERCISE : 1.1

1. Express each number as a product of its prime factors:

(i) 140

@: $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline \end{array}$$

(ii) 156

@: $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

$$\begin{array}{r|l} 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline & 13 \end{array}$$

(iii) 3825

@: $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

$$\begin{array}{r|l} 3 & 3825 \\ \hline 3 & 1275 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

(iv) 5005

@: $5005 = 5 \times 7 \times 11 \times 13$

$$\begin{array}{r|l} 5 & 5005 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

(v) 7429

@: $7429 = 17 \times 19 \times 23$

$$\begin{array}{r|l} 17 & 7429 \\ \hline 19 & 437 \\ \hline & 23 \\ \hline \end{array}$$

* Prove that PT $\sqrt{5}$ is an irrational number

sol :- We have to prove $\sqrt{5}$ is an (Q') number.

Let us assume that $\sqrt{5}$ is a Rational number

$\therefore \sqrt{5} = \frac{a}{b}$, $a, b \in Z$, $b \neq 0$ and a and b are co-primes.

Squaring on both sides

$(\sqrt{5})^2 = \left(\frac{a}{b}\right)^2$

Note :- 5 divides a అంటే a 5 యొక్క multiple అవుతుంది.

$5 = \frac{a^2}{b^2}$

$5b^2 = a^2$ —— ①

$b^2 = \frac{a^2}{5}$

5 divides a^2 $\therefore$ 5 divides a —— ②

$\therefore a = 5c$ for some c.

Substitude the value of a in ①

$5b^2 = (5c)^2 = 25c^2$

$\frac{5b^2}{25} = c^2$

$\frac{b^2}{5} = c^2$

$\therefore$ 5 divides b^2

5 divides b

$\therefore$ 5 is a common factor of a and b.

It is contradiction since a and b are our assumption,s false ~~co-primes~~.

$\therefore \sqrt{5}$ is an irrational numbers.

* PT $\sqrt{7}$ is an irrational number.

@:- we have to prove $\sqrt{7}$ is an irrational number.

let us assume that $\sqrt{7}$ is a Rational number

$\therefore \sqrt{7} = \frac{a}{b}$, $a, b \in Z$, $b \neq 0$ and a and b are co-primes.

Squaring on both sides

$(\sqrt{7})^2 = \left(\frac{a}{b}\right)^2$

$7 = \frac{a^2}{b^2}$

$7b^2 = a^2$ — ①

$b^2 = \frac{a^2}{7}$

7 divides a^2 $\therefore$ 7 divides a — ②

$\therefore a = 7c$ for some c.

Substitude the value of a in ①

$7b^2 = (7c)^2 = 49c^2$

$\frac{7b^2}{49} = c^2$

$\frac{b^2}{7} = c^2$ $\therefore$ 7 divides b^2

7 divide b

$\therefore$ 7 is a common factor of a and b.

It is contradiction since a and b are co-primes. $\therefore$ It is false

$\therefore \sqrt{7}$ is an irrational number.

2. Find the LCM and HCF of the followin[g] pairs of integers and verify that LCM [×] HCF = Product of the two numbers.

(i) 26 and 91

@: $26 = 2 \times 13$

$91 = 7 \times 13$

$HCF(26, 91) = 13$

$LCM(26, 91) = 2 \times 7 \times 13 = 182$

$LCM(26, 91) \times HCF(26, 91) = 182 \times 13 = 2366$

product of two numbers $= 26 \times 91 = 2366$

$\therefore LCM(26, 91) \times HCF(26, 91) = 26 \times 91$

(ii) 510 and 92

@: $510 = 2 \times 3 \times 5 \times 17$ $HCF(510, 92) = 2$

$92 = 2^2 \times 23$

$LCM(510, 92) = 2^2 \times 3 \times 5 \times 17 \times 23 = 4 \times 3 \times 5 \times 17 \times 2[3]$

$= 23460$

$LCM(510, 92) \times HCF(510, 92) = 23460 \times 2 = 4692[0]$

product of two numbers $= 510 \times 92 = 4692[0]$

$LCM(510, 92) \times HCF(510, 92) = 510 \times 92$

(iii) 336 and 54.

<u>sol</u>:- $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

$HCF(336, 54) = 2 \times 3 = 6$

$LCM(336, 54) = 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 = 3024$

$LCM \times HCF = \cancel{23460} \times \cancel{2} = \cancel{46920}$

$3024 \times 6 = 18144$

product of two numbers $= 336 \times 54 = 1814[4]$

$LCM(336, 54) \times HCF(336, 54) = 336 \times 54$

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Sol:- $12 = 2^2 \times 3^1$

$15 = 3^1 \times 5^1$

$21 = 3^1 \times 7^1$

LCM of 12, 15 and 21 $= 2^2 \times 3^1 \times 5^1 \times 7^1$

$= 4 \times 3 \times 5 \times 7 = 420$

HCF of 12, 15 and 21 $= 3^1 = 3$

(ii) 17, 23 and 29.

Sol: $17 = 17^1$

$23 = 23^1$

$29 = 29^1$

LCM $= 17 \times 23 \times 29 = 11339$

HCF $= 1$ (H.C.F of co-primes $= 1$)

(iii) 8, 9 and 25

Sol:- $8 = 2^3$

$9 = 3^2$

$25 = 5^2$

LCM $= 2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25 = 1800$

HCF $= 1$ (HCF of co-primes $= 1$)

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol:- $LCM(a,b) = \dfrac{a \times b}{HCF(a,b)}$

$$LCM\ (306, 657) = \frac{306 \times 657}{HCF\ (306, 657)} = \frac{306 \times 657}{9}$$

$$= 34 \times 657 = 22{,}338$$

5. check whether 6^n can end with the digit 0 for any natural number n.

Sol:- If the prime factorisation of a number contain 2 and 5 then the number ends with digit zero.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

Since 5 is not present in prime factorisation 6^n.

So 6^n cannot end with the digit zero for any natural number n.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol:- $7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78 = 2 \times 3 \times 13^2$

2, 3 and 13 are the factors of $7 \times 11 \times 13 + 13$.

So, $7 \times 11 \times 13 + 13$ is a composite numb

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

5, 1009 are factories of $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$.

So, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a Composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 min for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol:- Time taken by Sonia to drive one round = 18 minutes.

time taken by Ravi to drive one round = 12 min

Time taken by both to meet again = LCM (18, 12)

$18 = 2 \times 3^2$

$12 = 2^2 \times 3$

$LCM(18, 12) = 2^2 \times 3^2 = 4 \times 9 = 36$

After 36 minutes they will meet again at the starting point.

Revisiting Irrational Numbers :

(i) Rational Numbers (Q) : Numbers th can be Expressed as the quotient or fraction p/q, where p and q are integers and q is not Equal zero.

⇒ Irrational Numbers : numbers that cannot be Expressed as p/q whe are integers.

$\sqrt{2}, \sqrt{3}, \sqrt{15}, \pi, -\frac{\sqrt{2}}{\sqrt{3}}, 0.10110111011110$

(ii) $\sqrt{p}$ is irrational, where p is a prime.

Theorem 1.2 : Let p be a prime num If p divides a^2, then p divides where a is a positive integer.

Sol :- $a = P_1 P_2 \ldots P_n$ where $P_1, P_2 \ldots P_n$ are primes not necessarily distinct.

$a^2 = (P_1 P_2 \ldots P_n)(P_1 P_2 \ldots P_n) = P_1^2 P_2^2 \ldots P_n^2$.

If p divides a^2

From the fundamental theorem of Arithmetic, it follows that p is of the prime factors of a^2 so is one of $P_1, P_2 \ldots P_n$

⇒ p divides a

Theorem 1.3 : Prove that $\sqrt{2}$ is irrational.

Proof: Let us assume $\sqrt{2}$ is a rational

then $\sqrt{2} = \frac{a}{b}$ (a, b are co-primes)

Squaring on both sides we get

$2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \rightarrow (1)$

$= b^2 = \frac{a^2}{2}$

$\Rightarrow$ 2 divides a^2 $\therefore$ 2 divides a

we can write $a = 2c$ for some integer c

$\Rightarrow a^2 = 4c^2$

$\Rightarrow 2b^2 = 4c^2$ (from (1))

$\Rightarrow b^2 = 2c^2$

$\Rightarrow c^2 = \frac{b^2}{2}$

$\Rightarrow$ 2 divides b^2 $\therefore$ 2 divides b

$\therefore$ both a and b have 2 as a common factor. But this contradicts the fact that a and b are co-prime. thus our assumption is false

So, we conclude that $\sqrt{2}$ is irrational.

Example 5 : prove that $\sqrt{3}$ is irrational

sol:- let us assume $\sqrt{3}$ is rational number.

then $\sqrt{3} = \frac{a}{b}$ (a, b are co-primes)

Squaring on both sides we get

$3 = \frac{a^2}{b^2} \Rightarrow 3b^2 = a^2 \rightarrow (1)$

$= b^2 = \frac{a^2}{3}$

$\Rightarrow$ 3 divides a^2 $\therefore$ 3 divides a
we can write $a = 3c$ for some integer c

$= a^2 = 9c^2$

$= 3b^2 = 9c^2$ from (1)

$b^2 = 3c^2$

$= c^2 = \frac{b^2}{3}$

= 3 divides b^2 $\therefore$ 3 divides b
Therefore, both a and b have 3 as a co
factor. But this contradicts the fact th
a and b are co-prime. thus our
assumption is false.
So, we conclude that $\sqrt{3}$ is irration

Example 6 : show that $5-\sqrt{3}$ is irratio

sol:- Let us assume that $5-\sqrt{3}$ is (Q).

$5-\sqrt{3} = \frac{a}{b}$ (a, b are co-primes)

$5 - \frac{a}{b} = \sqrt{3}$

$\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$ .... (1)

since 5, a and b are integers the
RHS of (1) i.e $\frac{5b-a}{b}$ is rational
the L.H.S $= \sqrt{3}$ also rational
But this contradicts the fact tha
$\sqrt{3}$ is irrational.
Thus our assumption is false.
so, we conclude that $5-\sqrt{3}$ is
irrational.

$b^2 = 5c^2$

$= c^2 = \frac{b^2}{5}$

$= 5$ divides a^2 $\therefore$ 5 divides a

$\therefore$ Both a and b have 5 as a commo factor. But this contradicts the fac that a and b are co-prime. thus our assumption is false. so, we conclude that $\sqrt{5}$ is irratio

2. prove that $3+2\sqrt{5}$ is irrational.

Sol:- let us assume $3+2\sqrt{5}$ is rati

let $3+2\sqrt{5} = \frac{a}{b}$ (a, b are co-primes)

Squaring on both sides we get

$5 = \frac{a^2}{b^2} = 5b^2 = a^2 \rightarrow (1)$

$b^2 = \frac{a^2}{5}$

$= 5$ divides a^2 $\therefore$ 5 divides a.

$2\sqrt{5} = \frac{a}{b} - 3 = \frac{a-3b}{b}$

$\sqrt{5} = \frac{a-3b}{2b} \rightarrow (1)$

Since 2, 3, a and b are integers the R.H.S of (1) i.e $\frac{a-3b}{2b}$ is ratio so the L.H.S $\sqrt{5}$ also rational.

Coprimes అంటే no common factors or primes

Thus our assumption is false.
so, we conclude that $3+2\sqrt{5}$ is irrational.

Sir cheppina sum:—

* We have to prove $3+2\sqrt{5}$ is an irrational.

Let us assume that $3+2\sqrt{5}$ is a rational.

$\therefore 3+2\sqrt{5} = \frac{a}{b}$, $a,b \in Z$, and b are co-primes

$2\sqrt{5} = \frac{a}{b} - 3$ ← $\boxed{\text{Note ఇక్కడ } 3+2\sqrt{5} \text{ కాబట్టి ఈ మైనస్ లా + వస్తుంది.}}$

$2\sqrt{5} = \frac{a-3b}{b}$

$\sqrt{5} = \frac{a-3b}{2b}$

Since $a,b \in Z$, $\frac{a-3b}{2b}$ is a rational number

$\therefore \sqrt{5}$ is also rational number

$\therefore$ our assumption this contradiction.

$\therefore$ our assumption is false.

$\therefore 3+2\sqrt{5}$ is irrational number.

3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

Sol:- let us assume that $\frac{1}{\sqrt{2}}$ is rational.

Let $\frac{1}{\sqrt{2}} = \frac{a}{b}$ (a, b are coprimes)

$\sqrt{2} = \cancel{\frac{a}{b}}\ \frac{b}{a}$

since a and b are integers, $\frac{b}{a}$ is rational, and so $\sqrt{2}$ is rational.

⊛. Prove that $\sqrt{2}+3\sqrt{5}$ is an Irrational Number.

Sol:- let us assume $\sqrt{2}+3\sqrt{5}$ is a rational number.

$\therefore \sqrt{2}+3\sqrt{5} = \frac{a}{b}$, $a b \in Z$ and a and b are Co-primes.

$\therefore 3\sqrt{5} = \frac{a}{b} - \sqrt{2}$

Squaring on both sides

$(3\sqrt{5})^2 = \left(\frac{a}{b} - \sqrt{2}\right)^2$

$3^2 \cdot 5 = \left(\frac{a}{b}\right)^2 + (\sqrt{2})^2 - 2 \cdot \frac{a}{b} \cdot \sqrt{2}$ $\quad \therefore (a-b)^2 = a^2 - 2ab + b^2$

$45 = \frac{a^2}{b^2} + 2 - \frac{2\sqrt{2}a}{b}$

$\frac{2\sqrt{2}a}{b} = \frac{a^2}{b^2} + 2 - 45$ $\quad \therefore$ RHS to LHS

$\frac{2\sqrt{2}a}{b} = \frac{a^2}{b^2} - 43$

$= \frac{a^2 - 43b^2}{b^2}$

$\sqrt{2} = \frac{a^2 - 43b^2}{2ab}$

since $\frac{a - 43b^2}{2ab}$ rational $\sqrt{2}$ is also ration so this is the contradaction thus our assumption is wrong, $\sqrt{2}+3\sqrt{5}$ is a irrational number.