

iZ-Library
0.0.1

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Chapter 1

iZprime User Manual

iZprime is a C library for prime sieving, range counting/streaming, and random-prime generation.

1.1 What this manual covers

- Public API ([include/iZ_api.h](#))
- Core toolkit types and internals ([include/iZ_toolkit.h](#))
- Utility modules and shared data structures

1.2 Related project docs

- Algorithm pseudocode: docs/pseudocode.pdf
- CLI usage: docs/cli.md
- Testing guide: docs/tests.md
- Benchmark guide and results: docs/benchmarks.md
- Build system reference: docs/Makefile.md

Chapter 2

Benchmarks

This document captures benchmark methodology, current result snapshots, and plotting usage for the iZprime project.

2.1 1. Run Benchmarks

From repository root:

```
make benchmark-p_sieve
make benchmark-p_gen
make benchmark-SiZ_count
```

Save results into docs/test_results/:

```
make benchmark-p_sieve save-results
make benchmark-p_gen save-results
make benchmark-SiZ_count save-results
```

Save results and generate plot artifacts automatically:

```
make benchmark-p_sieve plot
make benchmark-p_gen plot
```

Notes:

- plot implies save-results.
- Dash-style flags also work with GNU make passthrough:

```
make -- benchmark-p_sieve --save-results --plot
make -- benchmark-p_gen --save-results --plot
make -- benchmark-SiZ_count --save-results
```

Note: plotting is currently wired for p_sieve and p_gen result formats.

2.2 2. Result Files Used In This Snapshot

Sieve benchmark snapshot:

- docs/test_results/psieve_10093442.txt

Prime generation benchmark snapshots:

- docs/test_results/p_gen_12094122.txt (1024-bit)
- docs/test_results/p_gen_12094126.txt (2048-bit)
- docs/test_results/p_gen_12094224.txt (4096-bit)

SiZ_count benchmark snapshot:

- docs/test_results/SiZ_count_19205134.txt

2.3 3. Sieve Benchmark Table and Plot

Times are in microseconds (us) from docs/test_results/psieve_10093442.txt.

| Algorithm | 10 ⁴ | 10 ⁵ | 10 ⁶ | 10 ⁷ | 10 ⁸ | 10 ⁹ | 10 ¹⁰ |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|
| SoE | 14 | 130 | 1136 | 28874 | 136817 | 2510303 | 29944939 |
| SSoE | 21 | 230 | 1826 | 16351 | 148737 | 1393887 | 13312626 |
| SoEu | 18 | 198 | 1567 | 14153 | 139020 | 1793877 | 18707288 |
| SoS | 14 | 159 | 1395 | 14393 | 142700 | 2414564 | 28599960 |
| SoA | 24 | 197 | 1645 | 14909 | 145648 | 3600075 | 46331079 |
| SiZ | 9 | 119 | 912 | 8924 | 87100 | 1436860 | 18074650 |
| SiZm | 12 | 139 | 931 | 8825 | 73872 | 709836 | 6917661 |
| SiZm_vy | 25 | 178 | 1061 | 7997 | 68305 | 684938 | 6385593 |

2.4 4. Prime Generation Benchmark Table

Average times are in seconds for each bit size from the corresponding docs/test_results/p_gen_*.txt files.

| Algorithm | 1024 | 2048 | 4096 |
|-----------------------|----------|----------|----------|
| vy_random_prime | 0.021182 | 0.063515 | 3.181408 |
| vx_random_prime | 0.008107 | 0.082872 | 1.220519 |
| iZ_random_next_prime | 0.023231 | 0.150618 | 0.578747 |
| gmp_random_next_prime | 0.009421 | 0.071797 | 1.666787 |
| BN_generate_prime_ex | 0.026731 | 0.422709 | 4.959384 |

2.5 5. Plot Artifacts

Current plot files:

- docs/test_results/p_gen_12094122_avg.svg
- docs/test_results/p_gen_12094126_avg.svg
- docs/test_results/p_gen_12094224_avg.svg

2.5.1 Prime generation plots

2.6 6. SiZ_count Benchmark Table

Times are in seconds from docs/test_results/SiZ_count_19205134.txt.

Window size: 10⁹ integers for each starting point.

| Start Point | Prime Count | Time (s) |
|------------------|-------------|----------|
| 10 ¹⁰ | 43336106 | 0.157103 |

| Start Point | Prime Count | Time (s) |
|-------------|-------------|------------|
| 10^20 | 21710426 | 45.271972 |
| 10^30 | 14473890 | 50.446129 |
| 10^40 | 10860250 | 75.721432 |
| 10^50 | 8684378 | 83.009872 |
| 10^60 | 7237481 | 107.691231 |
| 10^70 | 6205976 | 119.342140 |
| 10^80 | 5428021 | 167.147788 |
| 10^90 | 4825182 | 178.556225 |
| 10^100 | 4342289 | 218.545428 |

2.7 7. Plot From Terminal (Interactive Prompt)

You can generate/view plots directly from terminal. If no filepath is passed, scripts prompt for one:

```
python3 py_tools/plot_results.py
python3 py_tools/plot_p_sieve_results.py
python3 py_tools/plot_p_gen_results.py
```

Or pass a file explicitly:

```
python3 py_tools/plot_results.py output/psieve_10093442.txt --save
python3 py_tools/plot_results.py output/p_gen_12094224.txt --avg --save
```


Chapter 3

izprime CLI

The CLI binary is `izprime` and exposes task-oriented subcommands over the library API.

3.1 Build

```
make cli
./build/bin/izprime help
```

3.2 Numeric input syntax

Commands that accept numbers support:

- decimal integers: 1000000
- grouped decimals: 1,000,000
- powers: 10^6
- scientific shorthand: 1e6, 10e100
- additive expressions: 10e100 + 10e9

3.3 Commands

3.4 stream_primes

Streams primes in an inclusive range using [SiZ_stream](#).

```
izprime stream_primes --range "[LOWER, UPPER]" [--print | --stream-to FILE] [--mr-rounds N]
```

Examples:

```
izprime stream_primes --range "[0, 10^5]" --print
izprime stream_primes --range "[1,000,000, 1,001,000]" --stream-to output/range.txt
```

Alias: `sieve`.

3.5 count__primes

Counts primes in an inclusive range using [SiZ_count](#).

```
izprime count__primes --range "[LOWER, UPPER]" [--cores-number N] [--mr-rounds N]
```

Examples:

```
izprime count__primes --range "[0, 10^9]" --cores-number 8
izprime count__primes --range "10^100 + 10^9, 10^100 + 10^9 + 10^9" --cores-number 4
```

Alias: count.

Note: on platforms without fork, --cores-number is accepted but execution falls back to single-process mode.

3.6 next__prime

Finds the next prime after n using [iZ_next_prime](#).

```
izprime next__prime --n VALUE
```

Example:

```
izprime next__prime --n "10^12 + 39"
```

3.7 is__prime

Checks primality of n using [check_primality](#).

```
izprime is__prime --n VALUE [--rounds N]
```

Example:

```
izprime is__prime --n "10^61 + 1" --rounds 40
```

3.8 Existing maintenance commands

```
izprime test [--limit N]
izprime benchmark [--limit N] [--repeat N] [--algo NAME|all] [--save-results FILE]
izprime doctor
```

Use `izprime help <command>` for command-specific help.

Chapter 4

Contributing to iZprime

Thanks for contributing. This project is prepared for research-grade publication, so we hold changes to a strict quality bar.

4.1 Scope

Contributions are welcome for:

- correctness fixes,
- performance improvements,
- documentation quality,
- tests/benchmarks and tooling,
- new algorithms that align with the iZprime design.

4.2 Development Setup

Required dependencies:

- C compiler (GCC/Clang),
- make,
- GMP,
- OpenSSL.

Optional:

- pkg-config,
- Doxygen,
- Python packages in `py_tools/requirements.txt` for plots.

4.3 Code Style

- Keep modules focused and composable.
- Prefer explicit ownership and cleanup paths.
- Keep public API declarations in `include/`, implementations in `src/`.
- Document new public structs/functions with Doxygen comments.
- Avoid introducing hidden global state.

4.4 Validation Requirements

Before opening a pull request:

```
make clean
make test-unit
make test-integration
make -- test-all --verbose
```

If you touched performance-sensitive code, also run:

```
make benchmark-p_sieve save-results
make benchmark-p_gen save-results
make benchmark-SiZ_count save-results
```

If you changed API/docs:

```
make userManual
```

4.5 Benchmarking Expectations

- Include the exact command(s) used.
- Include hardware/OS/compiler context.
- Share result files from output/ when relevant.
- Prefer reporting both absolute timing and relative speedup/regression.

4.6 Pull Request Checklist

1. Explain the problem and the technical approach.
2. List behavioral changes and compatibility impact.
3. Link tests that validate the change.
4. Add or update benchmark evidence for performance claims.
5. Update docs (README.md, docs/*.md, Doxygen comments) when needed.

4.7 Algorithm Contributions

For new sieve/generation techniques:

- map design to existing toolkit abstractions ([IZM](#), [VX_SEG](#), bitmaps, arrays),
- provide pseudocode-level explanation in docs/pseudocode.pdf source pipeline,
- add integration tests versus trusted baselines,
- add benchmark hooks so users can compare against existing models.

4.8 Review Standard

PRs are reviewed for:

- correctness and edge-case behavior,
- memory/process safety,
- API clarity and consistency,
- test coverage,
- reproducibility of benchmark claims.

Chapter 5

Makefile documentation (iZprime)

This document describes the modular make workflow for this repository.

5.1 Quick start

- Build static library + CLI and run:
 - make
- Build library only:
 - make lib
- Check dependencies:
 - make doctor
- Create release source archive:
 - make dist VERSION=1.0.0
- Run unit + integration tests:
 - make test-all
- Run sieve benchmarks:
 - make benchmark-p_sieve
- Run prime-generation benchmarks:
 - make benchmark-p_gen

5.2 Dependencies

- C toolchain: gcc or clang
- Build tool: make
- Libraries:
 - GMP (-lgmp)
 - OpenSSL 3 (-lssl -lcrypto)

The Makefile prefers pkg-config for portability; on macOS it also supports Homebrew path fallbacks.

5.3 Targets

5.3.1 Build / run

- all (default): Builds the CLI and runs it
- lib: Builds build/lib/libizprime.a and shared-library artifacts (default)
- cli: Builds build/bin/izprime (or PROGRAM override)
- run: Runs the CLI binary (builds first if needed)
- debug: Builds with debug flags (-O0 -g) and logging enabled
- release: Builds an optimized binary and disables logging
- clean: Removes build artifacts and generated output/log directories
- help: Prints a concise help summary of targets and options
- install: Installs headers, static/shared libraries, CLI, and pkg-config file
- install-lib: Installs headers, static/shared libraries, and pkg-config file
- uninstall: Removes files installed by the targets above
- doctor: Verifies build dependencies with a compile/link smoke test
- dist: Builds source tarball and SHA256 checksum under dist/

5.3.2 Tests

The primary test runner is built at build/test/test_runner. It links against the static library target (build/lib/libizprime.a).

- test-all: Runs unit + integration tests
- test-unit: Runs unit tests only
- test-integration: Runs integration tests only

Notes:

- Test selection defaults to --unit --integration when no flags are provided.
- Test runner arguments can be extended via TEST_ARGS.

5.3.3 Benchmarks

Benchmarks are intentionally separate from tests, so they can be invoked explicitly and kept out of typical CI workflows.

- benchmark-p_sieve: Runs prime sieve model benchmarks
 - Invokes: ./build/test/test_runner --benchmark-p-sieve ...
- benchmark-p_gen: Runs random prime generation benchmarks
 - Invokes: ./build/test/test_runner --benchmark-p-gen ...
- benchmark-SiZ_count: Runs large-window SiZ_count benchmark
 - Invokes: ./build/test/test_runner --benchmark-siz-count ...

5.3.4 Examples

- examples: Builds all example programs into build/examples/
- run-example: Runs one example
 - Required variables:
 - * EX=<example_name> (name of the example source without extension)
 - * ARGS="..." (optional arguments passed to the example)

Example:

- make run-example EX=sieve_primes ARGS="SiZm 10000000 10"

5.4 Configuration variables

5.4.1 Local configuration file

- config.mk (optional, local)
 - If present, it overrides defaults in Makefile.
 - Start from:
 - * cp config.mk.example config.mk

5.4.2 Dependency discovery

- USE_PKG_CONFIG (default: 1)
 - When 1, attempts to use pkg-config to discover GMP and OpenSSL.
 - When 0, skips pkg-config and uses the Homebrew/generic fallbacks.
- PKG_CONFIG (default: pkg-config)
 - Override if you need a specific pkg-config binary.

5.4.3 Build toggles

- LOGGING (default: 1)
 - When 1, compiles with -DENABLE_LOGGING.
 - The release target forces LOGGING=0.
- PROGRAM (default: izprime)
 - Output name for the CLI binary under build/bin/.
- CLI_ENTRY (default: src/main.c)
 - Source file used as CLI entrypoint (main()).
- VERSION (default: 1.0.0)
 - Release/version metadata for the CLI and shared library.
- SOVERSION (default: first component of VERSION)
 - Shared-library ABI major version.
- BUILD_SHARED (default: 1)
 - Build shared library artifacts in addition to static library.

5.4.4 Test/benchmark options

- VERBOSE (default: 0)
 - When 1, adds `--verbose` to the test runner.
- SAVE_RESULTS (default: 0)
 - When 1, adds `--save-results` to benchmark runs (where supported).
- TEST_OUTPUT (default: empty)
 - When set, redirects test runner output to the given file.
- TEST_ARGS (default: empty)
 - Extra arguments appended to the test runner invocation.

5.5 Common recipes

- Run integration tests with verbose output:
 - `make test-integration VERBOSE=1`
- Save benchmark results:
 - `make benchmark-p_sieve SAVE_RESULTS=1`
 - `make benchmark-p_gen SAVE_RESULTS=1`
 - `make benchmark-SiZ_count SAVE_RESULTS=1`
- Redirect test output to a file:
 - `make test-all TEST_OUTPUT=output/test_all.txt`
- Pass extra args to the test runner:
 - `make test-all TEST_ARGS="--help"`
- Install under a local prefix:
 - `make install PREFIX=$HOME/.local`
- Build static-only artifacts:
 - `make lib BUILD_SHARED=0`
- Create a release tarball:
 - `make dist VERSION=1.0.0`

5.6 Notes on outputs

- Build outputs live under `build/`.
- Benchmark/test logs can be redirected via `TEST_OUTPUT`.
- Some benchmarks may be long-running; use `SAVE_RESULTS=1` to persist results when supported by that benchmark.

Chapter 6

Releasing iZprime

This document defines the release workflow for v1.x and later.

6.1 Versioning

- Use SemVer tags: vMAJOR.MINOR.PATCH (example: v1.0.0).
- VERSION in Makefile should match the release tag.
- SOVERSION defaults to the major version and should change only on ABI breaks.

6.2 Pre-release Gate

Run this checklist from a clean working tree:

```
make clean
make test-unit
make test-integration
make benchmark-p_sieve save-results
make benchmark-p_gen save-results
make benchmark-SiZ_count save-results
make userManual
make dist VERSION=1.0.0
```

Confirm:

- generated docs/userManual.pdf opens correctly,
- benchmark outputs are saved in output/,
- dist/ contains tarball + checksum,
- release tarball does not include transient artifacts (build/, logs/, output/, .git/).

Windows readiness (recommended for each release):

- run CI builds on windows-latest (MSYS2/MinGW toolchain),
- run at least make doctor, make test-unit, and make test-integration,
- confirm fallback behavior for multi-process APIs on non-fork platforms.

6.3 Changelog and Release Notes

ChangeLog should summarize user-visible changes.

Suggested generation workflow:

```
LAST_TAG=$(git describe --tags --abbrev=0 2>/dev/null || echo "")
if [ -n "$LAST_TAG" ]; then
  git log --pretty=format:"- %s (%h)" "$LAST_TAG"..HEAD
else
  git log --pretty=format:"- %s (%h)"
fi
```

Use the output as a draft, then curate into sections:

- Added
- Changed
- Fixed
- Performance
- Documentation

Release notes in GitHub/GitLab should include:

- highlights,
- migration notes (if any),
- validation commands used,
- benchmark artifacts summary.

6.4 Tag and Publish

```
git tag -a v1.0.0 -m "iZprime v1.0.0"
git push origin v1.0.0
```

Attach dist/izprime-<version>.tar.gz and checksum file to the release.

6.5 Post-release

- bump VERSION to next development version (for example 1.0.1-dev),
- update docs if commands/artifacts changed,
- track follow-up fixes as issues/milestone items.

Chapter 7

Tests

This document describes test targets, execution commands, and current pass snapshots.

7.1 1. Test Targets

From repository root:

```
make test-all
make test-unit
make test-integration
```

Verbose output:

```
make test-all verbose
make test-unit verbose
make test-integration verbose
```

GNU make passthrough form (dash flags):

```
make -- test-all --verbose
make -- test-unit --verbose
make -- test-integration --verbose
```

7.2 2. What Each Target Runs

- test-unit: bitmap/int-array/iZm/vx-seg module-level tests.
- test-integration: sieve hash integrity, range APIs, and prime-generation integration checks.
- test-all: unit + integration suites through the shared test runner.

7.3 3. Current Snapshot Results

Snapshot command set:

```
make test-unit
make test-integration
make test-all
```

Observed exit codes and summaries:

| Target | Exit code | Summary |
|----------------|-----------|-----------------------------------|
| make test-unit | 0 | 7/7 module groups passed (100.0%) |

| Target | Exit code | Summary |
|-----------------------|-----------|---|
| make test-integration | 0 | 6/6 integration groups passed (100.0%) |
| make test-all | 0 | full test runner completed successfully |

Expected success markers in output:

- Unit suite: [SUCCESS] ALL MODULE TESTS PASSED!
- Integration suite: [SUCCESS] ALL INTEGRATION TESTS PASSED!

7.4 4. Show Results In Terminal

Quick pass/fail runs:

```
make test-unit
make test-integration
```

Detailed run (prints per-test information and hashes):

```
make test-integration verbose
```

Capture logs for review:

```
make test-unit > /tmp/test_unit.log 2>&1
make test-integration > /tmp/test_integration.log 2>&1
```

Tail the summaries:

```
tail -n 40 /tmp/test_unit.log
tail -n 50 /tmp/test_integration.log
```

7.5 5. Notes

- The integrity suite intentionally skips [SiZm_vy](#) in hash equality checks because it emits unordered prime lists.
- The test runner exits non-zero when any group fails; make surfaces that as a target failure.

Chapter 8

Topic Index

8.1 Topics

Here is a list of all topics with brief descriptions:

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| Printer, implemented in toolkit/print_utils.c | 94 |
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Chapter 9

Data Structure Index

9.1 Data Structures

Here are the data structures with brief descriptions:

| | |
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Chapter 10

File Index

10.1 File List

Here is a list of all documented files with brief descriptions:

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| include/ int_arrays.h | Dynamic arrays for uint16_t, uint32_t, and uint64_t values | 123 |
| include/ iZ_api.h | Public API for prime sieving, range scans, and prime generation | 126 |
| include/ iZ_toolkit.h | Core iZ/iZm primitives used by SiZ-family algorithms | 128 |
| include/ logger.h | Header file for the logging system | 132 |
| include/ platform.h | Cross-platform system abstraction utilities | 135 |
| include/ test_api.h | Public test/benchmark entry points for the iZ library | 137 |
| include/ utils.h | Shared utilities and common includes for the iZprime library | 138 |
| src/ iZ_apps.c | High-level application routines built on top of the iZ_toolkit | 141 |
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| src/toolkit/ logger.c | Logging module | 184 |
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| src/toolkit/ print_utils.c | Print formatting helpers for tests, benchmarks, and CLI output | 189 |

| | |
|--|---------------------|
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| Stopwatch helpers based on monotonic wall-clock time | 191 |
| src/toolkit/utils.c | |
| Implementations for the shared utility layer | 192 |

Chapter 11

Topic Documentation

11.1 Bitmap Module

Packed bit-array primitives for sieve and toolkit modules.

Files

- file [bitmap.c](#)
Implementation of the bitmap module for efficient bit array operations.

Data Structures

- struct [BITMAP](#)
Packed bit-array with optional SHA-256 checksum.

Functions

- int [TEST_BITMAP](#) (int verbose)
Run bitmap module tests.

Lifecycle

- [BITMAP](#) * [bitmap_init](#) (size_t size, int set_bits)
Allocate a bitmap with size bits.
- void [bitmap_free](#) ([BITMAP](#) **bitmap)
Free a bitmap and set the caller pointer to NULL.
- [BITMAP](#) * [bitmap_clone](#) ([BITMAP](#) *src)
Deep-copy a bitmap, including data and checksum.

Single-bit Operations

- int [bitmap_get_bit](#) ([BITMAP](#) *bitmap, size_t idx)
Read bit value at idx.
- void [bitmap_set_bit](#) ([BITMAP](#) *bitmap, size_t idx)
Set bit at idx to 1.
- void [bitmap_flip_bit](#) ([BITMAP](#) *bitmap, size_t idx)
Toggle bit at idx.
- void [bitmap_clear_bit](#) ([BITMAP](#) *bitmap, size_t idx)
Clear bit at idx (set to 0).

Bulk Operations

- void [bitmap_set_all](#) ([BITMAP](#) *bitmap)
Set all bits to 1.
- void [bitmap_clear_all](#) ([BITMAP](#) *bitmap)
Clear all bits to 0.
- void [bitmap_clear_steps](#) ([BITMAP](#) *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
Clear every step-th bit from start_idx up to limit.
- void [bitmap_clear_steps_simd](#) ([BITMAP](#) *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
SIMD-accelerated variant of [bitmap_clear_steps\(\)](#).

Integrity and I/O

- void [bitmap_compute_hash](#) ([BITMAP](#) *bitmap)
Compute SHA-256 over bitmap data and store it in [BITMAP::sha256](#).
- int [bitmap_validate_hash](#) ([BITMAP](#) *bitmap)
Verify [BITMAP::sha256](#) against current bitmap data.
- int [bitmap_fwrite](#) ([BITMAP](#) *bitmap, FILE *file)
Write bitmap payload and checksum to a binary stream.
- [BITMAP](#) * [bitmap_fread](#) (FILE *file)
Read bitmap payload and checksum from a binary stream.

11.1.1 Detailed Description

Packed bit-array primitives for sieve and toolkit modules.

11.1.2 Function Documentation

11.1.2.1 `bitmap_clear_all()`

```
void bitmap_clear_all (
    BITMAP * bitmap)
```

Clear all bits to 0.

Parameters

| | |
|--------|-------------------|
| bitmap | Bitmap to modify. |
|--------|-------------------|

Clear all bits to 0.

Parameters

| | |
|--------|--|
| bitmap | A pointer to the BITMAP structure. |
|--------|--|

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c](#).

Definition at line [180](#) of file [bitmap.c](#).

References [BITMAP::data](#), and [BITMAP::size](#).

11.1.2.2 `bitmap_clear_bit()`

```
void bitmap_clear_bit (
    BITMAP * bitmap,
    size_t idx)
```

Clear bit at idx (set to 0).

Parameters

| | |
|--------|-----------------------|
| bitmap | Bitmap to modify. |
| idx | Zero-based bit index. |

Clear bit at idx (set to 0).

Parameters

| | |
|--------|---------------------------------------|
| bitmap | The BITMAP to modify. |
| idx | The index of the bit to clear. |

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c](#).

Definition at line [159](#) of file [bitmap.c](#).

References [BITMAP::data](#).

Referenced by [iZm_construct_vx_base\(\)](#), [SoEu\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

11.1.2.3 `bitmap_clear_steps()`

```
void bitmap_clear_steps (
    BITMAP * bitmap,
    uint64_t step,
    uint64_t start_idx,
    uint64_t limit)
```

Clear every step-th bit from start_idx up to limit.

This is the core primitive for marking composite progressions in sieve code.

Parameters

| | |
|-----------|---------------------------------------|
| bitmap | Bitmap to modify. |
| step | Index increment between cleared bits. |
| start_idx | First index to clear. |
| limit | Inclusive upper index bound. |

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 210 of file `bitmap.c`.

References `BITMAP::data`, `MIN`, and `BITMAP::size`.

Referenced by `vx_search_prime()`.

11.1.2.4 `bitmap_clear_steps_simd()`

```
void bitmap_clear_steps_simd (
    BITMAP * bitmap,
    uint64_t step,
    uint64_t start_idx,
    uint64_t limit)
```

SIMD-accelerated variant of `bitmap_clear_steps()`.

Parameters

| | |
|-----------|---------------------------------------|
| bitmap | Bitmap to modify. |
| step | Index increment between cleared bits. |
| start_idx | First index to clear. |
| limit | Inclusive upper index bound. |

SIMD-accelerated variant of `bitmap_clear_steps()`.

This function uses SIMD instructions (AVX2, SSE2, or NEON) to clear bits at regular intervals in the bitmap. It processes multiple bits in parallel for improved performance on large bitmaps.

Parameters

| | |
|--------|---------------------------------|
| bitmap | Pointer to the bitmap to modify |
|--------|---------------------------------|

| | |
|-----------|--|
| step | Interval between bits to clear (must be > 0) |
| start_idx | Starting bit index (inclusive) |
| limit | Upper bound for clearing (inclusive) |

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c](#).

Definition at line 233 of file [bitmap.c](#).

References [BITMAP::data](#), [MIN](#), and [BITMAP::size](#).

Referenced by [iZm_construct_vx_base\(\)](#), [process_iZ_bitmaps\(\)](#), [process_N_bitmap\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoA\(\)](#), [SoS\(\)](#), [SSoE\(\)](#), and [vx_det_sieve\(\)](#).

11.1.2.5 bitmap_clone()

```
BITMAP * bitmap_clone (
    BITMAP * src)
```

Deep-copy a bitmap, including data and checksum.

Parameters

| | |
|-----|----------------|
| src | Source bitmap. |
|-----|----------------|

Returns

Cloned bitmap, or NULL on failure.

Deep-copy a bitmap, including data and checksum.

Allocates a new bitmap with the same size as the source and copies all data including the bit array and SHA-256 hash. The returned bitmap is completely independent of the source and must be freed separately.

Parameters

| | |
|-----|---------------------------------------|
| src | Pointer to the source bitmap to clone |
|-----|---------------------------------------|

Returns

Pointer to newly allocated cloned [BITMAP](#), or NULL on failure

Return values

| | |
|------|--|
| NULL | if src is NULL, src->size is 0, or memory allocation fails |
|------|--|

Note

The clone includes a copy of the SHA-256 hash from the source
 Caller must free the returned bitmap using [bitmap_free\(\)](#)
 Changes to the clone do not affect the source and vice versa

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c.](#)

Definition at line 382 of file [bitmap.c](#).

References [bitmap_init\(\)](#), [BITMAP::byte_size](#), [BITMAP::data](#), [BITMAP::sha256](#), and [BITMAP::size](#).

Referenced by [iZm_clone\(\)](#), [SiZm\(\)](#), and [vx_init\(\)](#).

11.1.2.6 [bitmap_compute_hash\(\)](#)

```
void bitmap_compute_hash (
    BITMAP * bitmap)
```

Compute SHA-256 over bitmap data and store it in [BITMAP::sha256](#).

Parameters

| | |
|--------|-----------------|
| bitmap | Bitmap to hash. |
|--------|-----------------|

Compute SHA-256 over bitmap data and store it in [BITMAP::sha256](#).

Parameters

| | |
|--------|---|
| bitmap | The BITMAP for which the SHA-256 hash is generated. |
|--------|---|

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c.](#)

Definition at line 398 of file [bitmap.c](#).

References [BITMAP::byte_size](#), [BITMAP::data](#), and [BITMAP::sha256](#).

Referenced by [bitmap_fwrite\(\)](#).

11.1.2.7 [bitmap_flip_bit\(\)](#)

```
void bitmap_flip_bit (
    BITMAP * bitmap,
    size_t idx)
```

Toggle bit at idx.

Parameters

| | |
|--------|-------------------|
| bitmap | Bitmap to modify. |
|--------|-------------------|

| | |
|-----|-----------------------|
| idx | Zero-based bit index. |
|-----|-----------------------|

Toggle bit at idx.

Parameters

| | |
|--------|---------------------------------------|
| bitmap | The BITMAP to modify. |
| idx | The index of the bit to flip. |

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 148 of file [bitmap.c](#).

References [BITMAP::data](#).

Referenced by [SoA\(\)](#).

11.1.2.8 bitmap_fread()

[BITMAP](#) * bitmap_fread (
FILE * file)

Read bitmap payload and checksum from a binary stream.

Parameters

| | |
|------|------------------|
| file | Readable stream. |
|------|------------------|

Returns

Newly allocated bitmap, or NULL on parse/verification failure.

Read bitmap payload and checksum from a binary stream.

Parameters

| | |
|------|--------------------------------|
| file | The file pointer to read from. |
|------|--------------------------------|

Returns

[BITMAP](#)* A pointer to the newly created [BITMAP](#), or NULL on failure.

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 492 of file [bitmap.c](#).

References [bitmap_free\(\)](#), [bitmap_init\(\)](#), [bitmap_validate_hash\(\)](#), [BITMAP::byte_size](#), [BITMAP::data](#), [log_error\(\)](#), and [BITMAP::sha256](#).

11.1.2.9 `bitmap_free()`

```
void bitmap_free (
    BITMAP ** bitmap)
```

Free a bitmap and set the caller pointer to NULL.

Parameters

| | |
|--------|--|
| bitmap | Address of a BITMAP pointer. |
|--------|--|

Free a bitmap and set the caller pointer to NULL.

This function safely deallocates the bitmap's data array and the structure itself, then sets the pointer to NULL to prevent use-after-free bugs and double-free errors. Safe to call with NULL pointers.

Parameters

| | |
|--------|--|
| bitmap | Double pointer to the BITMAP to free |
|--------|--|

Postcondition

*bitmap is set to NULL after successful deallocation

Safe to call multiple times on the same pointer

Note

No operation performed if `bitmap == NULL` or `*bitmap == NULL`

This is the only correct way to free a [BITMAP](#); never use `free()` directly

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line [103](#) of file [bitmap.c](#).

Referenced by [bitmap_fread\(\)](#), [get_root_primes\(\)](#), [iZm_free\(\)](#), [SiZ\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoA\(\)](#), [SoE\(\)](#), [SoEu\(\)](#), [SoS\(\)](#), [SSoE\(\)](#), [vx_free\(\)](#), and [vx_search_prime\(\)](#).

11.1.2.10 `bitmap_fwrite()`

```
int bitmap_fwrite (
    BITMAP * bitmap,
    FILE * file)
```

Write bitmap payload and checksum to a binary stream.

Parameters

| | |
|--------|----------------------|
| bitmap | Bitmap to serialize. |
|--------|----------------------|

| | |
|------|------------------|
| file | Writable stream. |
|------|------------------|

Returns

1 on success, otherwise 0.

Write bitmap payload and checksum to a binary stream.

Parameters

| | |
|--------|--------------------------------------|
| bitmap | The BITMAP to write. |
| file | The file pointer to write to. |

Returns

int 1 on success, 0 on failure.

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 440 of file [bitmap.c](#).

References [bitmap_compute_hash\(\)](#), [BITMAP::byte_size](#), [BITMAP::data](#), [log_error\(\)](#), [BITMAP::sha256](#), and [BITMAP::size](#).

11.1.2.11 bitmap_get_bit()

```
int bitmap_get_bit (
    BITMAP * bitmap,
    size_t idx)
```

Read bit value at idx.

Parameters

| | |
|--------|-----------------------|
| bitmap | Bitmap to inspect. |
| idx | Zero-based bit index. |

Returns

1 if set, otherwise 0.

Read bit value at idx.

Parameters

| | |
|--------|--|
| bitmap | The BITMAP to read from. |
|--------|--|

| | |
|-----|-------------------------------|
| idx | The index of the bit to read. |
|-----|-------------------------------|

Returns

int 1 if the bit is set, 0 if unset, -1 on error.

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c](#).

Definition at line 137 of file [bitmap.c](#).

References [BITMAP::data](#).

Referenced by [iZ_next_prime\(\)](#), [process_iZ_bitmaps\(\)](#), [process_N_bitmap\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoA\(\)](#), [SoEu\(\)](#), [SoS\(\)](#), [SSoE\(\)](#), [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), [vx_prob_sieve\(\)](#), [vx_search_prime\(\)](#), and [vx_stream\(\)](#).

11.1.2.12 bitmap_init()

```
BITMAP * bitmap_init (
    size_t size,
    int set_bits)
```

Allocate a bitmap with size bits.

Parameters

| | |
|----------|---|
| size | Number of bits to allocate (must be > 0). |
| set_bits | Non-zero initializes all bits to 1, otherwise to 0. |

Returns

Newly allocated bitmap, or NULL on allocation failure.

Allocate a bitmap with size bits.

Allocates memory for both the [BITMAP](#) structure and its underlying data array. The data array is allocated using `calloc()`, ensuring all bits start at 0. The SHA-256 hash field is initialized to all zeros.

Parameters

| | |
|----------|--|
| size | The number of bits in the bitmap (must be > 0) |
| set_bits | If non-zero, initializes all bits to 1; otherwise, all bits are 0. |

Returns

Pointer to newly allocated [BITMAP](#) on success, NULL on failure

Return values

| | |
|------|--------------|
| NULL | if size is 0 |
|------|--------------|

| | |
|------|----------------------------|
| NULL | if memory allocation fails |
|------|----------------------------|

Note

Memory allocated: `sizeof(BITMAP) + size/8` bytes

On failure, any partially allocated memory is freed before returning

Warning

Caller must free returned bitmap using `bitmap_free()`, not `free()`

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 57 of file `bitmap.c`.

References `BITMAP::byte_size`, `BITMAP::data`, `log_error()`, `BITMAP::sha256`, and `BITMAP::size`.

Referenced by `bitmap_clone()`, `bitmap_fread()`, `get_root_primes()`, `iZm_init()`, `SiZ()`, `SiZm()`, `SiZm_vy()`, `SoA()`, `SoE()`, `SoEu()`, `SoS()`, `SSoE()`, and `vx_search_prime()`.

11.1.2.13 `bitmap_set_all()`

```
void bitmap_set_all (
    BITMAP * bitmap)
```

Set all bits to 1.

Parameters

| | |
|--------|-------------------|
| bitmap | Bitmap to modify. |
|--------|-------------------|

Set all bits to 1.

Parameters

| | |
|--------|------------------------------------|
| bitmap | The <code>BITMAP</code> to modify. |
|--------|------------------------------------|

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 170 of file `bitmap.c`.

References `BITMAP::data`, and `BITMAP::size`.

Referenced by `iZm_construct_vx_base()`, `SiZm_vy()`, and `SSoE()`.

11.1.2.14 `bitmap_set_bit()`

```
void bitmap_set_bit (
    BITMAP * bitmap,
    size_t idx)
```

Set bit at `idx` to 1.

Parameters

| | |
|--------|-------------------|
| bitmap | Bitmap to modify. |
|--------|-------------------|

| | |
|-----|-----------------------|
| idx | Zero-based bit index. |
|-----|-----------------------|

Set bit at idx to 1.

Parameters

| | |
|--------|---------------------------------------|
| bitmap | The BITMAP to modify. |
| idx | The index of the bit to set. |

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line [125](#) of file [bitmap.c](#).

References [BITMAP::data](#).

11.1.2.15 bitmap_validate_hash()

```
int bitmap_validate_hash (
    BITMAP * bitmap)
```

Verify [BITMAP::sha256](#) against current bitmap data.

Parameters

| | |
|--------|-------------------|
| bitmap | Bitmap to verify. |
|--------|-------------------|

Returns

1 if checksum matches, otherwise 0.

Verify [BITMAP::sha256](#) against current bitmap data.

Parameters

| | |
|--------|---|
| bitmap | The BITMAP whose hash is validated. |
|--------|---|

Returns

int 1 if the hash matches, 0 otherwise.

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line [412](#) of file [bitmap.c](#).

References [BITMAP::byte_size](#), [BITMAP::data](#), [log_error\(\)](#), and [BITMAP::sha256](#).

Referenced by [bitmap_fread\(\)](#).

11.1.2.16 TEST_BITMAP()

```
int TEST_BITMAP (  
    int verbose)
```

Run bitmap module tests.

Parameters

| | |
|---------|------------------------------------|
| verbose | Non-zero enables detailed logging. |
|---------|------------------------------------|

Returns

1 when all tests pass, otherwise 0.

11.2 Integer Arrays

Dynamic integer containers used by sieve and toolkit layers.

Files

- file [int_arrays.c](#)
Implementation of dynamic integer array module for 16-bit, 32-bit, and 64-bit unsigned integers.

Data Structures

- struct [UI16_ARRAY](#)
Dynamic array for uint16_t values.
- struct [UI32_ARRAY](#)
Dynamic array for uint32_t values.
- struct [UI64_ARRAY](#)
Dynamic array for uint64_t values.

Functions

- int [TEST_GENERIC_INT_ARRAYS](#) (int verbose)
Run generic dispatch tests for C11 _Generic helper macros.

UI16 API

- [UI16_ARRAY](#) * ui16_init (int capacity)
Allocate a UI16 array with an initial capacity.
- void ui16_free ([UI16_ARRAY](#) **array)
Free a UI16 array and null the caller pointer.
- void ui16_resize_to ([UI16_ARRAY](#) *array, int new_capacity)
Resize UI16 storage to new_capacity (must be >= count).
- void ui16_resize_to_fit ([UI16_ARRAY](#) *array)
Shrink UI16 storage so capacity equals count.
- void ui16_push ([UI16_ARRAY](#) *array, uint16_t element)
Append a uint16 value, growing storage if needed.
- void ui16_sort ([UI16_ARRAY](#) *array)
Sort values in ascending order.
- void ui16_pop ([UI16_ARRAY](#) *array)
Remove the last element if the array is non-empty.
- void ui16_compute_hash ([UI16_ARRAY](#) *array)
Compute SHA-256 checksum over active payload.
- int ui16_verify_hash ([UI16_ARRAY](#) *array)
Verify the stored checksum against current payload.
- int ui16_fwrite ([UI16_ARRAY](#) *array, FILE *file)
Serialize count, payload, and checksum to a binary stream.
- [UI16_ARRAY](#) * ui16_fread (FILE *file)
Deserialize a UI16 array from a binary stream.
- int TEST_UI16_ARRAY (int verbose)
Execute UI16 test suite.

UI32 API

- [UI32_ARRAY](#) * ui32_init (int capacity)
Allocate a UI32 array with an initial capacity.
- void ui32_free ([UI32_ARRAY](#) **array)
Free a UI32 array and null the caller pointer.
- void ui32_resize_to ([UI32_ARRAY](#) *array, int new_capacity)
Resize UI32 storage to new_capacity (must be >= count).
- void ui32_resize_to_fit ([UI32_ARRAY](#) *array)
Shrink UI32 storage so capacity equals count.
- void ui32_push ([UI32_ARRAY](#) *array, uint32_t element)
Append a uint32 value, growing storage if needed.
- void ui32_sort ([UI32_ARRAY](#) *array)
Sort values in ascending order.
- void ui32_pop ([UI32_ARRAY](#) *array)
Remove the last element if the array is non-empty.
- void ui32_compute_hash ([UI32_ARRAY](#) *array)
Compute SHA-256 checksum over active payload.
- int ui32_verify_hash ([UI32_ARRAY](#) *array)
Verify the stored checksum against current payload.
- int ui32_fwrite ([UI32_ARRAY](#) *array, FILE *file)
Serialize count, payload, and checksum to a binary stream.
- [UI32_ARRAY](#) * ui32_fread (FILE *file)
Deserialize a UI32 array from a binary stream.
- int TEST_UI32_ARRAY (int verbose)
Execute UI32 test suite.

UI64 API

- `UI64_ARRAY * ui64_init (int capacity)`
Allocate a UI64 array with an initial capacity.
- `void ui64_free (UI64_ARRAY **array)`
Free a UI64 array and null the caller pointer.
- `void ui64_resize_to (UI64_ARRAY *array, int new_capacity)`
Resize UI64 storage to new_capacity (must be >= count).
- `void ui64_resize_to_fit (UI64_ARRAY *array)`
Shrink UI64 storage so capacity equals count.
- `void ui64_push (UI64_ARRAY *array, uint64_t element)`
Append a uint64 value, growing storage if needed.
- `void ui64_sort (UI64_ARRAY *array)`
Sort values in ascending order.
- `void ui64_pop (UI64_ARRAY *array)`
Remove the last element if the array is non-empty.
- `void ui64_compute_hash (UI64_ARRAY *array)`
Compute SHA-256 checksum over active payload.
- `int ui64_verify_hash (UI64_ARRAY *array)`
Verify the stored checksum against current payload.
- `int ui64_fwrite (UI64_ARRAY *array, FILE *file)`
Serialize count, payload, and checksum to a binary stream.
- `UI64_ARRAY * ui64_fread (FILE *file)`
Deserialize a UI64 array from a binary stream.
- `int TEST_UI64_ARRAY (int verbose)`
Execute UI64 test suite.

11.2.1 Detailed Description

Dynamic integer containers used by sieve and toolkit layers.

11.2.2 Function Documentation

11.2.2.1 TEST_GENERIC_INT_ARRAYS()

```
int TEST_GENERIC_INT_ARRAYS (
    int verbose)
```

Run generic dispatch tests for C11 _Generic helper macros.

Parameters

| | |
|---------|------------------------------------|
| verbose | Non-zero enables detailed logging. |
|---------|------------------------------------|

Returns

1 when all tests pass, otherwise 0.

11.3 iZ Public API

< iZ/iZm toolkit structures and helpers.

Files

- file [iZ_apps.c](#)
High-level application routines built on top of the iZ_toolkit.
- file [prime_sieve.c](#)
Implementations of classical and SiZ-family sieve algorithms.

Data Structures

- struct [INPUT_SIEVE_RANGE](#)
Input parameters for range sieving/counting.

SiZ Range Variants

Count/stream primes over a numeric interval.

- typedef struct INPUT_SIEVE_RANGE [INPUT_SIEVE_RANGE](#)
Input parameters for range sieving/counting.
- uint64_t [SiZ_stream](#) ([INPUT_SIEVE_RANGE](#) *range)
Stream primes in a range to filepath (and return the count).
- uint64_t [SiZ_count](#) ([INPUT_SIEVE_RANGE](#) *input_range, int cores_num)
Count primes in a range using multiple worker processes.

Classic Prime Sieve Algorithms

Baseline sieves up to a numeric limit.

All classic sieve functions take a limit n and return an ascending list of primes $\leq n$.

- [UI64_ARRAY](#) * [SoE](#) (uint64_t n)
Optimized Sieve of Eratosthenes.
- [UI64_ARRAY](#) * [SSoE](#) (uint64_t n)
Segmented Sieve of Eratosthenes.
- [UI64_ARRAY](#) * [SoEu](#) (uint64_t n)
Euler (linear) sieve.
- [UI64_ARRAY](#) * [SoS](#) (uint64_t n)
Sieve of Sundaram.
- [UI64_ARRAY](#) * [SoA](#) (uint64_t n)
Sieve of Atkin.

iZ-based Sieve Algorithms

Sieve family operating in the $6x-1$ / $6x+1$ index space.

- [UI64_ARRAY](#) * [SiZ](#) (uint64_t n)
Solid Sieve-iZ (wheel 6, iZ index space).
- [UI64_ARRAY](#) * [SiZm](#) (uint64_t n)
Segmented Sieve-iZm (VX segmented, horizontal processing).
- [UI64_ARRAY](#) * [SiZm_vy](#) (uint64_t n)
Segmented Sieve-iZm (vertical processing; faster, unordered output).

Prime Generators

Probabilistic prime searches using iZm/VX machinery.

- int [vy_random_prime](#) (mpz_t p, int bit_size, int cores_num)
Search for a random prime using vertical (vy) sieving.
- int [vx_random_prime](#) (mpz_t p, int bit_size, int cores_num)
Search for a random prime using horizontal (vx) sieving.
- int [iZ_next_prime](#) (mpz_t p, mpz_t base, int forward)
Advance to the next (or previous) prime from a base value.

11.3.1 Detailed Description

< iZ/iZm toolkit structures and helpers.

< Common utilities, types, and dependencies.

High-level entry points for sieves and prime generation.

The API is organized into:

- Classic sieves: operate directly on the integer domain.
- SiZ / SiZm: operate in iZ index space ($6x-1$ and $6x+1$), optionally segmented.
- Range variants: count/stream primes over large intervals.
- Prime generators: probabilistic searches using the iZm/VX machinery.

11.3.2 Typedef Documentation

11.3.2.1 INPUT_SIEVE_RANGE

```
typedef struct INPUT_SIEVE_RANGE INPUT_SIEVE_RANGE
```

Input parameters for range sieving/counting.

The interval is interpreted as:

- Start Z_s from the decimal string start.
- End $Z_e = Z_s + \text{range} - 1$.

Forward declaration; concrete definition lives in [iZ_api.h](#).

11.3.3 Function Documentation

11.3.3.1 iZ_next_prime()

```
int iZ_next_prime (
    mpz_t p,
    mpz_t base,
    int forward)
```

Advance to the next (or previous) prime from a base value.

Find the next prime number after a given base.

Parameters

| | |
|---------|---|
| p | Output prime (initialized by caller). |
| base | Starting value. |
| forward | Non-zero to search forward, 0 to search backward. |

Returns

1 on success, 0 on failure.

Description: This function searches for the next/previous prime number after a given base using the iZ framework.

Parameters

| | |
|---------|--|
| p | The mpz_t variable to store the found prime number. |
| base | The base number to start the search from. |
| forward | If true, search for the next prime; if false, search for the previous prime. |

Returns

1 if a prime is found, 0 otherwise.

Definition at line 845 of file [iZ_apps.c](#).

References [IZM::base_x5](#), [IZM::base_x7](#), [bitmap_get_bit\(\)](#), [iZ_mpz\(\)](#), [iZm_free\(\)](#), [iZm_init\(\)](#), [log_debug\(\)](#), [log_error\(\)](#), [MR_ROUNDS](#), [VX5](#), and [VX6](#).

11.3.3.2 SiZ()

```
UI64_ARRAY * SiZ (
    uint64_t n)
```

Solid Sieve-iZ (wheel 6, iZ index space).

Classic Sieve-iZ algorithm for prime generation up to n.

Parameters

| | |
|---|--------------------------|
| n | Upper bound (inclusive). |
|---|--------------------------|

Returns

Heap-allocated prime list, or NULL on allocation failure.

Precondition

n is in $(10, 10^{12}]$.

The Sieve-iZ algorithm operates in the iZ index space of numbers of the form $6x \pm 1$, excluding all multiples of 2 and 3. It maintains two bitmaps, `x5` and `x7`, representing candidates of the form $6x - 1$ (iZ-) and $6x + 1$ (iZ+), respectively. For each index x whose bit remains set, the corresponding candidate is treated as prime and its composites are marked using the Xp identity in the iZ space.

This provides a wheel factorization of modulo 6, reducing the number of candidates by approximately a factor of 3 compared to a classical sieve over all integers. After sieving, any remaining set bits correspond to primes of the form $6x \pm 1$, which are mapped back to their integer values and appended to the output array alongside the primes 2 and 3.

Parameters

| | |
|----------------|---|
| <code>n</code> | Upper bound (inclusive) for prime generation. Must satisfy $10 < n \leq \text{pow}(10, 12)$. |
|----------------|---|

Returns

Pointer to a [UI64_ARRAY](#) containing all primes $\leq n$ on success, or NULL if allocation fails.

Definition at line 444 of file [prime_sieve.c](#).

References [UI64_ARRAY::array](#), [ASSERT_LIMIT](#), [bitmap_free\(\)](#), [bitmap_init\(\)](#), [UI64_ARRAY::count](#), [Pi](#), [process_iZ_bitmaps\(\)](#), [ui64_free\(\)](#), [ui64_init\(\)](#), [ui64_pop\(\)](#), [ui64_push\(\)](#), and [ui64_resize_to_fit\(\)](#).

Referenced by [compute_max_vx\(\)](#), [iZm_init\(\)](#), [SiZm\(\)](#), and [SiZm_vy\(\)](#).

11.3.3.3 SiZ_count()

```
uint64_t SiZ_count (
    INPUT_SIEVE_RANGE * input_range,
    int cores_num)
```

Count primes in a range using multiple worker processes.

Multi-process prime counting over an arbitrary numeric range using iZ toolkit.

Parameters

| | |
|--------------------------|--|
| <code>input_range</code> | Range configuration (output disabled). |
| <code>cores_num</code> | Requested worker process count. |

Returns

Prime count in the interval, or 0 on error.

This function parallelizes prime counting over the interval $[Zs, Ze]$ (interpreted from `input_range`) by partitioning the corresponding `iZ` index space into `VX` segments and distributing contiguous blocks of segments across multiple processes. Each child process returns a local prime count to the parent via a pipe.

The parent process aggregates all child counts, applies boundary corrections for endpoints that do not align exactly with $6x \pm 1$, and returns the total number of primes in $[Zs, Ze]$. Any failure to create pipes, fork children, or initialize per-process resources causes the function to log an error and return 0.

Parameters

| | |
|--------------------------|---|
| <code>input_range</code> | Pointer to an INPUT_SIEVE_RANGE structure describing the numeric interval and Miller–Rabin configuration. |
|--------------------------|---|

| | |
|-----------|--|
| cores_num | Requested number of worker processes. The actual number used is clamped to the available CPU cores and the number of VX segments in the range. |
|-----------|--|

Returns

The total number of primes found in $[Zs, Ze]$ on success, or 0 on any error or child-process failure.

Definition at line 184 of file `iZ_apps.c`.

References `UI64_ARRAY::array`, `compute_l2_vx()`, `UI64_ARRAY::count`, `get_cpu_cores_count()`, `iZ_mpz()`, `iZm_clone()`, `iZm_free()`, `iZm_init()`, `log_error()`, `log_info()`, `MAX`, `MIN`, `INPUT_SIEVE_RANGE::mr_rou`, `VX_SEG::p_count`, `INPUT_SIEVE_RANGE::range`, `range_info_free()`, `range_info_init()`, `SiZm()`, `INPUT_SIEVE_RANGE::start`, `ui64_free()`, `vx_free()`, `vx_full_sieve()`, `vx_init()`, `IZM_RANGE_INFO::Xe`, `IZM_RANGE_INFO::Xs`, `IZM_RANGE_INFO::y_range`, `IZM_RANGE_INFO::Ye`, `IZM_RANGE_INFO::Ys`, `IZM_RANGE_INFO::Ze`, and `IZM_RANGE_INFO::Zs`.

11.3.3.4 SiZ_stream()

```
uint64_t SiZ_stream (
    INPUT_SIEVE_RANGE * input_range)
```

Stream primes in a range to filepath (and return the count).

Stream primes in an arbitrary numeric range using iZ toolkit.

Parameters

| | |
|-------|--|
| range | Range configuration (must include filepath). |
|-------|--|

Returns

Prime count in the interval, or 0 on error.

This function counts and streams primes in the interval $[Zs, Ze]$ defined by the input range. It maps the numeric bounds into the iZ index space ($6x \pm 1$), partitions that space into y segments of length vx , and streams primes in each segment using the iZ toolkit.

When an output file path is provided in `input_range->filepath`, the function additionally streams all primes in the requested interval to that file in ascending order, reconstructing them from per-segment prime gaps. Otherwise, it operates in counting mode only.

Parameters

| | |
|-------------|--|
| input_range | Pointer to an <code>INPUT_SIEVE_RANGE</code> structure describing the start value (as a decimal string), the range length, the optional output filepath, and Miller–Rabin configuration. |
|-------------|--|

Returns

The total number of primes found in $[Zs, Ze]$ on success, or 0 on invalid input, allocation failure, or I/O error.

Definition at line 35 of file [iZ_apps.c](#).

References [UI64_ARRAY::array](#), [UI64_ARRAY::count](#), [INPUT_SIEVE_RANGE::filepath](#), [iZm_free\(\)](#), [iZm_init\(\)](#), [log_error\(\)](#), [MAX](#), [MIN](#), [INPUT_SIEVE_RANGE::mr_rounds](#), [VX_SEG::p_count](#), [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZm\(\)](#), [INPUT_SIEVE_RANGE::start](#), [ui64_free\(\)](#), [IZM_RANGE_INFO::vx](#), [VX6](#), [vx_free\(\)](#), [vx_init\(\)](#), [vx_stream\(\)](#), [IZM_RANGE_INFO::Xe](#), [IZM_RANGE_INFO::Xs](#), [IZM_RANGE_INFO::y_range](#), [IZM_RANGE_INFO::Ye](#), [IZM_RANGE_INFO::Ys](#), [IZM_RANGE_INFO::Ze](#), and [IZM_RANGE_INFO::Zs](#).

11.3.3.5 SiZm()

```
UI64_ARRAY * SiZm (
    uint64_t n)
```

Segmented Sieve-iZm (VX segmented, horizontal processing).

Segmented Sieve-iZm algorithm for prime generation up to n .

Parameters

| | |
|---|--------------------------|
| n | Upper bound (inclusive). |
|---|--------------------------|

Returns

Heap-allocated prime list, or NULL on allocation failure.

Precondition

n is in $(10, 10^{12}]$.

Description: The Sieve-iZm algorithm is a segmented, cache-aware variant of the Sieve-iZ that operates in the iZ index space of numbers of the form $6x \pm 1$. It partitions the search interval into segments of length vx , where vx is a product of small primes, and constructs a pre-sieved base segment using those small primes once at initialization time.

For each segment, Sieve-iZm clones the base bitmaps and only marks composites of the remaining root primes that do not divide vx . This amortizes the cost of small-prime sieving across all segments and keeps the working set small and highly cache-resident. After marking, any remaining set bits correspond to primes of the form $6x \pm 1$ within that segment, which are then mapped back to their integer values and appended to the output array.

Compared to the basic Sieve-iZ, Sieve-iZm achieves the same $O(n \log \log n)$ time complexity with effectively constant auxiliary memory (aside from the output) and significantly reduced constant factors, making it well-suited for enumerating primes up to large bounds.

Parameters

| | |
|---|---|
| n | Upper bound (inclusive) for prime generation. Must satisfy $10 < n \leq \text{pow}(10, 12)$. |
|---|---|

Returns

Pointer to a [UI64_ARRAY](#) containing all primes $\leq n$ on success, or NULL on allocation or initialization failure.

Definition at line 516 of file [prime_sieve.c](#).

References [UI64_ARRAY::array](#), [ASSERT_LIMIT](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_clone\(\)](#), [bitmap_free\(\)](#), [bitmap_get_bit\(\)](#), [bitmap_init\(\)](#), [BITMAP::byte_size](#), [compute_l2_vx\(\)](#), [UI64_ARRAY::count](#), [BITMAP::data](#), [iZ\(\)](#), [iZm_construct_vx_base\(\)](#), [iZm_solve_for_x0\(\)](#), [Pi](#), [process_iZ_bitmaps\(\)](#), [SiZ\(\)](#), [ui64_free\(\)](#), [ui64_init\(\)](#), [ui64_pop\(\)](#), [ui64_push\(\)](#), and [ui64_resize_to_fit\(\)](#).

Referenced by [SiZ_count\(\)](#), [SiZ_stream\(\)](#), and [vx_search_prime\(\)](#).

11.3.3.6 SiZm_vy()

```
UI64_ARRAY * SiZm_vy (
    uint64_t n)
```

Segmented Sieve-iZm (vertical processing; faster, unordered output).

Segmented Sieve-iZm with vertical (vy) traversal order.

This variant emphasizes throughput and may not preserve ascending order of returned primes.

Parameters

| | |
|---|--------------------------|
| n | Upper bound (inclusive). |
|---|--------------------------|

Returns

Heap-allocated prime list, or NULL on allocation failure.

Precondition

n is in $(10, 10^{12}]$.

This variant prioritizes throughput and returns primes in non-sorted order.

Parameters

| | |
|---|---|
| n | Inclusive upper bound for prime generation. |
|---|---|

Returns

Heap-allocated list of primes $\leq n$, or NULL on failure.

Definition at line 623 of file [prime_sieve.c](#).

References [UI64_ARRAY::array](#), [ASSERT_LIMIT](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_free\(\)](#), [bitmap_get_bit\(\)](#), [bitmap_init\(\)](#), [bitmap_set_all\(\)](#), [UI64_ARRAY::count](#), [gcd\(\)](#), [get_root_primes\(\)](#), [iZ\(\)](#), [iZm_solve_for_y0\(\)](#), [UI64_ARRAY::ordered](#), [Pi](#), [SiZ\(\)](#), [ui64_init\(\)](#), [ui64_push\(\)](#), and [ui64_resize_to_fit\(\)](#).

11.3.3.7 SoA()

[UI64_ARRAY](#) * SoA (
 uint64_t n)

Sieve of Atkin.

Sieve of Atkin: Generates a list of prime numbers up to a given limit using the Sieve of Atkin algorithm.

Parameters

| | |
|---|--------------------------|
| n | Upper bound (inclusive). |
|---|--------------------------|

Returns

Heap-allocated prime list, or NULL on allocation failure.

Precondition

n is in (10, 10¹²].

Description: This function implements the Sieve of Atkin, a modern algorithm to find all prime numbers up to a specified integer n. It initializes a bitmap to mark potential primes and applies the Atkin conditions to identify primes. The function also marks odd multiples of squares of primes as non-prime, then collects the remaining unmarked numbers as primes.

Parameters

| | |
|---|--|
| n | The upper limit (inclusive) up to which prime numbers are to be found. |
|---|--|

Returns

- [UI64_ARRAY](#)* A pointer to the [UI64_ARRAY](#) structure containing the list of primes up to n.
- NULL if memory allocation fails or if n is less than 10.

Definition at line 328 of file [prime_sieve.c](#).

References [ASSERT_LIMIT](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_flip_bit\(\)](#), [bitmap_free\(\)](#), [bitmap_get_bit\(\)](#), [bitmap_init\(\)](#), [Pi](#), [ui64_free\(\)](#), [ui64_init\(\)](#), [ui64_push\(\)](#), and [ui64_resize_to_fit\(\)](#).

11.3.3.8 SoE()

[UI64_ARRAY](#) * SoE (
 uint64_t n)

Optimized Sieve of Eratosthenes.

Optimized Sieve of Eratosthenes algorithm to find all primes up to n.

Parameters

| | |
|---|--------------------------|
| n | Upper bound (inclusive). |
|---|--------------------------|

Returns

Heap-allocated prime list, or NULL on allocation failure.

Precondition

n is in $(10, 10^{12}]$.

Description: This function applies common speedups to the Sieve of Eratosthenes algorithm. It works the same way as the classic sieve but skips checking or marking even numbers and starts with 3, incrementing by 2, it also starts the sieve from $p*p$.

Parameters

| | |
|----------------|---------------------------------|
| <code>n</code> | The upper limit to find primes. |
|----------------|---------------------------------|

Returns

- A pointer to a [UI64_ARRAY](#) structure containing the list of prime numbers up to n ,
- NULL if memory allocation fails or if n is less than 10.

Definition at line 72 of file [prime_sieve.c](#).

References [ASSERT_LIMIT](#), [bitmap_free\(\)](#), [bitmap_init\(\)](#), [Pi](#), [process_N_bitmap\(\)](#), [ui64_free\(\)](#), [ui64_init\(\)](#), and [ui64_resize_to_fit\(\)](#).

11.3.3.9 SoEu()

```
UI64_ARRAY * SoEu (
    uint64_t n)
```

Euler (linear) sieve.

Sieve of Euler: Generates a list of prime numbers up to a given limit using the Euler Sieve algorithm.

Parameters

| | |
|----------------|--------------------------|
| <code>n</code> | Upper bound (inclusive). |
|----------------|--------------------------|

Returns

Heap-allocated prime list, or NULL on allocation failure.

Precondition

n is in $(10, 10^{12}]$.

Description: This function uses the Euler Sieve algorithm to find all prime numbers up to a specified limit n . It marks each composite only once, allowing for a more efficient sieve process. It initializes a bitmap to track prime numbers and iterates through the numbers, marking distinct multiples of each prime found. The function also handles memory allocation and resizing of the primes array as needed.

Parameters

| | |
|-----|--|
| n | The upper limit up to which prime numbers are to be found. |
|-----|--|

Returns

- `UI64_ARRAY*` A pointer to the `UI64_ARRAY` structure containing the list of primes up to n .
- `NULL` if memory allocation fails or if n is less than 10.

Definition at line 200 of file `prime_sieve.c`.

References `UI64_ARRAY::array`, `ASSERT_LIMIT`, `bitmap_clear_bit()`, `bitmap_free()`, `bitmap_get_bit()`, `bitmap_init()`, `UI64_ARRAY::count`, `Pi`, `ui64_free()`, `ui64_init()`, `ui64_push()`, and `ui64_resize_to_fit()`.

11.3.3.10 SoS()

`UI64_ARRAY * SoS (`
`uint64_t n)`

Sieve of Sundaram.

Sieve of Sundaram: Generates a list of prime numbers up to a given limit using the Sieve of Sundaram algorithm.

Parameters

| | |
|-----|--------------------------|
| n | Upper bound (inclusive). |
|-----|--------------------------|

Returns

Heap-allocated prime list, or `NULL` on allocation failure.

Precondition

n is in $(10, 10^{12}]$.

Description: This function implements the Sieve of Sundaram algorithm to find all prime numbers up to a specified limit n . The Sieve of Sundaram works by eliminating numbers of the form $i + j + 2ij$ from a list of integers from 1 to $k = (n-1)/2$. If an integer x in this range is not eliminated, then $2x+1$ is a prime number. The prime number 2 is handled as a special case, as the sieve only generates odd primes.

Parameters

| | |
|-----|--|
| n | The upper limit (inclusive) up to which prime numbers are to be found. |
|-----|--|

Returns

- `UI64_ARRAY*` A pointer to the `UI64_ARRAY` structure containing the list of primes up to `n`.
- `NULL` if memory allocation fails or if `n` is less than 10.

Definition at line 264 of file `prime_sieve.c`.

References `ASSERT_LIMIT`, `bitmap_clear_steps_simd()`, `bitmap_free()`, `bitmap_get_bit()`, `bitmap_init()`, `Pi`, `ui64_free()`, `ui64_init()`, `ui64_push()`, and `ui64_resize_to_fit()`.

11.3.3.11 SSoE()

`UI64_ARRAY *` SSoE (
`uint64_t n`)

Segmented Sieve of Eratosthenes.

Segmented Sieve of Eratosthenes algorithm to find all primes up to `n`.

Parameters

| | |
|----------------|--------------------------|
| <code>n</code> | Upper bound (inclusive). |
|----------------|--------------------------|

Returns

Heap-allocated prime list, or `NULL` on allocation failure.

Precondition

`n` is in $(10, 10^{12}]$.

Description: This function implements the Segmented Sieve of Eratosthenes algorithm to find all prime numbers up to a given limit `n`. It divides the range into segments and uses a bitmap to mark prime numbers in each segment. The function returns a pointer to a `UI64_ARRAY` structure containing the list of prime numbers found. The function also handles memory allocation and resizing of the primes array as needed.

Parameters

| | |
|----------------|---------------------------------|
| <code>n</code> | The upper limit to find primes. |
|----------------|---------------------------------|

Returns

- A pointer to the `UI64_ARRAY` structure containing the list of primes up to `n`,
- `NULL` if memory allocation fails or if `n` is less than 1000.

Definition at line 114 of file `prime_sieve.c`.

References `UI64_ARRAY::array`, `ASSERT_LIMIT`, `bitmap_clear_steps_simd()`, `bitmap_free()`, `bitmap_get_bit()`, `bitmap_init()`, `bitmap_set_all()`, `UI64_ARRAY::count`, `MAX`, `Pi`, `process_N_bitmap()`, `ui64_free()`, `ui64_init()`, `ui64_push()`, and `ui64_resize_to_fit()`.

11.3.3.12 vx_random_prime()

```
int vx_random_prime (
    mpz_t p,
    int bit_size,
    int cores_num)
```

Search for a random prime using horizontal (vx) sieving.

Generates a random prime candidate using the vx_search_prime routine.

Parameters

| | |
|-----------|---------------------------------------|
| p | Output prime (initialized by caller). |
| bit_size | Target size of p in bits. |
| cores_num | Requested worker processes. |

Returns

1 on success, 0 on failure.

Description: This function generates a random prime of a given bit size using the vx_search_prime routine. It initializes the random base and sets up the search parameters. The function also allows for parallel processing of the search using multiple child processes. The generated prime is stored in the provided mpz_t p variable.

Parameters

| | |
|-----------|---|
| p | The mpz_t variable to store the generated prime number. |
| bit_size | The target bit size of the prime. |
| cores_num | The number of cores to use for parallel processing. |

Returns

1 if a prime is found, 0 otherwise.

Definition at line 712 of file iZ_apps.c.

References [log_error\(\)](#), [log_info\(\)](#), [MAX](#), [VX5](#), [VX6](#), and [vx_search_prime\(\)](#).

11.3.3.13 vy_random_prime()

```
int vy_random_prime (
    mpz_t p,
    int bit_size,
    int cores_num)
```

Search for a random prime using vertical (vy) sieving.

Generates a random prime candidate using the vy_search_prime routine.

Parameters

| | |
|---|---------------------------------------|
| p | Output prime (initialized by caller). |
|---|---------------------------------------|

| | |
|-----------|-----------------------------|
| bit_size | Target size of p in bits. |
| cores_num | Requested worker processes. |

Returns

1 on success, 0 on failure.

Description: This function generates a random prime of a given bit size using the `vy_search_prime` routine. It initializes the random base and sets up the search parameters. The function also allows for parallel processing of the search using multiple child processes. The generated prime is stored in the provided `mpz_t p` variable.

Parameters

| | |
|-----------|--|
| p | The <code>mpz_t</code> variable to store the generated prime number. |
| bit_size | The target bit size of the prime. |
| cores_num | The number of cores to use for parallel processing. |

Returns

1 if a prime is found, 0 otherwise.

Definition at line 558 of file `iZ_apps.c`.

References `compute_max_vx()`, `log_error()`, `log_info()`, `MAX`, and `vy_search_prime()`.

11.4 Toolkit (iZ/iZm)

Internal building blocks for SiZ and SiZm implementations.

Files

- file `iZ_toolkit.c`
Implementation of iZ index space helpers and iZm/VX segment machinery.

Data Structures

- struct `IZM`
Precomputed iZm assets for repeated VX-segment sieving.
- struct `VX_SEG`
Runtime state for one VX segment at a specific y.
- struct `IZM_RANGE_INFO`
Precomputed iZm coordinates for an inclusive numeric interval.

Macros

- `#define MR_ROUNDS 25`

Functions

- `void iZm_construct_vx_base (uint64_t vx, BITMAP *base_x5, BITMAP *base_x7)`
Build pre-sieved base bitmaps for a VX segment.
- `IZM_RANGE_INFO range_info_init (INPUT_SIEVE_RANGE *input_range, int vx)`
Map a decimal range input into iZm coordinates and segment bounds.
- `void range_info_free (IZM_RANGE_INFO *info)`
Clear all GMP fields owned by info.

iZ Mapping Helpers

- `uint64_t iZ (uint64_t x, int i)`
Map iZ coordinates to an integer: $6*x + i$.
- `void iZ_mpz (mpz_t z, mpz_t x, int i)`
GMP variant of `iZ()`.
- `void process_iZ_bitmaps (UI64_ARRAY *primes, BITMAP *x5, BITMAP *x7, uint64_t x_limit)`
Traverse iZ bitmaps, emit surviving primes, and mark composites.
- `void get_root_primes (UI64_ARRAY *primes, uint64_t limit)`
Generate primes up to limit for deterministic sieving.
- `int check_primality (mpz_t n, int rounds)`
Check the primality of a number using GMP's probabilistic test.

IZM Lifecycle

- `IZM * iZm_init (size_t vx)`
Allocate and initialize an `IZM` object for a given VX.
- `IZM * iZm_clone (IZM *src)`
Deep-copy an `IZM` object for per-worker ownership.
- `void iZm_free (IZM **iZm)`
Release an `IZM` object and set the caller pointer to NULL.

VX Selection Helpers

- `uint64_t compute_vx_k (int k)`
Calculate `VX_{ k }`.
- `uint64_t compute_l2_vx (uint64_t n)`
Choose VX using an L2-cache-aware heuristic.
- `void compute_max_vx (mpz_t vx, int bit_size)`
Compute largest VX below $2^{\text{bit_size}}$.

Modular Hit Solvers

- `uint64_t iZm_solve_for_x0` (int m_id, uint64_t p, uint64_t vx, uint64_t y)
Solve first x-hit of prime p for line m_id in segment y.
- `uint64_t iZm_solve_for_x0_mpz` (int m_id, uint64_t p, uint64_t vx, mpz_t y)
GMP variant of `iZm_solve_for_x0()` for very large y.
- `int64_t iZm_solve_for_y0` (int m_id, uint64_t p, uint64_t vx, uint64_t x)
Solve first y-hit for fixed x in vertical (vy) scanning.

VX Segment Lifecycle and Execution

- `VX_SEG * vx_init` (IZM *iZm, int start_x, int end_x, char *y_str, int mr_rounds)
Initialize and deterministically sieve one VX segment.
- `void vx_free` (VX_SEG **vx_obj)
Free a VX segment and all owned resources.
- `void vx_collect_p_gaps` (VX_SEG *vx_obj)
Extract prime-gap encoding from a fully sieved segment.
- `void vx_full_sieve` (VX_SEG *vx_obj, int collect_p_gaps)
Complete segment processing (probabilistic stage and optional gaps).
- `void vx_stream` (VX_SEG *vx_obj, FILE *output)
Stream segment primes to an output stream.

Random Prime Search Routines

- `int vx_search_prime` (mpz_t p, int m_id, int vx, int bit_size)
Horizontal iZm/VX random-prime search.
- `int vy_search_prime` (mpz_t p, int m_id, mpz_t vx)
Vertical iZm/VY random-prime search.

Toolkit Tests

- `int TEST_IZM` (int verbose)
Run IZM construction and solver tests.
- `int TEST_VX_SEG` (int verbose)
Run VX segment tests.

Standard VX Sizes (primorial products excluding 2,3)

- `#define VX2` (5 * 7ULL)
- `#define VX3` (VX2 * 11ULL)
- `#define VX4` (VX3 * 13ULL)
- `#define VX5` (VX4 * 17ULL)
- `#define VX6` (VX5 * 19ULL)
- `#define VX7` (VX6 * 23ULL)
- `#define VX8` (VX7 * 29ULL)

11.4.1 Detailed Description

Internal building blocks for SiZ and SiZm implementations.

11.4.2 Macro Definition Documentation

11.4.2.1 MR_ROUNDS

```
#define MR_ROUNDS 25
```

Default Miller-Rabin rounds used by toolkit search/sieve helpers.

Definition at line 79 of file [iZ_toolkit.h](#).

Referenced by [iZ_next_prime\(\)](#), [vx_init\(\)](#), [vx_search_prime\(\)](#), and [vy_search_prime\(\)](#).

11.4.2.2 VX2

```
#define VX2 (5 * 7ULL)
```

35

Definition at line 69 of file [iZ_toolkit.h](#).

11.4.2.3 VX3

```
#define VX3 (VX2 * 11ULL)
```

385

Definition at line 70 of file [iZ_toolkit.h](#).

11.4.2.4 VX4

```
#define VX4 (VX3 * 13ULL)
```

5005

Definition at line 71 of file [iZ_toolkit.h](#).

11.4.2.5 VX5

```
#define VX5 (VX4 * 17ULL)
```

85085

Definition at line 72 of file [iZ_toolkit.h](#).

Referenced by [iZ_next_prime\(\)](#), and [vx_random_prime\(\)](#).

11.4.2.6 VX6

```
#define VX6 (VX5 * 19ULL)
```

1616615

Definition at line 73 of file [iZ_toolkit.h](#).

Referenced by [iZ_next_prime\(\)](#), [SiZ_stream\(\)](#), and [vx_random_prime\(\)](#).

11.4.2.7 VX7

```
#define VX7 (VX6 * 23ULL)
```

37260615

Definition at line 74 of file [iZ_toolkit.h](#).

11.4.2.8 VX8

```
#define VX8 (VX7 * 29ULL)
```

1080558835

Definition at line 75 of file [iZ_toolkit.h](#).

11.4.3 Function Documentation

11.4.3.1 [check_primality\(\)](#)

```
int check_primality (
    mpz_t n,
    int rounds)
```

Check the primality of a number using GMP's probabilistic test.

Parameters

| | |
|--------|--------------------------------|
| n | Number to check. |
| rounds | Number of Miller-Rabin rounds. |

Returns

Non-zero if probably prime, 0 if composite.

Parameters

| | |
|---|------------------|
| n | Number to check. |
|---|------------------|

| | |
|--------|--------------------------------|
| rounds | Number of Miller-Rabin rounds. |
|--------|--------------------------------|

Returns

Non-zero if probably prime, 0 if composite.

This function serves as a single source of truth for primality testing, wrapping GMP's `mpz_probab_prime_p` function, allowing for changes to the underlying primality testing method in the future without affecting the API.

Definition at line 153 of file `iZ_toolkit.c`.

Referenced by `vx_prob_sieve()`, `vx_search_prime()`, `vx_stream()`, and `vy_search_prime()`.

11.4.3.2 `compute_l2_vx()`

```
uint64_t compute_l2_vx (
    uint64_t n)
```

Choose VX using an L2-cache-aware heuristic.

Choose a cache-aware VX size based on detected L2 capacity.

Parameters

| | |
|---|-----------------------|
| n | Target numeric limit. |
|---|-----------------------|

Returns

Selected VX size.

Parameters

| | |
|---|-----------------------------|
| n | Target numeric sieve bound. |
|---|-----------------------------|

Returns

L2 compatible VX size.

Definition at line 306 of file `iZ_toolkit.c`.

References `get_cpu_L2_cache_size_bits()`, `MIN`, and `s_primes`.

Referenced by `SiZ_count()`, and `SiZm()`.

11.4.3.3 `compute_max_vx()`

```
void compute_max_vx (
    mpz_t vx,
    int bit_size)
```

Compute largest VX below $2^{\text{bit_size}}$.

Compute the maximum vx such that $vx < 2^{\text{bit_size}}$.

Parameters

| | |
|----|-----------------------------|
| vx | Output mpz_t containing VX. |
|----|-----------------------------|

| | |
|----------|-------------------|
| bit_size | Bit-size ceiling. |
|----------|-------------------|

Parameters:

Parameters

| | |
|----------|--|
| vx | mpz_t variable to store the computed maximum vx value. |
| bit_size | Integer representing the bit size limit for vx. |

Definition at line 331 of file [iZ_toolkit.c](#).

References [UI64_ARRAY::array](#), and [SiZ\(\)](#).

Referenced by [vy_random_prime\(\)](#).

11.4.3.4 compute_vx_k()

```
uint64_t compute_vx_k (
    int k)
```

Calculate $VX_{\{k\}}$.

Calculate a $VX_{\{k\}}$ as the product of the first k primes > 3 .

Parameters

| | |
|---|---|
| k | Number of multiplicative prime factors (> 3). |
|---|---|

Returns

Selected VX size.

Parameters

| | |
|---|---|
| k | number of consecutive primes (> 3) to include in the product. |
|---|---|

Returns

Computed VX_k value.

Definition at line 286 of file [iZ_toolkit.c](#).

References [s_primes](#).

11.4.3.5 get_root_primes()

```
void get_root_primes (
    UI64_ARRAY * primes,
    uint64_t limit)
```

Generate primes up to limit for deterministic sieving.

Generate root primes up to limit using iZ bitmap traversal.

Parameters

| | |
|--------|--|
| primes | Output array, appended in ascending order. |
|--------|--|

| | |
|--------|----------------------|
| limit | Numeric upper bound. |
| primes | Destination array. |
| limit | Numeric upper bound. |

Definition at line 112 of file [iZ_toolkit.c](#).

References [bitmap_free\(\)](#), [bitmap_init\(\)](#), [process_iZ_bitmaps\(\)](#), [ui64_free\(\)](#), and [ui64_push\(\)](#).

Referenced by [SiZm_vy\(\)](#).

11.4.3.6 iZ()

```
uint64_t iZ (
    uint64_t x,
    int i)    [inline]
```

Map iZ coordinates to an integer: $6*x + i$.

Parameters

| | |
|---|--|
| x | iZ x-coordinate. |
| i | Matrix identifier, typically -1 or +1. |

Returns

Mapped integer value.

Map iZ coordinates to an integer: $6*x + i$.

Parameters:

Parameters

| | |
|---|---|
| x | (uint64_t) The value of x in $6x + i$. |
| i | (int) The value of i in $6x + i$. |

Returns

The computed value $6x + i$ as a 64-bit unsigned integer.

Definition at line 24 of file [iZ_toolkit.c](#).

Referenced by [process_iZ_bitmaps\(\)](#), [SiZm\(\)](#), and [SiZm_vy\(\)](#).

11.4.3.7 iZ_mpz()

```
void iZ_mpz (
    mpz_t z,
    mpz_t x,
    int i)
```

GMP variant of [iZ\(\)](#).

Parameters

| | |
|---|---------------|
| z | Output value. |
|---|---------------|

| | |
|---|--|
| x | Input x-coordinate. |
| i | Matrix identifier, typically -1 or +1. |

GMP variant of [iZ\(\)](#).

Parameters:

Parameters

| | |
|---|--|
| z | (mpz_t) The result of the calculation $6x + i$. |
| x | (mpz_t) The input value of x in $6x + i$. |
| i | (int) The value of i in $6x + i$. |

Definition at line 37 of file [iZ_toolkit.c](#).

Referenced by [iZ_next_prime\(\)](#), [SiZ_count\(\)](#), [vx_prob_sieve\(\)](#), [vx_search_prime\(\)](#), [vx_set_base_values\(\)](#), [vx_stream\(\)](#), and [vy_search_prime\(\)](#).

11.4.3.8 iZm_clone()

```
IZM * iZm_clone (
    IZM * src)
```

Deep-copy an [IZM](#) object for per-worker ownership.

Deep-copy an [IZM](#) object for independent worker usage.

Parameters

| | |
|-----|--------------------------------------|
| src | Source IZM instance. |
|-----|--------------------------------------|

Returns

Newly allocated clone, or NULL on failure.

Parameters

| | |
|-----|------------------------------------|
| src | Source IZM object. |
|-----|------------------------------------|

Returns

Cloned [IZM](#) object, or NULL on failure.

Definition at line 232 of file [iZ_toolkit.c](#).

References [UI64_ARRAY::array](#), [IZM::base_x5](#), [IZM::base_x7](#), [bitmap_clone\(\)](#), [UI64_ARRAY::capacity](#), [UI64_ARRAY::count](#), [iZm_free\(\)](#), [IZM::k_vx](#), [log_error\(\)](#), [IZM::root_primes](#), [ui64_init\(\)](#), and [IZM::vx](#).

Referenced by [SiZ_count\(\)](#).

11.4.3.9 `iZm_construct_vx_base()`

```
void iZm_construct_vx_base (
    uint64_t vx,
    BITMAP * base_x5,
    BITMAP * base_x7)
```

Build pre-sieved base bitmaps for a VX segment.

Constructs a pre-sieved iZm base segment of size vx.

Parameters

| | |
|---------|------------------------------------|
| vx | Segment width. |
| base_x5 | Output bitmap for 6x-1 candidates. |
| base_x7 | Output bitmap for 6x+1 candidates. |

Description: This function constructs a pre-sieved iZm base segment of size vx. It marks all composites of small primes that divide vx in the bitmaps base_x5 (iZ-) and base_x7 (iZ+). This pre-sieved base can then be used as a template for sieving all vx segments.

Parameters:

Parameters

| | |
|-----|---|
| iZm | Pointer to the IZM structure containing the base segment bitmaps. |
|-----|---|

Definition at line 363 of file [iZ_toolkit.c](#).

References [bitmap_clear_bit\(\)](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_set_all\(\)](#), [s_primes](#), and [s_primes_count](#).

Referenced by [iZm_init\(\)](#), and [SiZm\(\)](#).

11.4.3.10 `iZm_free()`

```
void iZm_free (
    IZM ** iZm)
```

Release an [IZM](#) object and set the caller pointer to NULL.

Free the memory allocated for an [IZM](#) structure.

Parameters

| | |
|-----|--|
| iZm | Address of an IZM pointer. |
|-----|--|

Parameters:

Parameters

| | |
|-----|---|
| iZm | A pointer to the IZM structure to be freed. |
|-----|---|

Definition at line 266 of file [iZ_toolkit.c](#).

References [bitmap_free\(\)](#), and [ui64_free\(\)](#).

Referenced by [iZ_next_prime\(\)](#), [iZm_clone\(\)](#), [iZm_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

11.4.3.11 iZm_init()

```
IZM * iZm_init (
    size_t vx)
```

Allocate and initialize an [IZM](#) object for a given VX.

Initialize an iZm structure for a given vx size.

Parameters

| | |
|----|---|
| vx | Segment width; must be odd, not divisible by 3, and ≥ 35 . |
|----|---|

Returns

Initialized [IZM](#) object, or NULL on failure.

This function initializes an iZm structure for a specified vx size. It allocates memory for the structure, generates root primes for deterministic sieving, initializes base bitmaps, and constructs pre-sieved base segments for iZm5 and iZm7.

Parameters:

Parameters

| | |
|----|--|
| vx | The size of the segment for the iZm structure. |
|----|--|

Returns

A pointer to the initialized [IZM](#) structure.

Definition at line 184 of file [iZ_toolkit.c](#).

References [IZM::base_x5](#), [IZM::base_x7](#), [bitmap_init\(\)](#), [compute_k_vx\(\)](#), [iZm_construct_vx_base\(\)](#), [iZm_free\(\)](#), [IZM::k_vx](#), [log_error\(\)](#), [IZM::root_primes](#), [SiZ\(\)](#), [ui64_free\(\)](#), and [IZM::vx](#).

Referenced by [iZ_next_prime\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

11.4.3.12 iZm_solve_for_x0()

```
uint64_t iZm_solve_for_x0 (
    int m_id,
    uint64_t p,
    uint64_t vx,
    uint64_t y)
```

Solve first x-hit of prime p for line m_id in segment y.

Compute the first composite hit of p in the yth vx segment in iZm index space.

Parameters

| | |
|------|---------------------------------|
| m_id | Line id (-1 for x5, +1 for x7). |
|------|---------------------------------|

| | |
|----|-------------------------|
| p | Prime used for marking. |
| vx | Segment width. |
| y | Segment index. |

Returns

First x index to clear for this prime/line.

Description: This function solves for the smallest x that satisfies: $(x + vx * y) \equiv x_p \pmod{p}$, where x_p is normalized based on the `m_id` and `p` to either x_p or $p - x_p$, then computes x and returns it.

Parameters:

Parameters

| | |
|------|--|
| m_id | int matrix identifier (-1 for iZm5, 1 for iZm7). |
| p | Unsigned 64-bit integer parameter. |
| vx | size_t parameter representing the vx value. |
| y | Unsigned 64-bit integer parameter. |

Returns

The computed x value as a 64-bit unsigned integer.

Definition at line 415 of file [iZ_toolkit.c](#).

Referenced by [SiZm\(\)](#), and [vx_det_sieve\(\)](#).

11.4.3.13 iZm_solve_for_x0_mpz()

```
uint64_t iZm_solve_for_x0_mpz (
    int m_id,
    uint64_t p,
    uint64_t vx,
    mpz_t y)
```

GMP variant of [iZm_solve_for_x0\(\)](#) for very large y.

Solve for x given `m_id`, `p`, `vx`, and `y` using GMP. Same as above but using GMP. Suitable for arbitrary y values.

Parameters

| | |
|------|---------------------------------|
| m_id | Line id (-1 for x5, +1 for x7). |
| p | Prime used for marking. |
| vx | Segment width. |
| y | Segment index as mpz. |

Returns

First x index to clear for this prime/line.

Parameters:

Parameters

| | |
|------|--|
| m_id | int matrix identifier (-1 for iZm5, 1 for iZm7). |
|------|--|

| | |
|----|---|
| p | Unsigned 64-bit integer parameter. |
| vx | Size_t parameter representing the vx value. |
| y | mpz_t parameter representing the y value. |

Returns

The computed x value as a 64-bit unsigned integer.

Definition at line 449 of file [iZ_toolkit.c](#).

Referenced by [vx_det_sieve\(\)](#), and [vx_search_prime\(\)](#).

11.4.3.14 iZm_solve_for_y0()

```
int64_t iZm_solve_for_y0 (
    int m_id,
    uint64_t p,
    uint64_t vx,
    uint64_t x)
```

Solve first y-hit for fixed x in vertical (vy) scanning.

Compute the first composite hit of p in the xth vy segment in iZm of width vx.

Parameters

| | |
|------|---------------------------------|
| m_id | Line id (-1 for x5, +1 for x7). |
| p | Prime used for marking. |
| vx | Segment width. |
| x | Fixed x-coordinate. |

Returns

y index on success, -1 when no modular solution exists.

Description: This function solves for the smallest y that satisfies the equation $(x + vx * y) \equiv x_p \pmod{p}$, where x_p is normalized based on the m_id and p. The function checks if p and vx are co-primes, and if not, it returns -1 indicating no solution can be found. If they are co-primes, it calculates the multiplicative inverse of vx modulo p and computes y using the formula $y = (\text{delta} * vx_inv) \% p$, where delta is the difference between x_p and x modulo p.

Parameters:

Parameters

| | |
|------|--|
| m_id | int matrix identifier (-1 for iZm5, 1 for iZm7). |
| p | Unsigned 64-bit integer parameter. |

| | |
|----|---|
| vx | Size_t parameter representing the vx value. |
| x | Unsigned 64-bit integer parameter. |

Returns

The computed y value as a 64-bit unsigned integer.

Definition at line 490 of file [iZ_toolkit.c](#).

References [gcd\(\)](#), and [modular_inverse\(\)](#).

Referenced by [SiZm_vy\(\)](#).

11.4.3.15 process_iZ_bitmaps()

```
void process_iZ_bitmaps (
    UI64_ARRAY * primes,
    BITMAP * x5,
    BITMAP * x7,
    uint64_t x_limit)
```

Traverse iZ bitmaps, emit surviving primes, and mark composites.

Consume iZ candidate bitmaps, emit primes, and mark root composites.

Parameters

| | |
|---------|--|
| primes | Output prime array. |
| x5 | Bitmap for 6x-1 candidates. |
| x7 | Bitmap for 6x+1 candidates. |
| x_limit | Exclusive x upper bound. |
| primes | Destination array for discovered primes. |
| x5 | Bitmap for 6x-1 candidates. |
| x7 | Bitmap for 6x+1 candidates. |
| x_limit | Exclusive x upper bound. |

Definition at line 70 of file [iZ_toolkit.c](#).

References [bitmap_clear_steps_simd\(\)](#), [bitmap_get_bit\(\)](#), [iZ\(\)](#), and [ui64_push\(\)](#).

Referenced by [get_root_primes\(\)](#), [SiZ\(\)](#), and [SiZm\(\)](#).

11.4.3.16 range_info_free()

```
void range_info_free (
    IZM_RANGE_INFO * info)
```

Clear all GMP fields owned by info.

Definition at line 989 of file [iZ_toolkit.c](#).

References [IZM_RANGE_INFO::Xe](#), [IZM_RANGE_INFO::Xs](#), [IZM_RANGE_INFO::Ye](#), [IZM_RANGE_INFO::Ys](#), [IZM_RANGE_INFO::Ze](#), and [IZM_RANGE_INFO::Zs](#).

Referenced by [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

11.4.3.17 `range_info_init()`

```
IZM_RANGE_INFO range_info_init (
    INPUT_SIEVE_RANGE * input_range,
    int vx)
```

Map a decimal range input into iZm coordinates and segment bounds.

Parameters

| | |
|--------------------------|-----------------------------|
| <code>input_range</code> | Range input (start, range). |
| <code>vx</code> | iZm segment width. |

Returns

Initialized range-info object; `y_range < 0` indicates invalid input or an unsupported y-span for the current implementation.

Definition at line 944 of file [iZ_toolkit.c](#).

References [log_error\(\)](#), [INPUT_SIEVE_RANGE::range](#), [INPUT_SIEVE_RANGE::start](#), [IZM_RANGE_INFO::vx](#), [IZM_RANGE_INFO::Xe](#), [IZM_RANGE_INFO::Xs](#), [IZM_RANGE_INFO::y_range](#), [IZM_RANGE_INFO::Ye](#), [IZM_RANGE_INFO::Ys](#), [IZM_RANGE_INFO::Ze](#), and [IZM_RANGE_INFO::Zs](#).

Referenced by [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

11.4.3.18 `vx_collect_p_gaps()`

```
void vx_collect_p_gaps (
    VX_SEG * vx_obj)
```

Extract prime-gap encoding from a fully sieved segment.

Convert survivor bits into compact prime-gap encoding.

Parameters

| | |
|---------------------|-----------------|
| <code>vx_obj</code> | Segment object. |
|---------------------|-----------------|

This routine must run after deterministic/probabilistic sieving is complete. It populates [VX_SEG::p_gaps](#) for downstream streaming and gap traversal.

Parameters

| | |
|---------------------|--|
| <code>vx_obj</code> | Segment object with validated prime survivors. |
|---------------------|--|

Definition at line 787 of file [iZ_toolkit.c](#).

References [bitmap_get_bit\(\)](#), [VX_SEG::is_large_limit](#), [VX_SEG::p_count](#), [VX_SEG::p_gaps](#), [VX_SEG::start_x](#), [ui16_init\(\)](#), [ui16_push\(\)](#), [vx_full_sieve\(\)](#), [VX_SEG::x5](#), [VX_SEG::x7](#), and [VX_SEG::y](#).

Referenced by [vx_full_sieve\(\)](#).

11.4.3.19 vx_free()

```
void vx_free (
    VX_SEG ** vx_obj)
```

Free a VX segment and all owned resources.

Free all memory owned by a VX segment object.

Parameters

| | |
|--------|---|
| vx_obj | Address of a VX_SEG pointer. |
| vx_obj | Address of the VX_SEG pointer to release. |

Definition at line [759](#) of file [iZ_toolkit.c](#).

References [bitmap_free\(\)](#), and [ui16_free\(\)](#).

Referenced by [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

11.4.3.20 vx_full_sieve()

```
void vx_full_sieve (
    VX_SEG * vx_obj,
    int collect_p_gaps)
```

Complete segment processing (probabilistic stage and optional gaps).

This function performs the sieve process on a given vx and y defined in the [VX_SEG](#) structure, and stores the primes gaps in the vx_obj->p_gaps array.

Parameters

| | |
|----------------|---|
| vx_obj | Segment object. |
| collect_p_gaps | Non-zero to populate VX_SEG::p_gaps . |

Description: This function combines deterministic sieving and probabilistic primality tests to identify prime candidates in a standard VX segment of a specific y in the iZ-Matrix. It could be used to only count primes in the vx segment, and optionally populates the vx_obj->p_gaps array with prime gaps between consecutive primes detected in the segment.

Parameters

| | |
|----------------|---|
| vx_obj | The VX_SEG to be processed. |
| collect_p_gaps | 0 only counts primes and doesn't store prime gaps, other non-zero int values populates vx_obj->p_gaps with detected prime gaps. |

Definition at line [847](#) of file [iZ_toolkit.c](#).

References [VX_SEG::is_large_limit](#), [vx_collect_p_gaps\(\)](#), and [vx_prob_sieve\(\)](#).

Referenced by [SiZ_count\(\)](#), and [vx_collect_p_gaps\(\)](#).

11.4.3.21 vx_init()

```

VX_SEG * vx_init (
    IZM * iZm,
    int start_x,
    int end_x,
    char * y_str,
    int mr_rounds)

```

Initialize and deterministically sieve one VX segment.

Initialize the members of the **VX_SEG** structure with the given parameters and defaults.

Parameters

| | |
|-----------|---------------------------------------|
| iZm | Initialized toolkit context. |
| start_x | Inclusive x start index. |
| end_x | Inclusive x end index. |
| y_str | Segment index y as a decimal string. |
| mr_rounds | Miller-Rabin rounds (0 uses default). |

Returns

Initialized and partially processed segment, or NULL on failure.

Description: This function allocates memory for a **VX_SEG** structure and initializes its members. The count is initialized to 0, and the p_gaps array is allocated with an initial size of (vx/2).

Parameters:

Parameters

| | |
|-------|--|
| y_str | A character pointer representing a numeric string. |
|-------|--|

Returns

VX_SEG* A pointer to the initialized **VX_SEG** structure. NULL if memory allocation fails or y_str is not a numeric string.

Definition at line 713 of file [iz_toolkit.c](#).

References [IZM::base_x5](#), [IZM::base_x7](#), [VX_SEG::bit_ops](#), [bitmap_clone\(\)](#), [VX_SEG::end_x](#), [log_error\(\)](#), [MAX](#), [MIN](#), [MR_ROUNDS](#), [VX_SEG::mr_rounds](#), [VX_SEG::p_count](#), [VX_SEG::p_gaps](#), [VX_SEG::p_test_ops](#), [VX_SEG::start_x](#), [IZM::vx](#), [VX_SEG::vx](#), [vx_det_sieve\(\)](#), [vx_set_base_values\(\)](#), [VX_SEG::x5](#), and [VX_SEG::x7](#).

Referenced by [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

11.4.3.22 vx_search_prime()

```
int vx_search_prime (
    mpz_t p,
    int m_id,
    int vx,
    int bit_size)
```

Horizontal iZm/VX random-prime search.

horizontal search routine for generating a random prime.

Parameters

| | |
|----------|---|
| p | Output prime. |
| m_id | Requested line id (-1, +1, or random when other value). |
| vx | Segment width. |
| bit_size | Target bit size. |

Returns

1 on success, otherwise 0.

Description: This function searches for a prime number using the given parameters. It combines the iZ-Matrix filtering techniques with the Miller-Rabin primality test. The search is performed by finding suitable x values that do not correspond to composites of primes that divide vx. Then, it iterates over y values in the equation $p = iZ(x + vx * y)$ until a prime is found.

Parameters

| | |
|----------|---|
| p | The prime number found in the search. |
| m_id | The identifier (1 or -1) for the iZ matrix. |
| vx | The horizontal vector of the iZ matrix. |
| bit_size | The target bit size of the prime. |

Returns

1 if a prime is found, 0 otherwise.

Definition at line 1017 of file [iZ_toolkit.c](#).

References [UI64_ARRAY::array](#), [bitmap_clear_steps\(\)](#), [bitmap_free\(\)](#), [bitmap_get_bit\(\)](#), [bitmap_init\(\)](#), [check_primality\(\)](#), [UI64_ARRAY::count](#), [gmp_seed_randstate\(\)](#), [iZ_mpz\(\)](#), [iZm_solve_for_x0_mpz\(\)](#), [log_error\(\)](#), [MAX](#), [MR_ROUNDS](#), [SiZm\(\)](#), and [ui64_free\(\)](#).

Referenced by [vx_random_prime\(\)](#).

11.4.3.23 vx_stream()

```
void vx_stream (  
    VX\_SEG * vx_obj,  
    FILE * output)
```

Stream segment primes to an output stream.

Stream segment primes to an output stream in traversal order.

Parameters

| | |
|--------|-----------------|
| vx_obj | Segment object. |
|--------|-----------------|

| | |
|--------|---|
| output | Writable output stream (e.g. stdout or a file). |
| vx_obj | Segment object. |
| output | Destination stream (stdout or file). |

Definition at line 866 of file [iZ_toolkit.c](#).

References [bitmap_clear_bit\(\)](#), [bitmap_get_bit\(\)](#), [check_primalty\(\)](#), [VX_SEG::is_large_limit](#), [iZ_mpz\(\)](#), [VX_SEG::mr_rounds](#), [VX_SEG::p_count](#), [VX_SEG::p_test_ops](#), [VX_SEG::start_x](#), [VX_SEG::x5](#), [VX_SEG::x7](#), and [VX_SEG::yvx](#).

Referenced by [SiZ_stream\(\)](#).

11.4.3.24 vy_search_prime()

```
int vy_search_prime (
    mpz_t p,
    int m_id,
    mpz_t vx)
```

Vertical iZm/VY random-prime search.

vertical search routine for generating a random prime.

Parameters

| | |
|------|---|
| p | Output prime. |
| m_id | Requested line id (-1, +1, or random when other value). |
| vx | Segment width. |

Returns

1 on success, otherwise 0.

Description: This function searches for a prime number using the given parameters. It combines the iZ-Matrix filtering techniques with the Miller-Rabin primality test. The search is performed by finding a suitable x value that does not correspond to a composite of a prime that divides vx. Then, it iterates over y value in the equation $p = iZ(x + vx * y, m_id)$ until a prime is found.

Parameters

| | |
|------|---|
| p | The prime number found in the search. |
| m_id | The identifier (1 or -1) for the iZ matrix. |
| vx | The horizontal vector of the iZ matrix. |

Returns

1 if a prime is found, 0 otherwise.

Definition at line 1114 of file [iZ_toolkit.c](#).

References [check_primalty\(\)](#), [gmp_seed_randstate\(\)](#), [iZ_mpz\(\)](#), and [MR_ROUNDS](#).

Referenced by [vy_random_prime\(\)](#).

11.5 Logging

Thread-safe runtime logging helpers.

Files

- file [logger.c](#)
Logging module.

Macros

- `#define LOGGER_FORMAT_PRINTF(fmt_idx, arg_idx)`
Compiler attribute wrapper for printf-style format checking.
- `#define LOG_DIR "logs/"`
Directory where logs are stored.
- `#define LOG_FILE LOG_DIR "log.txt"`
Default log file.
- `#define LOG_MAX_SIZE 1024 * 1024 * 5`
Maximum log file size (5 MB).

Enumerations

- enum [LogLevel](#) {
[LOG_DEBUG](#) , [LOG_INFO](#) , [LOG_WARNING](#) , [LOG_ERROR](#) ,
[LOG_FATAL](#) }
Enumeration of log levels.

Functions

- const char * [log_level_to_string](#) ([LogLevel](#) level)
Returns a string representation of the log level.
- void [log_init](#) (const char *log_file)
Initializes the logging system.
- void [log_shutdown](#) (void)
Shuts down the logging system and cleans up resources.
- void [log_set_log_level](#) ([LogLevel](#) level)
Sets the current log level. Messages below this level will not be logged.
- void [log_message](#) ([LogLevel](#) level, const char *format,...) [LOGGER_FORMAT_PRINTF](#)(2)
Logs a formatted message at the given log level.
- void [log_message_extended](#) ([LogLevel](#) level, const char *file_name, int line_number, const char *format,...) [LOGGER_FORMAT_PRINTF](#)(4)
Logs a formatted message with extended information (file name, line number).
- void [log_console](#) (const char *format,...) [LOGGER_FORMAT_PRINTF](#)(1)
Logs a timestamped message to the console only, without requiring a log level.
- void [log_debug](#) (const char *format,...) [LOGGER_FORMAT_PRINTF](#)(1)
Logs a debug message.
- void [log_info](#) (const char *format,...) [LOGGER_FORMAT_PRINTF](#)(1)
Logs an info message.
- void [log_warn](#) (const char *format,...) [LOGGER_FORMAT_PRINTF](#)(1)
Logs a warning message.
- void [log_error](#) (const char *format,...) [LOGGER_FORMAT_PRINTF](#)(1)
Logs an error message.
- void [log_fatal](#) (const char *format,...) [LOGGER_FORMAT_PRINTF](#)(1)
Logs a fatal message.

11.5.1 Detailed Description

Thread-safe runtime logging helpers.

11.5.2 Macro Definition Documentation

11.5.2.1 LOG_DIR

```
#define LOG_DIR "logs/"
```

Directory where logs are stored.

Definition at line 54 of file [logger.h](#).

11.5.2.2 LOG_FILE

```
#define LOG_FILE LOG_DIR "log.txt"
```

Default log file.

Definition at line 55 of file [logger.h](#).

11.5.2.3 LOG_MAX_SIZE

```
#define LOG_MAX_SIZE 1024 * 1024 * 5
```

Maximum log file size (5 MB).

Definition at line 56 of file [logger.h](#).

11.5.2.4 LOGGER_FORMAT_PRINTF

```
#define LOGGER_FORMAT_PRINTF(  
    fmt_idx,  
    arg_idx)
```

Compiler attribute wrapper for printf-style format checking.

Definition at line 50 of file [logger.h](#).

Referenced by [log_console\(\)](#), [log_debug\(\)](#), [log_error\(\)](#), [log_fatal\(\)](#), [log_info\(\)](#), [log_message\(\)](#), [log_message_extended\(\)](#), and [log_warn\(\)](#).

11.5.3 Enumeration Type Documentation

11.5.3.1 LogLevel

enum [LogLevel](#)

Enumeration of log levels.

This enumeration defines the different log levels that can be used in the logging system. Each level corresponds to a severity of importance.

Enumerator

| | |
|-----------|--------------|
| LOG_DEBUG | Debug level. |
|-----------|--------------|

| | |
|-------------|----------------|
| LOG_INFO | Info level. |
| LOG_WARNING | Warning level. |
| LOG_ERROR | Error level. |
| LOG_FATAL | Fatal level. |

Definition at line 65 of file [logger.h](#).

11.5.4 Function Documentation

11.5.4.1 log_console()

```
void void void log_console (
    const char * format,
    ...)
```

Logs a timestamped message to the console only, without requiring a log level.

This can be used for debugging or tracking specific points in the code.

Parameters

| | |
|--------|--|
| format | The format string for the log message. |
|--------|--|

References [log_console\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [log_console\(\)](#).

11.5.4.2 log_debug()

```
void void void void log_debug (
    const char * format,
    ...)
```

Logs a debug message.

Parameters

| | |
|--------|--|
| format | The format string for the log message. |
|--------|--|

References [log_debug\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [iZ_next_prime\(\)](#), and [log_debug\(\)](#).

11.5.4.3 `log_error()`

```
void void void void void void void log_error (
    const char * format,
    ...)
```

Logs an error message.

Parameters

| | |
|--------|--|
| format | The format string for the log message. |
|--------|--|

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c](#).

References [log_error\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [bitmap_fread\(\)](#), [bitmap_fwrite\(\)](#), [bitmap_init\(\)](#), [bitmap_validate_hash\(\)](#), [create_dir\(\)](#), [iZ_next_prime\(\)](#), [iZm_clone\(\)](#), [iZm_init\(\)](#), [log_error\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), [SiZ_stream\(\)](#), [vx_init\(\)](#), [vx_random_prime\(\)](#), [vx_search_prime\(\)](#), [vx_set_base_values\(\)](#), and [vy_random_prime\(\)](#).

11.5.4.4 `log_fatal()`

```
void void void void void void void void log_fatal (
    const char * format,
    ...)
```

Logs a fatal message.

Parameters

| | |
|--------|--|
| format | The format string for the log message. |
|--------|--|

References [log_fatal\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [log_fatal\(\)](#).

11.5.4.5 `log_info()`

```
void void void void void void log_info (
    const char * format,
    ...)
```

Logs an info message.

Parameters

| | |
|--------|--|
| format | The format string for the log message. |
|--------|--|

References [log_info\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [log_info\(\)](#), [SiZ_count\(\)](#), [vx_prob_sieve\(\)](#), [vx_random_prime\(\)](#), and [vy_random_prime\(\)](#).

11.5.4.6 log_init()

```
void log_init (
    const char * log_file)
```

Initializes the logging system.

Creates the log directory if it doesn't exist and rotates logs if necessary.

Parameters

| | |
|----------|---------------------------|
| log_file | The path to the log file. |
|----------|---------------------------|

11.5.4.7 log_level_to_string()

```
const char * log_level_to_string (
    LogLevel level)
```

Returns a string representation of the log level.

Parameters

| | |
|-------|----------------|
| level | The log level. |
|-------|----------------|

Returns

A string corresponding to the log level.

11.5.4.8 log_message()

```
void log_message (
    LogLevel level,
    const char * format,
    ...)
```

Logs a formatted message at the given log level.

This function logs messages to the log file in a thread-safe manner.

Parameters

| | |
|--------|--|
| level | The log level for the message. |
| format | The format string for the log message. |

References [LOGGER_FORMAT_PRINTF](#).

11.5.4.9 log_message_extended()

```
void void log_message_extended (  
    LogLevel level,  
    const char * file_name,  
    int line_number,  
    const char * format,  
    ...)
```

Logs a formatted message with extended information (file name, line number).

This function logs messages to the log file in a thread-safe manner.

Parameters

| | |
|-------|--------------------------------|
| level | The log level for the message. |
|-------|--------------------------------|

| | |
|-------------|--|
| file_name | The source file where the log was generated. |
| line_number | The line number in the source file. |
| format | The format string for the log message. |

References [log_message_extended\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [log_message_extended\(\)](#).

11.5.4.10 log_set_log_level()

```
void log_set_log_level (
    LogLevel level)
```

Sets the current log level. Messages below this level will not be logged.

Parameters

| | |
|-------|-----------------------|
| level | The log level to set. |
|-------|-----------------------|

11.5.4.11 log_warn()

```
void void void void void log_warn (
    const char * format,
    ...)
```

Logs a warning message.

Parameters

| | |
|--------|--|
| format | The format string for the log message. |
|--------|--|

References [log_warn\(\)](#), and [LOGGER_FORMAT_PRINTF](#).

Referenced by [log_warn\(\)](#).

11.6 Platform Abstraction

OS-adaptation helpers used by utility and API layers.

Files

- file [platform.c](#)
Platform abstraction implementation.

Functions

- int [iz_platform_create_dir](#) (const char *dir)
Create a directory if it does not already exist.
- int [iz_platform_fill_random](#) (void *buffer, size_t bytes)
Fill a buffer with random bytes.
- int [iz_platform_cpu_cores_count](#) (void)
Return number of online logical CPU cores (≥ 1).
- int [iz_platform_l2_cache_size_bits](#) (void)
Best-effort L2 cache size query in bits.
- double [iz_platform_monotonic_seconds](#) (void)
Return a monotonic timestamp in seconds.

11.6.1 Detailed Description

OS-adaptation helpers used by utility and API layers.

11.6.2 Function Documentation

11.6.2.1 [iz_platform_cpu_cores_count\(\)](#)

```
int iz_platform_cpu_cores_count (
    void )
```

Return number of online logical CPU cores (≥ 1).

Returns

Core count.

Definition at line [57](#) of file [platform.c](#).

Referenced by [get_cpu_cores_count\(\)](#).

11.6.2.2 [iz_platform_create_dir\(\)](#)

```
int iz_platform_create_dir (
    const char * dir)
```

Create a directory if it does not already exist.

Parameters

| | |
|-----|-----------------|
| dir | Directory path. |
|-----|-----------------|

Returns

0 on success, -1 on failure.

Definition at line [16](#) of file [platform.c](#).

Referenced by [create_dir\(\)](#).

11.6.2.3 iz_platform_fill_random()

```
int iz_platform_fill_random (  
    void * buffer,  
    size_t bytes)
```

Fill a buffer with random bytes.

Parameters

| | |
|--------|---------------------|
| buffer | Destination buffer. |
|--------|---------------------|

| | |
|-------|--------------------------|
| bytes | Number of bytes to fill. |
|-------|--------------------------|

Returns

1 on success, 0 on failure.

Definition at line 31 of file [platform.c](#).

Referenced by [gmp_seed_randstate\(\)](#).

11.6.2.4 [iz_platform_l2_cache_size_bits\(\)](#)

```
int iz_platform_l2_cache_size_bits (
    void )
```

Best-effort L2 cache size query in bits.

Returns

L2 size in bits, or a conservative fallback.

Definition at line 70 of file [platform.c](#).

Referenced by [get_cpu_L2_cache_size_bits\(\)](#).

11.6.2.5 [iz_platform_monotonic_seconds\(\)](#)

```
double iz_platform_monotonic_seconds (
    void )
```

Return a monotonic timestamp in seconds.

Returns

Monotonic seconds.

Definition at line 104 of file [platform.c](#).

Referenced by [sw_elapsed_now_seconds\(\)](#).

11.7 Tests and Benchmarks

Test harness entry points.

Data Structures

- struct [SIEVE_LIMIT](#)
- struct [SIEVE_MODEL](#)

Typedefs

- typedef [UI64_ARRAY](#) [*\(* SIEVE_FN\)](#) (uint64_t)

Sieve model tests/benchmarks

- int TEST_UTILS (int verbose)
Validate utility helpers (numeric/range parsers and shared utilities).
- int TEST_SIEVE_MODELS_INTEGRITY (int verbose)
Run correctness checks across all registered sieve models.
- void BENCHMARK_SIEVE_MODELS (int save_results)
Benchmark registered sieve models and optionally persist results.

Range sieve tests/benchmarks

- int TEST_SiZ_stream (int verbose)
Validate [SiZ_stream](#) behavior and output formatting.
- int TEST_SiZ_count (int verbose)
Validate [SiZ_count](#) correctness across worker counts.
- void BENCHMARK_SiZ_count (int save_results)
Benchmark [SiZ_count](#) over increasing ranges starting from 10^{10} , 10^{20} , ..., 10^{100} using max cores.

Prime generation tests/benchmarks

- int TEST_iZ_next_prime (int verbose)
Validate [iZ_next_prime](#) for forward/backward traversal cases.
- int TEST_vy_random_prime (int verbose)
Validate [vy_random_prime](#) for primality and bit-length constraints.
- int TEST_vx_random_prime (int verbose)
Validate [vx_random_prime](#) for primality and bit-length constraints.
- int BENCHMARK_P_GEN_ALGORITHMS (int bit_size, int test_rounds, int save_results)
Benchmark random-prime generation routines over repeated trials.

11.7.1 Detailed Description

Test harness entry points.

11.7.2 Typedef Documentation

11.7.2.1 SIEVE_FN

typedef [UI64_ARRAY](#) [*\(* SIEVE_FN\)](#) (uint64_t)

Sieve function type: takes a limit and returns a prime list.

Definition at line 20 of file [test_api.h](#).

11.8 Utilities

Cross-cutting helpers used across modules.

Files

- file [utils.c](#)
Implementations for the shared utility layer.

Macros

- `#define MIN(a, b)`
Return the smaller of two expressions.
- `#define MAX(a, b)`
Return the larger of two expressions.
- `#define DIR_output "/output"`
- `#define MAX_CORES get_cpu_cores_count()`

Functions

- `int create_dir (const char *dir)`
Create a directory if it does not exist.
- `int is_numeric_str (const char *str)`
Return non-zero if str contains only ASCII digits.
- `int parse_numeric_expr_mpz (mpz_t out, const char *expr)`
Parse an integer expression into an mpz value.
- `int parse_numeric_expr_u64 (const char *expr, uint64_t *out)`
Parse an integer expression into uint64_t.
- `int parse_inclusive_range_mpz (const char *range_expr, mpz_t lower, mpz_t upper)`
Parse an inclusive range expression into lower/upper bounds.
- `uint64_t gcd (uint64_t a, uint64_t b)`
Compute the greatest common divisor of a and b.
- `uint64_t modular_inverse (uint64_t a, uint64_t m)`
Compute the modular inverse of a modulo m.
- `void gmp_seed_randstate (gmp_randstate_t state)`
Seed a GMP random state.
- `int get_cpu_cores_count (void)`
Get the number of online CPU cores (≥ 1).
- `int get_cpu_L2_cache_size_bits (void)`
Return the CPU L2 cache size in bits (best effort).

11.8.1 Detailed Description

Cross-cutting helpers used across modules.

11.8.2 Macro Definition Documentation

11.8.2.1 DIR_output

```
#define DIR_output "/output"
```

Default directory for output artifacts produced by examples/tests.

Definition at line 47 of file [utils.h](#).

11.8.2.2 MAX

```
#define MAX(  
    a,  
    b)
```

Value:

$((a) > (b)) ? (a) : (b)$

Return the larger of two expressions.

Definition at line 43 of file [utils.h](#).

Referenced by [SiZ_count\(\)](#), [SiZ_stream\(\)](#), [SSoE\(\)](#), [vx_init\(\)](#), [vx_random_prime\(\)](#), [vx_search_prime\(\)](#), and [vy_random_prime\(\)](#).

11.8.2.3 MAX_CORES

```
#define MAX_CORES get\_cpu\_cores\_count\(\)
```

Convenience macro: number of online CPU cores.

Definition at line 49 of file [utils.h](#).

11.8.2.4 MIN

```
#define MIN(  
    a,  
    b)
```

Value:

$((a) < (b)) ? (a) : (b)$

Return the smaller of two expressions.

Examples

[/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c](#).

Definition at line 39 of file [utils.h](#).

Referenced by [bitmap_clear_steps\(\)](#), [bitmap_clear_steps_simd\(\)](#), [compute_l2_vx\(\)](#), [SiZ_count\(\)](#), [SiZ_stream\(\)](#), and [vx_init\(\)](#).

11.8.3 Function Documentation

11.8.3.1 `create_dir()`

```
int create_dir (  
    const char * dir)
```

Create a directory if it does not exist.

Parameters

| | |
|-----|-----------------|
| dir | Directory path. |
|-----|-----------------|

Returns

0 on success (including already exists), -1 on failure.

Parameters

| | |
|-----|---------------------|
| dir | The directory path. |
|-----|---------------------|

Returns

int 0 if the directory already exists or was successfully created, -1 otherwise.

Definition at line [402](#) of file [utils.c](#).

References [iz_platform_create_dir\(\)](#), and [log_error\(\)](#).

11.8.3.2 `gcd()`

```
uint64_t gcd (  
    uint64_t a,  
    uint64_t b)
```

Compute the greatest common divisor of a and b.

Compute the greatest common divisor of a and b.

Parameters

| | |
|---|---------------------|
| a | The first integer. |
| b | The second integer. |

Returns

int The GCD of a and b.

Definition at line [419](#) of file [utils.c](#).

Referenced by [iZm_solve_for_y0\(\)](#), and [SiZm_vy\(\)](#).

11.8.3.3 `get_cpu_cores_count()`

```
int get_cpu_cores_count (  
    void )
```

Get the number of online CPU cores (≥ 1).

Get the number of online CPU cores (≥ 1).

Returns

int The number of CPU cores.

Definition at line 495 of file [utils.c](#).

References [iz_platform_cpu_cores_count\(\)](#).

Referenced by [SiZ_count\(\)](#).

11.8.3.4 `get_cpu_L2_cache_size_bits()`

```
int get_cpu_L2_cache_size_bits (  
    void )
```

Return the CPU L2 cache size in bits (best effort).

Returns

Cache size in bits, or a conservative default.

Return the CPU L2 cache size in bits (best effort).

Description: This function retrieves the L2 cache size using platform-specific methods:

- Linux: Reads from sysfs (`/sys/devices/system/cpu/cpu0/cache/index2/size`)
- macOS/BSD: Uses `sysctl` to query `hw.l2cachesize`
- Fallback: Returns 256 Kbits as a conservative default

The function attempts multiple detection methods to ensure robustness across different architectures and operating systems.

Returns

int The L2 cache size in bits, or 256 Kbits if unable to determine.

Definition at line 514 of file [utils.c](#).

References [iz_platform_l2_cache_size_bits\(\)](#).

Referenced by [compute_l2_vx\(\)](#).

11.8.3.5 gmp_seed_randstate()

```
void gmp_seed_randstate (  
    gmp_randstate_t state)
```

Seed a GMP random state.

Uses platform entropy sources (OpenSSL-backed) with a time-based fallback.

Seed a GMP random state.

Description: This function seeds the GMP random state using platform entropy (OpenSSL-backed), with a time-based fallback.

Parameters

| | |
|-------|-----------------------|
| state | The GMP random state. |
|-------|-----------------------|

Definition at line 481 of file [utils.c](#).

References [iz_platform_fill_random\(\)](#).

Referenced by [vx_search_prime\(\)](#), and [vy_search_prime\(\)](#).

11.8.3.6 is_numeric_str()

```
int is_numeric_str (  
    const char * str)
```

Return non-zero if str contains only ASCII digits.

Parameters

| | |
|-----|---------------|
| str | Input string. |
|-----|---------------|

Returns

1 if numeric, 0 otherwise.

Return non-zero if str contains only ASCII digits.

Parameters

| | |
|-----|------------------------|
| str | Pointer to the string. |
|-----|------------------------|

Returns

int 1 if the string is numeric, 0 otherwise.

Definition at line 258 of file [utils.c](#).

11.8.3.7 modular_inverse()

```
uint64_t modular_inverse (
    uint64_t a,
    uint64_t m)
```

Compute the modular inverse of a modulo m.

Returns

The inverse in $[0, m-1]$ if it exists; otherwise an implementation-defined value.

Compute the modular inverse of a modulo m.

Description: This function computes the modular inverse of a modulo m using the Extended Euclidean Algorithm. The function takes two integers a and m as input and returns the modular inverse of a modulo m. If a and m are not co-prime, the function returns -1 indicating no modular inverse exists. The function also handles the case where m is 1, returning 0 as the modular inverse.

Parameters:

Parameters

| | |
|---|--|
| a | The integer for which the modular inverse is to be computed. |
| m | The modulus. |

Returns

The modular inverse of a modulo m, or -1 if no inverse exists.

Definition at line 447 of file [utils.c](#).

Referenced by [iZm_solve_for_y0\(\)](#).

11.8.3.8 parse_inclusive_range_mpz()

```
int parse_inclusive_range_mpz (
    const char * range_expr,
    mpz_t lower,
    mpz_t upper)
```

Parse an inclusive range expression into lower/upper bounds.

Accepted forms:

- L,R
- [L, R]
- range[L, R]
- L..R
- L:R

Each bound accepts the same numeric formats as [parse_numeric_expr_mpz](#).

Parameters

| | |
|------------|--------------------------|
| range_expr | Range expression string. |
|------------|--------------------------|

| | |
|-------|---------------------|
| lower | Output lower bound. |
| upper | Output upper bound. |

Returns

1 on success, 0 on parse failure.

Definition at line 335 of file [utils.c](#).

11.8.3.9 `parse_numeric_expr_mpz()`

```
int parse_numeric_expr_mpz (
    mpz_t out,
    const char * expr)
```

Parse an integer expression into an mpz value.

Supported term formats:

- plain decimal integer (1000000, 1,000,000)
- power notation (10^6)
- scientific shorthand (1e6, 10e100)
- additive expressions ($10e100 + 10e9$)

Parameters

| | |
|------|--------------------------|
| out | Output mpz value. |
| expr | Input expression string. |

Returns

1 on success, 0 on parse failure.

Definition at line 272 of file [utils.c](#).

Referenced by [parse_numeric_expr_u64\(\)](#).

11.8.3.10 `parse_numeric_expr_u64()`

```
int parse_numeric_expr_u64 (
    const char * expr,
    uint64_t * out)
```

Parse an integer expression into uint64_t.

Parameters

| | |
|------|--------------------------|
| expr | Input expression string. |
|------|--------------------------|

| | |
|-----|---------------|
| out | Output value. |
|-----|---------------|

Returns

1 on success, 0 on parse failure or overflow.

Definition at line 314 of file [utils.c](#).

References [parse_numeric_expr_mpz\(\)](#).

11.9 Printer, implemented in toolkit/print_utils.c

Console formatting helpers used by tests and benchmark runners.

Files

- file [print_utils.c](#)
Print formatting helpers for tests, benchmarks, and CLI output.

Functions

- void [print_sha256_hash](#) (const unsigned char *hash)
Print a SHA-256 digest as hex to stdout.
- void [print_line](#) (int length, char fill_char)
Print a repeated-character horizontal line.
- void [print_centered_text](#) (const char *text, int line_length, char fill_char)
Print text centered inside a padded line.
- void [print_test_table_header](#) (void)
Print the generic test runner header.
- void [print_test_module_header](#) (char *module_name)
Print a test-suite header banner for a module.
- void [print_test_fn_header](#) (char *fn_name)
Print a formatted header for a test function.
- void [print_test_module_result](#) (int result, int test_id, const char *unit_name, const char *format,...)
Print a single test-row result.
- void [print_test_summary](#) (char *module_name, int passed, int failed, int verbose)
Print module-level test summary.

11.9.1 Detailed Description

Console formatting helpers used by tests and benchmark runners.

11.9.2 Function Documentation

11.9.2.1 print_centered_text()

```
void print_centered_text (  
    const char * text,  
    int line_length,  
    char fill_char)
```

Print text centered inside a padded line.

Print centered text within a line of specified length.

Parameters

| | |
|------|---------------|
| text | Text payload. |
|------|---------------|

| | |
|-------------|--|
| line_length | Target total line width. |
| fill_char | Padding character. |
| text | Text to center. |
| line_length | Total length of the line. |
| fill_char | Character to use for padding (' ' is used when zero). |

Definition at line 30 of file [print_utils.c](#).

Referenced by [print_test_fn_header\(\)](#).

11.9.2.2 print_line()

```
void print_line (
    int length,
    char fill_char)
```

Print a repeated-character horizontal line.

Parameters

| | |
|-----------|--|
| length | Number of characters to print. |
| fill_char | Character to repeat ('-' is used when zero). |

Definition at line 14 of file [print_utils.c](#).

Referenced by [print_test_fn_header\(\)](#), [print_test_module_header\(\)](#), [print_test_summary\(\)](#), and [print_test_table_header\(\)](#).

11.9.2.3 print_sha256_hash()

```
void print_sha256_hash (
    const unsigned char * hash)
```

Print a SHA-256 digest as hex to stdout.

Print the SHA-256 hash in hexadecimal format.

Parameters

| | |
|------|------------------------------------|
| hash | Pointer to the SHA-256 hash array. |
|------|------------------------------------|

Definition at line 58 of file [print_utils.c](#).

11.9.2.4 print_test_fn_header()

```
void print_test_fn_header (
    char * fn_name)
```

Print a formatted header for a test function.

Definition at line 148 of file [print_utils.c](#).

References [print_centered_text\(\)](#), and [print_line\(\)](#).

11.9.2.5 print_test_module_header()

```
void print_test_module_header (
    char * module_name)
```

Print a test-suite header banner for a module.

Print module-level banner for test output.

Parameters

| | |
|-------------|------------------------------------|
| module_name | Display name of the tested module. |
|-------------|------------------------------------|

Definition at line 70 of file [print_utils.c](#).

References [print_line\(\)](#).

11.9.2.6 print_test_module_result()

```
void print_test_module_result (
    int result,
    int test_id,
    const char * unit_name,
    const char * format,
    ...)
```

Print a single test-row result.

Print a single test unit result row with formatted details.

Parameters

| | |
|-----------|---|
| result | 1 for pass, 0 for fail. |
| test_id | Monotonic case index. |
| unit_name | Short test unit label. |
| format | printf-style detail format. |
| result | 1 for pass, 0 for fail. |
| test_id | Monotonic case index. |
| unit_name | Short test unit label. |
| format | printf-style detail format string. |
| ... | Arguments for the detail format string. |

Definition at line 96 of file [print_utils.c](#).

11.9.2.7 print_test_summary()

```
void print_test_summary (  
    char * module_name,  
    int passed,  
    int failed,  
    int verbose)
```

Print module-level test summary.

Print a summary of test results for a module.

Parameters

| | |
|-------------|-------------------|
| module_name | Test module name. |
|-------------|-------------------|

| | |
|-------------|---|
| passed | Number of passing tests. |
| failed | Number of failing tests. |
| verbose | Non-zero enables richer output hints. |
| module_name | Name of the tested module. |
| passed | Number of passing tests. |
| failed | Number of failing tests. |
| verbose | If non-zero, prints additional hints about the results. |

Definition at line 126 of file [print_utils.c](#).

References [print_line\(\)](#).

11.9.2.8 print_test_table_header()

```
void print_test_table_header (
    void )
```

Print the generic test runner header.

Print standard test-table column headers.

Definition at line 80 of file [print_utils.c](#).

References [print_line\(\)](#).

11.10 Stopwatch

Monotonic wall-clock timing helpers for tests and benchmarks.

Files

- file [stopwatch.c](#)
Stopwatch helpers based on monotonic wall-clock time.

Data Structures

- struct [STOPWATCH](#)
Stopwatch state for elapsed wall-clock measurements.

Functions

- void [sw_start](#) ([STOPWATCH](#) *sw)
Start or restart a stopwatch.
- void [sw_stop](#) ([STOPWATCH](#) *sw)
Stop a running stopwatch, capturing the elapsed time in seconds.
- double [sw_elapsed_seconds](#) (const [STOPWATCH](#) *sw)
Return elapsed seconds for the stopwatch.
- double [sw_elapsed_now_seconds](#) (void)
Capture the current monotonic time in seconds.

11.10.1 Detailed Description

Monotonic wall-clock timing helpers for tests and benchmarks.

11.10.2 Function Documentation

11.10.2.1 `sw_elapsed_now_seconds()`

```
double sw_elapsed_now_seconds (
    void )
```

Capture the current monotonic time in seconds.

Returns

Current monotonic timestamp in seconds.

Definition at line 68 of file [stopwatch.c](#).

References [iz_platform_monotonic_seconds\(\)](#).

11.10.2.2 `sw_elapsed_seconds()`

```
double sw_elapsed_seconds (
    const STOPWATCH * sw)
```

Return elapsed seconds for the stopwatch.

If the stopwatch is still running, elapsed time is computed against the current monotonic timestamp.

Parameters

| | |
|----|-------------------|
| sw | Stopwatch object. |
|----|-------------------|

Returns

Elapsed seconds.

Definition at line 57 of file [stopwatch.c](#).

References [STOPWATCH::elapsed_sec](#), [STOPWATCH::running](#), and [STOPWATCH::start_time](#).

11.10.2.3 `sw_start()`

```
void sw_start (
    STOPWATCH * sw)
```

Start or restart a stopwatch.

Parameters

| | |
|----|-------------------|
| sw | Stopwatch object. |
|----|-------------------|

Definition at line 39 of file [stopwatch.c](#).

References [STOPWATCH::end_time](#), [STOPWATCH::running](#), and [STOPWATCH::start_time](#).

11.10.2.4 `sw_stop()`

```
void sw_stop (  
    STOPWATCH * sw)
```

Stop a running stopwatch, capturing the elapsed time in seconds.

Parameters

| | |
|----|-------------------|
| sw | Stopwatch object. |
|----|-------------------|

Definition at line [47](#) of file [stopwatch.c](#).

References [STOPWATCH::elapsed_sec](#), [STOPWATCH::end_time](#), [STOPWATCH::running](#), and [STOPWATCH::start_time](#).

Chapter 12

Data Structure Documentation

12.1 BITMAP Struct Reference

Packed bit-array with optional SHA-256 checksum.

```
#include <bitmap.h>
```

Data Fields

- `size_t` [size](#)
- `size_t` [byte_size](#)
- `unsigned char *` [data](#)
- `unsigned char` [sha256](#) [SHA256_DIGEST_LENGTH]

12.1.1 Detailed Description

Packed bit-array with optional SHA-256 checksum.

[size](#) is measured in bits and [byte_size](#) is the backing storage in bytes. The checksum is maintained explicitly via [bitmap_compute_hash\(\)](#) and verified via [bitmap_validate_hash\(\)](#).

Examples

```
/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c.
```

Definition at line [26](#) of file [bitmap.h](#).

12.1.2 Field Documentation

12.1.2.1 `byte_size`

`size_t` BITMAP::byte_size

Number of bytes in [data](#).

Examples

```
/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c.
```

Definition at line [29](#) of file [bitmap.h](#).

Referenced by [bitmap_clone\(\)](#), [bitmap_compute_hash\(\)](#), [bitmap_fread\(\)](#), [bitmap_fwrite\(\)](#), [bitmap_init\(\)](#), [bitmap_validate_hash\(\)](#), and [SiZm\(\)](#).

12.1.2.2 data

unsigned char* BITMAP::data

Packed bits (LSB-first per byte).

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 30 of file [bitmap.h](#).

Referenced by [bitmap_clear_all\(\)](#), [bitmap_clear_bit\(\)](#), [bitmap_clear_steps\(\)](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_clone\(\)](#), [bitmap_compute_hash\(\)](#), [bitmap_flip_bit\(\)](#), [bitmap_fread\(\)](#), [bitmap_fwrite\(\)](#), [bitmap_get_bit\(\)](#), [bitmap_init\(\)](#), [bitmap_set_all\(\)](#), [bitmap_set_bit\(\)](#), [bitmap_validate_hash\(\)](#), and [SiZm\(\)](#).

12.1.2.3 sha256

unsigned char BITMAP::sha256[SHA256_DIGEST_LENGTH]

Cached SHA-256 checksum.

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 31 of file [bitmap.h](#).

Referenced by [bitmap_clone\(\)](#), [bitmap_compute_hash\(\)](#), [bitmap_fread\(\)](#), [bitmap_fwrite\(\)](#), [bitmap_init\(\)](#), and [bitmap_validate_hash\(\)](#).

12.1.2.4 size

size_t BITMAP::size

Number of addressable bits.

Examples

</Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/bitmap.c>.

Definition at line 28 of file [bitmap.h](#).

Referenced by [bitmap_clear_all\(\)](#), [bitmap_clear_steps\(\)](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_clone\(\)](#), [bitmap_fwrite\(\)](#), [bitmap_init\(\)](#), and [bitmap_set_all\(\)](#).

The documentation for this struct was generated from the following file:

- [include/bitmap.h](#)

12.2 INPUT_SIEVE_RANGE Struct Reference

Input parameters for range sieving/counting.

```
#include <iZ_api.h>
```

Data Fields

- `char * start`
Start of range as a base-10 numeric string.
- `uint64_t range`
Interval size (number of integers to cover).
- `int mr_rounds`
Miller–Rabin rounds for large primality checks.
- `char * filepath`
Output path for streaming primes (NULL to disable output).

12.2.1 Detailed Description

Input parameters for range sieving/counting.

The interval is interpreted as:

- Start Z_s from the decimal string `start`.
- End $Z_e = Z_s + \text{range} - 1$.

Definition at line 124 of file `iZ_api.h`.

12.2.2 Field Documentation

12.2.2.1 filepath

```
char* INPUT_SIEVE_RANGE::filepath
```

Output path for streaming primes (NULL to disable output).

Definition at line 129 of file `iZ_api.h`.

Referenced by `SiZ_stream()`.

12.2.2.2 mr_rounds

```
int INPUT_SIEVE_RANGE::mr_rounds
```

Miller–Rabin rounds for large primality checks.

Definition at line 128 of file `iZ_api.h`.

Referenced by `SiZ_count()`, and `SiZ_stream()`.

12.2.2.3 range

uint64_t INPUT_SIEVE_RANGE::range

Interval size (number of integers to cover).

Definition at line 127 of file [iZ_api.h](#).

Referenced by [range_info_init\(\)](#), and [SiZ_count\(\)](#).

12.2.2.4 start

char* INPUT_SIEVE_RANGE::start

Start of range as a base-10 numeric string.

Definition at line 126 of file [iZ_api.h](#).

Referenced by [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

The documentation for this struct was generated from the following file:

- [include/iZ_api.h](#)

12.3 IZM Struct Reference

Precomputed iZm assets for repeated VX-segment sieving.

```
#include <iZ_toolkit.h>
```

Data Fields

- int [vx](#)
- int [k_vx](#)
- BITMAP * [base_x5](#)
- BITMAP * [base_x7](#)
- UI64_ARRAY * [root_primes](#)

12.3.1 Detailed Description

Precomputed iZm assets for repeated VX-segment sieving.

Definition at line 84 of file [iZ_toolkit.h](#).

12.3.2 Field Documentation

12.3.2.1 base_x5

[BITMAP](#)* IZM::base_x5

Pre-sieved base bitmap for 6x-1 line.

Definition at line 88 of file [iZ_toolkit.h](#).

Referenced by [iZ_next_prime\(\)](#), [iZm_clone\(\)](#), [iZm_init\(\)](#), and [vx_init\(\)](#).

12.3.2.2 base_x7

[BITMAP](#)* IZM::base_x7

Pre-sieved base bitmap for 6x+1 line.

Definition at line 89 of file [iZ_toolkit.h](#).

Referenced by [iZ_next_prime\(\)](#), [iZm_clone\(\)](#), [iZm_init\(\)](#), and [vx_init\(\)](#).

12.3.2.3 k_vx

int IZM::k_vx

Count of small primes dividing vx (excluding 2,3).

Definition at line 87 of file [iZ_toolkit.h](#).

Referenced by [iZm_clone\(\)](#), [iZm_init\(\)](#), and [vx_det_sieve\(\)](#).

12.3.2.4 root_primes

[UI64_ARRAY](#)* IZM::root_primes

Root primes used for deterministic marking.

Definition at line 90 of file [iZ_toolkit.h](#).

Referenced by [iZm_clone\(\)](#), [iZm_init\(\)](#), and [vx_det_sieve\(\)](#).

12.3.2.5 vx

int IZM::vx

Segment width in iZ x-units.

Definition at line 86 of file [iZ_toolkit.h](#).

Referenced by [compute_k_vx\(\)](#), [iZm_clone\(\)](#), [iZm_init\(\)](#), and [vx_init\(\)](#).

The documentation for this struct was generated from the following file:

- [include/iZ_toolkit.h](#)

12.4 IZM_RANGE_INFO Struct Reference

Precomputed iZm coordinates for an inclusive numeric interval.

```
#include <iZ_toolkit.h>
```

Data Fields

- int [vx](#)
- mpz_t [Zs](#)
- mpz_t [Ze](#)
- mpz_t [Xs](#)
- mpz_t [Xe](#)
- mpz_t [Ys](#)
- mpz_t [Ye](#)
- int [y_range](#)

12.4.1 Detailed Description

Precomputed iZm coordinates for an inclusive numeric interval.

This structure maps an input range [[Zs](#), [Ze](#)] into the iZ index space used by segmented SiZ routines:

- X^* are integer-domain indices such that $Z \sim 6X \pm 1$.
- Y^* are segment indices (X / vx).
- [y_range](#) = [Ye](#) - [Ys](#) and is used to drive segment iteration counts.

A negative [y_range](#) marks an invalid/unusable mapping (bad input or range outside current implementation bounds).

Definition at line [254](#) of file [iZ_toolkit.h](#).

12.4.2 Field Documentation

12.4.2.1 vx

```
int IZM_RANGE_INFO::vx
```

iZm segment width.

Definition at line [256](#) of file [iZ_toolkit.h](#).

Referenced by [range_info_init\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.2 Xe

mpz_t IZM_RANGE_INFO::Xe

$x_Ze = Ze / 6$.

Definition at line 260 of file [iZ_toolkit.h](#).

Referenced by [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.3 Xs

mpz_t IZM_RANGE_INFO::Xs

$x_Zs = Zs / 6$.

Definition at line 259 of file [iZ_toolkit.h](#).

Referenced by [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.4 y_range

int IZM_RANGE_INFO::y_range

Number of y-steps minus one ([Ye](#) - [Ys](#)), or -1 when invalid.

Definition at line 263 of file [iZ_toolkit.h](#).

Referenced by [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.5 Ye

mpz_t IZM_RANGE_INFO::Ye

$y_Ze = Xe / vx$.

Definition at line 262 of file [iZ_toolkit.h](#).

Referenced by [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.6 Ys

mpz_t IZM_RANGE_INFO::Ys

$y_Zs = Xs / vx$.

Definition at line 261 of file [iZ_toolkit.h](#).

Referenced by [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.7 Ze

mpz_t IZM_RANGE_INFO::Ze

Inclusive upper bound of the search interval.

Definition at line 258 of file [iZ_toolkit.h](#).

Referenced by [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

12.4.2.8 Zs

mpz_t IZM_RANGE_INFO::Zs

Inclusive lower bound of the search interval.

Definition at line 257 of file [iZ_toolkit.h](#).

Referenced by [range_info_free\(\)](#), [range_info_init\(\)](#), [SiZ_count\(\)](#), and [SiZ_stream\(\)](#).

The documentation for this struct was generated from the following file:

- [include/iZ_toolkit.h](#)

12.5 SIEVE_LIMIT Struct Reference

```
#include <test_api.h>
```

Data Fields

- int [base](#)
- int [exp](#)

12.5.1 Detailed Description

Limit specification as base^{exp} .

Definition at line 23 of file [test_api.h](#).

12.5.2 Field Documentation

12.5.2.1 base

int SIEVE_LIMIT::base

Base value in base^{exp} limit notation.

Definition at line 25 of file [test_api.h](#).

12.5.2.2 exp

int SIEVE_LIMIT::exp

Exponent value in base^exp limit notation.

Definition at line 26 of file [test_api.h](#).

The documentation for this struct was generated from the following file:

- include/[test_api.h](#)

12.6 SIEVE_MODEL Struct Reference

```
#include <test_api.h>
```

Data Fields

- [SIEVE_FN](#) function
- const char [name](#) [32]

12.6.1 Detailed Description

Pair a sieve function with a human-readable name (for reports/benchmarks).

Definition at line 30 of file [test_api.h](#).

12.6.2 Field Documentation

12.6.2.1 function

[SIEVE_FN](#) SIEVE_MODEL::function

Implementation entry point.

Definition at line 32 of file [test_api.h](#).

12.6.2.2 name

const char SIEVE_MODEL::name[32]

Display name used in test/benchmark output.

Definition at line 33 of file [test_api.h](#).

The documentation for this struct was generated from the following file:

- include/[test_api.h](#)

12.7 STOPWATCH Struct Reference

Stopwatch state for elapsed wall-clock measurements.

```
#include <utils.h>
```

Data Fields

- struct timespec [start_time](#)
- struct timespec [end_time](#)
- int [running](#)
- double [elapsed_sec](#)

12.7.1 Detailed Description

Stopwatch state for elapsed wall-clock measurements.

Definition at line [200](#) of file [utils.h](#).

12.7.2 Field Documentation

12.7.2.1 elapsed_sec

double STOPWATCH::elapsed_sec

Elapsed seconds, captured at stop.

Definition at line [205](#) of file [utils.h](#).

Referenced by [sw_elapsed_seconds\(\)](#), and [sw_stop\(\)](#).

12.7.2.2 end_time

struct timespec STOPWATCH::end_time

Captured stop timestamp.

Definition at line [203](#) of file [utils.h](#).

Referenced by [sw_start\(\)](#), and [sw_stop\(\)](#).

12.7.2.3 running

int STOPWATCH::running

Non-zero when timing is active.

Definition at line [204](#) of file [utils.h](#).

Referenced by [sw_elapsed_seconds\(\)](#), [sw_start\(\)](#), and [sw_stop\(\)](#).

12.7.2.4 start_time

struct timespec STOPWATCH::start_time

Captured start timestamp.

Definition at line 202 of file [utils.h](#).

Referenced by [sw_elapsed_seconds\(\)](#), [sw_start\(\)](#), and [sw_stop\(\)](#).

The documentation for this struct was generated from the following file:

- include/[utils.h](#)

12.8 UI16_ARRAY Struct Reference

Dynamic array for uint16_t values.

```
#include <int_arrays.h>
```

Data Fields

- int [capacity](#)
- int [count](#)
- uint16_t * [array](#)
- int [ordered](#)
- unsigned char [sha256](#) [SHA256_DIGEST_LENGTH]

12.8.1 Detailed Description

Dynamic array for uint16_t values.

Definition at line 19 of file [int_arrays.h](#).

12.8.2 Field Documentation

12.8.2.1 array

uint16_t* UI16_ARRAY::array

Contiguous element storage.

Definition at line 23 of file [int_arrays.h](#).

12.8.2.2 capacity

`int UI16_ARRAY::capacity`

Allocated element capacity.

Definition at line 21 of file [int_arrays.h](#).

12.8.2.3 count

`int UI16_ARRAY::count`

Number of valid elements.

Definition at line 22 of file [int_arrays.h](#).

12.8.2.4 ordered

`int UI16_ARRAY::ordered`

Flag indicating if the array is sorted.

Definition at line 24 of file [int_arrays.h](#).

12.8.2.5 sha256

`unsigned char UI16_ARRAY::sha256[SHA256_DIGEST_LENGTH]`

SHA-256 over used payload.

Definition at line 25 of file [int_arrays.h](#).

The documentation for this struct was generated from the following file:

- [include/int_arrays.h](#)

12.9 UI32_ARRAY Struct Reference

Dynamic array for `uint32_t` values.

`#include <int_arrays.h>`

Data Fields

- `int` [capacity](#)
- `int` [count](#)
- `uint32_t *` [array](#)
- `int` [ordered](#)
- `unsigned char` [sha256](#) [SHA256_DIGEST_LENGTH]

12.9.1 Detailed Description

Dynamic array for uint32_t values.

Definition at line 29 of file [int_arrays.h](#).

12.9.2 Field Documentation

12.9.2.1 array

uint32_t* UI32_ARRAY::array

Contiguous element storage.

Definition at line 33 of file [int_arrays.h](#).

12.9.2.2 capacity

int UI32_ARRAY::capacity

Allocated element capacity.

Definition at line 31 of file [int_arrays.h](#).

12.9.2.3 count

int UI32_ARRAY::count

Number of valid elements.

Definition at line 32 of file [int_arrays.h](#).

12.9.2.4 ordered

int UI32_ARRAY::ordered

Flag indicating if the array is sorted.

Definition at line 34 of file [int_arrays.h](#).

12.9.2.5 sha256

unsigned char UI32_ARRAY::sha256[SHA256_DIGEST_LENGTH]

SHA-256 over used payload.

Definition at line 35 of file [int_arrays.h](#).

The documentation for this struct was generated from the following file:

- [include/int_arrays.h](#)

12.10 UI64_ARRAY Struct Reference

Dynamic array for uint64_t values.

```
#include <int_arrays.h>
```

Data Fields

- int [capacity](#)
- int [count](#)
- uint64_t * [array](#)
- int [ordered](#)
- unsigned char [sha256](#) [SHA256_DIGEST_LENGTH]

12.10.1 Detailed Description

Dynamic array for uint64_t values.

Definition at line 39 of file [int_arrays.h](#).

12.10.2 Field Documentation

12.10.2.1 array

```
uint64_t* UI64_ARRAY::array
```

Contiguous element storage.

Definition at line 43 of file [int_arrays.h](#).

Referenced by [compute_max_vx\(\)](#), [iZm_clone\(\)](#), [SiZ\(\)](#), [SiZ_count\(\)](#), [SiZ_stream\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoEu\(\)](#), [SSoE\(\)](#), [vx_det_sieve\(\)](#), and [vx_search_prime\(\)](#).

12.10.2.2 capacity

```
int UI64_ARRAY::capacity
```

Allocated element capacity.

Definition at line 41 of file [int_arrays.h](#).

Referenced by [iZm_clone\(\)](#).

12.10.2.3 count

```
int UI64_ARRAY::count
```

Number of valid elements.

Definition at line 42 of file [int_arrays.h](#).

Referenced by [iZm_clone\(\)](#), [SiZ\(\)](#), [SiZ_count\(\)](#), [SiZ_stream\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoEu\(\)](#), [SSoE\(\)](#), [vx_det_sieve\(\)](#), and [vx_search_prime\(\)](#).

12.10.2.4 ordered

int UI64_ARRAY::ordered

Flag indicating if the array is sorted.

Definition at line 44 of file [int_arrays.h](#).

Referenced by [SiZm_vy\(\)](#).

12.10.2.5 sha256

unsigned char UI64_ARRAY::sha256[SHA256_DIGEST_LENGTH]

SHA-256 over used payload.

Definition at line 45 of file [int_arrays.h](#).

The documentation for this struct was generated from the following file:

- include/[int_arrays.h](#)

12.11 VX_SEG Struct Reference

Runtime state for one VX segment at a specific y.

#include <iZ_toolkit.h>

Data Fields

- int [vx](#)
- mpz_t [y](#)
- mpz_t [yvx](#)
- mpz_t [root_limit](#)
- int [is_large_limit](#)
- int [mr_rounds](#)
- int [start_x](#)
- int [end_x](#)
- BITMAP * [x5](#)
- BITMAP * [x7](#)
- int [p_count](#)
- UI16_ARRAY * [p_gaps](#)
- int [bit_ops](#)
- int [p_test_ops](#)

12.11.1 Detailed Description

Runtime state for one VX segment at a specific y.

Definition at line 184 of file [iZ_toolkit.h](#).

12.11.2 Field Documentation

12.11.2.1 bit_ops

int VX_SEG::bit_ops

Approximate deterministic mark operations.

Definition at line 198 of file [iZ_toolkit.h](#).

Referenced by [vx_det_sieve\(\)](#), and [vx_init\(\)](#).

12.11.2.2 end_x

int VX_SEG::end_x

Inclusive end x for this segment.

Definition at line 193 of file [iZ_toolkit.h](#).

Referenced by [vx_det_sieve\(\)](#), [vx_init\(\)](#), and [vx_prob_sieve\(\)](#).

12.11.2.3 is_large_limit

int VX_SEG::is_large_limit

Non-zero => requires probabilistic primality checks.

Definition at line 190 of file [iZ_toolkit.h](#).

Referenced by [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), [vx_full_sieve\(\)](#), [vx_prob_sieve\(\)](#), [vx_set_base_values\(\)](#), and [vx_stream\(\)](#).

12.11.2.4 mr_rounds

int VX_SEG::mr_rounds

Miller-Rabin rounds when probabilistic checks are used.

Definition at line 191 of file [iZ_toolkit.h](#).

Referenced by [vx_init\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

12.11.2.5 p_count

int VX_SEG::p_count

Count of primes found in this segment.

Definition at line 196 of file [iZ_toolkit.h](#).

Referenced by [SiZ_count\(\)](#), [SiZ_stream\(\)](#), [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), [vx_init\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

12.11.2.6 p_gaps

[UI16_ARRAY*](#) VX_SEG::p_gaps

Optional prime-gap encoding for streamed output.

Definition at line 197 of file [iZ_toolkit.h](#).

Referenced by [vx_collect_p_gaps\(\)](#), and [vx_init\(\)](#).

12.11.2.7 p_test_ops

int VX_SEG::p_test_ops

Probabilistic primality tests executed.

Definition at line 199 of file [iZ_toolkit.h](#).

Referenced by [vx_init\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

12.11.2.8 root_limit

mpz_t VX_SEG::root_limit

$\sqrt{iZ(yvx + vx, +1)}$.

Definition at line 189 of file [iZ_toolkit.h](#).

Referenced by [vx_det_sieve\(\)](#), and [vx_set_base_values\(\)](#).

12.11.2.9 start_x

int VX_SEG::start_x

Inclusive start x for this segment.

Definition at line 192 of file [iZ_toolkit.h](#).

Referenced by [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), [vx_init\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

12.11.2.10 vx

int VX_SEG::vx

Segment width.

Definition at line 186 of file [iZ_toolkit.h](#).

Referenced by [vx_det_sieve\(\)](#), [vx_init\(\)](#), and [vx_set_base_values\(\)](#).

12.11.2.11 x5

[BITMAP*](#) [VX_SEG::x5](#)

Candidate bitmap for $6x-1$.

Definition at line 194 of file [iZ_toolkit.h](#).

Referenced by [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), [vx_init\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

12.11.2.12 x7

[BITMAP*](#) [VX_SEG::x7](#)

Candidate bitmap for $6x+1$.

Definition at line 195 of file [iZ_toolkit.h](#).

Referenced by [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), [vx_init\(\)](#), [vx_prob_sieve\(\)](#), and [vx_stream\(\)](#).

12.11.2.13 y

[mpz_t](#) [VX_SEG::y](#)

Segment index y .

Definition at line 187 of file [iZ_toolkit.h](#).

Referenced by [vx_collect_p_gaps\(\)](#), [vx_det_sieve\(\)](#), and [vx_set_base_values\(\)](#).

12.11.2.14 yvx

[mpz_t](#) [VX_SEG::yvx](#)

Cached product $y*vx$.

Definition at line 188 of file [iZ_toolkit.h](#).

Referenced by [vx_prob_sieve\(\)](#), [vx_set_base_values\(\)](#), and [vx_stream\(\)](#).

The documentation for this struct was generated from the following file:

- [include/iZ_toolkit.h](#)

Chapter 13

File Documentation

13.1 include/bitmap.h File Reference

Bitmap container and bit operations used by sieve implementations.

#include <utils.h>

Data Structures

- struct [BITMAP](#)
Packed bit-array with optional SHA-256 checksum.

Functions

- int [TEST_BITMAP](#) (int verbose)
Run bitmap module tests.

Lifecycle

- [BITMAP * bitmap_init](#) (size_t size, int set_bits)
Allocate a bitmap with size bits.
- void [bitmap_free](#) ([BITMAP **](#)bitmap)
Free a bitmap and set the caller pointer to NULL.
- [BITMAP * bitmap_clone](#) ([BITMAP *src](#))
Deep-copy a bitmap, including data and checksum.

Single-bit Operations

- int [bitmap_get_bit](#) ([BITMAP *](#)bitmap, size_t idx)
Read bit value at idx.
- void [bitmap_set_bit](#) ([BITMAP *](#)bitmap, size_t idx)
Set bit at idx to 1.
- void [bitmap_flip_bit](#) ([BITMAP *](#)bitmap, size_t idx)
Toggle bit at idx.
- void [bitmap_clear_bit](#) ([BITMAP *](#)bitmap, size_t idx)
Clear bit at idx (set to 0).

Bulk Operations

- void [bitmap_set_all](#) (BITMAP *bitmap)
Set all bits to 1.
- void [bitmap_clear_all](#) (BITMAP *bitmap)
Clear all bits to 0.
- void [bitmap_clear_steps](#) (BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
Clear every step-th bit from start_idx up to limit.
- void [bitmap_clear_steps_simd](#) (BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
SIMD-accelerated variant of [bitmap_clear_steps](#)().

Integrity and I/O

- void [bitmap_compute_hash](#) (BITMAP *bitmap)
Compute SHA-256 over bitmap data and store it in [BITMAP::sha256](#).
- int [bitmap_validate_hash](#) (BITMAP *bitmap)
Verify [BITMAP::sha256](#) against current bitmap data.
- int [bitmap_fwrite](#) (BITMAP *bitmap, FILE *file)
Write bitmap payload and checksum to a binary stream.
- BITMAP * [bitmap_fread](#) (FILE *file)
Read bitmap payload and checksum from a binary stream.

13.1.1 Detailed Description

Bitmap container and bit operations used by sieve implementations.

The bitmap API is intentionally minimal and fast: a packed bit-array plus helpers for per-bit operations, stepped clearing (for sieving), hashing, and binary serialization.

Definition in file [bitmap.h](#).

13.2 bitmap.h

[Go to the documentation of this file.](#)

```

00001
00009
00010 #ifndef BITMAP_H
00011 #define BITMAP_H
00012
00013 #include <utils.h>
00014
00018
00026 typedef struct
00027 {
00028     size_t size;
00029     size_t byte_size;
00030     unsigned char *data;
00031     unsigned char sha256[SHA256_DIGEST_LENGTH];
00032 } BITMAP;
00033
00042 BITMAP *bitmap_init(size_t size, int set_bits);
00043
00048 void bitmap_free(BITMAP **bitmap);
00049
00055 BITMAP *bitmap_clone(BITMAP *src);
00057
00066 int bitmap_get_bit(BITMAP *bitmap, size_t idx);
00067
00073 void bitmap_set_bit(BITMAP *bitmap, size_t idx);
00074
00080 void bitmap_flip_bit(BITMAP *bitmap, size_t idx);
00081

```

```

00087 void bitmap\_clear\_bit(BITMAP *bitmap, size_t idx);
00089
00096 void bitmap\_set\_all(BITMAP *bitmap);
00097
00102 void bitmap\_clear\_all(BITMAP *bitmap);
00103
00114 void bitmap\_clear\_steps(BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit);
00115
00123 void bitmap\_clear\_steps\_simd(BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit);
00125
00132 void bitmap\_compute\_hash(BITMAP *bitmap);
00133
00139 int bitmap\_validate\_hash(BITMAP *bitmap);
00140
00147 int bitmap\_fwrite(BITMAP *bitmap, FILE *file);
00148
00154 BITMAP *bitmap\_fread(FILE *file);
00156
00162 int TEST\_BITMAP(int verbose);
00163
00165
00166 #endif // BITMAP_H

```

13.3 include/int_arrays.h File Reference

Dynamic arrays for uint16_t, uint32_t, and uint64_t values.

```
#include <utils.h>
```

Data Structures

- struct [UI16_ARRAY](#)
Dynamic array for uint16_t values.
- struct [UI32_ARRAY](#)
Dynamic array for uint32_t values.
- struct [UI64_ARRAY](#)
Dynamic array for uint64_t values.

Functions

- int [TEST_GENERIC_INT_ARRAYS](#) (int verbose)
Run generic dispatch tests for C11 _Generic helper macros.

UI16 API

- [UI16_ARRAY](#) * [ui16_init](#) (int capacity)
Allocate a UI16 array with an initial capacity.
- void [ui16_free](#) ([UI16_ARRAY](#) **array)
Free a UI16 array and null the caller pointer.
- void [ui16_resize_to](#) ([UI16_ARRAY](#) *array, int new_capacity)
Resize UI16 storage to new_capacity (must be >= count).
- void [ui16_resize_to_fit](#) ([UI16_ARRAY](#) *array)
Shrink UI16 storage so capacity equals count.
- void [ui16_push](#) ([UI16_ARRAY](#) *array, uint16_t element)
Append a uint16 value, growing storage if needed.
- void [ui16_sort](#) ([UI16_ARRAY](#) *array)
Sort values in ascending order.
- void [ui16_pop](#) ([UI16_ARRAY](#) *array)

- Remove the last element if the array is non-empty.
- `void ui16_compute_hash (UI16_ARRAY *array)`
Compute SHA-256 checksum over active payload.
- `int ui16_verify_hash (UI16_ARRAY *array)`
Verify the stored checksum against current payload.
- `int ui16_fwrite (UI16_ARRAY *array, FILE *file)`
Serialize count, payload, and checksum to a binary stream.
- `UI16_ARRAY * ui16_fread (FILE *file)`
Deserialize a UI16 array from a binary stream.
- `int TEST_UI16_ARRAY (int verbose)`
Execute UI16 test suite.

UI32 API

- `UI32_ARRAY * ui32_init (int capacity)`
Allocate a UI32 array with an initial capacity.
- `void ui32_free (UI32_ARRAY **array)`
Free a UI32 array and null the caller pointer.
- `void ui32_resize_to (UI32_ARRAY *array, int new_capacity)`
Resize UI32 storage to new_capacity (must be >= count).
- `void ui32_resize_to_fit (UI32_ARRAY *array)`
Shrink UI32 storage so capacity equals count.
- `void ui32_push (UI32_ARRAY *array, uint32_t element)`
Append a uint32 value, growing storage if needed.
- `void ui32_sort (UI32_ARRAY *array)`
Sort values in ascending order.
- `void ui32_pop (UI32_ARRAY *array)`
Remove the last element if the array is non-empty.
- `void ui32_compute_hash (UI32_ARRAY *array)`
Compute SHA-256 checksum over active payload.
- `int ui32_verify_hash (UI32_ARRAY *array)`
Verify the stored checksum against current payload.
- `int ui32_fwrite (UI32_ARRAY *array, FILE *file)`
Serialize count, payload, and checksum to a binary stream.
- `UI32_ARRAY * ui32_fread (FILE *file)`
Deserialize a UI32 array from a binary stream.
- `int TEST_UI32_ARRAY (int verbose)`
Execute UI32 test suite.

UI64 API

- `UI64_ARRAY * ui64_init (int capacity)`
Allocate a UI64 array with an initial capacity.
- `void ui64_free (UI64_ARRAY **array)`
Free a UI64 array and null the caller pointer.
- `void ui64_resize_to (UI64_ARRAY *array, int new_capacity)`
Resize UI64 storage to new_capacity (must be >= count).
- `void ui64_resize_to_fit (UI64_ARRAY *array)`
Shrink UI64 storage so capacity equals count.
- `void ui64_push (UI64_ARRAY *array, uint64_t element)`
Append a uint64 value, growing storage if needed.
- `void ui64_sort (UI64_ARRAY *array)`
Sort values in ascending order.
- `void ui64_pop (UI64_ARRAY *array)`
Remove the last element if the array is non-empty.
- `void ui64_compute_hash (UI64_ARRAY *array)`
Compute SHA-256 checksum over active payload.
- `int ui64_verify_hash (UI64_ARRAY *array)`

- Verify the stored checksum against current payload.
- int ui64_fwrite ([UI64_ARRAY](#) *array, FILE *file)
Serialize count, payload, and checksum to a binary stream.
- [UI64_ARRAY](#) * ui64_fread (FILE *file)
Deserialize a UI64 array from a binary stream.
- int TEST_UI64_ARRAY (int verbose)
Execute UI64 test suite.

13.3.1 Detailed Description

Dynamic arrays for uint16_t, uint32_t, and uint64_t values.

The module provides uniform typed arrays with append/pop operations, deterministic resizing, optional SHA-256 integrity checks, and binary I/O.

Definition in file [int_arrays.h](#).

13.4 int_arrays.h

[Go to the documentation of this file.](#)

```

00001
00008
00009 #ifndef INT_ARRAYS_H
00010 #define INT_ARRAYS_H
00011
00012 #include <utils.h>
00013
00017
00019 typedef struct
00020 {
00021     int capacity;
00022     int count;
00023     uint16_t *array;
00024     int ordered;
00025     unsigned char sha256[SHA256_DIGEST_LENGTH];
00026 } UI16_ARRAY;
00027
00029 typedef struct
00030 {
00031     int capacity;
00032     int count;
00033     uint32_t *array;
00034     int ordered;
00035     unsigned char sha256[SHA256_DIGEST_LENGTH];
00036 } UI32_ARRAY;
00037
00039 typedef struct
00040 {
00041     int capacity;
00042     int count;
00043     uint64_t *array;
00044     int ordered;
00045     unsigned char sha256[SHA256_DIGEST_LENGTH];
00046 } UI64_ARRAY;
00047
00051 UI16_ARRAY *ui16_init(int capacity);
00053 void ui16_free(UI16_ARRAY **array);
00055 void ui16_resize_to(UI16_ARRAY *array, int new_capacity);
00057 void ui16_resize_to_fit(UI16_ARRAY *array);
00059 void ui16_push(UI16_ARRAY *array, uint16_t element);
00061 void ui16_sort(UI16_ARRAY *array);
00063 void ui16_pop(UI16_ARRAY *array);
00065 void ui16_compute_hash(UI16_ARRAY *array);
00067 int ui16_verify_hash(UI16_ARRAY *array);
00069 int ui16_fwrite(UI16_ARRAY *array, FILE *file);
00071 UI16_ARRAY *ui16_fread(FILE *file);
00073 int TEST_UI16_ARRAY(int verbose);
00075
00079 UI32_ARRAY *ui32_init(int capacity);
00081 void ui32_free(UI32_ARRAY **array);
00083 void ui32_resize_to(UI32_ARRAY *array, int new_capacity);

```

```

00085 void ui32_resize_to_fit(UI32_ARRAY *array);
00087 void ui32_push(UI32_ARRAY *array, uint32_t element);
00089 void ui32_sort(UI32_ARRAY *array);
00091 void ui32_pop(UI32_ARRAY *array);
00093 void ui32_compute_hash(UI32_ARRAY *array);
00095 int ui32_verify_hash(UI32_ARRAY *array);
00097 int ui32_fwrite(UI32_ARRAY *array, FILE *file);
00099 UI32_ARRAY *ui32_fread(FILE *file);
00101 int TEST_UI32_ARRAY(int verbose);
00103
00107 UI64_ARRAY *ui64_init(int capacity);
00109 void ui64_free(UI64_ARRAY **array);
00111 void ui64_resize_to(UI64_ARRAY *array, int new_capacity);
00113 void ui64_resize_to_fit(UI64_ARRAY *array);
00115 void ui64_push(UI64_ARRAY *array, uint64_t element);
00117 void ui64_sort(UI64_ARRAY *array);
00119 void ui64_pop(UI64_ARRAY *array);
00121 void ui64_compute_hash(UI64_ARRAY *array);
00123 int ui64_verify_hash(UI64_ARRAY *array);
00125 int ui64_fwrite(UI64_ARRAY *array, FILE *file);
00127 UI64_ARRAY *ui64_fread(FILE *file);
00129 int TEST_UI64_ARRAY(int verbose);
00131
00137 int TEST_GENERIC_INT_ARRAYS(int verbose);
00138
00145 #if __STDC_VERSION__ >= 201112L
00146
00148 #define int_array_free(arr) _Generic((arr), \
00149     UI16_ARRAY *: ui16_free, \
00150     UI32_ARRAY *: ui32_free, \
00151     UI64_ARRAY *: ui64_free)(arr)
00152
00154 #define int_array_resize_to(arr, cap) _Generic((arr), \
00155     UI16_ARRAY *: ui16_resize_to, \
00156     UI32_ARRAY *: ui32_resize_to, \
00157     UI64_ARRAY *: ui64_resize_to)(arr, cap)
00158
00160 #define int_array_resize_to_fit(arr) _Generic((arr), \
00161     UI16_ARRAY *: ui16_resize_to_fit, \
00162     UI32_ARRAY *: ui32_resize_to_fit, \
00163     UI64_ARRAY *: ui64_resize_to_fit)(arr)
00164
00166 #define int_array_push(arr, val) _Generic((arr), \
00167     UI16_ARRAY *: ui16_push, \
00168     UI32_ARRAY *: ui32_push, \
00169     UI64_ARRAY *: ui64_push)(arr, val)
00170
00172 #define int_array_sort(arr) _Generic((arr), \
00173     UI16_ARRAY *: ui16_sort, \
00174     UI32_ARRAY *: ui32_sort, \
00175     UI64_ARRAY *: ui64_sort)(arr)
00176
00178 #define int_array_pop(arr) _Generic((arr), \
00179     UI16_ARRAY *: ui16_pop, \
00180     UI32_ARRAY *: ui32_pop, \
00181     UI64_ARRAY *: ui64_pop)(arr)
00182
00184 #define int_array_compute_hash(arr) _Generic((arr), \
00185     UI16_ARRAY *: ui16_compute_hash, \
00186     UI32_ARRAY *: ui32_compute_hash, \
00187     UI64_ARRAY *: ui64_compute_hash)(arr)
00188
00190 #define int_array_verify_hash(arr) _Generic((arr), \
00191     UI16_ARRAY *: ui16_verify_hash, \
00192     UI32_ARRAY *: ui32_verify_hash, \
00193     UI64_ARRAY *: ui64_verify_hash)(arr)
00194
00196 #define int_array_fwrite(arr, file) _Generic((arr), \
00197     UI16_ARRAY *: ui16_fwrite, \
00198     UI32_ARRAY *: ui32_fwrite, \
00199     UI64_ARRAY *: ui64_fwrite)(arr, file)
00200
00201 #endif // __STDC_VERSION__ >= 201112L
00203
00205
00206 #endif // INT_ARRAYS_H

```

13.5 include/iZ_api.h File Reference

Public API for prime sieving, range scans, and prime generation.

```
#include <utils.h>
#include <iZ_toolkit.h>
```

Data Structures

- struct [INPUT_SIEVE_RANGE](#)
Input parameters for range sieving/counting.

Functions

Classic Prime Sieve Algorithms

Baseline sieves up to a numeric limit.

All classic sieve functions take a limit n and return an ascending list of primes $\leq n$.

- [UI64_ARRAY](#) * [SoE](#) (uint64_t n)
Optimized Sieve of Eratosthenes.
- [UI64_ARRAY](#) * [SSoE](#) (uint64_t n)
Segmented Sieve of Eratosthenes.
- [UI64_ARRAY](#) * [SoEu](#) (uint64_t n)
Euler (linear) sieve.
- [UI64_ARRAY](#) * [SoS](#) (uint64_t n)
Sieve of Sundaram.
- [UI64_ARRAY](#) * [SoA](#) (uint64_t n)
Sieve of Atkin.

iZ-based Sieve Algorithms

Sieve family operating in the $6x-1$ / $6x+1$ index space.

- [UI64_ARRAY](#) * [SiZ](#) (uint64_t n)
Solid Sieve-iZ (wheel 6, iZ index space).
- [UI64_ARRAY](#) * [SiZm](#) (uint64_t n)
Segmented Sieve-iZm (VX segmented, horizontal processing).
- [UI64_ARRAY](#) * [SiZm_vy](#) (uint64_t n)
Segmented Sieve-iZm (vertical processing; faster, unordered output).

Prime Generators

Probabilistic prime searches using iZm/VX machinery.

- int [vy_random_prime](#) (mpz_t p, int bit_size, int cores_num)
Search for a random prime using vertical (vy) sieving.
- int [vx_random_prime](#) (mpz_t p, int bit_size, int cores_num)
Search for a random prime using horizontal (vx) sieving.
- int [iZ_next_prime](#) (mpz_t p, mpz_t base, int forward)
Advance to the next (or previous) prime from a base value.

SiZ Range Variants

Count/stream primes over a numeric interval.

- typedef struct [INPUT_SIEVE_RANGE](#) [INPUT_SIEVE_RANGE](#)
Input parameters for range sieving/counting.
- uint64_t [SiZ_stream](#) ([INPUT_SIEVE_RANGE](#) *range)
Stream primes in a range to filepath (and return the count).
- uint64_t [SiZ_count](#) ([INPUT_SIEVE_RANGE](#) *input_range, int cores_num)
Count primes in a range using multiple worker processes.

13.5.1 Detailed Description

Public API for prime sieving, range scans, and prime generation.

This header provides the stable, high-level entry points for the iZ library. Most functions return a heap-allocated `UI64_ARRAY*` that must be released with `ui64_free()`.

Definition in file `iZ_api.h`.

13.6 iZ_api.h

[Go to the documentation of this file.](#)

```

00001
00009
00010 #ifndef IZ_API_H
00011 #define IZ_API_H
00012
00013 #include <utils.h>
00014 #include <iZ_toolkit.h>
00015
00026
00034
00041 UI64_ARRAY *SoE(uint64_t n);
00042
00049 UI64_ARRAY *SSoE(uint64_t n);
00050
00057 UI64_ARRAY *SoEu(uint64_t n);
00058
00065 UI64_ARRAY *SoS(uint64_t n);
00066
00073 UI64_ARRAY *SoA(uint64_t n);
00074
00076
00081
00088 UI64_ARRAY *SiZ(uint64_t n);
00089
00096 UI64_ARRAY *SiZm(uint64_t n);
00097
00108 UI64_ARRAY *SiZm_vy(uint64_t n);
00109
00111
00116
00124 typedef struct INPUT_SIEVE_RANGE
00125 {
00126     char *start;
00127     uint64_t range;
00128     int mr_rounds;
00129     char *filepath;
00130 } INPUT_SIEVE_RANGE;
00131
00137 uint64_t SiZ_stream(INPUT_SIEVE_RANGE *range);
00138
00145 uint64_t SiZ_count(INPUT_SIEVE_RANGE *input_range, int cores_num);
00146
00148
00153
00161 int vy_random_prime(mpz_t p, int bit_size, int cores_num);
00162
00170 int vx_random_prime(mpz_t p, int bit_size, int cores_num);
00171
00179 int iZ_next_prime(mpz_t p, mpz_t base, int forward);
00180
00182
00184
00185 #endif // IZ_API_H

```

13.7 include/iZ_toolkit.h File Reference

Core iZ/iZm primitives used by SiZ-family algorithms.

```

#include <utils.h>
#include <int_arrays.h>
#include <bitmap.h>

```

Data Structures

- struct [IZM](#)
Precomputed iZm assets for repeated VX-segment sieving.
- struct [VX_SEG](#)
Runtime state for one VX segment at a specific y.
- struct [IZM_RANGE_INFO](#)
Precomputed iZm coordinates for an inclusive numeric interval.

Macros

- `#define` [MR_ROUNDS](#) 25

Standard VX Sizes (primorial products excluding 2,3)

- `#define` [VX2](#) (5 * 7ULL)
- `#define` [VX3](#) ([VX2](#) * 11ULL)
- `#define` [VX4](#) ([VX3](#) * 13ULL)
- `#define` [VX5](#) ([VX4](#) * 17ULL)
- `#define` [VX6](#) ([VX5](#) * 19ULL)
- `#define` [VX7](#) ([VX6](#) * 23ULL)
- `#define` [VX8](#) ([VX7](#) * 29ULL)

Functions

- void [iZm_construct_vx_base](#) (uint64_t vx, [BITMAP](#) *base_x5, [BITMAP](#) *base_x7)
Build pre-sieved base bitmaps for a VX segment.
- [IZM_RANGE_INFO range_info_init](#) ([INPUT_SIEVE_RANGE](#) *input_range, int vx)
Map a decimal range input into iZm coordinates and segment bounds.
- void [range_info_free](#) ([IZM_RANGE_INFO](#) *info)
Clear all GMP fields owned by info.

iZ Mapping Helpers

- uint64_t [iZ](#) (uint64_t x, int i)
Map iZ coordinates to an integer: 6*x + i.
- void [iZ_mpz](#) (mpz_t z, mpz_t x, int i)
GMP variant of [iZ\(\)](#).
- void [process_iZ_bitmaps](#) ([UI64_ARRAY](#) *primes, [BITMAP](#) *x5, [BITMAP](#) *x7, uint64_t x↔_limit)
Traverse iZ bitmaps, emit surviving primes, and mark composites.
- void [get_root_primes](#) ([UI64_ARRAY](#) *primes, uint64_t limit)
Generate primes up to limit for deterministic sieving.
- int [check_primality](#) (mpz_t n, int rounds)
Check the primality of a number using GMP's probabilistic test.

IZM Lifecycle

- [IZM](#) * [iZm_init](#) (size_t vx)
Allocate and initialize an [IZM](#) object for a given VX.
- [IZM](#) * [iZm_clone](#) ([IZM](#) *src)
Deep-copy an [IZM](#) object for per-worker ownership.
- void [iZm_free](#) ([IZM](#) **iZm)
Release an [IZM](#) object and set the caller pointer to NULL.

VX Selection Helpers

- `uint64_t compute_vx_k` (int k)
Calculate $VX_{\{k\}}$.
- `uint64_t compute_l2_vx` (uint64_t n)
Choose VX using an L2-cache-aware heuristic.
- `void compute_max_vx` (mpz_t vx, int bit_size)
Compute largest VX below $2^{\text{bit_size}}$.

Modular Hit Solvers

- `uint64_t iZm_solve_for_x0` (int m_id, uint64_t p, uint64_t vx, uint64_t y)
Solve first x-hit of prime p for line m_id in segment y.
- `uint64_t iZm_solve_for_x0_mpz` (int m_id, uint64_t p, uint64_t vx, mpz_t y)
GMP variant of `iZm_solve_for_x0()` for very large y.
- `int64_t iZm_solve_for_y0` (int m_id, uint64_t p, uint64_t vx, uint64_t x)
Solve first y-hit for fixed x in vertical (vy) scanning.

VX Segment Lifecycle and Execution

- `VX_SEG * vx_init` (IZM *iZm, int start_x, int end_x, char *y_str, int mr_rounds)
Initialize and deterministically sieve one VX segment.
- `void vx_free` (VX_SEG **vx_obj)
Free a VX segment and all owned resources.
- `void vx_collect_p_gaps` (VX_SEG *vx_obj)
Extract prime-gap encoding from a fully sieved segment.
- `void vx_full_sieve` (VX_SEG *vx_obj, int collect_p_gaps)
Complete segment processing (probabilistic stage and optional gaps).
- `void vx_stream` (VX_SEG *vx_obj, FILE *output)
Stream segment primes to an output stream.

Random Prime Search Routines

- `int vx_search_prime` (mpz_t p, int m_id, int vx, int bit_size)
Horizontal iZm/VX random-prime search.
- `int vy_search_prime` (mpz_t p, int m_id, mpz_t vx)
Vertical iZm/VY random-prime search.

Toolkit Tests

- `int TEST_IZM` (int verbose)
Run IZM construction and solver tests.
- `int TEST_VX_SEG` (int verbose)
Run VX segment tests.

13.7.1 Detailed Description

Core iZ/iZm primitives used by SiZ-family algorithms.

This layer exposes index-space mapping helpers, VX sizing utilities, pre-sieved iZm base construction, per-segment sieve objects, and internal prime-search routines used by high-level APIs in [iZ_api.h](#).

Definition in file [iZ_toolkit.h](#).

13.8 iZ_toolkit.h

[Go to the documentation of this file.](#)

```

00001
00009
00010 #ifndef IZ_TOOLKIT_H
00011 #define IZ_TOOLKIT_H
00012
00013 #include <utils.h> // Common utilities, types, and dependencies.
00014 #include <int_arrays.h> // Integer array containers.
00015 #include <bitmap.h> // Packed bit-array utilities.
00016
00018 typedef struct INPUT_SIEVE_RANGE INPUT_SIEVE_RANGE;
00019
00023
00032 uint64_t iZ(uint64_t x, int i);
00033
00040 void iZ_mpz(mpz_t z, mpz_t x, int i);
00041
00049 void process_iZ_bitmaps(UI64_ARRAY *primes, BITMAP *x5, BITMAP *x7, uint64_t x_limit);
00050
00056 void get_root_primes(UI64_ARRAY *primes, uint64_t limit);
00057
00064 int check_primality(mpz_t n, int rounds);
00066
00069 #define VX2 (5 * 7ULL)
00070 #define VX3 (VX2 * 11ULL)
00071 #define VX4 (VX3 * 13ULL)
00072 #define VX5 (VX4 * 17ULL)
00073 #define VX6 (VX5 * 19ULL)
00074 #define VX7 (VX6 * 23ULL)
00075 #define VX8 (VX7 * 29ULL)
00077
00079 #define MR_ROUNDS 25
00080
00084 typedef struct
00085 {
00086     int vx;
00087     int k_vx;
00088     BITMAP *base_x5;
00089     BITMAP *base_x7;
00090     UI64_ARRAY *root_primes;
00091 } IZM;
00092
00100 IZM *iZm_init(size_t vx);
00101
00107 IZM *iZm_clone(IZM *src);
00108
00113 void iZm_free(IZM **iZm);
00115
00123 uint64_t compute_vx_k(int k);
00124
00130 uint64_t compute_l2_vx(uint64_t n);
00131
00137 void compute_max_vx(mpz_t vx, int bit_size);
00139
00146 void iZm_construct_vx_base(uint64_t vx, BITMAP *base_x5, BITMAP *base_x7);
00147
00158 uint64_t iZm_solve_for_x0(int m_id, uint64_t p, uint64_t vx, uint64_t y);
00159
00168 uint64_t iZm_solve_for_x0_mpz(int m_id, uint64_t p, uint64_t vx, mpz_t y);
00169
00178 int64_t iZm_solve_for_y0(int m_id, uint64_t p, uint64_t vx, uint64_t x);
00180
00184 typedef struct
00185 {
00186     int vx;
00187     mpz_t y;
00188     mpz_t yvx;
00189     mpz_t root_limit;
00190     int is_large_limit;
00191     int mr_rounds;
00192     int start_x;
00193     int end_x;
00194     BITMAP *x5;
00195     BITMAP *x7;
00196     int p_count;
00197     UI64_ARRAY *p_gaps;
00198     int bit_ops;
00199     int p_test_ops;
00200 } VX_SEG;
00201
00213 VX_SEG *vx_init(IZM *iZm, int start_x, int end_x, char *y_str, int mr_rounds);
00214
00219 void vx_free(VX_SEG **vx_obj);

```

```

00220
00225 void vx_collect_p_gaps(VX_SEG *vx_obj);
00226
00232 void vx_full_sieve(VX_SEG *vx_obj, int collect_p_gaps);
00233
00239 void vx_stream(VX_SEG *vx_obj, FILE *output);
00241
00254 typedef struct
00255 {
00256     int vx;
00257     mpz_t Zs;
00258     mpz_t Ze;
00259     mpz_t Xs;
00260     mpz_t Xe;
00261     mpz_t Ys;
00262     mpz_t Ye;
00263     int y_range;
00264 } IZM_RANGE_INFO;
00265
00273 IZM_RANGE_INFO range_info_init(INPUT_SIEVE_RANGE *input_range, int vx);
00275 void range_info_free(IZM_RANGE_INFO *info);
00276
00287 int vx_search_prime(mpz_t p, int m_id, int vx, int bit_size);
00288
00296 int vy_search_prime(mpz_t p, int m_id, mpz_t vx);
00298
00302 int TEST_IZM(int verbose);
00304 int TEST_VX_SEG(int verbose);
00306
00308
00309 #endif // IZ_TOOLKIT_H

```

13.9 include/logger.h File Reference

Header file for the logging system.

```

#include <stdio.h>
#include <stdlib.h>
#include <string.h>

```

Macros

- #define `LOGGER_FORMAT_PRINTF`(fmt_idx, arg_idx)
Compiler attribute wrapper for printf-style format checking.
- #define `LOG_DIR` "logs/"
Directory where logs are stored.
- #define `LOG_FILE LOG_DIR` "log.txt"
Default log file.
- #define `LOG_MAX_SIZE` 1024 * 1024 * 5
Maximum log file size (5 MB).

Enumerations

- enum `LogLevel` {
 `LOG_DEBUG` , `LOG_INFO` , `LOG_WARNING` , `LOG_ERROR` ,
 `LOG_FATAL` }
Enumeration of log levels.

Functions

- `const char * log_level_to_string (LogLevel level)`
Returns a string representation of the log level.
- `void log_init (const char *log_file)`
Initializes the logging system.
- `void log_shutdown (void)`
Shuts down the logging system and cleans up resources.
- `void log_set_log_level (LogLevel level)`
Sets the current log level. Messages below this level will not be logged.
- `void log_message (LogLevel level, const char *format,...) LOGGER_FORMAT_PRINTF(2)`
Logs a formatted message at the given log level.
- `void log_message_extended (LogLevel level, const char *file_name, int line_number, const char *format,...) LOGGER_FORMAT_PRINTF(4)`
Logs a formatted message with extended information (file name, line number).
- `void log_console (const char *format,...) LOGGER_FORMAT_PRINTF(1)`
Logs a timestamped message to the console only, without requiring a log level.
- `void log_debug (const char *format,...) LOGGER_FORMAT_PRINTF(1)`
Logs a debug message.
- `void log_info (const char *format,...) LOGGER_FORMAT_PRINTF(1)`
Logs an info message.
- `void log_warn (const char *format,...) LOGGER_FORMAT_PRINTF(1)`
Logs a warning message.
- `void log_error (const char *format,...) LOGGER_FORMAT_PRINTF(1)`
Logs an error message.
- `void log_fatal (const char *format,...) LOGGER_FORMAT_PRINTF(1)`
Logs a fatal message.

13.9.1 Detailed Description

Header file for the logging system.

This file contains function declarations and macros for a logging system that provides various logging levels (DEBUG, INFO, WARNING, ERROR, FATAL). The logging system supports thread-safe logging, log rotation, and console output. It also includes convenience functions for logging messages with extended information (file name, line number).

Note

This module is part of a larger project that includes various components requiring logging functionality. It is designed to be flexible and easy to use, allowing developers to log messages at different levels and with different formats.

API:

- `log_init`: Initializes the logging system.
- `log_shutdown`: Shuts down the logging system.
- `log_set_log_level`: Sets the current log level.
- `log_message`: Logs a formatted message at the given log level.

- `log_message_extended`: Logs a formatted message with extended information.
- `log_console`: Logs a formatted message to the console.
- `log_debug`: Logs a debug message.
- `log_info`: Logs an info message.
- `log_warn`: Logs a warning message.
- `log_error`: Logs an error message.
- `log_fatal`: Logs a fatal message.

Definition in file [logger.h](#).

13.10 logger.h

[Go to the documentation of this file.](#)

```

00001
00031
00032 #ifndef LOGGER_H
00033 #define LOGGER_H
00034
00035 #include <stdio.h>
00036 #include <stdlib.h>
00037 #include <string.h>
00038
00042
00047 #if defined(__GNUC__) || defined(__clang__)
00048 #define LOGGER_FORMAT_PRINTF(fmt_idx, arg_idx) __attribute__((format(printf, fmt_idx, arg_idx)))
00049 #else
00050 #define LOGGER_FORMAT_PRINTF(fmt_idx, arg_idx)
00051 #endif
00052
00053 // Log directory and file configuration
00054 #define LOG_DIR "logs/"
00055 #define LOG_FILE LOG_DIR "log.txt"
00056 #define LOG_MAX_SIZE 1024 * 1024 * 5
00057
00065 typedef enum
00066 {
00067     LOG_DEBUG,
00068     LOG_INFO,
00069     LOG_WARNING,
00070     LOG_ERROR,
00071     LOG_FATAL
00072 } LogLevel;
00073
00074 // Function declarations
00075
00082 const char *log_level_to_string(LogLevel level);
00083
00091 void log_init(const char *log_file);
00092
00096 void log_shutdown(void);
00097
00103 void log_set_log_level(LogLevel level);
00104
00113 void log_message(LogLevel level, const char *format, ...) LOGGER_FORMAT_PRINTF(2, 3);
00114
00125 void log_message_extended(LogLevel level, const char *file_name, int line_number, const char *format, ...)
    LOGGER_FORMAT_PRINTF(4, 5);
00126
00134 void log_console(const char *format, ...) LOGGER_FORMAT_PRINTF(1, 2);
00135
00136 // Convenience logging functions for different log levels
00142 void log_debug(const char *format, ...) LOGGER_FORMAT_PRINTF(1, 2);
00143
00149 void log_info(const char *format, ...) LOGGER_FORMAT_PRINTF(1, 2);
00150
00156 void log_warn(const char *format, ...) LOGGER_FORMAT_PRINTF(1, 2);
00157
00163 void log_error(const char *format, ...) LOGGER_FORMAT_PRINTF(1, 2);
00164
00170 void log_fatal(const char *format, ...) LOGGER_FORMAT_PRINTF(1, 2);
00171
00173
00174 #endif // LOGGER_H

```

13.11 include/platform.h File Reference

Cross-platform system abstraction utilities.

```
#include <stddef.h>
#include <stdint.h>
#include <time.h>
#include <sys/types.h>
#include <sys/stat.h>
#include <fcntl.h>
#include <unistd.h>
#include <signal.h>
#include <sys/wait.h>
```

Macros

- `#define IZ_PLATFORM_WINDOWS 0`
- `#define IZ_PLATFORM_POSIX 1`
- `#define IZ_PLATFORM_HAS_FORK 1`

Functions

- `int iz_platform_create_dir (const char *dir)`
Create a directory if it does not already exist.
- `int iz_platform_fill_random (void *buffer, size_t bytes)`
Fill a buffer with random bytes.
- `int iz_platform_cpu_cores_count (void)`
Return number of online logical CPU cores (≥ 1).
- `int iz_platform_l2_cache_size_bits (void)`
Best-effort L2 cache size query in bits.
- `double iz_platform_monotonic_seconds (void)`
Return a monotonic timestamp in seconds.

13.11.1 Detailed Description

Cross-platform system abstraction utilities.

This header centralizes OS-dependent capabilities used by iZprime. It provides a single layer for process capability flags, entropy, monotonic timing, directory creation, and hardware query helpers.

Definition in file [platform.h](#).

13.11.2 Macro Definition Documentation

13.11.2.1 IZ_PLATFORM_HAS_FORK

```
#define IZ_PLATFORM_HAS_FORK 1
```

Definition at line 34 of file [platform.h](#).

13.11.2.2 IZ_PLATFORM_POSIX

```
#define IZ_PLATFORM_POSIX 1
```

Definition at line 33 of file [platform.h](#).

13.11.2.3 IZ_PLATFORM_WINDOWS

```
#define IZ_PLATFORM_WINDOWS 0
```

Definition at line 32 of file [platform.h](#).

13.12 platform.h

[Go to the documentation of this file.](#)

```
00001
00009
00010 #ifndef IZ_PLATFORM_H
00011 #define IZ_PLATFORM_H
00012
00013 #include <stddef.h>
00014 #include <stdint.h>
00015 #include <time.h>
00016
00017 #if defined(_WIN32) || defined(_WIN64)
00018 #define IZ_PLATFORM_WINDOWS 1
00019 #define IZ_PLATFORM_POSIX 0
00020 #define IZ_PLATFORM_HAS_FORK 0
00021
00022 #ifndef WIN32_LEAN_AND_MEAN
00023 #define WIN32_LEAN_AND_MEAN
00024 #endif
00025 #ifndef NOMINMAX
00026 #define NOMINMAX
00027 #endif
00028 #include <windows.h>
00029 #include <direct.h>
00030
00031 #else
00032 #define IZ_PLATFORM_WINDOWS 0
00033 #define IZ_PLATFORM_POSIX 1
00034 #define IZ_PLATFORM_HAS_FORK 1
00035
00036 #include <sys/types.h>
00037 #include <sys/stat.h>
00038 #include <fcntl.h>
00039 #include <unistd.h>
00040 #include <signal.h>
00041 #include <sys/wait.h>
00042
00043 #if defined(__APPLE__) || defined(__FreeBSD__) || defined(__OpenBSD__) || defined(__NetBSD__)
00044 #include <sys/sysctl.h>
00045 #endif
00046 #endif
00047
00052
00058 int iz_platform_create_dir(const char *dir);
00059
00066 int iz_platform_fill_random(void *buffer, size_t bytes);
00067
00072 int iz_platform_cpu_cores_count(void);
00073
00078 int iz_platform_l2_cache_size_bits(void);
00079
00084 double iz_platform_monotonic_seconds(void);
00085
00087
00088 #endif // IZ_PLATFORM_H
```

13.13 include/test_api.h File Reference

Public test/benchmark entry points for the iZ library.

```
#include <iZ_api.h>
```

Data Structures

- struct [SIEVE_LIMIT](#)
- struct [SIEVE_MODEL](#)

Typedefs

- typedef [UI64_ARRAY](#) [*\(* SIEVE_FN\)](#) (uint64_t)

Functions

Sieve model tests/benchmarks

- int [TEST_UTILS](#) (int verbose)
Validate utility helpers (numeric/range parsers and shared utilities).
- int [TEST_SIEVE_MODELS_INTEGRITY](#) (int verbose)
Run correctness checks across all registered sieve models.
- void [BENCHMARK_SIEVE_MODELS](#) (int save_results)
Benchmark registered sieve models and optionally persist results.

Range sieve tests/benchmarks

- int [TEST_SiZ_stream](#) (int verbose)
Validate [SiZ_stream](#) behavior and output formatting.
- int [TEST_SiZ_count](#) (int verbose)
Validate [SiZ_count](#) correctness across worker counts.
- void [BENCHMARK_SiZ_count](#) (int save_results)
Benchmark [SiZ_count](#) over increasing ranges starting from 10^4 , 10^6 , ..., 10^{100} using max cores.

Prime generation tests/benchmarks

- int [TEST_iZ_next_prime](#) (int verbose)
Validate [iZ_next_prime](#) for forward/backward traversal cases.
- int [TEST_vy_random_prime](#) (int verbose)
Validate [vy_random_prime](#) for primality and bit-length constraints.
- int [TEST_vx_random_prime](#) (int verbose)
Validate [vx_random_prime](#) for primality and bit-length constraints.
- int [BENCHMARK_P_GEN_ALGORITHMS](#) (int bit_size, int test_rounds, int save_results)
Benchmark random-prime generation routines over repeated trials.

13.13.1 Detailed Description

Public test/benchmark entry points for the iZ library.

Doxygen can include this header when documenting the project to expose the test harness API and benchmark entry points.

Definition in file [test_api.h](#).

13.14 test_api.h

[Go to the documentation of this file.](#)

```

00001
00008
00009 #ifndef TESTS_H
00010 #define TESTS_H
00011
00016
00017 #include <iZ_api.h>
00018
00020 typedef UI64_ARRAY *(*SIEVE_FN)(uint64_t);
00021
00023 typedef struct
00024 {
00025     int base;
00026     int exp;
00027 } SIEVE_LIMIT;
00028
00030 typedef struct
00031 {
00032     SIEVE_FN function;
00033     const char name[32];
00034 } SIEVE_MODEL;
00035
00038
00039 int TEST_UTILS(int verbose);
00041 int TEST_SIEVE_MODELS_INTEGRITY(int verbose);
00043 void BENCHMARK_SIEVE_MODELS(int save_results);
00045
00048
00049 int TEST_SiZ_stream(int verbose);
00051 int TEST_SiZ_count(int verbose);
00053 void BENCHMARK_SiZ_count(int save_results);
00055
00058
00059 int TEST_iZ_next_prime(int verbose);
00061 int TEST_vy_random_prime(int verbose);
00063 int TEST_vx_random_prime(int verbose);
00064
00066 int BENCHMARK_P_GEN_ALGORITHMS(int bit_size, int test_rounds, int save_results);
00068
00070
00071 #endif // TESTS_H

```

13.15 include/Utils.h File Reference

Shared utilities and common includes for the iZprime library.

```

#include <stdlib.h>
#include <stdint.h>
#include <stddef.h>
#include <string.h>
#include <assert.h>
#include <math.h>
#include <stdio.h>
#include <stdarg.h>
#include <time.h>
#include <platform.h>
#include <logger.h>
#include <gmp.h>
#include <openssl/sha.h>

```

Data Structures

- struct [STOPWATCH](#)
Stopwatch state for elapsed wall-clock measurements.

Macros

- `#define MIN(a, b)`
Return the smaller of two expressions.
- `#define MAX(a, b)`
Return the larger of two expressions.
- `#define DIR_output "/output"`
- `#define MAX_CORES get_cpu_cores_count()`

Functions

- `int create_dir (const char *dir)`
Create a directory if it does not exist.
- `int is_numeric_str (const char *str)`
Return non-zero if str contains only ASCII digits.
- `int parse_numeric_expr_mpz (mpz_t out, const char *expr)`
Parse an integer expression into an mpz value.
- `int parse_numeric_expr_u64 (const char *expr, uint64_t *out)`
Parse an integer expression into uint64_t.
- `int parse_inclusive_range_mpz (const char *range_expr, mpz_t lower, mpz_t upper)`
Parse an inclusive range expression into lower/upper bounds.
- `uint64_t gcd (uint64_t a, uint64_t b)`
Compute the greatest common divisor of a and b.
- `uint64_t modular_inverse (uint64_t a, uint64_t m)`
Compute the modular inverse of a modulo m.
- `void gmp_seed_randstate (gmp_randstate_t state)`
Seed a GMP random state.
- `int get_cpu_cores_count (void)`
Get the number of online CPU cores (≥ 1).
- `int get_cpu_L2_cache_size_bits (void)`
Return the CPU L2 cache size in bits (best effort).
- `void print_sha256_hash (const unsigned char *hash)`
Print a SHA-256 digest as hex to stdout.
- `void print_line (int length, char fill_char)`
Print a repeated-character horizontal line.
- `void print_centered_text (const char *text, int line_length, char fill_char)`
Print text centered inside a padded line.
- `void print_test_table_header (void)`
Print the generic test runner header.
- `void print_test_module_header (char *module_name)`
Print a test-suite header banner for a module.
- `void print_test_fn_header (char *fn_name)`
Print a formatted header for a test function.
- `void print_test_module_result (int result, int test_id, const char *unit_name, const char *format,...)`
Print a single test-row result.
- `void print_test_summary (char *module_name, int passed, int failed, int verbose)`
Print module-level test summary.
- `void sw_start (STOPWATCH *sw)`
Start or restart a stopwatch.
- `void sw_stop (STOPWATCH *sw)`

- Stop a running stopwatch, capturing the elapsed time in seconds.
- double `sw_elapsed_seconds` (const `STOPWATCH *sw`)
Return elapsed seconds for the stopwatch.
- double `sw_elapsed_now_seconds` (void)
Capture the current monotonic time in seconds.

13.15.1 Detailed Description

Shared utilities and common includes for the iZprime library.

This header centralizes cross-cutting helpers (string checks, arithmetic, formatting, GMP helpers, and system queries).

Definition in file [utils.h](#).

13.16 `utils.h`

[Go to the documentation of this file.](#)

```

00001
00008
00009 #ifndef IZ_UTILS_H
00010 #define IZ_UTILS_H
00011
00012 // Standard library includes
00013 #include <stdlib.h> // For malloc, free, etc.
00014 #include <stdint.h> // For fixed-width integer types like uint64_t
00015 #include <stddef.h> // For size_t
00016 #include <string.h> // For string manipulation functions like snprintf
00017 #include <assert.h> // For assertions
00018 #include <math.h> // For math functions like sqrt
00019 #include <stdio.h> // For printf, FILE, fopen, fwrite, fread, etc.
00020 #include <stdarg.h> // For variadic functions (va_list, va_start, va_end)
00021 #include <time.h> // For struct timespec and time-based helpers
00022
00023 #include <platform.h> // OS abstraction layer (process/timing/system helpers)
00024
00025 // Internal includes
00026 #include <logger.h> // Logger module for logging timestamped messages.
00027
00028 // Third-party libraries
00029 #include <gmp.h> // GMP library for arbitrary precision arithmetic
00030 #include <openssl/sha.h> // For SHA-256 hashing
00031
00036
00037 #ifndef MIN
00039 #define MIN(a, b) (((a) < (b)) ? (a) : (b))
00040 #endif
00041 #ifndef MAX
00043 #define MAX(a, b) (((a) > (b)) ? (a) : (b))
00044 #endif
00045
00047 #define DIR_output "/output"
00049 #define MAX_CORES get_cpu_cores_count()
00050
00051 // file utilities
00057 int create_dir(const char *dir);
00058
00059 // string utilities
00065 int is_numeric_str(const char *str);
00066
00080 int parse_numeric_expr_mpz(mpz_t out, const char *expr);
00081
00088 int parse_numeric_expr_u64(const char *expr, uint64_t *out);
00089
00107 int parse_inclusive_range_mpz(const char *range_expr, mpz_t lower, mpz_t upper);
00108
00109 // math utilities
00111 uint64_t gcd(uint64_t a, uint64_t b);
00116 uint64_t modular_inverse(uint64_t a, uint64_t m);
00117
00118 // gmp utilities

```



```

00124 void gmp_seed_randstate(gmp_randstate_t state);
00125
00126 // system utilities
00128 int get_cpu_cores_count(void);
00133 int get_cpu_L2_cache_size_bits(void);
00134
00136
00141
00146 void print_sha256_hash(const unsigned char *hash);
00147
00153 void print_line(int length, char fill_char);
00154
00161 void print_centered_text(const char *text, int line_length, char fill_char);
00162
00164 void print_test_table_header(void);
00165
00167 void print_test_module_header(char *module_name);
00168
00170 void print_test_fn_header(char *fn_name);
00171
00179 void print_test_module_result(int result, int test_id, const char *unit_name, const char *format, ...);
00180
00188 void print_test_summary(char *module_name, int passed, int failed, int verbose);
00189
00191
00196
00200 typedef struct
00201 {
00202     struct timespec start_time;
00203     struct timespec end_time;
00204     int running;
00205     double elapsed_sec;
00206 } STOPWATCH;
00207
00212 void sw_start(STOPWATCH *sw);
00213
00218 void sw_stop(STOPWATCH *sw);
00219
00229 double sw_elapsed_seconds(const STOPWATCH *sw);
00230
00235 double sw_elapsed_now_seconds(void);
00236
00238
00239 #endif // IZ_UTILS_H

```

13.17 src/iZ_apps.c File Reference

High-level application routines built on top of the iZ_toolkit.

```
#include <iZ_api.h>
```

Functions

SiZ Range Variants

Count/stream primes over a numeric interval.

- uint64_t `SiZ_stream` (`INPUT_SIEVE_RANGE` *range)
Stream primes in a range to filepath (and return the count).
- uint64_t `SiZ_count` (`INPUT_SIEVE_RANGE` *input_range, int cores_num)
Count primes in a range using multiple worker processes.

Prime Generators

Probabilistic prime searches using iZm/VX machinery.

- int `vy_random_prime` (mpz_t p, int bit_size, int cores_num)
Search for a random prime using vertical (vy) sieving.
- int `vx_random_prime` (mpz_t p, int bit_size, int cores_num)
Search for a random prime using horizontal (vx) sieving.
- int `iZ_next_prime` (mpz_t p, mpz_t base, int forward)
Advance to the next (or previous) prime from a base value.

13.17.1 Detailed Description

High-level application routines built on top of the `iZ_toolkit`.

This module contains range-oriented entry points (stream/count) and probabilistic prime generation wrappers.

Definition in file `iZ_apps.c`.

13.18 iZ_apps.c

[Go to the documentation of this file.](#)

```

00001
00009
00010 #include <iZ_api.h>
00011
00012 // =====
00013 // * SiZ Range Variants
00014 // =====
00015
00035 uint64_t SiZ_stream(INPUT_SIEVE_RANGE *input_range)
00036 {
00037     assert(input_range && input_range->start && "Invalid INPUT_SIEVE_RANGE passed to SiZ_stream.");
00038
00039     int has_output_file = (input_range->filepath && input_range->filepath[0] != '\0');
00040     FILE *output = stdout; // default to stdout if no valid filepath is provided
00041
00042     if (has_output_file)
00043     {
00044         output = fopen(input_range->filepath, "w");
00045         if (output == NULL)
00046         {
00047             log_error("Failed to open output file: %s", input_range->filepath);
00048             return 0;
00049         }
00050     }
00051
00052     uint64_t total = 0; // output: total prime count
00053
00054     int vx = VX6; // Use VX6 segment size (1,616,615 bits) for optimal results
00055     // Miller-Rabin rounds, bounded [5, 50]
00056     int mr_rounds = MIN(MAX(input_range->mr_rounds, 5), 50);
00057
00058     IZM_RANGE_INFO info = range_info_init(input_range, vx);
00059     if (info.y_range < 0)
00060     {
00061         if (has_output_file)
00062             fclose(output);
00063         range_info_free(&info);
00064         return 0;
00065     }
00066
00067     IZM *iZm = NULL;
00068     mpz_t current_y;
00069     mpz_init(current_y);
00070     mpz_set(current_y, info.Ys);
00071     int start_x = mpz_fdiv_ui(info.Xs, info.vx);
00072     int end_x = mpz_fdiv_ui(info.Xe, info.vx);
00073
00074     // if Ys = 0, use SiZm for the first segment
00075     if (mpz_cmp_ui(current_y, 0) == 0)
00076     {
00077         uint64_t limit = mpz_cmp_ui(info.Ye, 0) > 0 ? info.vx : end_x;
00078         UI64_ARRAY *primes = SiZm(limit * 6 + 1);
00079         if (!primes)
00080         {
00081             total = 0;
00082             goto stream_cleanup;
00083         }
00084
00085         uint64_t s = mpz_get_ui(info.Zs);
00086         uint64_t e = mpz_get_ui(info.Ze);
00087
00088         for (int i = 0; i < primes->count; i++)
00089         {
00090             // only primes in [Zs, Ze]
00091             if (primes->array[i] > s && primes->array[i] <= e)

```

```

00092     {
00093         total++;
00094         fprintf(output, "%llu ", primes->array[i]);
00095     }
00096 }
00097
00098     start_x = 1; // next segment starts at x=1
00099     mpz_add_ui(current_y, current_y, 1); // increment Ys for the next segment
00100
00101     ui64_free(&primes);
00102 }
00103
00104 // No remaining vx segments after the initial one.
00105 if (mpz_cmp(current_y, info.Ye) > 0)
00106 {
00107     goto stream_cleanup;
00108 }
00109
00110 // Initialize iZm structure for vx segments
00111 iZm = iZm_init(vx);
00112 if (!iZm)
00113 {
00114     total = 0;
00115     goto stream_cleanup;
00116 }
00117
00118 // Process remaining segments for y in [current_y:Ye]
00119 int first_segment = 1;
00120 while (mpz_cmp(current_y, info.Ye) <= 0)
00121 {
00122     int seg_start_x = first_segment ? start_x : 1;
00123     int seg_end_x = (mpz_cmp(current_y, info.Ye) == 0) ? end_x : vx;
00124     char *y_str = mpz_get_str(NULL, 10, current_y);
00125     if (!y_str)
00126     {
00127         total = 0;
00128         goto stream_cleanup;
00129     }
00130
00131     VX_SEG *vx_obj = vx_init(iZm, seg_start_x, seg_end_x, y_str, mr_rounds);
00132     free(y_str);
00133     if (!vx_obj)
00134     {
00135         // check logs for errors
00136         total = 0;
00137         goto stream_cleanup;
00138     }
00139
00140     vx_stream(vx_obj, output);
00141     total += vx_obj->p_count; // accumulate prime count
00142
00143     vx_free(&vx_obj);
00144     first_segment = 0;
00145     mpz_add_ui(current_y, current_y, 1); // increment Ys for the next segment
00146 }
00147
00148 stream_cleanup:
00149     iZm_free(&iZm);
00150     range_info_free(&info);
00151     mpz_clear(current_y);
00152     if (has_output_file)
00153         fclose(output);
00154     else
00155         fflush(stdout);
00156
00157     return total;
00158 }
00159
00184 uint64_t SiZ_count(INPUT_SIEVE_RANGE *input_range, int cores_num)
00185 {
00186     assert(input_range && input_range->start && input_range->range > 100 &&
00187         "Invalid INPUT_SIEVE_RANGE passed to SiZ_count.");
00188
00189     uint64_t total = 0;
00190     int vx = compute_l2_vx(pow(10, 9)); // Use a segment width that balances workload and overhead for 10^9 range
00191     cores_num = MAX(1, MIN(cores_num, get_cpu_cores_count()));
00192 #if !IZ_PLATFORM_HAS_FORK
00193     if (cores_num > 1)
00194     {
00195         log_info("SiZ_count: multi-process mode is unavailable on this platform; using single-process mode.");
00196         cores_num = 1;
00197     }
00198 #endif
00199     IZM *iZm = NULL;
00200 #if IZ_PLATFORM_HAS_FORK
00201     int (*pipe_fds)[2] = NULL;
00202     pid_t *pids = NULL;

```

```

00203 #endif
00204
00205 IZM_RANGE_INFO info = range_info_init(input_range, vx);
00206 if (info.y_range < 0)
00207 {
00208     range_info_free(&info);
00209     return 0;
00210 }
00211
00212 mpz_t current_y;
00213 mpz_init(current_y);
00214 mpz_set(current_y, info.Ys);
00215 int start_x = mpz_fdiv_ui(info.Xs, vx);
00216 int end_x = mpz_fdiv_ui(info.Xe, vx);
00217
00218 // if current_y = 0, use sieve_iZm for the first segment
00219 if (mpz_cmp_ui(current_y, 0) == 0)
00220 {
00221     uint64_t limit = mpz_cmp_ui(info.Ye, 0) > 0 ? vx : end_x;
00222     UI64_ARRAY *primes = SiZm(limit * 6 + 1);
00223     if (!primes)
00224     {
00225         total = 0;
00226         goto count_cleanup;
00227     }
00228
00229     total += primes->count;
00230     uint64_t s = mpz_get_ui(info.Zs);
00231     uint64_t e = mpz_get_ui(info.Ze);
00232
00233     for (int i = 0; i < primes->count; i++)
00234     {
00235         // Exclude values outside [Zs, Ze] from the first solid segment.
00236         if (primes->array[i] < s || primes->array[i] > e)
00237         {
00238             total--;
00239         }
00240     }
00241
00242     ui64_free(&primes);
00243     start_x = 1;
00244     mpz_add_ui(current_y, current_y, 1); // increment Ys for the next segment
00245 }
00246
00247 if (mpz_cmp(current_y, info.Ye) > 0)
00248 {
00249     goto count_cleanup;
00250 }
00251
00252 iZm = iZm_init(vx);
00253 if (!iZm)
00254 {
00255     total = 0;
00256     goto count_cleanup;
00257 }
00258
00259 // Handle edge cases:
00260 mpz_t prime_z;
00261 mpz_init(prime_z);
00262 // if Ys > 0 and Zs % 6 <= 1
00263 if (mpz_cmp_ui(current_y, 0) > 0 && mpz_fdiv_ui(info.Zs, 6) <= 1)
00264 {
00265     // if iZ(Xs, -1) < Zs and prime, decrement total
00266     iZ_mpz(prime_z, info.Xs, -1);
00267     if (mpz_cmp(prime_z, info.Zs) < 0)
00268     {
00269         if (mpz_probab_prime_p(prime_z, 25))
00270         {
00271             total--;
00272         }
00273     }
00274 }
00275 // if Ye > 0 and Ze % 6 <= 1
00276 if (mpz_cmp_ui(info.Ye, 0) > 0 && mpz_fdiv_ui(info.Ze, 6) <= 1)
00277 {
00278     // if iZ(Xe, 1) > Ze and prime, decrement total
00279     iZ_mpz(prime_z, info.Xe, 1);
00280     if (mpz_cmp(prime_z, info.Ze) > 0)
00281     {
00282         if (mpz_probab_prime_p(prime_z, 25))
00283         {
00284             total--;
00285         }
00286     }
00287 }
00288 mpz_clear(prime_z);
00289

```

```

00290 int total_segments = (int)(mpz_get_ui(info.Ye) - mpz_get_ui(current_y) + 1);
00291 if (total_segments <= 0)
00292 {
00293     goto count_cleanup;
00294 }
00295
00296 // Single-process processing of all segments
00297 if (cores_num == 1)
00298 {
00299     int first_segment = 1;
00300     for (int i = 0; i < total_segments; i++)
00301     {
00302         int seg_start_x = first_segment ? start_x : 1;
00303         int seg_end_x = (i == total_segments - 1) ? end_x : vx;
00304         char *y_str = mpz_get_str(NULL, 10, current_y);
00305         if (!y_str)
00306         {
00307             total = 0;
00308             goto count_cleanup;
00309         }
00310
00311         VX_SEG *vx_obj = vx_init(iZm, seg_start_x, seg_end_x, y_str, input_range->mr_rounds);
00312         free(y_str);
00313         if (!vx_obj)
00314         {
00315             total = 0;
00316             goto count_cleanup;
00317         }
00318
00319         vx_full_sieve(vx_obj, 0);
00320         total += vx_obj->p_count;
00321
00322         vx_free(&vx_obj);
00323         first_segment = 0;
00324         mpz_add_ui(current_y, current_y, 1); // increment Ys for the next segment
00325     }
00326     goto count_cleanup;
00327 }
00328
00329 #if IZ_PLATFORM_HAS_FORK
00330 // Multi-process processing of remaining segments
00331 // if total_segments < cores_num, adjust cores_num
00332 if (total_segments < cores_num)
00333 {
00334     cores_num = total_segments;
00335 }
00336
00337 int segments_per_core = total_segments / cores_num;
00338 int remainder_segments = total_segments % cores_num;
00339
00340 pipe_fds = malloc((size_t)cores_num * sizeof(*pipe_fds));
00341 pids = malloc((size_t)cores_num * sizeof(*pids));
00342 if (!pipe_fds || !pids)
00343 {
00344     log_error("SiZ_count: Failed to allocate process bookkeeping arrays.");
00345     total = 0;
00346     goto count_cleanup;
00347 }
00348
00349 for (int i = 0; i < cores_num; i++)
00350 {
00351     pids[i] = -1;
00352     pipe_fds[i][0] = -1;
00353     pipe_fds[i][1] = -1;
00354 }
00355
00356 for (int core = 0; core < cores_num; core++)
00357 {
00358     if (pipe(pipe_fds[core]) == -1)
00359     {
00360         log_error("SiZ_count: Failed to create pipe for core %d. Aborting.", core);
00361         // cleanup and abort on any pipe error
00362         for (int j = 0; j <= core; j++)
00363         {
00364             if (pipe_fds[j][0] != -1)
00365                 close(pipe_fds[j][0]);
00366             if (pipe_fds[j][1] != -1)
00367                 close(pipe_fds[j][1]);
00368         }
00369         total = 0;
00370         goto count_cleanup;
00371     }
00372
00373     pids[core] = fork();
00374     if (pids[core] < 0)
00375     {
00376         log_error("SiZ_count: Failed to fork process for core %d. Aborting.", core);

```

```

00377 // close this core's pipe and any previous ones, then exit
00378 for (int j = 0; j <= core; j++)
00379 {
00380     if (pipe_fds[j][0] != -1)
00381         close(pipe_fds[j][0]);
00382     if (pipe_fds[j][1] != -1)
00383         close(pipe_fds[j][1]);
00384 }
00385 total = 0;
00386 goto count_cleanup;
00387 }
00388
00389 if (pids[core] == 0)
00390 {
00391     // Child process
00392     // Close unrelated pipes
00393     for (int j = 0; j < cores_num; j++)
00394     {
00395         if (j != core && pipe_fds[j][0] != -1)
00396             close(pipe_fds[j][0]);
00397         if (j != core && pipe_fds[j][1] != -1)
00398             close(pipe_fds[j][1]);
00399     }
00400     close(pipe_fds[core][0]); // close read end
00401
00402     // Compute this child's starting Y and number of segments
00403     int offset = core * segments_per_core + (core < remainder_segments ? core : remainder_segments);
00404     int local_segments = segments_per_core + (core < remainder_segments ? 1 : 0);
00405
00406     uint64_t child_total = 0;
00407
00408     if (local_segments > 0)
00409     {
00410         mpz_t local_Ys;
00411         mpz_init(local_Ys);
00412         mpz_set(local_Ys, current_y);
00413         mpz_add_ui(local_Ys, local_Ys, offset);
00414
00415         // Each child has its own IZM to avoid data races
00416         iZm *iZm_local = iZm_clone(iZm);
00417         if (!iZm_local)
00418         {
00419             mpz_clear(local_Ys);
00420             close(pipe_fds[core][1]);
00421             exit(1);
00422         }
00423
00424         for (int i = 0; i < local_segments; i++)
00425         {
00426             int global_segment = offset + i;
00427             int seg_start_x = (global_segment == 0) ? start_x : 1;
00428             int seg_end_x = (global_segment == total_segments - 1) ? end_x : vx;
00429             char *y_str = mpz_get_str(NULL, 10, local_Ys);
00430             if (!y_str)
00431             {
00432                 iZm_free(&iZm_local);
00433                 mpz_clear(local_Ys);
00434                 close(pipe_fds[core][1]);
00435                 exit(1);
00436             }
00437
00438             VX_SEG *vx_obj = vx_init(iZm_local, seg_start_x, seg_end_x, y_str, input_range->mr_rounds);
00439             free(y_str);
00440             if (!vx_obj)
00441             {
00442                 iZm_free(&iZm_local);
00443                 mpz_clear(local_Ys);
00444                 close(pipe_fds[core][1]);
00445                 exit(1);
00446             }
00447
00448             vx_full_sieve(vx_obj, 0);
00449             child_total += vx_obj->p_count;
00450
00451             vx_free(&vx_obj);
00452             mpz_add_ui(local_Ys, local_Ys, 1);
00453         }
00454
00455         iZm_free(&iZm_local);
00456         mpz_clear(local_Ys);
00457     }
00458
00459     // Send result to parent
00460     ssize_t written = write(pipe_fds[core][1], &child_total, sizeof(child_total));
00461     if (written != sizeof(child_total))
00462     {
00463         log_error("SiZ_count: Child %d failed to write result.", core);

```

```

00464     }
00465
00466     close(pipe_fds[core][1]);
00467     exit(0);
00468 }
00469 else
00470 {
00471     // Parent process
00472     close(pipe_fds[core][1]); // close write end
00473     pipe_fds[core][1] = -1;
00474 }
00475 }
00476
00477 // Collect results
00478 for (int core = 0; core < cores_num; core++)
00479 {
00480     if (pids[core] > 0 && pipe_fds[core][0] != -1)
00481     {
00482         uint64_t child_total = 0;
00483         ssize_t bytes_read = read(pipe_fds[core][0], &child_total, sizeof(child_total));
00484         if (bytes_read == sizeof(child_total))
00485         {
00486             total += child_total;
00487         }
00488         else if (bytes_read == 0)
00489         {
00490             log_error("SiZ_count: Child %d closed pipe without sending result.", core);
00491         }
00492         else if (bytes_read < 0)
00493         {
00494             log_error("SiZ_count: Failed to read result from child %d.", core);
00495         }
00496         else
00497         {
00498             log_error("SiZ_count: Partial read from child %d (got %zd bytes).", core, bytes_read);
00499         }
00500
00501         close(pipe_fds[core][0]);
00502         pipe_fds[core][0] = -1;
00503
00504         int status;
00505         waitpid(pids[core], &status, 0);
00506         if (WIFEXITED(status) && WEXITSTATUS(status) != 0)
00507         {
00508             log_error("SiZ_count: Child %d exited with status %d.", core, WEXITSTATUS(status));
00509             // propagate child failure as an overall error
00510             total = 0;
00511             goto count_cleanup;
00512         }
00513         else if (WIFSIGNALED(status))
00514         {
00515             log_error("SiZ_count: Child %d terminated by signal %d.", core, WTERMSIG(status));
00516             total = 0;
00517             goto count_cleanup;
00518         }
00519     }
00520     else if (pipe_fds[core][0] != -1)
00521     {
00522         close(pipe_fds[core][0]);
00523         pipe_fds[core][0] = -1;
00524     }
00525 }
00526 #endif
00527
00528 count_cleanup:
00529 #if IZ_PLATFORM_HAS_FORK
00530     free(pids);
00531     free(pipe_fds);
00532 #endif
00533     range_info_free(&info);
00534     iZm_free(&iZm);
00535     mpz_clear(current_y);
00536
00537     return total;
00538 }
00539
00540 // =====
00541 // * Random Prime Generation
00542 // =====
00543
00558 int vy_random_prime(mpz_t p, int bit_size, int cores_num)
00559 {
00560     int found = 0; // flag to indicate if a prime was found
00561     bit_size = MAX(bit_size, 10); // Set minimum bit size
00562
00563     // 1. Compute max vx for the given bit-size
00564     mpz_t vx;

```

```

00565     mpz_init(vx);
00566     compute_max_vx(vx, bit_size);
00567
00568     // 2. If < 2 cores, run the search in-process
00569     if (cores_num < 2)
00570     {
00571         found = vy_search_prime(p, 0, vx);
00572         mpz_clear(vx);
00573         return found;
00574     }
00575
00576 #if !IZ_PLATFORM_HAS_FORK
00577     log_info("vy_random_prime: multi-process mode is unavailable on this platform; using single-process mode.");
00578     found = vy_search_prime(p, 0, vx);
00579     mpz_clear(vx);
00580     return found;
00581 #else
00582     // 3. Else, fork multiple processes to search for a prime
00583     // Create a pipe for inter-process communication.
00584     int fd[2];
00585     if (pipe(fd) == -1)
00586     {
00587         log_error("Failed to create pipe in vy_random_prime. Falling back to in-process search.");
00588         found = vy_search_prime(p, 0, vx);
00589         mpz_clear(vx);
00590         return found;
00591     }
00592
00593     pid_t pids[cores_num];
00594     int forks_created = 0;
00595
00596     // Fork child processes.
00597     for (int i = 0; i < cores_num; i++)
00598     {
00599         pid_t pid = fork();
00600         if (pid < 0)
00601         {
00602             log_error("Failed to fork process %d in vy_random_prime", i);
00603             continue; // Skip this fork failure and continue creating other forks
00604         }
00605         else if (pid == 0)
00606         {
00607             // Child process: close the read-end.
00608             close(fd[0]);
00609
00610             mpz_t local_p; // local prime candidate
00611             mpz_init(local_p);
00612
00613             // Search for a candidate prime
00614             found = vy_search_prime(local_p, 0, vx);
00615
00616             // If a candidate is found, send it via the pipe.
00617             if (found)
00618             {
00619                 char *p_str = mpz_get_str(NULL, 10, local_p);
00620                 if (p_str)
00621                 {
00622                     write(fd[1], p_str, strlen(p_str) + 1);
00623                     free(p_str);
00624                 }
00625             }
00626             mpz_clear(local_p);
00627
00628             close(fd[1]); // Close the write-end.
00629             exit(0);
00630         }
00631         else
00632         {
00633             // Parent process saves child's PID.
00634             pids[i] = pid;
00635             forks_created++;
00636         }
00637     }
00638
00639     // 4. Parent reads first result from the pipe.
00640     close(fd[1]); // Close the write-end.
00641
00642     if (forks_created == 0)
00643     {
00644         log_error("No child processes were created in vy_random_prime, falling back to in-process search");
00645         close(fd[0]);
00646         found = vy_search_prime(p, 0, vx);
00647         mpz_clear(vx);
00648         return found;
00649     }
00650
00651     // Allocate a sufficiently large buffer to hold the prime string + null terminator

```



```

00652 // bit_size * 0.302 is approx digits, add extra padding for safety
00653 size_t buf_size = (size_t)(bit_size * 0.4);
00654 char *buf = malloc(buf_size);
00655 if (!buf)
00656 {
00657     log_error("Failed to allocate buffer in vy_random_prime");
00658     found = 0;
00659 }
00660 else
00661 {
00662     ssize_t n = read(fd[0], buf, buf_size - 1);
00663     if (n == -1)
00664     {
00665         log_error("Failed to read from pipe in vx_random_prime");
00666         found = 0;
00667     }
00668     else if (n > 0)
00669     {
00670         buf[n] = '\0'; // Ensure null-termination
00671         if (mpz_set_str(p, buf, 10) == 0)
00672         {
00673             found = 1;
00674         }
00675         else
00676         {
00677             log_error("Failed to set prime from buffer in vx_random_prime");
00678             found = 0;
00679         }
00680     }
00681     free(buf);
00682 }
00683 close(fd[0]); // Close the read-end of the pipe.
00684
00685 // 5. Terminate all child processes
00686 for (int i = 0; i < forks_created; i++)
00687 {
00688     kill(pids[i], SIGTERM); // Terminate child process
00689     waitpid(pids[i], NULL, 0); // Wait for child process to terminate
00690 }
00691 mpz_clear(vx);
00692
00693 return found;
00694 #endif
00695 }
00696
00712 int vx_random_prime(mpz_t p, int bit_size, int cores_num)
00713 {
00714     int found = 0;
00715     bit_size = MAX(bit_size, 10);
00716     int vx = bit_size <= 2048 ? VX5 : VX6;
00717
00718     // 2. If < 2 cores, run the search in-process
00719     if (cores_num < 2)
00720     {
00721         found = vx_search_prime(p, 0, vx, bit_size);
00722         return found;
00723     }
00724
00725 #if !IZ_PLATFORM_HAS_FORK
00726     log_info("vx_random_prime: multi-process mode is unavailable on this platform; using single-process mode.");
00727     found = vx_search_prime(p, 0, vx, bit_size);
00728     return found;
00729 #else
00730     // 3. Multi-core: fork processes
00731     int fd[2];
00732     if (pipe(fd) == -1)
00733     {
00734         log_error("Failed to create pipe in vx_random_prime. Falling back to in-process search.");
00735         found = vx_search_prime(p, 0, vx, bit_size);
00736         return found;
00737     }
00738
00739     pid_t pids[cores_num];
00740     int forks_created = 0;
00741
00742     for (int i = 0; i < cores_num; i++)
00743     {
00744         pid_t pid = fork();
00745         if (pid < 0)
00746         {
00747             log_error("Failed to fork process %d in vx_random_prime", i);
00748             continue;
00749         }
00750         else if (pid == 0)
00751         {
00752             // CHILD PROCESS: Initialize everything fresh

```

```

00753     close(fd[0]);
00754
00755     // Search for prime with child's own objects
00756     mpz_t local_p;
00757     mpz_init(local_p);
00758     int child_found = vx_search_prime(local_p, 0, vx, bit_size);
00759
00760     if (child_found)
00761     {
00762         char *p_str = mpz_get_str(NULL, 10, local_p);
00763         if (p_str != NULL)
00764         {
00765             write(fd[1], p_str, strlen(p_str) + 1);
00766             free(p_str);
00767         }
00768     }
00769
00770     // Cleanup child resources
00771     mpz_clear(local_p);
00772     close(fd[1]);
00773     exit(0);
00774 }
00775 else
00776 {
00777     pids[i] = pid;
00778     forks_created++;
00779 }
00780 }
00781
00782 // 4. Parent process reads result
00783 close(fd[1]);
00784
00785 if (forks_created == 0)
00786 {
00787     log_error("No child processes were created in vx_random_prime, falling back to in-process search");
00788     close(fd[0]);
00789     // fallback to in-process search
00790     found = vx_search_prime(p, 0, vx, bit_size);
00791     return found;
00792 }
00793
00794 // Allocate a sufficiently large buffer to hold the prime string + null terminator
00795 size_t buf_size = (size_t)(bit_size * 0.4);
00796 char *buf = malloc(buf_size);
00797 if (!buf)
00798 {
00799     log_error("Failed to allocate buffer in vx_random_prime");
00800     found = 0;
00801 }
00802 else
00803 {
00804     ssize_t n = read(fd[0], buf, buf_size - 1);
00805     if (n > 0)
00806     {
00807         buf[n] = '\0'; // Ensure null-termination
00808         if (mpz_set_str(p, buf, 10) == 0)
00809         {
00810             found = 1;
00811         }
00812         else
00813         {
00814             log_error("Failed to set prime from buffer in vx_random_prime");
00815             found = 0;
00816         }
00817     }
00818     free(buf);
00819 }
00820
00821 close(fd[0]); // Close the read-end of the pipe.
00822
00823 // Terminate children
00824 for (int i = 0; i < forks_created; i++)
00825 {
00826     kill(pids[i], SIGTERM);
00827     waitpid(pids[i], NULL, 0);
00828 }
00829
00830 return found;
00831 #endif
00832 }
00833
00845 int iz_next_prime(mpz_t p, mpz_t base, int forward)
00846 {
00847     // 1. Initialization
00848     int found = 0; // flag to indicate if a prime was found
00849
00850     // tmp variable to hold the value until a prime is found

```

```

00851     mpz_t z;
00852     mpz_init_set(z, base); // set tmp = base
00853
00854     // a. Edge cases:
00855     // if forward and base is iZ-, check next iZ+
00856     if (mpz_fdiv_ui(z, 6) == 5 && forward)
00857     {
00858         mpz_add_ui(z, z, 2); // increment tmp by 2
00859         if (mpz_probab_prime_p(z, MR_ROUNDS))
00860         {
00861             mpz_set(p, z); // set p = tmp + 2
00862             mpz_clear(z);
00863             return 1;
00864         }
00865     }
00866     // if backward and base is iZ+, check previous iZ-
00867     else if (mpz_fdiv_ui(z, 6) == 1 && !forward)
00868     {
00869         mpz_sub_ui(z, z, 2); // decrement tmp by 2
00870         if (mpz_probab_prime_p(z, MR_ROUNDS))
00871         {
00872             mpz_set(p, z); // set p = tmp - 2
00873             mpz_clear(z);
00874             return 1;
00875         }
00876     }
00877
00878     int vx = (mpz_sizeinbase(base, 2) > 2048) ? VX6 : VX5;
00879     iZM *iZm = iZm_init(vx);
00880     if (!iZm)
00881     {
00882         log_error("iZm initialization failed in iZ_next_prime.");
00883         mpz_clear(z);
00884         return 0;
00885     }
00886
00887     // c. Initialize and set y and yvx
00888     mpz_t y, yvx, x_p;
00889     mpz_init(y);
00890     mpz_init(yvx);
00891     mpz_init(x_p);
00892
00893     mpz_div_ui(y, base, 6 * vx); // compute y = base / 6 * vx
00894     mpz_mul_ui(yvx, y, vx); // compute yvx = y * vx
00895     mpz_div_ui(x_p, z, 6); // compute x_p = tmp / 6
00896
00897     // 2. Iterate over the x5 and x7 bitmaps to find a prime
00898     // set start_x = x_p % vx +/- 1
00899     int step = forward ? 1 : -1;
00900     int start_x = mpz_fdiv_ui(x_p, vx) + step;
00901     int end_x = forward ? vx : 1;
00902
00903     int i = 0; // segment counter
00904
00905     while (!found)
00906     {
00907         if (forward)
00908         {
00909             if (i > 0)
00910                 start_x = 1; // start from the beginning of the bitmap
00911
00912             for (int x = start_x; x <= end_x; x++)
00913             {
00914                 // check if x5[x] is set
00915                 if (bitmap_get_bit(iZm->base_x5, x))
00916                 {
00917                     mpz_add_ui(x_p, yvx, x); // set x_p = yvx + x
00918                     iZ_mpz(z, x_p, 1); // compute p = iZ(x_p, 1)
00919                     // check if tmp is prime
00920                     found = mpz_probab_prime_p(z, MR_ROUNDS);
00921
00922                     if (found)
00923                         break;
00924                 }
00925
00926                 // check if x7[x] is set
00927                 if (bitmap_get_bit(iZm->base_x7, x))
00928                 {
00929                     mpz_add_ui(x_p, yvx, x); // set x_p = yvx + x
00930                     iZ_mpz(z, x_p, 1); // compute tmp = iZ(x_p, 1)
00931                     // check if tmp is prime
00932                     found = mpz_probab_prime_p(z, MR_ROUNDS);
00933
00934                     if (found)
00935                         break;
00936                 }
00937             }

```

```

00938
00939     mpz_add_ui(yvx, yvx, vx); // increment yvx by vx for next segment
00940 }
00941 else // backward search
00942 {
00943     if (i > 0)
00944         start_x = vx; // start from the end of the bitmaps
00945
00946     // check iZ+ first if backward
00947     for (int x = start_x; x >= end_x; x--)
00948     {
00949         // check if x7[x] is set
00950         if (bitmap_get_bit(iZm->base_x7, x))
00951         {
00952             mpz_add_ui(x_p, yvx, x); // set x_p = yvx + x
00953             iZ_mpz(z, x_p, 1); // compute tmp = iZ(x_p, 1)
00954             // check if tmp is prime
00955             found = mpz_probab_prime_p(z, MR_ROUNDS);
00956
00957             if (found)
00958                 break;
00959         }
00960
00961         // check iZ-
00962         if (bitmap_get_bit(iZm->base_x5, x))
00963         {
00964             mpz_add_ui(x_p, yvx, x); // set x_p = yvx + x
00965             iZ_mpz(z, x_p, -1); // compute p = iZ(x_p, -1)
00966             // check if tmp is prime
00967             found = mpz_probab_prime_p(z, MR_ROUNDS);
00968
00969             if (found)
00970                 break;
00971         }
00972     }
00973
00974     mpz_sub_ui(yvx, yvx, vx); // decrement yvx by vx for next segment
00975 }
00976
00977 i++; // increment segment counter
00978 }
00979
00980 // 3. Set the found prime
00981 if (found)
00982     mpz_set(p, z); // set p = tmp
00983 else
00984     log_debug("No prime found :/");
00985
00986 // cleanup
00987 iZm_free(&iZm);
00988 mpz_clears(y, yvx, x_p, z, NULL);
00989
00990 return found;
00991 }

```

13.19 src/playground.c File Reference

Scratch space for testing new ideas.

```
#include <iZ_api.h>
```

13.19.1 Detailed Description

Scratch space for testing new ideas.

Definition in file [playground.c](#).

13.20 playground.c

[Go to the documentation of this file.](#)

```

00001
00005
00006 #include <iZ_api.h>

```

13.21 src/prime_sieve.c File Reference

Implementations of classical and SiZ-family sieve algorithms.

```
#include <iZ_api.h>
```

Macros

- `#define N_LIMIT (1000000000000ULL)`
- `#define ASSERT_LIMIT(n)`
Assert that input `n` is within the valid range for sieve functions.
- `#define Pi(n)`
`Pi(n)` is an approximation of the number of primes up to `n`, used for initial array sizing.

Functions

- static void `process_N_bitmap (UI64_ARRAY *primes, BITMAP *sieve_bitmap, uint64_t n)`
A helper function to process the bitmap for the Sieve of Eratosthenes and collect primes.

Classic Prime Sieve Algorithms

Baseline sieves up to a numeric limit.

All classic sieve functions take a limit `n` and return an ascending list of primes $\leq n$.

- `UI64_ARRAY * SoE (uint64_t n)`
Optimized Sieve of Eratosthenes.
- `UI64_ARRAY * SSoE (uint64_t n)`
Segmented Sieve of Eratosthenes.
- `UI64_ARRAY * SoEu (uint64_t n)`
Euler (linear) sieve.
- `UI64_ARRAY * SoS (uint64_t n)`
Sieve of Sundaram.
- `UI64_ARRAY * SoA (uint64_t n)`
Sieve of Atkin.

iZ-based Sieve Algorithms

Sieve family operating in the $6x-1$ / $6x+1$ index space.

- `UI64_ARRAY * SiZ (uint64_t n)`
Solid Sieve-iZ (wheel 6, iZ index space).
- `UI64_ARRAY * SiZm (uint64_t n)`
Segmented Sieve-iZm (VX segmented, horizontal processing).
- `UI64_ARRAY * SiZm_vy (uint64_t n)`
Segmented Sieve-iZm (vertical processing; faster, unordered output).

13.21.1 Detailed Description

Implementations of classical and SiZ-family sieve algorithms.

This file contains the implementations of various prime sieving algorithms, including classical algorithms (SoE/SSoE/SoEu/SoS/SoA) as well as SiZ-family algorithms (SiZ/SiZm/SiZm_vy). All functions are single-threaded, take an upper limit `n` and return a pointer to a `UI64_ARRAY` containing the prime numbers up to `n`.

Definition in file `prime_sieve.c`.

13.21.2 Macro Definition Documentation

13.21.2.1 ASSERT_LIMIT

```
#define ASSERT_LIMIT(  
    n)
```

Value:

```
assert((n) > 10 && (n) <= N_LIMIT && "Input must be in the range (10, 10^12).")
```

Assert that input n is within the valid range for sieve functions.

Definition at line 23 of file [prime_sieve.c](#).

Referenced by [SiZ\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoA\(\)](#), [SoE\(\)](#), [SoEu\(\)](#), [SoS\(\)](#), and [SSoE\(\)](#).

13.21.2.2 N_LIMIT

```
#define N_LIMIT (1000000000000ULL)
```

Maximum supported sieve limit for standard entry points (10^{12}).

Definition at line 20 of file [prime_sieve.c](#).

13.21.2.3 Pi

```
#define Pi(  
    n)
```

Value:

```
((n / log(n)))
```

[Pi\(n\)](#) is an approximation of the number of primes up to n, used for initial array sizing.

Definition at line 26 of file [prime_sieve.c](#).

Referenced by [SiZ\(\)](#), [SiZm\(\)](#), [SiZm_vy\(\)](#), [SoA\(\)](#), [SoE\(\)](#), [SoEu\(\)](#), [SoS\(\)](#), and [SSoE\(\)](#).

13.21.3 Function Documentation

13.21.3.1 process_N_bitmap()

```
void process_N_bitmap (  
    UI64_ARRAY * primes,  
    BITMAP * sieve_bitmap,  
    uint64_t n) [static]
```

A helper function to process the bitmap for the Sieve of Eratosthenes and collect primes.

Parameters

| | |
|--------|---------------------|
| primes | Output prime array. |
|--------|---------------------|

| | |
|--------------|--|
| sieve_bitmap | Candidate bitmap. |
| n | Inclusive numeric upper bound represented in sieve_bitmap. |

Definition at line 38 of file [prime_sieve.c](#).

References [bitmap_clear_steps_simd\(\)](#), [bitmap_get_bit\(\)](#), and [ui64_push\(\)](#).

Referenced by [SoE\(\)](#), and [SSoE\(\)](#).

13.22 prime_sieve.c

[Go to the documentation of this file.](#)

```

00001
00012
00013 #include <iZ_api.h>
00014
00015 // =====
00016 // * Internal helper macros:
00017 // =====
00018
00020 #define N_LIMIT (1000000000000ULL)
00021
00023 #define ASSERT_LIMIT(n) assert((n) > 10 && (n) <= N_LIMIT && "Input must be in the range (10, 10^12).")
00024
00026 #define Pi(n) ((n / log(n)))
00027
00028 // =====
00029 // * Classic Sieve Algorithms
00030 // =====
00031
00033 static void process_N_bitmap(UI64_ARRAY *primes, BITMAP *sieve_bitmap, uint64_t n)
00034 {
00040     ui64_push(primes, 2); // Add 2 to primes to focus on odd candidates
00041     uint64_t n_sqrt = sqrt(n);
00042
00043     // Sieve logic:
00044     // Iterate through odd numbers i starting from 3,
00045     // check if i is marked as prime in the bitmap,
00046     // if so, collect i as a prime and mark its odd multiples if i <= sqrt(n).
00047     for (uint64_t i = 3; i <= n; i += 2)
00048     {
00049         if (bitmap_get_bit(sieve_bitmap, i))
00050         {
00051             ui64_push(primes, i);
00052             if (i <= n_sqrt)
00053                 bitmap_clear_steps_simd(sieve_bitmap, 2 * i, i * i, n + 1);
00054         }
00055     }
00056 }
00057
00072 UI64_ARRAY *SoE(uint64_t n)
00073 {
00074     ASSERT_LIMIT(n); // Validate input limit
00075
00076     // Initialize the primes object with an estimated capacity
00077     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00078     assert(primes && "Memory allocation failed for primes array.");
00079
00080     // Create a bitmap to mark prime numbers
00081     BITMAP *sieve = bitmap_init(n + 1, 1);
00082     if (!sieve)
00083     {
00084         ui64_free(&primes);
00085         return NULL;
00086     }
00087
00088     // Sieve logic
00089     process_N_bitmap(primes, sieve, n);
00090
00091     // cleanup and finalize
00092     bitmap_free(&sieve); // free bitmap
00093     ui64_resize_to_fit(primes); // Trim excess memory in primes array
00094
00095     return primes;

```

```

00096 }
00097
00114 UI64_ARRAY *SSoE(uint64_t n)
00115 {
00116     ASSERT_LIMIT(n); // Validate input limit
00117
00118     // Initialize UI64_ARRAY with an estimated capacity
00119     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00120     assert(primes && "Memory allocation failed for primes array.");
00121
00122     // Define the segment size; can be tuned based on memory constraints
00123     uint64_t segment_size = (uint64_t)sqrt(n);
00124
00125     // * Step 1: Sieve small primes up to sqrt(n) using the traditional sieve
00126     BITMAP *sieve = bitmap_init(segment_size + 8, 1);
00127     if (!sieve)
00128     {
00129         ui64_free(&primes);
00130         return NULL;
00131     }
00132
00133     // process first segment to collect root primes
00134     process_N_bitmap(primes, sieve, segment_size);
00135
00136     // * Step 2: Segmented sieve
00137     uint64_t low = segment_size + 1;
00138     uint64_t high = low + segment_size - 1;
00139
00140     // Iterate over segments
00141     while (low <= n)
00142     {
00143         bitmap_set_all(sieve); // Reset segment bitmap
00144         uint64_t root_limit = sqrt(high);
00145
00146         // Sieve the current segment using primes <= sqrt(high)
00147         for (int i = 1; i < primes->count; i++) // skip 2
00148         {
00149             uint64_t p = primes->array[i];
00150             if (p > root_limit)
00151                 break;
00152
00153             // Find the minimum number in [low, high] that is a multiple of p
00154             uint64_t start = (low / p) * p;
00155             start += (start < low) ? p : 0; // ensure start >= low
00156             start += (start % 2 == 0) ? p : 0; // ensure start is odd
00157             start = MAX(p * p, start); // start from p^2 or higher
00158
00159             // Mark multiples of p within the segment
00160             bitmap_clear_steps_simd(sieve, 2 * p, start - low, high - low + 1);
00161         }
00162
00163         // Collect primes from the current segment
00164         uint64_t i = (low % 2 == 0) ? low + 1 : low;
00165         for (; i <= high; i += 2) // skip even numbers
00166         {
00167             if (bitmap_get_bit(sieve, i - low))
00168                 ui64_push(primes, i);
00169         }
00170
00171         // Move to the next segment
00172         low += segment_size;
00173         high += segment_size;
00174         if (high > n)
00175             high = n;
00176     }
00177
00178     // * Step 3: Cleanup and finalize
00179     bitmap_free(&sieve); // free bitmap
00180     ui64_resize_to_fit(primes); // Trim excess memory in primes array
00181
00182     return primes;
00183 }
00184
00200 UI64_ARRAY *SoEu(uint64_t n)
00201 {
00202     ASSERT_LIMIT(n); // Validate input limit
00203
00204     // initialization
00205     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00206     assert(primes && "Memory allocation failed for primes array.");
00207
00208     BITMAP *sieve = bitmap_init(n + 1, 1);
00209     if (sieve == NULL)
00210     {
00211         ui64_free(&primes);
00212         return NULL;
00213     }

```



```

00214
00215 // starting the prime list with 2 to skip reading even numbers
00216 ui64_push(primes, 2);
00217
00218 // sieve logic: iterate through odd numbers, marking composites by clearing multiples of primes
00219 for (uint64_t i = 3; i <= n; i += 2)
00220 {
00221     if (bitmap_get_bit(sieve, i))
00222         ui64_push(primes, i);
00223
00224     // Mark multiples of the current prime
00225     for (int j = 1; j < primes->count; ++j)
00226     {
00227         uint64_t p = primes->array[j];
00228
00229         if (p * i > n)
00230             break;
00231
00232         bitmap_clear_bit(sieve, p * i);
00233
00234         if (i % p == 0)
00235             break;
00236     }
00237 }
00238
00239 // cleanup
00240 bitmap_free(&sieve);
00241
00242 // Resize primes array to fit the exact number of primes found
00243 ui64_resize_to_fit(primes);
00244
00245 return primes;
00246 }
00247
00264 UI64_ARRAY *SoS(uint64_t n)
00265 {
00266     ASSERT_LIMIT(n); // Validate input limit
00267
00268     // Calculate k as the odd numbers up to n.
00269     uint64_t k = (n - 1) / 2 + 1;
00270
00271     // Initialize the primes object with an estimated capacity
00272     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00273     assert(primes && "Memory allocation failed for primes array.");
00274
00275     ui64_push(primes, 2);
00276
00277     // Create a bitmap with size k+1.
00278     BITMAP *sieve = bitmap_init(k + 8, 1);
00279     if (sieve == NULL)
00280     {
00281         ui64_free(&primes);
00282         return NULL;
00283     }
00284
00285     uint64_t n_sqrt = sqrt(n) + 1;
00286
00287     // iterate through odd numbers
00288     for (uint64_t i = 1; i < k; ++i)
00289     {
00290         if (bitmap_get_bit(sieve, i))
00291         {
00292             uint64_t p = 2 * i + 1;
00293             ui64_push(primes, p);
00294             if (p < n_sqrt)
00295             {
00296                 // First composite mark xp in the bitmap is given by:
00297                 //  $xp = 2 * i^2 + 2 * i = p * i + i$ , corresponding to  $p^2$  in the odd set
00298                 uint64_t xp = p * i + i;
00299                 // Mark composites of p additively in the bitmap using step size p
00300                 bitmap_clear_steps_simd(sieve, p, xp, k);
00301             }
00302         }
00303     }
00304
00305     // Cleanup
00306     bitmap_free(&sieve);
00307
00308     // Resize primes array to fit the exact number of primes found
00309     ui64_resize_to_fit(primes);
00310
00311     return primes;
00312 }
00313
00328 UI64_ARRAY *SoA(uint64_t n)
00329 {
00330     ASSERT_LIMIT(n); // Validate input limit

```

```

00331
00332 UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00333 assert(primes && "Memory allocation failed for primes array.");
00334
00335 // Create a bitmap to mark potential primes
00336 BITMAP *sieve = bitmap_init(n + 1, 0);
00337 if (sieve == NULL)
00338 {
00339     ui64_free(&primes);
00340     return NULL;
00341 }
00342
00343 // Add 2 and 3 to primes
00344 ui64_push(primes, 2);
00345 ui64_push(primes, 3);
00346
00347 // 1. Mark potential primes in the bitmap using the Atkin conditions
00348 // Condition 1: for all  $4x^2 + y^2 \leq n$ 
00349 for (uint64_t x = 1; 4 * x * x < n; x++)
00350 {
00351     uint64_t a = 4 * x * x;
00352     for (uint64_t y = 1; a + y * y <= n; ++y)
00353     {
00354         uint64_t b = a + y * y;
00355         // if  $4x^2 + y^2 \equiv 1 \text{ or } 5 \pmod{12}$ , flip the bit
00356         if (b % 12 == 1 || b % 12 == 5)
00357             bitmap_flip_bit(sieve, b);
00358     }
00359 }
00360
00361 // Condition 2: for all  $3x^2 + y^2 \leq n$ 
00362 for (uint64_t x = 1; 3 * x * x < n; x++)
00363 {
00364     uint64_t a = 3 * x * x;
00365     for (uint64_t y = 1; a + y * y <= n; ++y)
00366     {
00367         uint64_t b = a + y * y;
00368         // if  $3x^2 + y^2 \equiv 7 \pmod{12}$ , flip the bit
00369         if (b % 12 == 7)
00370             bitmap_flip_bit(sieve, b);
00371     }
00372 }
00373
00374 // Condition 3: for all  $3x^2 - y^2 \leq n$  and  $x > y$ 
00375 for (uint64_t x = 1; 2 * x * x < n; x++) // Approximation for loop bound
00376 {
00377     uint64_t a = 3 * x * x;
00378     for (uint64_t y = x - 1; y > 0; y--)
00379     {
00380         uint64_t b = a - y * y;
00381         if (b > n)
00382             break; // b increases as y decreases
00383
00384         // if  $3x^2 - y^2 \equiv 11 \pmod{12}$ , flip the bit
00385         if (b % 12 == 11)
00386             bitmap_flip_bit(sieve, b);
00387     }
00388 }
00389
00390 // 2. Remove remaining composites by sieving out multiples of squares of root primes
00391 uint64_t n_sqrt = sqrt(n);
00392 for (uint64_t p = 5; p <= n_sqrt; p += 2)
00393 {
00394     if (bitmap_get_bit(sieve, p))
00395     {
00396         // Mark odd multiples of  $p^2$  as non-prime
00397         // starting at  $p^2$  with step size  $2p^2$ 
00398         bitmap_clear_steps_simd(sieve, 2 * p * p, p * p, n + 1);
00399     }
00400 }
00401
00402 // 3. Collect primes from the bitmap
00403 for (uint64_t p = 5; p <= n; p += 2)
00404 {
00405     if (bitmap_get_bit(sieve, p))
00406         ui64_push(primes, p);
00407 }
00408
00409 // cleanup
00410 bitmap_free(&sieve);
00411
00412 // Resize primes array to fit the exact number of primes found
00413 ui64_resize_to_fit(primes);
00414
00415 return primes;
00416 }
00417

```

```

00418 // =====
00419 // * Sieve-iZ Algorithms
00420 // =====
00421
00444 UI64_ARRAY *SiZ(uint64_t n)
00445 {
00446     ASSERT_LIMIT(n); // Validate input limit
00447
00448     // Initialize primes object with enough initial estimation
00449     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00450     assert(primes && "Memory allocation failed for primes array.");
00451
00452     // Add 2, 3 to primes
00453     ui64_push(primes, 2);
00454     ui64_push(primes, 3);
00455
00456     // Calculate x_n, max x value in iZ space for given n
00457     uint64_t x_n = n / 6 + 1;
00458
00459     // Create bitmap X-Arrays x5, x7, each of size x_n + 1 bits
00460     BITMAP *x5 = bitmap_init(x_n + 1, 1);
00461     BITMAP *x7 = bitmap_init(x_n + 1, 1);
00462
00463     // Memory allocation failed, check logs
00464     if (!x5 || !x7)
00465     {
00466         ui64_free(&primes);
00467         return NULL;
00468     }
00469
00470     // Sieve logic
00471     process_iZ_bitmaps(primes, x5, x7, x_n);
00472
00473     // Cleanup: free memory of x5, x7
00474     bitmap_free(&x5);
00475     bitmap_free(&x7);
00476
00477     // Handle edge case: if last prime > n, remove it
00478     if (primes->array[primes->count - 1] > n)
00479         ui64_pop(primes);
00480
00481     // Trim unused memory in primes object
00482     ui64_resize_to_fit(primes);
00483
00484     return primes;
00485 }
00486
00516 UI64_ARRAY *SiZm(uint64_t n)
00517 {
00518     ASSERT_LIMIT(n); // Validate input limit
00519
00520     // if n < 10000, return SiZ(n), doesn't worth segmenting
00521     if (n < 10000)
00522         return SiZ(n);
00523
00524     // * 1. Initialization:
00525     // Initialize primes array with enough capacity to avoid reallocs
00526     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4); // 40% over-estimation to avoid reallocs
00527     assert(primes && "Memory allocation failed for primes array in SiZm.");
00528
00529     // Compute vx wheel size that fits in L2 cache
00530     int vx = compute_l2_vx(n);
00531
00532     // Initialize and construct base bitmaps for iZm/vx
00533     BITMAP *base_x5 = bitmap_init(vx + 8, 1);
00534     BITMAP *base_x7 = bitmap_init(vx + 8, 1);
00535     if (!base_x5 || !base_x7)
00536     {
00537         ui64_free(&primes);
00538         return NULL;
00539     }
00540     iZm_construct_vx_base(vx, base_x5, base_x7);
00541
00542     // Add the pre-sieved k primes to primes array
00543     const uint64_t base_primes[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
00544     int k = 0;
00545     while ((6 * vx) % base_primes[k] == 0)
00546         ui64_push(primes, base_primes[k++]);
00547
00548     // * 2. Process first segment (y = 0) to collect root primes:
00549     uint64_t x_n = n / 6 + 1; // max x value up to n
00550     // uint64_t root_limit = sqrt(6 * x_limit) + 1;
00551     // Initialize active sieve bitmaps from base
00552     BITMAP *x5 = bitmap_clone(base_x5);
00553     BITMAP *x7 = bitmap_clone(base_x7);
00554     if (!x5 || !x7)
00555     {

```

```

00556     ui64_free(&primes);
00557     bitmap_free(&base_x5);
00558     bitmap_free(&base_x7);
00559     return NULL;
00560 }
00561 process_iZ_bitmaps(primes, x5, x7, vx + 1);
00562
00563 // * 3. Process remaining segments (y >= 1) to collect primes:
00564 int y_limit = x_n / vx; // number of full segments to process
00565 uint64_t yvx = vx; // current base value (y * vx)
00566 for (int y = 1; y <= y_limit; y++)
00567 {
00568     // * a. Reset active bitmaps to base state
00569     memcpy(x5->data, base_x5->data, x5->byte_size);
00570     memcpy(x7->data, base_x7->data, x7->byte_size);
00571
00572     int x_limit = (y < y_limit) ? vx : x_n % vx; // local x limit adjusted for last segment
00573     uint64_t root_limit = sqrt(6 * (yvx + x_limit)) + 1; // local root limit for current segment
00574
00575     // * b. Mark composites of root primes in current segment
00576     for (int i = k; i < primes->count; i++)
00577     {
00578         uint64_t p = primes->array[i];
00579         if (p > root_limit)
00580             break;
00581
00582         // mark composites of p in current segment
00583         bitmap_clear_steps_simd(x5, p, iZm_solve_for_x0(-1, p, vx, y), x_limit);
00584         bitmap_clear_steps_simd(x7, p, iZm_solve_for_x0(1, p, vx, y), x_limit);
00585     }
00586
00587     // * c. Collect unmarked indices as primes in current segment
00588     for (int x = 2; x <= x_limit; x++)
00589     {
00590         if (bitmap_get_bit(x5, x)) // i.e. iZ- prime
00591             ui64_push(primes, iZ(yvx + x, -1));
00592
00593         if (bitmap_get_bit(x7, x)) // i.e. iZ+ prime
00594             ui64_push(primes, iZ(yvx + x, 1));
00595     }
00596
00597     yvx += vx; // advance yvx for next segment
00598 }
00599
00600 // * 4. Clean up and finalize
00601 bitmap_free(&x5);
00602 bitmap_free(&x7);
00603 bitmap_free(&base_x5);
00604 bitmap_free(&base_x7);
00605
00606 // Handle edge case: if last prime > n, remove it
00607 if (primes->array[primes->count - 1] > n)
00608     ui64_pop(primes);
00609
00610 ui64_resize_to_fit(primes); // Trim excess memory in primes array
00611 return primes;
00612 }
00613
00623 UI64_ARRAY *SiZm_vy(uint64_t n)
00624 {
00625     ASSERT_LIMIT(n); // Validate input limit
00626
00627     // if n is less than 10000, return SiZ(n)
00628     if (n < 10000)
00629         return SiZ(n);
00630
00631     // * 1. Initialization:
00632     // Initialize primes array with enough capacity to avoid reallocs
00633     UI64_ARRAY *primes = ui64_init(Pi(n) * 1.4);
00634     assert(primes && "Memory allocation failed for primes array in SiZm.");
00635
00636     uint64_t x_n = n / 6 + 1; // iZ limit for n
00637     uint64_t root_limit = sqrt(n) + 1;
00638
00639     get_root_primes(primes, root_limit);
00640     int root_count = primes->count;
00641
00642     int k = 4; // pointing at 11 in root_primes
00643     int vx = 35;
00644     if (n >= pow(10, 9))
00645     {
00646         vx *= 11;
00647         k++;
00648     }
00649     if (n >= pow(10, 11))
00650     {
00651         vx *= 13;

```

```

00652     k++;
00653 }
00654
00655 int vy = x_n / vx;
00656
00657 BITMAP *sieve = bitmap_init(vy + 8, 1);
00658
00659 // * 2. Sieve logic: Process iZm's columns as segments
00660 for (int x = 2; x <= vx; x++)
00661 {
00662     // Handle iZ- case:
00663     // crucial, ensure iZ(x, -1) is coprime to vx before sieving
00664     if (gcd(iZ(x, -1), vx) == 1)
00665     {
00666         // * a. reset sieve bitmap for new segment
00667         bitmap_set_all(sieve); // set all bits
00668
00669         // * b. mark composites of root primes in sieve
00670         for (int i = k; i < root_count; i++)
00671         {
00672             uint64_t p = primes->array[i];
00673             int64_t y_0 = iZm_solve_for_y0(-1, p, vx, x);
00674             bitmap_clear_steps_simd(sieve, p, y_0, vy);
00675         }
00676
00677         // * c. collect primes from sieve up to y = vy-1
00678         for (int y = 0; y < vy; y++)
00679         {
00680             if (bitmap_get_bit(sieve, y))
00681                 ui64_push(primes, iZ(y * vx + x, -1));
00682         }
00683         // handle partial last row where y = vy (check if p < n before pushing)
00684         if (bitmap_get_bit(sieve, vy))
00685         {
00686             uint64_t p = iZ(vy * vx + x, -1);
00687             if (p < n)
00688                 ui64_push(primes, p);
00689         }
00690     }
00691
00692     // Handle iZ+ case similarly:
00693     if (gcd(iZ(x, 1), vx) == 1)
00694     {
00695         bitmap_set_all(sieve); // reset sieve bitmap for new segment
00696         for (int i = k; i < root_count; i++)
00697         {
00698             uint64_t p = primes->array[i];
00699             int64_t y_0 = iZm_solve_for_y0(1, p, vx, x);
00700             bitmap_clear_steps_simd(sieve, p, y_0, vy);
00701         }
00702
00703         for (int y = 0; y < vy; y++)
00704         {
00705             if (bitmap_get_bit(sieve, y))
00706                 ui64_push(primes, iZ(y * vx + x, 1));
00707         }
00708         if (bitmap_get_bit(sieve, vy))
00709         {
00710             uint64_t p = iZ(vy * vx + x, 1);
00711             if (p < n)
00712                 ui64_push(primes, p);
00713         }
00714     }
00715 }
00716
00717 // * 3. Clean up and finalize
00718 bitmap_free(&sieve);
00719 ui64_resize_to_fit(primes); // Trim excess memory in primes array
00720
00721 primes->ordered = 0; // Mark the array as unordered
00722 return primes;
00723 }

```

13.23 src/toolkit/bitmap.c File Reference

Implementation of the bitmap module for efficient bit array operations.

#include <bitmap.h>

Functions

- **BITMAP * bitmap_init** (size_t size, int set_bits)
Creates and initializes a new bitmap with the specified number of bits.
- void **bitmap_free** (BITMAP **bitmap)
Frees all memory associated with a bitmap and nullifies the pointer.
- void **bitmap_set_bit** (BITMAP *bitmap, size_t idx)
Sets a specific bit to 1.
- int **bitmap_get_bit** (BITMAP *bitmap, size_t idx)
Gets the value of a specific bit.
- void **bitmap_flip_bit** (BITMAP *bitmap, size_t idx)
Flips the value of a specific bit.
- void **bitmap_clear_bit** (BITMAP *bitmap, size_t idx)
Clears (sets to 0) a specific bit.
- void **bitmap_set_all** (BITMAP *bitmap)
Sets all bits in the array to 1.
- void **bitmap_clear_all** (BITMAP *bitmap)
Clears all bits in the bitmap (sets them to 0).
- void **bitmap_clear_steps** (BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
Clear every step-th bit from start_idx up to limit.
- void **bitmap_clear_steps_simd** (BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
SIMD-optimized version of bitmap_clear_steps.
- **BITMAP * bitmap_clone** (BITMAP *src)
Creates a deep copy (clone) of an existing bitmap.
- void **bitmap_compute_hash** (BITMAP *bitmap)
Generates a SHA-256 hash for the bitmap->data.
- int **bitmap_validate_hash** (BITMAP *bitmap)
Validates the SHA-256 hash stored in bitmap->sha256.
- int **bitmap_fwrite** (BITMAP *bitmap, FILE *file)
Writes the bitmap to a binary file.
- **BITMAP * bitmap_fread** (FILE *file)
Reads a bitmap from a binary file.

13.23.1 Detailed Description

Implementation of the bitmap module for efficient bit array operations.

This file provides the complete implementation of bitmap functions including memory management, bit manipulation, data integrity verification via SHA-256 hashing, and binary file I/O with automatic hash validation.

13.23.1.1 Implementation Notes

- All functions include NULL pointer and bounds checking
- File I/O operations verify all read/write operations succeed
- Overflow protection is implemented in loop-based operations
- Memory is managed with proper cleanup on errors
- SHA-256 hashing uses OpenSSL library

13.23.1.2 Performance Characteristics

- [bitmap_init\(\)](#): $O(n/8)$ where n is number of bits
- Individual bit ops: $O(1)$
- Bulk operations: $O(n/8)$
- Hash computation: $O(n/8)$
- File I/O: $O(n/8)$

Author

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Date

October 2025

Version

1.0

See also

[bitmap.h](#) for API documentation
[test/test_bitmap.c](#) for test suite

Definition in file [bitmap.c](#).

13.24 bitmap.c

[Go to the documentation of this file.](#)

```

00001
00030
00031 #include <bitmap.h>
00032
00033 #ifdef __aarch64__
00034 #include <arm_neon.h>
00035 #elif defined(__x86_64__) || defined(_M_X64)
00036 #include <immintrin.h>
00037 #endif
00038
00057 BITMAP *bitmap_init(size_t size, int set_bits)
00058 {
00059     assert(size > 0 && "Bitmap size must be positive");
00060
00061     BITMAP *bitmap = (BITMAP *)malloc(sizeof(BITMAP));
00062     if (bitmap == NULL)
00063     {
00064         log_error("Memory allocation failed for BITMAP struct");
00065         return NULL;
00066     }
00067
00068     bitmap->size = size;
00069     bitmap->byte_size = (size + 7) / 8;
00070     bitmap->data = (unsigned char *)malloc(bitmap->byte_size);
00071     if (bitmap->data == NULL)
00072     {
00073         free(bitmap);
00074         log_error("bitmap_create: Memory allocation failed for BITMAP");

```

```

00075     return NULL;
00076 }
00077
00078 if (set_bits)
00079     memset(bitmap->data, 0xFF, bitmap->byte_size);
00080 else
00081     memset(bitmap->data, 0x00, bitmap->byte_size);
00082
00083 memset(bitmap->sha256, 0, SHA256_DIGEST_LENGTH); // Initialize SHA-256 to zero
00084
00085 return bitmap;
00086 }
00087
00103 void bitmap_free(BITMAP **bitmap)
00104 {
00105     // Safe guard against double free and NULL pointer
00106     if (bitmap && *bitmap)
00107     {
00108         if ((*bitmap)->data)
00109         {
00110             free((*bitmap)->data);
00111             (*bitmap)->data = NULL;
00112         }
00113
00114         free(*bitmap);
00115         *bitmap = NULL;
00116     }
00117 }
00118
00125 void bitmap_set_bit(BITMAP *bitmap, size_t idx)
00126 {
00127     bitmap->data[idx / 8] |= (1 « (idx % 8));
00128 }
00129
00137 int bitmap_get_bit(BITMAP *bitmap, size_t idx)
00138 {
00139     return (bitmap->data[idx / 8] & (1 « (idx % 8))) != 0;
00140 }
00141
00148 void bitmap_flip_bit(BITMAP *bitmap, size_t idx)
00149 {
00150     bitmap->data[idx / 8] ^= (1 « (idx % 8));
00151 }
00152
00159 void bitmap_clear_bit(BITMAP *bitmap, size_t idx)
00160 {
00161     // Clear the bit by ANDing with the negation of the bit mask
00162     bitmap->data[idx / 8] &= ~(1 « (idx % 8));
00163 }
00164
00170 void bitmap_set_all(BITMAP *bitmap)
00171 {
00172     memset(bitmap->data, 0xFF, (bitmap->size + 7) / 8);
00173 }
00174
00180 void bitmap_clear_all(BITMAP *bitmap)
00181 {
00182     memset(bitmap->data, 0x00, (bitmap->size + 7) / 8);
00183 }
00184
00210 void bitmap_clear_steps(BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
00211 {
00212     assert(step > 0 && "Step must be positive in bitmap_clear_steps.");
00213     limit = MIN(limit, bitmap->size - 1);
00214
00215     for (uint64_t idx = start_idx; idx <= limit; idx += step)
00216     {
00217         bitmap->data[idx / 8] &= ~(1 « (idx % 8));
00218     }
00219 }
00220
00233 void bitmap_clear_steps_simd(BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
00234 {
00235     assert(step > 0 && "Step must be positive in bitmap_clear_steps_simd.");
00236     limit = MIN(limit, bitmap->size - 1);
00237
00238 #ifdef __aarch64__
00239     // NEON implementation for ARM64
00240     uint64_t idx = start_idx;
00241
00242     // Process 4 steps at a time
00243     // Condition: idx + 3*step <= limit
00244
00245     if (limit >= 3 * step && idx <= limit - 3 * step)
00246     {
00247         uint64x2_t v_step2 = vdupq_n_u64(2 * step);
00248         uint64x2_t v_step4 = vdupq_n_u64(4 * step);

```



```

00249
00250 // Initialize indices: [idx, idx+step] and [idx+2step, idx+3step]
00251 uint64x2_t v_idx_01 = vdupq_n_u64(idx);
00252 v_idx_01 = vsetq_lane_u64(idx + step, v_idx_01, 1);
00253
00254 uint64x2_t v_idx_23 = vaddq_u64(v_idx_01, v_step2);
00255
00256 while (idx <= limit - 3 * step)
00257 {
00258     // Extract indices and perform operations
00259     // Note: NEON doesn't have efficient scatter for bytes, so we extract and do scalar stores
00260
00261     uint64_t i0 = vgetq_lane_u64(v_idx_01, 0);
00262     uint64_t i1 = vgetq_lane_u64(v_idx_01, 1);
00263     uint64_t i2 = vgetq_lane_u64(v_idx_23, 0);
00264     uint64_t i3 = vgetq_lane_u64(v_idx_23, 1);
00265
00266     bitmap->data[i0 / 8] &= ~(1 « (i0 % 8));
00267     bitmap->data[i1 / 8] &= ~(1 « (i1 % 8));
00268     bitmap->data[i2 / 8] &= ~(1 « (i2 % 8));
00269     bitmap->data[i3 / 8] &= ~(1 « (i3 % 8));
00270
00271     // Advance indices
00272     v_idx_01 = vaddq_u64(v_idx_01, v_step4);
00273     v_idx_23 = vaddq_u64(v_idx_23, v_step4);
00274
00275     idx += 4 * step;
00276 }
00277 }
00278
00279 // Handle remaining steps
00280 for (; idx <= limit; idx += step)
00281 {
00282     bitmap->data[idx / 8] &= ~(1 « (idx % 8));
00283 }
00284
00285 #elif defined(__AVX2__)
00286 // AVX2 implementation for x86_64
00287 uint64_t idx = start_idx;
00288
00289 // Process 4 steps at a time
00290 if (limit >= 3 * step && idx <= limit - 3 * step)
00291 {
00292     __m256i v_step4 = _mm256_set1_epi64x(4 * step);
00293
00294     // Initialize indices: [idx, idx+step, idx+2step, idx+3step]
00295     // Note: _mm256_set_epi64x takes arguments in reverse order (e3, e2, e1, e0)
00296     __m256i v_idx = _mm256_set_epi64x(idx + 3 * step, idx + 2 * step, idx + step, idx);
00297
00298     while (idx <= limit - 3 * step)
00299     {
00300         // Extract indices
00301         uint64_t i0 = _mm256_extract_epi64(v_idx, 0);
00302         uint64_t i1 = _mm256_extract_epi64(v_idx, 1);
00303         uint64_t i2 = _mm256_extract_epi64(v_idx, 2);
00304         uint64_t i3 = _mm256_extract_epi64(v_idx, 3);
00305
00306         bitmap->data[i0 / 8] &= ~(1 « (i0 % 8));
00307         bitmap->data[i1 / 8] &= ~(1 « (i1 % 8));
00308         bitmap->data[i2 / 8] &= ~(1 « (i2 % 8));
00309         bitmap->data[i3 / 8] &= ~(1 « (i3 % 8));
00310
00311         // Advance indices
00312         v_idx = _mm256_add_epi64(v_idx, v_step4);
00313         idx += 4 * step;
00314     }
00315 }
00316
00317 // Handle remaining steps
00318 for (; idx <= limit; idx += step)
00319 {
00320     bitmap->data[idx / 8] &= ~(1 « (idx % 8));
00321 }
00322
00323 #elif defined(__SSE2__)
00324 // SSE2 implementation for x86_64
00325 uint64_t idx = start_idx;
00326
00327 // Process 2 steps at a time
00328 if (limit >= step && idx <= limit - step)
00329 {
00330     __m128i v_step2 = _mm_set1_epi64x(2 * step);
00331     // Note: _mm_set_epi64x takes arguments in reverse order (e1, e0)
00332     __m128i v_idx = _mm_set_epi64x(idx + step, idx);
00333
00334     while (idx <= limit - step)
00335     {

```

```

00336         // Extract indices
00337         // _mm_cvtsi128_si64 extracts the lower 64 bits (element 0)
00338         uint64_t i0 = _mm_cvtsi128_si64(v_idx);
00339         // Unpack high to low to extract element 1
00340         uint64_t i1 = _mm_cvtsi128_si64(_mm_unpackhi_epi64(v_idx, v_idx));
00341
00342         bitmap->data[i0 / 8] &= ~(1 « (i0 % 8));
00343         bitmap->data[i1 / 8] &= ~(1 « (i1 % 8));
00344
00345         // Advance indices
00346         v_idx = _mm_add_epi64(v_idx, v_step2);
00347         idx += 2 * step;
00348     }
00349 }
00350
00351 // Handle remaining steps
00352 for (; idx <= limit; idx += step)
00353 {
00354     bitmap->data[idx / 8] &= ~(1 « (idx % 8));
00355 }
00356
00357 #else
00358 // Fallback to scalar implementation
00359 for (uint64_t idx = start_idx; idx <= limit; idx += step)
00360 {
00361     bitmap->data[idx / 8] &= ~(1 « (idx % 8));
00362 }
00363 #endif
00364 }
00365
00382 BITMAP *bitmap_clone(BITMAP *src)
00383 {
00384     assert(src && src->data && "Invalid source bitmap passed to bitmap_clone.");
00385
00386     BITMAP *dest = bitmap_init(src->size, 0);
00387     memcpy(dest->data, src->data, src->byte_size);
00388     memcpy(dest->sha256, src->sha256, SHA256_DIGEST_LENGTH);
00389
00390     return dest;
00391 }
00392
00398 void bitmap_compute_hash(BITMAP *bitmap)
00399 {
00400     assert(bitmap && bitmap->data && "Invalid bitmap passed to bitmap_compute_hash.");
00401
00402     // Generate SHA-256 hash and store it in the struct
00403     SHA256((unsigned char *)bitmap->data, bitmap->byte_size, bitmap->sha256);
00404 }
00405
00412 int bitmap_validate_hash(BITMAP *bitmap)
00413 {
00414     assert(bitmap && bitmap->data && "Invalid bitmap");
00415
00416     unsigned char correct_hash[SHA256_DIGEST_LENGTH]; // Buffer to hold the computed hash
00417
00418     // Generate SHA-256 hash and store it in correct_hash
00419     SHA256((unsigned char *)bitmap->data, bitmap->byte_size, correct_hash);
00420
00421     // Compare actual_hash with the stored hash in bitmap->sha256
00422     if (memcmp(correct_hash, bitmap->sha256, SHA256_DIGEST_LENGTH) == 0)
00423     {
00424         return 1; // SHA-256 match
00425     }
00426     else
00427     {
00428         log_error("SHA-256 checksum validation failed.");
00429         return 0; // SHA-256 mismatch
00430     }
00431 }
00432
00440 int bitmap_fwrite(BITMAP *bitmap, FILE *file)
00441 {
00442     assert(bitmap && bitmap->data && "Invalid bitmap passed to bitmap_fwrite.");
00443     assert(file && "File pointer is NULL in bitmap_fwrite.");
00444
00445     // Write size
00446     if (fwrite(&bitmap->size, sizeof(size_t), 1, file) != 1)
00447     {
00448         log_error("Failed to write bitmap size to file.");
00449         return 0;
00450     }
00451
00452     // Write data
00453     if (fwrite(bitmap->data, sizeof(unsigned char), bitmap->byte_size, file) != bitmap->byte_size)
00454     {
00455         log_error("Failed to write bitmap data to file.");
00456         return 0;

```

```

00457     }
00458
00459     // Compute SHA-256 hash if not already computed
00460     // Check if hash is all zeros by examining all bytes
00461     int hash_is_zero = 1;
00462     for (size_t i = 0; i < SHA256_DIGEST_LENGTH; i++)
00463     {
00464         if (bitmap->sha256[i] != 0)
00465         {
00466             hash_is_zero = 0;
00467             break;
00468         }
00469     }
00470
00471     if (hash_is_zero)
00472     {
00473         bitmap_compute_hash(bitmap);
00474     }
00475
00476     // Write SHA-256 hash
00477     if (fwrite(bitmap->sha256, 1, SHA256_DIGEST_LENGTH, file) != SHA256_DIGEST_LENGTH)
00478     {
00479         log_error("Failed to write SHA-256 hash to file.");
00480         return 0;
00481     }
00482
00483     return 1; // Success
00484 }
00485
00492 BITMAP *bitmap_fread(FILE *file)
00493 {
00494     assert(file && "File pointer is NULL in bitmap_fread.");
00495
00496     // Read size
00497     size_t size;
00498     if (fread(&size, sizeof(size_t), 1, file) != 1)
00499     {
00500         log_error("Failed to read bitmap size from file.");
00501         return NULL;
00502     }
00503
00504     BITMAP *bitmap = bitmap_init(size, 0);
00505     if (bitmap == NULL)
00506     {
00507         log_error("Failed to initialize bitmap.");
00508         return NULL;
00509     }
00510
00511     // Read data directly into the already-allocated buffer
00512     if (fread(bitmap->data, sizeof(unsigned char), bitmap->byte_size, file) != bitmap->byte_size)
00513     {
00514         log_error("Failed to read complete bitmap data from file.");
00515         bitmap_free(&bitmap);
00516         return NULL;
00517     }
00518
00519     // Read SHA-256 hash
00520     if (fread(bitmap->sha256, 1, SHA256_DIGEST_LENGTH, file) != SHA256_DIGEST_LENGTH)
00521     {
00522         log_error("Failed to read SHA-256 hash from file.");
00523         bitmap_free(&bitmap);
00524         return NULL;
00525     }
00526
00527     if (!bitmap_validate_hash(bitmap))
00528     {
00529         log_error("SHA-256 hash validation failed.");
00530         bitmap_free(&bitmap);
00531         return NULL;
00532     }
00533
00534     return bitmap;
00535 }

```

13.25 src/toolkit/int_arrays.c File Reference

Implementation of dynamic integer array module for 16-bit, 32-bit, and 64-bit unsigned integers.

```

#include <int_arrays.h>
#include "templates/int_array_impl.inc"

```

13.25.1 Detailed Description

Implementation of dynamic integer array module for 16-bit, 32-bit, and 64-bit unsigned integers.

This file provides the complete implementation of [UI16_ARRAY](#), [UI32_ARRAY](#), and [UI64_ARRAY](#) functions including memory management, automatic capacity growth, data integrity verification via SHA-256 hashing, and binary file I/O with automatic hash validation.

13.25.1.1 Implementation Notes

- All functions include NULL pointer checking
- Automatic capacity doubling (2x growth) ensures amortized O(1) append operations
- File I/O operations automatically compute and validate SHA-256 hashes
- Memory is managed with proper cleanup on errors
- SHA-256 hashing uses OpenSSL library

13.25.1.2 Performance Characteristics

- `init()`: O(1)
- `push()` (amortized): O(1)
- `push()` (worst case): O(n) during resize
- Hash operations: O(n)
- File I/O: O(n)

Author

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Date

October 2025

Version

1.0

See also

[int_arrays.h](#) for API documentation

Definition in file [int_arrays.c](#).

13.26 int_arrays.c

[Go to the documentation of this file.](#)

```
00001
00029
00030 #include <int_arrays.h>
00031
00032 //
=====
00033 // UI16_ARRAY IMPLEMENTATION
00034 //
=====
00036 #define TEMPLATE_TYPE uint16_t
00037 #define TEMPLATE_STRUCT UI16_ARRAY
00038 #define TEMPLATE_FUNC(name) ui16_##name
00039 #define TEMPLATE_NAME_STR "UI16_ARRAY"
00041 #include "templates/int_array_impl.inc"
00043 #undef TEMPLATE_TYPE
00044 #undef TEMPLATE_STRUCT
00045 #undef TEMPLATE_FUNC
00046 #undef TEMPLATE_NAME_STR
00048
00049 //
=====
00050 // UI32_ARRAY IMPLEMENTATION
00051 //
=====
00053 #define TEMPLATE_TYPE uint32_t
00054 #define TEMPLATE_STRUCT UI32_ARRAY
00055 #define TEMPLATE_FUNC(name) ui32_##name
00056 #define TEMPLATE_NAME_STR "UI32_ARRAY"
00058 #include "templates/int_array_impl.inc"
00060 #undef TEMPLATE_TYPE
00061 #undef TEMPLATE_STRUCT
00062 #undef TEMPLATE_FUNC
00063 #undef TEMPLATE_NAME_STR
00065
00066 //
=====
00067 // UI64_ARRAY IMPLEMENTATION
00068 //
=====
00070 #define TEMPLATE_TYPE uint64_t
00071 #define TEMPLATE_STRUCT UI64_ARRAY
00072 #define TEMPLATE_FUNC(name) ui64_##name
00073 #define TEMPLATE_NAME_STR "UI64_ARRAY"
00075 #include "templates/int_array_impl.inc"
00077 #undef TEMPLATE_TYPE
00078 #undef TEMPLATE_STRUCT
00079 #undef TEMPLATE_FUNC
00080 #undef TEMPLATE_NAME_STR
```

13.27 src/toolkit/iZ_toolkit.c File Reference

Implementation of iZ index space helpers and iZm/VX segment machinery.

```
#include <iZ_api.h>
```

Functions

- `uint64_t iZ (uint64_t x, int i)`
Computes $6x + i$ for a given x and i .
- `void iZ_mpz (mpz_t z, mpz_t x, int i)`
Computes $6x + i$ for arbitrary precision values using GMP.
- `static int compute_k_vx (IZM *iZm)`
Count how many small primes (>3) divide vx .
- `void iZm_construct_vx_base (uint64_t vx, BITMAP *base_x5, BITMAP *base_x7)`

- Build pre-sieved base bitmaps for a VX segment.
- static int `vx_set_base_values` (`VX_SEG` *vx_obj, char *y_str)
 - Initialize mpz-dependent base fields for a VX segment object.
- static void `vx_det_sieve` (`IZM` *iZm, `VX_SEG` *vx_obj)
 - Deterministic phase: mark composites using root primes.
- static void `vx_prob_sieve` (`VX_SEG` *vx_obj)
 - Perform probabilistic sieve cleanup for large numeric ranges.
- `IZM_RANGE_INFO` `range_info_init` (`INPUT_SIEVE_RANGE` *input_range, int vx)
 - Map a decimal range input into iZm coordinates and segment bounds.
- void `range_info_free` (`IZM_RANGE_INFO` *info)
 - Clear all GMP fields owned by info.

iZ Mapping Helpers

- void `process_iZ_bitmaps` (`UI64_ARRAY` *primes, `BITMAP` *x5, `BITMAP` *x7, uint64_t x↔_limit)
 - Traverse iZ bitmaps, emit surviving primes, and mark composites.
- void `get_root_primes` (`UI64_ARRAY` *primes, uint64_t limit)
 - Generate primes up to limit for deterministic sieving.
- int `check_primalty` (mpz_t n, int rounds)
 - Check the primality of a number using GMP's probabilistic test.

IZM Lifecycle

- `IZM` * `iZm_init` (size_t vx)
 - Allocate and initialize an `IZM` object for a given VX.
- `IZM` * `iZm_clone` (`IZM` *src)
 - Deep-copy an `IZM` object for per-worker ownership.
- void `iZm_free` (`IZM` **iZm)
 - Release an `IZM` object and set the caller pointer to NULL.

VX Selection Helpers

- uint64_t `compute_vx_k` (int k)
 - Calculate `VX_{ k }`.
- uint64_t `compute_l2_vx` (uint64_t n)
 - Choose VX using an L2-cache-aware heuristic.
- void `compute_max_vx` (mpz_t vx, int bit_size)
 - Compute largest VX below $2^{\text{bit_size}}$.

Modular Hit Solvers

- uint64_t `iZm_solve_for_x0` (int m_id, uint64_t p, uint64_t vx, uint64_t y)
 - Solve first x-hit of prime p for line m_id in segment y.
- uint64_t `iZm_solve_for_x0_mpz` (int m_id, uint64_t p, uint64_t vx, mpz_t y)
 - GMP variant of `iZm_solve_for_x0()` for very large y.
- int64_t `iZm_solve_for_y0` (int m_id, uint64_t p, uint64_t vx, uint64_t x)
 - Solve first y-hit for fixed x in vertical (vy) scanning.

VX Segment Lifecycle and Execution

- `VX_SEG` * `vx_init` (`IZM` *iZm, int start_x, int end_x, char *y_str, int mr_rounds)
 - Initialize and deterministically sieve one VX segment.
- void `vx_free` (`VX_SEG` **vx_obj)

- Free a VX segment and all owned resources.
- void [vx_collect_p_gaps](#) ([VX_SEG](#) *vx_obj)
Extract prime-gap encoding from a fully sieved segment.
- void [vx_full_sieve](#) ([VX_SEG](#) *vx_obj, int collect_p_gaps)
Complete segment processing (probabilistic stage and optional gaps).
- void [vx_stream](#) ([VX_SEG](#) *vx_obj, FILE *output)
Stream segment primes to an output stream.

Random Prime Search Routines

- int [vx_search_prime](#) (mpz_t p, int m_id, int vx, int bit_size)
Horizontal iZm/VX random-prime search.
- int [vy_search_prime](#) (mpz_t p, int m_id, mpz_t vx)
Vertical iZm/VY random-prime search.

Variables

- static const int [s_primes](#) []
- static int [s_primes_count](#) = sizeof([s_primes](#)) / sizeof(int)

13.27.1 Detailed Description

Implementation of iZ index space helpers and iZm/VX segment machinery.

Definition in file [iZ_toolkit.c](#).

13.27.2 Function Documentation

13.27.2.1 compute_k_vx()

```
int compute_k_vx (
    IZM * iZm) [static]
```

Count how many small primes (>3) divide vx.

Parameters

| | |
|-----|------------------------------|
| iZm | Initialized toolkit context. |
|-----|------------------------------|

Returns

Number of pre-sieved small primes encoded in vx.

Definition at line 52 of file [iZ_toolkit.c](#).

References [s_primes](#), and [IZM::vx](#).

Referenced by [iZm_init\(\)](#).

13.27.2.2 vx_det_sieve()

```
void vx_det_sieve (
    IZM * iZm,
    VX\_SEG * vx_obj) [static]
```

Deterministic phase: mark composites using root primes.

Parameters

| | |
|-----|---|
| iZm | Toolkit context containing root-prime table and base bitmaps. |
|-----|---|

| | |
|---------------------|---------------------------|
| <code>vx_obj</code> | Segment object to update. |
|---------------------|---------------------------|

Definition at line 566 of file [iZ_toolkit.c](#).

References [UI64_ARRAY::array](#), [VX_SEG::bit_ops](#), [bitmap_clear_steps_simd\(\)](#), [bitmap_get_bit\(\)](#), [UI64_ARRAY::count](#), [VX_SEG::end_x](#), [VX_SEG::is_large_limit](#), [iZm_solve_for_x0\(\)](#), [iZm_solve_for_x0_mpz\(\)](#), [IZM::k_vx](#), [VX_SEG::p_count](#), [VX_SEG::root_limit](#), [IZM::root_primes](#), [VX_SEG::start_x](#), [VX_SEG::vx](#), [VX_SEG::x5](#), [VX_SEG::x7](#), and [VX_SEG::y](#).

Referenced by [vx_init\(\)](#).

13.27.2.3 vx_prob_sieve()

```
void vx_prob_sieve (
    VX\_SEG * vx_obj) [static]
```

Perform probabilistic sieve cleanup for large numeric ranges.

Parameters

| | |
|---------------------|--|
| <code>vx_obj</code> | Segment object containing deterministic survivors. |
|---------------------|--|

Definition at line 633 of file [iZ_toolkit.c](#).

References [bitmap_clear_bit\(\)](#), [bitmap_get_bit\(\)](#), [check_primalty\(\)](#), [VX_SEG::end_x](#), [VX_SEG::is_large_limit](#), [iZ_mpz\(\)](#), [log_info\(\)](#), [VX_SEG::mr_rounds](#), [VX_SEG::p_count](#), [VX_SEG::p_test_ops](#), [VX_SEG::start_x](#), [VX_SEG::x5](#), [VX_SEG::x7](#), and [VX_SEG::yvx](#).

Referenced by [vx_full_sieve\(\)](#).

13.27.2.4 vx_set_base_values()

```
int vx_set_base_values (
    VX\_SEG * vx_obj,
    char * y_str) [static]
```

Initialize mpz-dependent base fields for a VX segment object.

Parameters

| | |
|---------------------|---------------------------------------|
| <code>vx_obj</code> | Segment object to populate. |
| <code>y_str</code> | Decimal y-coordinate for the segment. |

Returns

1 on success, 0 on parse/initialization failure.

Definition at line 529 of file [iZ_toolkit.c](#).

References [VX_SEG::is_large_limit](#), [iZ_mpz\(\)](#), [log_error\(\)](#), [VX_SEG::root_limit](#), [VX_SEG::vx](#), [VX_SEG::y](#), and [VX_SEG::yvx](#).

Referenced by [vx_init\(\)](#).

13.27.3 Variable Documentation

13.27.3.1 s_primes

```
const int s_primes[] [static]
```

Initial value:

```
= {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,  
   43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}
```

Small primes used to compute and construct wheel structures.

Definition at line 10 of file [iZ_toolkit.c](#).

Referenced by [compute_k_vx\(\)](#), [compute_l2_vx\(\)](#), [compute_vx_k\(\)](#), and [iZm_construct_vx_base\(\)](#).

13.27.3.2 s_primes_count

```
int s_primes_count = sizeof(s_primes) / sizeof(int) [static]
```

Number of entries available in [s_primes](#).

Definition at line 13 of file [iZ_toolkit.c](#).

Referenced by [iZm_construct_vx_base\(\)](#).

13.28 iZ_toolkit.c

[Go to the documentation of this file.](#)

```
00001
00006
00007 #include <iZ_api.h>
00008
00010 static const int s_primes[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
00011                               43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97};
00013 static int s_primes_count = sizeof(s_primes) / sizeof(int);
00014
00024 inline uint64_t iZ(uint64_t x, int i)
00025 {
00026     return 6 * x + i;
00027 }
00028
00037 void iZ_mpz(mpz_t z, mpz_t x, int i)
00038 {
00039     mpz_mul_ui(z, x, 6); // z = 6 * x
00040
00041     if (i > 0)
00042         mpz_add_ui(z, z, i); // z = z + i
00043     else
00044         mpz_sub_ui(z, z, -i); // z = z - i
00045 }
00046
00052 static int compute_k_vx(IZM *iZm)
00053 {
00054     int k = 0;
00055     while (iZm->vx % s_primes[k + 2] == 0)
00056     {
00057         k++;
00058     }
00059     return k;
00060 }
00061
00070 void process_iZ_bitmaps(UI64_ARRAY *primes, BITMAP *x5, BITMAP *x7, uint64_t x_limit)
00071 {
00072     uint64_t root_limit = sqrt(6 * x_limit) + 1;
00073 }
```

```

00074 // Iterate through x values in range 0 < x < x_n
00075 for (uint64_t x = 1; x < x_limit; x++)
00076 {
00077     // if x5[x], implying it's iZ- prime
00078     if (bitmap_get_bit(x5, x))
00079     {
00080         uint64_t p = iZ(x, -1); // compute p = iZ(x, -1)
00081         ui64_push(primes, p); // add p to primes
00082
00083         // if p is root prime, mark its multiples in x5, x7
00084         if (p < root_limit)
00085         {
00086             bitmap_clear_steps_simd(x5, p, p * x + x, x_limit);
00087             bitmap_clear_steps_simd(x7, p, p * x - x, x_limit);
00088         }
00089     }
00090
00091     // Do the same if x7[x], inverting the xp signs
00092     if (bitmap_get_bit(x7, x))
00093     {
00094         uint64_t p = iZ(x, 1);
00095         ui64_push(primes, p);
00096
00097         if (p < root_limit)
00098         {
00099             bitmap_clear_steps_simd(x5, p, p * x - x, x_limit);
00100             bitmap_clear_steps_simd(x7, p, p * x + x, x_limit);
00101         }
00102     }
00103 }
00104 }
00105
00112 void get_root_primes(UI64_ARRAY *primes, uint64_t limit)
00113 {
00114     // Add 2, 3 to primes
00115     ui64_push(primes, 2);
00116     ui64_push(primes, 3);
00117
00118     // Calculate x_n, max x value in iZ space for given n
00119     uint64_t x_n = limit / 6 + 1;
00120
00121     // Create bitmap X-Arrays x5, x7, each of size x_n + 1 bits
00122     BITMAP *x5 = bitmap_init(x_n + 1, 1);
00123     BITMAP *x7 = bitmap_init(x_n + 1, 1);
00124
00125     // Memory allocation failed, check logs
00126     if (!x5 || !x7)
00127     {
00128         ui64_free(&primes);
00129         return;
00130     }
00131
00132     // Sieve logic: mark composites and collect primes up to limit
00133     process_iZ_bitmaps(primes, x5, x7, x_n);
00134
00135     // Cleanup: free memory of x5, x7
00136     bitmap_free(&x5);
00137     bitmap_free(&x7);
00138
00139     return;
00140 }
00141
00153 int check_primality(mpz_t n, int rounds)
00154 {
00155     // GMP's mpz_probab_prime_p returns:
00156     // 0 if n is composite,
00157     // 1 if n is probably prime (without being certain),
00158     // 2 if n is definitely prime (only for small n).
00159     int is_prime = mpz_probab_prime_p(n, rounds);
00160     // ToDo: implement Baillie-PSW primality test for a deterministic alternative to Miller-Rabin, which would return a
00161     // boolean result instead of a probability.
00162     return is_prime;
00163 }
00164
00165 // =====
00166 // * iZm structure:
00167 // =====
00168
00184 IZM *iZm_init(size_t vx)
00185 {
00186     // validate vx
00187     assert(vx >= 35 && "vx must be at least 35.");
00188
00189     IZM *iZm = malloc(sizeof(IZM));
00190     if (!iZm)
00191     {

```

```

00192     log_error("Memory allocation failed for iZm.");
00193     return NULL;
00194 }
00195
00196 iZm->vx = vx;
00197 // get root primes for deterministic sieving
00198 iZm->root_primes = SiZ(vx);
00199 // iZm->root_primes = root_limit < pow(10, 7) ? SiZ(root_limit) : SiZm(root_limit);
00200 if (!iZm->root_primes)
00201 {
00202     log_error("Root primes generation failed for iZm.");
00203     free(iZm);
00204     return NULL;
00205 }
00206
00207 iZm->k_vx = compute_k_vx(iZm);
00208
00209 // initialize base bitmaps
00210 iZm->base_x5 = bitmap_init(vx + 10, 1);
00211 iZm->base_x7 = bitmap_init(vx + 10, 1);
00212 if (!iZm->base_x5 || !iZm->base_x7)
00213 {
00214     log_error("Bitmap initialization failed for iZm base segment.");
00215     iZm_free(&iZm);
00216     ui64_free(&iZm->root_primes);
00217     return NULL;
00218 }
00219
00220 // construct pre-sieved base_x5, base_x7 bitmaps
00221 iZm_construct_vx_base(vx, iZm->base_x5, iZm->base_x7);
00222
00223 return iZm;
00224 }
00225
00232 IZM *iZm_clone(IZM *src)
00233 {
00234     assert(src && "Invalid source IZM structure for cloning.");
00235
00236     IZM *clone = malloc(sizeof(IZM));
00237     assert(clone && "Memory allocation failed for iZm clone.");
00238
00239     clone->vx = src->vx;
00240     clone->k_vx = src->k_vx;
00241     clone->root_primes = ui64_init(src->root_primes->capacity);
00242     if (clone->root_primes == NULL)
00243     {
00244         log_error("Memory allocation failed for root primes in iZm clone.");
00245         iZm_free(&clone);
00246         return NULL;
00247     }
00248     // copy root primes
00249     memcpy(clone->root_primes->array, src->root_primes->array, src->root_primes->count * sizeof(uint64_t));
00250     clone->root_primes->count = src->root_primes->count;
00251
00252     // clone base bitmaps
00253     clone->base_x5 = bitmap_clone(src->base_x5);
00254     clone->base_x7 = bitmap_clone(src->base_x7);
00255
00256     return clone;
00257 }
00258
00266 void iZm_free(IZM **iZm)
00267 {
00268     if (iZm == NULL || *iZm == NULL)
00269         return;
00270
00271     // Free root primes object allocated in iZm_init
00272     ui64_free(&(*iZm)->root_primes);
00273     bitmap_free(&(*iZm)->base_x5);
00274     bitmap_free(&(*iZm)->base_x7);
00275     free(*iZm);
00276     *iZm = NULL;
00277 }
00278
00286 uint64_t compute_vx_k(int k)
00287 {
00288     uint64_t vx = 1;
00289     k += 2; // adjust k to account for skipping 2 and 3
00290     int i = 2; // start from prime 5
00291
00292     while (i < k && s_primes[i] * vx < UINT64_MAX)
00293     {
00294         vx *= s_primes[i++];
00295     }
00296
00297     return vx;
00298 }

```

```

00299
00306 uint64_t compute_l2_vx(uint64_t n)
00307 {
00308     uint64_t l2 = get_cpu_L2_cache_size_bits();
00309     uint64_t x_n = n / 6;
00310     uint64_t vx = 35; // minimum useful vx size
00311     int k = 4;        // pointing to prime 11 in s_primes
00312
00313     // while vx * k-th prime doesn't exceed x_n and l2 cache size
00314     while (vx * s_primes[k] < MIN(l2, x_n))
00315     {
00316         vx *= s_primes[k];
00317         k++;
00318     }
00319
00320     return vx;
00321 }
00322
00331 void compute_max_vx(mpz_t vx, int bit_size)
00332 {
00333     // get some primes to compute the primorial vx
00334     UI64_ARRAY *primes = SIZ(10000);
00335     assert(primes && "Failed to generate primes");
00336
00337     int i = 2; // to skip 2, 3
00338     mpz_set_ui(vx, primes->array[i]); // set vx = 5
00339
00340     // while vx * primes->array[i] < 2^bit_size
00341     while (mpz_sizeinbase(vx, 2) < (size_t)bit_size)
00342     {
00343         i++;
00344         mpz_mul_ui(vx, vx, primes->array[i]);
00345     }
00346
00347     mpz_div_ui(vx, vx, primes->array[i]); // divide by the last prime
00348 }
00349
00363 void iZm_construct_vx_base(uint64_t vx, BITMAP *base_x5, BITMAP *base_x7)
00364 {
00365     assert(base_x5 && base_x7 && "Invalid bitmaps passed to iZm_construct_vx_base.");
00366
00367     bitmap_set_all(base_x5);
00368     bitmap_set_all(base_x7);
00369     bitmap_clear_bit(base_x5, 0); // clear the 0th bit
00370     bitmap_clear_bit(base_x7, 0); // clear the 0th bit
00371
00372     // iterate through small primes that divide vx
00373     for (int i = 2; i < s_primes_count; i++)
00374     {
00375         int p = s_primes[i];
00376         if (vx % p != 0)
00377             continue;
00378
00379         int ip = (p % 6 == 1) ? 1 : -1;
00380         int xp = (p + 1) / 6;
00381         // clear composites in iZ- and iZ+
00382         if (ip == -1)
00383         {
00384             bitmap_clear_bit(base_x5, xp); // clear the prime itself
00385             bitmap_clear_steps_simd(base_x5, p, p * xp + xp, vx); // mark composites in iZ-
00386             bitmap_clear_steps_simd(base_x7, p, p * xp - xp, vx); // mark composites in iZ+
00387         }
00388         else
00389         {
00390             bitmap_clear_bit(base_x7, xp); // clear the prime itself
00391             bitmap_clear_steps_simd(base_x5, p, p * xp - xp, vx); // mark composites in iZ-
00392             bitmap_clear_steps_simd(base_x7, p, p * xp + xp, vx); // mark composites in iZ+
00393         }
00394     }
00395 }
00396
00415 uint64_t iZm_solve_for_x0(int m_id, uint64_t p, uint64_t vx, uint64_t y)
00416 {
00417     uint64_t xp = (p + 1) / 6;
00418     int ip = (p % 6 == 1) ? 1 : -1;
00419
00420     // Immediate solution check for y = 0
00421     if (y == 0)
00422     {
00423         // compute p * xp + m_id * ip * xp
00424         return p * xp + m_id * ip * xp;
00425     }
00426
00427     // Normalize x_p to x_p if p_id = m_id, else to p - x_p
00428     xp = (m_id == ip) ? xp : p - xp;
00429     uint64_t yvx = vx * y;
00430

```

```

00431 // Compute the first composite hit of p in the yth vx segment in iZm index space
00432 uint64_t x = p < vx ? p - (yvx - xp) % p : (yvx - xp) % p;
00433 return x;
00434 }
00435
00449 uint64_t iZm_solve_for_x0_mpz(int m_id, uint64_t p, uint64_t vx, mpz_t y)
00450 {
00451     mpz_t tmp;
00452     mpz_init(tmp);
00453
00454     // 1. Normalize x_p to x_p if p_id = m_id, else to p - x_p
00455     uint64_t xp = (p + 1) / 6;
00456     int ip = (p % 6 == 1) ? 1 : -1;
00457     xp = (m_id == ip) ? xp : p - xp;
00458
00459     mpz_mul_ui(tmp, y, vx);
00460     mpz_sub_ui(tmp, tmp, xp);
00461     mpz_mod_ui(tmp, tmp, p);
00462
00463     uint64_t x = p < vx ? p - mpz_get_ui(tmp) : mpz_get_ui(tmp);
00464
00465     mpz_clear(tmp);
00466     return x;
00467 }
00468
00490 int64_t iZm_solve_for_y0(int m_id, uint64_t p, uint64_t vx, uint64_t x)
00491 {
00492     // No solution check, if vx and p are not coprime
00493     if (gcd(vx, p) != 1)
00494     {
00495         printf("There's no solution for the given parameters\n");
00496         return -1; // No solution can be found
00497     }
00498
00499     uint64_t xp = (p + 1) / 6;
00500     int ip = (p % 6 == 1) ? 1 : -1;
00501     xp = (m_id == ip) ? xp : p - xp;
00502
00503     // Immediate solution check
00504     if (x % p == xp)
00505         return 0;
00506
00507     // Compute delta
00508     uint64_t x_mod_p = x % p;
00509     uint64_t delta = (xp + p - x_mod_p) % p;
00510
00511     // Find modular inverse
00512     uint64_t vx_inv = modular_inverse(vx % p, p);
00513
00514     // Compute and return y
00515     uint64_t y = (delta * vx_inv) % p;
00516     return y;
00517 }
00518
00519 // =====
00520 // * VX_SEG structure:
00521 // =====
00522
00529 static int vx_set_base_values(VX_SEG *vx_obj, char *y_str)
00530 {
00531
00532     // assert y_str is numeric
00533     mpz_t y_tmp;
00534     mpz_init(y_tmp);
00535     if (mpz_set_str(y_tmp, y_str, 10) != 0)
00536     {
00537         log_error("Invalid numeric string for y in vx_set_base_values\n");
00538         mpz_clear(y_tmp);
00539         return 0;
00540     }
00541
00542     mpz_init_set(vx_obj->y, y_tmp);
00543     mpz_clear(y_tmp);
00544
00545     // Compute yvx = y * vx
00546     mpz_init(vx_obj->yvx);
00547     mpz_mul_ui(vx_obj->yvx, vx_obj->y, vx_obj->vx);
00548
00549     // Compute root_limit = sqrt(iZ(vx * (y+1), 1))
00550     mpz_init(vx_obj->root_limit);
00551     mpz_add_ui(vx_obj->root_limit, vx_obj->yvx, vx_obj->vx);
00552     iZ_mpz(vx_obj->root_limit, vx_obj->root_limit, 1);
00553     mpz_sqrt(vx_obj->root_limit, vx_obj->root_limit);
00554
00555     // Set is_large_limit to determine if probabilistic primality test is needed
00556     // if root_limit > vx
00557     vx_obj->is_large_limit = mpz_cmp_ui(vx_obj->root_limit, vx_obj->vx) > 0;

```

```

00558     return 1;
00559 }
00560
00566 static void vx_det_sieve(IZM *iZm, VX_SEG *vx_obj)
00567 {
00568     assert(iZm && "iZm is NULL in vx_det_sieve");
00569     assert(vx_obj && "vx_obj is NULL in vx_det_sieve");
00570
00571     int vx = vx_obj->vx; // segment size
00572
00573     // * Deterministic Sieve: Mark composites of primes < vx in x5, x7
00574     int start_x = vx_obj->start_x;
00575     int end_x = vx_obj->end_x;
00576     int k = 2 + iZm->k_vx; // skip 2, 3 and pre-sieved k_vx primes
00577     UI64_ARRAY *root_primes = iZm->root_primes;
00578
00579     // if y < 2^64, use iZm_solve_for_x0 version for efficiency
00580     if (mpz_sizeinbase(vx_obj->y, 2) <= 64)
00581     {
00582         uint64_t y = mpz_get_ui(vx_obj->y);
00583         uint64_t root_limit = mpz_get_ui(vx_obj->root_limit);
00584
00585         // Iterate through root primes, skipping the first k pre-sieved primes
00586         for (int i = k; i < root_primes->count; i++)
00587         {
00588             uint64_t p = root_primes->array[i];
00589
00590             if (p > root_limit)
00591                 break;
00592
00593             // Mark composites of p in x5 and x7
00594             bitmap_clear_steps_simd(vx_obj->x5, p, iZm_solve_for_x0(-1, p, vx, y), end_x);
00595             bitmap_clear_steps_simd(vx_obj->x7, p, iZm_solve_for_x0(1, p, vx, y), end_x);
00596
00597             vx_obj->bit_ops += (2 * end_x) / p; // approximate number of bit operations
00598         }
00599     }
00600     else
00601     {
00602         // the same as above but using iZm_solve_for_x0_mpz version
00603         for (int i = k; i < root_primes->count; i++)
00604         {
00605             int p = root_primes->array[i];
00606
00607             bitmap_clear_steps_simd(vx_obj->x5, p, iZm_solve_for_x0_mpz(-1, p, vx, vx_obj->y), end_x);
00608             bitmap_clear_steps_simd(vx_obj->x7, p, iZm_solve_for_x0_mpz(1, p, vx, vx_obj->y), end_x);
00609
00610             vx_obj->bit_ops += (2 * end_x) / p;
00611         }
00612     }
00613
00614     // if no further probabilistic testing is needed,
00615     // count unmarked bits in x5 and x7 as p_count
00616     if (!vx_obj->is_large_limit)
00617     {
00618         vx_obj->p_count = 0;
00619         for (int x = start_x; x <= end_x; x++)
00620         {
00621             if (bitmap_get_bit(vx_obj->x5, x))
00622                 vx_obj->p_count++;
00623             if (bitmap_get_bit(vx_obj->x7, x))
00624                 vx_obj->p_count++;
00625         }
00626     }
00627 }
00628
00633 static void vx_prob_sieve(VX_SEG *vx_obj)
00634 {
00635     // If is_large_limit is false, no need for probabilistic testing
00636     if (!vx_obj->is_large_limit)
00637     {
00638         log_info("vx_obj->is_large_limit is false, no need for probabilistic testing");
00639         return;
00640     }
00641
00642     // Initialize GMP reusable variables p, x_p
00643     mpz_t p, x_p;
00644     mpz_init(p);
00645     mpz_init(x_p);
00646
00647     int r = vx_obj->mr_rounds;
00648     int s = vx_obj->start_x <= 1 ? 1 : vx_obj->start_x;
00649
00650     // Iterate through x values in the range start_x <= x <= end_x
00651     for (int x = s; x <= vx_obj->end_x; x++)
00652     {
00653         // Check if iZ(vx * y + x, -1) is prime, if not, clear x in x5

```

```

00654     if (bitmap_get_bit(vx_obj->x5, x))
00655     {
00656         // Compute x_p = yvx + x
00657         mpz_add_ui(x_p, vx_obj->yvx, x);
00658         iZ_mpz(p, x_p, -1); // Compute p = iZ(x_p, -1)
00659         int is_prime = check_primality(p, r);
00660         vx_obj->p_test_ops++;
00661
00662         // if is_prime, increment count, else clear composite from x5
00663         if (is_prime)
00664         {
00665             vx_obj->p_count++;
00666         }
00667         else
00668         {
00669             bitmap_clear_bit(vx_obj->x5, x); // Clear composite from x5
00670         }
00671     }
00672
00673     // same for the 6x+1 candidate
00674     if (bitmap_get_bit(vx_obj->x7, x))
00675     {
00676         mpz_add_ui(x_p, vx_obj->yvx, x);
00677         iZ_mpz(p, x_p, 1); // Compute p = iZ(x_p, 1)
00678         int is_prime = check_primality(p, r);
00679         vx_obj->p_test_ops++;
00680
00681         if (is_prime)
00682         {
00683             vx_obj->p_count++;
00684         }
00685         else
00686         {
00687             bitmap_clear_bit(vx_obj->x7, x); // Clear composite from x7
00688         }
00689     }
00690 }
00691
00692 vx_obj->is_large_limit = 0; // all composites cleared
00693
00694 // Cleanup
00695 mpz_clears(p, x_p, NULL);
00696 }
00697
00713 VX_SEG *vx_init(IZM *iZm, int start_x, int end_x, char *y_str, int mr_rounds)
00714 {
00715     // assert vx > 5 and not a multiple of 2 or 3
00716     assert(iZm && "iZm is NULL in vx_init");
00717     assert(y_str && "y_str is NULL in vx_init");
00718
00719     VX_SEG *vx_obj = malloc(sizeof(VX_SEG));
00720     if (vx_obj == NULL)
00721     {
00722         log_error("Memory allocation failed in vx_init\n");
00723         return NULL;
00724     }
00725
00726     // Initialize struct members
00727     vx_obj->vx = iZm->vx;
00728
00729     // Set base values
00730     if (!vx_set_base_values(vx_obj, y_str))
00731     {
00732         // check logs
00733         free(vx_obj);
00734         return NULL;
00735     }
00736
00737     vx_obj->mr_rounds = (mr_rounds == 0) ? MR_ROUNDS : mr_rounds; // default 25 rounds
00738
00739     vx_obj->start_x = MAX(start_x, 1);
00740     vx_obj->end_x = MIN(end_x, vx_obj->vx);
00741     vx_obj->x5 = bitmap_clone(iZm->base_x5);
00742     vx_obj->x7 = bitmap_clone(iZm->base_x7);
00743     vx_obj->p_count = 0;
00744     vx_obj->p_gaps = NULL;
00745     vx_obj->bit_ops = 0;
00746     vx_obj->p_test_ops = 0;
00747
00748     // perform deterministic sieving to prepare for probabilistic sieving or streaming
00749     vx_det_sieve(iZm, vx_obj);
00750
00751     return vx_obj;
00752 }
00753
00759 void vx_free(VX_SEG **vx_obj)
00760 {

```

```

00761     if (vx_obj == NULL || *vx_obj == NULL)
00762         return;
00763
00764     // clear bitmaps
00765     bitmap_free(&(*vx_obj)->x5);
00766     bitmap_free(&(*vx_obj)->x7);
00767
00768     // clear p_gaps array
00769     ui16_free(&(*vx_obj)->p_gaps);
00770
00771     // clear mpz_t variables
00772     mpz_clears((*vx_obj)->y, (*vx_obj)->yvx, (*vx_obj)->root_limit, NULL);
00773
00774     free(*vx_obj);
00775     *vx_obj = NULL;
00776 }
00777
00787 void vx_collect_p_gaps(VX_SEG *vx_obj)
00788 {
00789     assert(vx_obj && vx_obj->p_count > 0 && "Invalid vx_obj in vx_collect_p_gaps");
00790     assert(mpz_cmp_ui(vx_obj->y, 0) > 0 && "First segment requires special handling in vx_collect_p_gaps");
00791     if (vx_obj->is_large_limit)
00792     {
00793         vx_full_sieve(vx_obj, 0);
00794     }
00795
00796     vx_obj->p_gaps = ui16_init(vx_obj->p_count + 2);
00797     assert(vx_obj->p_gaps && "Memory allocation failed for vx_obj->p_gaps in vx_collect_p_gaps");
00798
00799     // Initialize gap counter
00800     int gap = 0;
00801
00802     // Iterate through x values in the range start_x <= x <= end_x
00803     for (int x = vx_obj->start_x; x <= vx_obj->end_x; x++)
00804     {
00805         // Increment gap by 4 since: iZ(x, -1) - iZ(x-1, 1) = 4
00806         gap += 4;
00807
00808         // Check if iZ(vx * y + x, -1) is prime
00809         if (bitmap_get_bit(vx_obj->x5, x))
00810         {
00811             // Append gap to p_gaps
00812             ui16_push(vx_obj->p_gaps, gap);
00813             gap = 0; // Reset gap
00814         }
00815
00816         // Increment gap by 2
00817         gap += 2;
00818
00819         // Check if iZ(vx * y + x, 1) is prime
00820         if (bitmap_get_bit(vx_obj->x7, x))
00821         {
00822             // Append gap to p_gaps
00823             ui16_push(vx_obj->p_gaps, gap);
00824             gap = 0; // Reset gap
00825         }
00826     }
00827
00828     // append final gap for backward calculations
00829     ui16_push(vx_obj->p_gaps, gap);
00830 }
00831
00847 void vx_full_sieve(VX_SEG *vx_obj, int collect_p_gaps)
00848 {
00849     assert(vx_obj && "vx_obj is NULL in vx_full_sieve");
00850
00851     // If is_large_limit is true, perform probabilistic primality tests
00852     if (vx_obj->is_large_limit)
00853         vx_prob_sieve(vx_obj);
00854
00855     // If collect_p_gaps is true, populate the p_gaps array
00856     if (collect_p_gaps)
00857         vx_collect_p_gaps(vx_obj);
00858 }
00859
00866 void vx_stream(VX_SEG *vx_obj, FILE *output)
00867 {
00868     assert(vx_obj && "vx_obj is NULL in vx_stream");
00869     assert(output && "output stream is NULL in vx_stream");
00870
00871     // Initialize GMP reusable variables p, x_p
00872     mpz_t p, x_p;
00873     mpz_init(p);
00874     mpz_init(x_p);
00875
00876     int r = vx_obj->mr_rounds;
00877

```



```

00878 // Iterate through x values in the range start_x <= x <= end_x
00879 for (int x = vx_obj->start_x; x <= vx_obj->end_x; x++)
00880 {
00881     // Check if iZ(vx * y + x, -1) is prime, if not, clear x in x5
00882     if (bitmap_get_bit(vx_obj->x5, x))
00883     {
00884         // Compute x_p = yvx + x
00885         mpz_add_ui(x_p, vx_obj->yvx, x);
00886         iZ_mpz(p, x_p, -1); // Compute p = iZ(x_p, -1)
00887         int is_prime = 1;
00888
00889         if (vx_obj->is_large_limit)
00890         {
00891             vx_obj->p_test_ops++;
00892             is_prime = check_primal(p, r);
00893         }
00894
00895         if (is_prime)
00896         {
00897             if (vx_obj->is_large_limit)
00898             {
00899                 vx_obj->p_count++; // otherwise already counted in det_sieve
00900             }
00901             gmp_printf(output, "%Zd ", p);
00902         }
00903         else
00904         {
00905             bitmap_clear_bit(vx_obj->x5, x); // Clear composite from x5
00906         }
00907     }
00908
00909     // test iZ(vx * y + x, 1) for primality
00910     if (bitmap_get_bit(vx_obj->x7, x))
00911     {
00912         mpz_add_ui(x_p, vx_obj->yvx, x);
00913         iZ_mpz(p, x_p, 1); // Compute p = iZ(x_p, 1)
00914         int is_prime = 1;
00915
00916         if (vx_obj->is_large_limit)
00917         {
00918             vx_obj->p_test_ops++;
00919             is_prime = check_primal(p, r);
00920         }
00921
00922         if (is_prime)
00923         {
00924             if (vx_obj->is_large_limit)
00925             {
00926                 vx_obj->p_count++;
00927             }
00928             gmp_printf(output, "%Zd ", p);
00929         }
00930         else
00931         {
00932             bitmap_clear_bit(vx_obj->x7, x); // Clear composite from x7
00933         }
00934     }
00935 }
00936
00937 mpz_clears(p, x_p, NULL);
00938 }
00939
00940 // =====
00941 // * IZM_RANGE_INFO structure:
00942 // =====
00943
00944 IZM_RANGE_INFO range_info_init(INPUT_SIEVE_RANGE *input_range, int vx)
00945 {
00946     IZM_RANGE_INFO info = {0};
00947     info.vx = vx;
00948     info.y_range = -1;
00949
00950     mpz_inits(info.Zs, info.Ze, info.Xs, info.Xe, info.Ys, info.Ye, NULL);
00951
00952     if (!input_range || !input_range->start || vx < 35 || mpz_set_str(info.Zs, input_range->start, 10) != 0)
00953     {
00954         log_error("range_info_init: invalid input.");
00955         return info;
00956     }
00957
00958     if (input_range->range == 0)
00959     {
00960         mpz_set(info.Ze, info.Zs);
00961     }
00962     else
00963     {
00964         // inclusive upper bound for range size semantics: [start, start + range - 1]

```

```

00965     mpz_add_ui(info.Ze, info.Zs, input_range->range - 1);
00966 }
00967
00968 mpz_fdiv_q_ui(info.Xs, info.Zs, 6);
00969 mpz_fdiv_q_ui(info.Xe, info.Ze, 6);
00970 mpz_fdiv_q_ui(info.Ys, info.Xs, vx);
00971 mpz_fdiv_q_ui(info.Ye, info.Xe, vx);
00972
00973 mpz_t y_delta;
00974 mpz_init(y_delta);
00975 mpz_sub(y_delta, info.Ye, info.Ys);
00976
00977 if (mpz_sgn(y_delta) < 0 || !mpz_fits_sint_p(y_delta))
00978 {
00979     log_error("range_info_init: computed y-range is out of supported bounds.");
00980     mpz_clear(y_delta);
00981     return info;
00982 }
00983
00984 info.y_range = (int)mpz_get_si(y_delta);
00985 mpz_clear(y_delta);
00986
00987 return info;
00988 }
00989 void range_info_free(IZM_RANGE_INFO *info)
00990 {
00991     if (info == NULL)
00992         return;
00993
00994     mpz_clears(info->Zs, info->Ze, info->Xs, info->Xe, info->Ys, info->Ye, NULL);
00995 }
00996
00997 // =====
00998 // * iZm Search primes routines:
00999 // =====
01000
01017 int vx_search_prime(mpz_t p, int m_id, int vx, int bit_size)
01018 {
01019     // assert p is not NULL
01020     assert(p && "p cannot be NULL in vx_search_prime.");
01021
01022     bit_size = MAX(bit_size, 10);
01023
01024     // set m_id randomly if not provided
01025     if (m_id != -1 && m_id != 1)
01026         m_id = (rand() % 2) ? 1 : -1;
01027
01028     gmp_randstate_t state;
01029     gmp_randinit_default(state);
01030     gmp_seed_randstate(state);
01031
01032     UI64_ARRAY *root_primes = SiZm(vx);
01033     if (!root_primes)
01034     {
01035         log_error("Failed to initialize root primes in vx_search_prime");
01036         gmp_randclear(state);
01037         return 0;
01038     }
01039
01040     mpz_t z, x_z, y, yvx;
01041     mpz_init(z);
01042     mpz_init(x_z);
01043     mpz_init(y);
01044     mpz_init(yvx);
01045
01046     mpz_urandomb(y, state, bit_size);
01047     mpz_fdiv_q_ui(y, y, 6 * vx);
01048     mpz_mul_ui(yvx, y, vx); // compute yvx = vx * y
01049
01050     int found = 0;
01051     while (!found)
01052     {
01053         BITMAP *bitmap = bitmap_init(vx + 10, 1);
01054
01055         // sieve the bitmap with root primes skipping 2 and 3
01056         for (int i = 2; i < root_primes->count; i++)
01057         {
01058             uint64_t q = root_primes->array[i];
01059             // mark composites of q in the bitmap
01060             bitmap_clear_steps(bitmap, q, iZm_solve_for_x0_mpz(m_id, q, vx, y), vx);
01061         }
01062
01063         int random_x = rand() % (vx / 2); // random x < vx/2
01064         for (int x = random_x; x < vx; x++)
01065         {
01066             // check if z is prime candidate in bitmap
01067             if (bitmap_get_bit(bitmap, x))

```

```

01068     {
01069         // compute x_z = vx * y + x
01070         mpz_add_ui(x_z, yvx, x);
01071         iZ_mpz(z, x_z, m_id);
01072         // check if z is prime
01073         found = check_primalty(z, MR_ROUNDS);
01074
01075         // if z is prime, set p = z
01076         if (found)
01077         {
01078             mpz_set(p, z);
01079             break;
01080         }
01081     }
01082 }
01083
01084 // cleanup
01085 bitmap_free(&bitmap);
01086
01087 // increment y by 1 for next vx segment
01088 mpz_add_ui(y, y, 1);
01089 mpz_add_ui(yvx, yvx, vx);
01090 }
01091
01092 ui64_free(&root_primes);
01093 mpz_clears(z, x_z, y, yvx, NULL);
01094 gmp_randclear(state);
01095
01096 return found;
01097 }
01098
01114 int vy_search_prime(mpz_t p, int m_id, mpz_t vx)
01115 {
01116     assert(p && vx && "Input parameters cannot be NULL");
01117
01118     int found = 0; // flag to indicate if a prime was found
01119
01120     // set m_id randomly if not provided
01121     if (m_id != -1 && m_id != 1)
01122         m_id = (rand() % 2) ? 1 : -1;
01123
01124     gmp_randstate_t state;
01125     gmp_randinit_default(state);
01126     gmp_seed_randstate(state); // seed random state
01127
01128     mpz_t z, g;
01129     mpz_init(z);
01130     mpz_init(g);
01131
01132     // set random x value in the range of vx
01133     mpz_urandomm(z, state, vx);
01134     // compute z = 6 * z + i
01135     iZ_mpz(z, z, m_id);
01136
01137     // search for x value such that gcd(iZ(x, m_id), z) = 1
01138     for (;;)
01139     {
01140         // increment z by 6 to advance x by 1
01141         mpz_add_ui(z, z, 6);
01142
01143         // Compute g = gcd(vx, z)
01144         mpz_gcd(g, vx, z);
01145
01146         // break if g = 1,
01147         // implying current x value can yield primes of the form iZ(x + vx * y, p_id)
01148         if (mpz_cmp_ui(g, 1) == 0)
01149             break;
01150     }
01151
01152     // set g = 6 * vx to use as a step
01153     mpz_mul_ui(g, vx, 6);
01154     // add z by rand * g
01155     int rand_steps = rand() % 100;
01156     mpz_addmul_ui(z, g, rand_steps);
01157
01158     while (!found)
01159     {
01160         // increment z by 6 * vx to advance y by 1
01161         mpz_add(z, z, g);
01162
01163         // check if z is prime
01164         found = check_primalty(z, MR_ROUNDS);
01165
01166         // if z is prime, set p = z
01167         if (found)
01168             mpz_set(p, z);
01169     }

```

```

01170
01171 // cleanup
01172 gmp_randclear(state);
01173 mpz_clears(z, g, NULL);
01174
01175 return found;
01176 }

```

13.29 src/toolkit/logger.c File Reference

Logging module.

13.29.1 Detailed Description

Logging module.

This module provides functions for logging messages at different levels.

Definition in file [logger.c](#).

13.30 logger.c

[Go to the documentation of this file.](#)

```

00001
00008 #ifdef ENABLE_LOGGING
00009
00010 #include "logger.h"
00011 #include <stdio.h>
00012 #include <time.h>
00013 #include <stdarg.h>
00014 #include <string.h>
00015 #include <sys/stat.h>
00016 #include <pthread.h>
00017 #include <stdlib.h>
00018 #include <errno.h>
00019
00020 // Initialize the mutex for thread-safe logging
00021 static pthread_mutex_t log_mutex = PTHREAD_MUTEX_INITIALIZER;
00022 static LogLevel current_log_level = LOG_DEBUG; // Default log level
00023
00030 const char *log_level_to_string(LogLevel level)
00031 {
00032     switch (level)
00033     {
00034     case LOG_DEBUG:
00035         return "DEBUG";
00036     case LOG_INFO:
00037         return "INFO";
00038     case LOG_WARNING:
00039         return "WARNING";
00040     case LOG_ERROR:
00041         return "ERROR";
00042     case LOG_FATAL:
00043         return "FATAL";
00044     default:
00045         return "UNKNOWN";
00046     }
00047 }
00048
00055 void get_current_timestamp(char *buffer, size_t size)
00056 {
00057     time_t now = time(NULL);
00058     struct tm *t = localtime(&now);
00059     strftime(buffer, size, "%Y-%m-%d %H:%M:%S", t);
00060 }
00061
00072 void log_rotate(const char *log_file, size_t max_size)
00073 {
00074     struct stat st;

```

```

00075     if (stat(log_file, &st) == 0 && st.st_size >= (off_t)max_size) // Cast max_size to off_t to match types
00076     {
00077         char old_log_file[256];
00078         for (int i = 5; i > 0; --i)
00079         {
00080             snprintf(old_log_file, sizeof(old_log_file), "%s.%d", log_file, i);
00081             if (i == 5)
00082             {
00083                 remove(old_log_file); // Remove the oldest log file
00084             }
00085             else
00086             {
00087                 char prev_log_file[256];
00088                 snprintf(prev_log_file, sizeof(prev_log_file), "%s.%d", log_file, i + 1);
00089                 rename(old_log_file, prev_log_file);
00090             }
00091         }
00092
00093         char new_log_file[256];
00094         snprintf(new_log_file, sizeof(new_log_file), "%s.1", log_file);
00095         rename(log_file, new_log_file);
00096     }
00097 }
00098
00106 void log_init(const char *log_file)
00107 {
00108     struct stat st = {0};
00109     if (stat(LOG_DIR, &st) == -1)
00110     {
00111         mkdir(LOG_DIR, 0700); // Create log directory if it doesn't exist
00112     }
00113
00114     log_rotate(log_file, LOG_MAX_SIZE); // Rotate logs if needed
00115 }
00116
00120 void log_shutdown(void)
00121 {
00122     pthread_mutex_destroy(&log_mutex);
00123 }
00124
00130 void log_set_log_level(LogLevel level)
00131 {
00132     current_log_level = level;
00133 }
00134
00145 static void log_message_va(LogLevel level, const char *format, va_list args)
00146 {
00147     if (level < current_log_level)
00148         return; // Don't log messages below the current log level
00149
00150     char message[1024];
00151     // Format string comes from a variadic function parameter - suppress warning
00152     #pragma clang diagnostic push
00153     #pragma clang diagnostic ignored "-Wformat-nonliteral"
00154     vsnprintf(message, sizeof(message), format, args);
00155     #pragma clang diagnostic pop
00156
00157     char timestamp[20];
00158     get_current_timestamp(timestamp, sizeof(timestamp));
00159
00160     pthread_mutex_lock(&log_mutex);
00161
00162     FILE *file = fopen(LOG_FILE, "a");
00163     if (!file)
00164     {
00165         perror("Failed to open log file");
00166         pthread_mutex_unlock(&log_mutex); // Ensure mutex is unlocked
00167         return;
00168     }
00169
00170     fprintf(file, "[%s] [%s] %s\n", timestamp, log_level_to_string(level), message);
00171     fclose(file);
00172
00173     pthread_mutex_unlock(&log_mutex);
00174 }
00175
00185 void log_message(LogLevel level, const char *format, ...)
00186 {
00187     va_list args;
00188     va_start(args, format);
00189     log_message_va(level, format, args);
00190     va_end(args);
00191 }
00192
00201 void log_message_extended(LogLevel level, const char *file_name, int line_number, const char *format, ...)
00202 {
00203     if (level < current_log_level)

```

```

00204     return; // Don't log messages below the current log level
00205
00206     va_list args;
00207     va_start(args, format);
00208     char message[1024];
00209     // Format string comes from a variadic function parameter - suppress warning
00210     #pragma clang diagnostic push
00211     #pragma clang diagnostic ignored "-Wformat-nonliteral"
00212     vsnprintf(message, sizeof(message), format, args);
00213     #pragma clang diagnostic pop
00214     va_end(args);
00215
00216     char timestamp[20];
00217     get_current_timestamp(timestamp, sizeof(timestamp));
00218
00219     pthread_mutex_lock(&log_mutex);
00220
00221     FILE *file = fopen(LOG_FILE, "a");
00222     if (!file)
00223     {
00224         perror("Failed to open log file");
00225         pthread_mutex_unlock(&log_mutex);
00226         return;
00227     }
00228
00229     fprintf(file, "[%s] [%s] %s (File: %s, Line: %d)\n",
00230             timestamp, log_level_to_string(level), message, file_name, line_number);
00231     fclose(file);
00232
00233     pthread_mutex_unlock(&log_mutex);
00234 }
00235
00243 void log_console(const char *format, ...)
00244 {
00245     va_list args;
00246     va_start(args, format);
00247     char message[1024];
00248     // Format string comes from a variadic function parameter - suppress warning
00249     #pragma clang diagnostic push
00250     #pragma clang diagnostic ignored "-Wformat-nonliteral"
00251     vsnprintf(message, sizeof(message), format, args);
00252     #pragma clang diagnostic pop
00253     va_end(args);
00254
00255     char timestamp[20];
00256     get_current_timestamp(timestamp, sizeof(timestamp));
00257
00258     // Print the message with a timestamp to the console
00259     printf("[%s] %s\n", timestamp, message);
00260 }
00261
00267 void log_debug(const char *format, ...)
00268 {
00269     va_list args;
00270     va_start(args, format);
00271     log_message_va(LOG_DEBUG, format, args);
00272     va_end(args);
00273 }
00274
00280 void log_info(const char *format, ...)
00281 {
00282     va_list args;
00283     va_start(args, format);
00284     log_message_va(LOG_INFO, format, args);
00285     va_end(args);
00286 }
00287
00293 void log_warn(const char *format, ...)
00294 {
00295     va_list args;
00296     va_start(args, format);
00297     log_message_va(LOG_WARNING, format, args);
00298     va_end(args);
00299 }
00300
00306 void log_error(const char *format, ...)
00307 {
00308     va_list args;
00309     va_start(args, format);
00310     log_message_va(LOG_ERROR, format, args);
00311     va_end(args);
00312 }
00313
00319 void log_fatal(const char *format, ...)
00320 {
00321     va_list args;
00322     va_start(args, format);

```

```

00323     log_message_va(LOG_FATAL, format, args);
00324     va_end(args);
00325 }
00326
00327 #endif // ENABLE_LOGGING

```

13.31 src/toolkit/platform.c File Reference

Platform abstraction implementation.

```

#include <platform.h>
#include <errno.h>
#include <limits.h>
#include <stdio.h>
#include <string.h>
#include <openssl/rand.h>

```

Functions

- int [iz_platform_create_dir](#) (const char *dir)
Create a directory if it does not already exist.
- int [iz_platform_fill_random](#) (void *buffer, size_t bytes)
Fill a buffer with random bytes.
- int [iz_platform_cpu_cores_count](#) (void)
Return number of online logical CPU cores (≥ 1).
- int [iz_platform_l2_cache_size_bits](#) (void)
Best-effort L2 cache size query in bits.
- double [iz_platform_monotonic_seconds](#) (void)
Return a monotonic timestamp in seconds.

13.31.1 Detailed Description

Platform abstraction implementation.

Definition in file [platform.c](#).

13.32 platform.c

[Go to the documentation of this file.](#)

```

00001
00006
00007 #include <platform.h>
00008
00009 #include <errno.h>
00010 #include <limits.h>
00011 #include <stdio.h>
00012 #include <string.h>
00013
00014 #include <openssl/rand.h>
00015
00016 int iz_platform_create_dir(const char *dir)
00017 {
00018     if (dir == NULL || dir[0] == '\0')
00019         return -1;

```

```

00020
00021 #if IZ_PLATFORM_WINDOWS
00022     if (_mkdir(dir) == 0 || errno == EEXIST)
00023         return 0;
00024 #else
00025     if (mkdir(dir, 0700) == 0 || errno == EEXIST)
00026         return 0;
00027 #endif
00028     return -1;
00029 }
00030
00031 int iz_platform_fill_random(void *buffer, size_t bytes)
00032 {
00033     if (buffer == NULL)
00034         return 0;
00035
00036     if (bytes == 0)
00037         return 1;
00038
00039     unsigned char *out = (unsigned char *)buffer;
00040     size_t offset = 0;
00041
00042     while (offset < bytes)
00043     {
00044         size_t chunk = bytes - offset;
00045         if (chunk > (size_t)INT_MAX)
00046             chunk = (size_t)INT_MAX;
00047
00048         if (RAND_bytes(out + offset, (int)chunk) != 1)
00049             return 0;
00050
00051         offset += chunk;
00052     }
00053
00054     return 1;
00055 }
00056
00057 int iz_platform_cpu_cores_count(void)
00058 {
00059     #if IZ_PLATFORM_WINDOWS
00060         SYSTEM_INFO info;
00061         GetSystemInfo(&info);
00062         int cores = (int)info.dwNumberOfProcessors;
00063         return (cores > 0) ? cores : 1;
00064     #else
00065         long cores = sysconf(_SC_NPROCESSORS_ONLN);
00066         return (cores > 0) ? (int)cores : 1;
00067     #endif
00068 }
00069
00070 int iz_platform_l2_cache_size_bits(void)
00071 {
00072     #if defined(__linux__)
00073         FILE *fp = fopen("/sys/devices/system/cpu/cpu0/cache/index2/size", "r");
00074         if (fp != NULL)
00075         {
00076             char buffer[32];
00077             if (fgets(buffer, sizeof(buffer), fp) != NULL)
00078             {
00079                 int size_kb = 0;
00080                 if (sscanf(buffer, "%dK", &size_kb) == 1 && size_kb > 0)
00081                 {
00082                     fclose(fp);
00083                     return size_kb * 1024 * 8;
00084                 }
00085             }
00086             fclose(fp);
00087         }
00088     #endif
00089
00090     #if defined(__APPLE__) || defined(__FreeBSD__) || defined(__OpenBSD__) || defined(__NetBSD__)
00091         size_t size_bytes = 0;
00092         size_t len = sizeof(size_bytes);
00093         if (sysctlbyname("hw.l2cachesize", &size_bytes, &len, NULL, 0) == 0 && size_bytes > 0)
00094             return (int)(size_bytes * 8);
00095
00096         len = sizeof(size_bytes);
00097         if (sysctlbyname("machdep.cpu.cache.L2_cache_size", &size_bytes, &len, NULL, 0) == 0 && size_bytes > 0)
00098             return (int)(size_bytes * 8);
00099     #endif
00100
00101     return 256 * 1024 * 8;
00102 }
00103
00104 double iz_platform_monotonic_seconds(void)
00105 {
00106     #if IZ_PLATFORM_WINDOWS

```



```

00107  LARGE_INTEGER counter;
00108  LARGE_INTEGER frequency;
00109  if (QueryPerformanceFrequency(&frequency) && QueryPerformanceCounter(&counter) && frequency.QuadPart > 0)
00110  {
00111      return (double)counter.QuadPart / (double)frequency.QuadPart;
00112  }
00113  return (double)GetTickCount64() / 1000.0;
00114 #else
00115  struct timespec ts;
00116  if (clock_gettime(CLOCK_MONOTONIC, &ts) == 0)
00117      return (double)ts.tv_sec + (double)ts.tv_nsec / 1000000000.0;
00118
00119  timespec_get(&ts, TIME_UTC);
00120  return (double)ts.tv_sec + (double)ts.tv_nsec / 1000000000.0;
00121 #endif
00122 }

```

13.33 src/toolkit/print_utils.c File Reference

Print formatting helpers for tests, benchmarks, and CLI output.

#include <utils.h>

Functions

- void [print_line](#) (int length, char fill_char)
Print a repeated-character horizontal line.
- void [print_centered_text](#) (const char *text, int line_length, char fill_char)
Print text centered inside a padded line.
- void [print_sha256_hash](#) (const unsigned char *hash)
Print a SHA-256 digest as hex to stdout.
- void [print_test_module_header](#) (char *module_name)
Print a test-suite header banner for a module.
- void [print_test_table_header](#) (void)
Print the generic test runner header.
- void [print_test_module_result](#) (int result, int test_id, const char *unit_name, const char *format,...)
Print a single test-row result.
- void [print_test_summary](#) (char *module_name, int passed, int failed, int verbose)
Print module-level test summary.
- void [print_test_fn_header](#) (char *fn_name)
Print a formatted header for a test function.

13.33.1 Detailed Description

Print formatting helpers for tests, benchmarks, and CLI output.

Definition in file [print_utils.c](#).

13.34 print_utils.c

[Go to the documentation of this file.](#)

```

00001
00006
00007 #include <utils.h>
00008
00014 void print_line(int length, char fill_char)
00015 {
00016     if (fill_char == '\0')
00017         fill_char = '-';
00018
00019     for (int i = 0; i < length; ++i)
00020         printf("%c", fill_char);
00021     printf("\n");
00022 }
00023
00030 void print_centered_text(const char *text, int line_length, char fill_char)
00031 {
00032     if (text == NULL)
00033         text = "";
00034
00035     int text_length = (int)strlen(text);
00036     if (text_length >= line_length)
00037     {
00038         printf("%s\n", text);
00039         return;
00040     }
00041
00042     int left_padding = (line_length - text_length) / 2;
00043     int right_padding = line_length - text_length - left_padding;
00044
00045     for (int i = 0; i < left_padding; ++i)
00046         printf("%c", fill_char);
00047     printf("%s", text);
00048     for (int i = 0; i < right_padding; ++i)
00049         printf("%c", fill_char);
00050     printf("\n");
00051 }
00052
00058 void print_sha256_hash(const unsigned char *hash)
00059 {
00060     for (int i = 0; i < SHA256_DIGEST_LENGTH; i++)
00061         printf("%02x", hash[i]);
00062
00063     printf("\n");
00064 }
00065
00070 void print_test_module_header(char *module_name)
00071 {
00072     print_line(60, '*');
00073     printf("** %s MODULE TEST SUITE\n", module_name);
00074     print_line(60, '*');
00075 }
00076
00080 void print_test_table_header(void)
00081 {
00082     print_line(92, '-');
00083     printf("[%s] %-30s %s %-66s\n", "ID", "Unit Name", "Result", "Details");
00084     print_line(92, '-');
00085 }
00086
00096 void print_test_module_result(int result, int test_id, const char *unit_name, const char *format, ...)
00097 {
00098     va_list args;
00099     va_start(args, format);
00100
00101     char message[1024];
00102     #pragma clang diagnostic push
00103     #pragma clang diagnostic ignored "-Wformat-nonliteral"
00104     vsnprintf(message, sizeof(message), format, args);
00105     #pragma clang diagnostic pop
00106     va_end(args);
00107
00108     if (result)
00109     {
00110         printf("[%02d] %-30s [PASS] %s\n", test_id, unit_name, message);
00111     }
00112     else
00113     {
00114         printf("[%02d] %-30s [FAIL] %s\n", test_id, unit_name, message);
00115     }
00116 }
00117
00126 void print_test_summary(char *module_name, int passed, int failed, int verbose)

```

```

00127 {
00128     (void)verbose;
00129
00130     print_line(60, '*');
00131     printf("Results Summary for %s\n", module_name);
00132     print_line(60, '-');
00133     printf("%-32s: %d\n", "Total Tests", passed + failed);
00134     printf("%-32s: %d\n", "Passed", passed);
00135     printf("%-32s: %d\n", "Failed", failed);
00136     print_line(60, '-');
00137     if (failed == 0)
00138     {
00139         printf("[SUCCESS] ALL %s TESTS PASSED!\n", module_name);
00140     }
00141     else
00142     {
00143         printf("[FAILURE] SOME %s TESTS FAILED :\\n", module_name);
00144     }
00145     print_line(60, '*');
00146 }
00147
00148 void print_test_fn_header(char *fn_name)
00149 {
00150     char header[64];
00151     snprintf(header, sizeof(header), " Testing %s ", fn_name);
00152
00153     print_line(60, '*');
00154     print_centered_text(header, 60, '=');
00155     print_line(60, '*');
00156     printf("\n");
00157 }

```

13.35 src/toolkit/stopwatch.c File Reference

Stopwatch helpers based on monotonic wall-clock time.

#include <utils.h>

Functions

- void [sw_start](#) (STOPWATCH *sw)
Start or restart a stopwatch.
- void [sw_stop](#) (STOPWATCH *sw)
Stop a running stopwatch, capturing the elapsed time in seconds.
- double [sw_elapsed_seconds](#) (const STOPWATCH *sw)
Return elapsed seconds for the stopwatch.
- double [sw_elapsed_now_seconds](#) (void)
Capture the current monotonic time in seconds.

13.35.1 Detailed Description

Stopwatch helpers based on monotonic wall-clock time.

Definition in file [stopwatch.c](#).

13.36 stopwatch.c

[Go to the documentation of this file.](#)

```

00001
00006
00007 #include <utils.h>
00008
00010 static struct timespec sw_now_timespec(void)
00011 {
00012     double now = iz_platform_monotonic_seconds();
00013     if (now < 0.0)
00014         now = 0.0;
00015
00016     struct timespec ts;
00017     ts.tv_sec = (time_t)now;
00018     ts.tv_nsec = (long)((now - (double)ts.tv_sec) * 1000000000.0);
00019     if (ts.tv_nsec < 0)
00020         ts.tv_nsec = 0;
00021     if (ts.tv_nsec >= 1000000000L)
00022         ts.tv_nsec = 999999999L;
00023     return ts;
00024 }
00025
00026 static double sw_elapsed_between(struct timespec start, struct timespec end)
00027 {
00028     time_t sec = end.tv_sec - start.tv_sec;
00029     long nsec = end.tv_nsec - start.tv_nsec;
00030     if (nsec < 0)
00031     {
00032         sec -= 1;
00033         nsec += 1000000000L;
00034     }
00035     return (double)sec + (double)nsec / 1000000000.0;
00036 }
00038
00039 void sw_start(STOPWATCH *sw)
00040 {
00041     assert(sw && "Invalid stopwatch passed to sw_start.");
00042     sw->start_time = sw_now_timespec();
00043     sw->end_time = sw->start_time;
00044     sw->running = 1;
00045 }
00046
00047 void sw_stop(STOPWATCH *sw)
00048 {
00049     assert(sw && "Invalid stopwatch passed to sw_stop.");
00050     if (!sw->running)
00051         return;
00052     sw->end_time = sw_now_timespec();
00053     sw->running = 0;
00054     sw->elapsed_sec = sw_elapsed_between(sw->start_time, sw->end_time);
00055 }
00056
00057 double sw_elapsed_seconds(const STOPWATCH *sw)
00058 {
00059     assert(sw && "Invalid stopwatch passed to sw_elapsed_seconds.");
00060     if (sw->running)
00061     {
00062         struct timespec now = sw_now_timespec();
00063         return sw_elapsed_between(sw->start_time, now);
00064     }
00065     return sw->elapsed_sec;
00066 }
00067
00068 double sw_elapsed_now_seconds(void)
00069 {
00070     return iz_platform_monotonic_seconds();
00071 }

```

13.37 src/toolkit/utils.c File Reference

Implementations for the shared utility layer.

```

#include <utils.h>
#include <ctype.h>

```

Functions

- int [is_numeric_str](#) (const char *str)
Check if a string is numeric.
- int [parse_numeric_expr_mpz](#) (mpz_t out, const char *expr)
Parse an integer expression into an mpz value.
- int [parse_numeric_expr_u64](#) (const char *expr, uint64_t *out)
Parse an integer expression into uint64_t.
- int [parse_inclusive_range_mpz](#) (const char *range_expr, mpz_t lower, mpz_t upper)
Parse an inclusive range expression into lower/upper bounds.
- int [create_dir](#) (const char *dir)
Create a directory if it does not exist.
- uint64_t [gcd](#) (uint64_t a, uint64_t b)
Compute the greatest common divisor (GCD) of two integers.
- uint64_t [modular_inverse](#) (uint64_t a, uint64_t m)
Compute the modular inverse of a modulo m.
- void [gmp_seed_randstate](#) (gmp_randstate_t state)
Seed the GMP random state.
- int [get_cpu_cores_count](#) (void)
Compute the number of CPU cores available.
- int [get_cpu_L2_cache_size_bits](#) (void)
Get the CPU L2 cache size in bits.

13.37.1 Detailed Description

Implementations for the shared utility layer.

Definition in file [utils.c](#).

13.38 utils.c

[Go to the documentation of this file.](#)

```

00001
00006
00007 #include <utils.h>
00008 #include <ctype.h>
00009
00011 static const char *skip_spaces(const char *s)
00012 {
00013     while (*s && isspace((unsigned char)*s))
00014         s++;
00015     return s;
00016 }
00017
00018 static char *dup_trimmed(const char *src)
00019 {
00020     if (src == NULL)
00021         return NULL;
00022
00023     const char *start = skip_spaces(src);
00024     const char *end = start + strlen(start);
00025     while (end > start && isspace((unsigned char)end[-1]))
00026         --end;
00027
00028     size_t len = (size_t)(end - start);
00029     char *out = malloc(len + 1);
00030     if (!out)
00031         return NULL;
00032     memcpy(out, start, len);
00033     out[len] = '\0';

```

```

00034     return out;
00035 }
00036
00037 static char *normalize_decimal_token(const char *token)
00038 {
00039     char *trimmed = dup_trimmed(token);
00040     if (trimmed == NULL)
00041         return NULL;
00042
00043     const char *s = trimmed;
00044     if (*s == '+')
00045         ++s;
00046
00047     if (*s == '\0')
00048     {
00049         free(trimmed);
00050         return NULL;
00051     }
00052
00053     size_t cap = strlen(s) + 1;
00054     char *normalized = malloc(cap);
00055     if (!normalized)
00056     {
00057         free(trimmed);
00058         return NULL;
00059     }
00060
00061     size_t n = 0;
00062     int has_comma = strchr(s, ',') != NULL;
00063
00064     if (!has_comma)
00065     {
00066         for (const char *p = s; *p; ++p)
00067         {
00068             if (*p == '_')
00069                 continue;
00070             if (!isdigit((unsigned char)*p))
00071             {
00072                 free(normalized);
00073                 free(trimmed);
00074                 return NULL;
00075             }
00076             normalized[n++] = *p;
00077         }
00078     }
00079     else
00080     {
00081         const char *seg = s;
00082         int group_idx = 0;
00083         while (1)
00084         {
00085             const char *comma = strchr(seg, ',');
00086             size_t seg_len = comma ? (size_t)(comma - seg) : strlen(seg);
00087             if (seg_len == 0)
00088             {
00089                 free(normalized);
00090                 free(trimmed);
00091                 return NULL;
00092             }
00093
00094             if (group_idx == 0)
00095             {
00096                 if (seg_len < 1 || seg_len > 3)
00097                 {
00098                     free(normalized);
00099                     free(trimmed);
00100                     return NULL;
00101                 }
00102             }
00103             else if (seg_len != 3)
00104             {
00105                 free(normalized);
00106                 free(trimmed);
00107                 return NULL;
00108             }
00109
00110             for (size_t i = 0; i < seg_len; ++i)
00111             {
00112                 if (!isdigit((unsigned char)seg[i]))
00113                 {
00114                     free(normalized);
00115                     free(trimmed);
00116                     return NULL;
00117                 }
00118                 normalized[n++] = seg[i];
00119             }
00120

```

```

00121         group_idx++;
00122         if (comma == NULL)
00123             break;
00124         seg = comma + 1;
00125     }
00126 }
00127
00128 normalized[n] = '\0';
00129 free(trimmed);
00130 return normalized;
00131 }
00132
00133 static int parse_integer_token_mpz(mpz_t out, const char *token)
00134 {
00135     char *normalized = normalize_decimal_token(token);
00136     if (normalized == NULL)
00137         return 0;
00138
00139     int ok = (mpz_set_str(out, normalized, 10) == 0);
00140     free(normalized);
00141     return ok;
00142 }
00143
00144 static int parse_exponent_u64(const char *token, unsigned long *exp_out)
00145 {
00146     mpz_t exp_mpz;
00147     mpz_init(exp_mpz);
00148     int ok = parse_integer_token_mpz(exp_mpz, token);
00149     if (!ok || !mpz_fits_ulong_p(exp_mpz))
00150     {
00151         mpz_clear(exp_mpz);
00152         return 0;
00153     }
00154     *exp_out = mpz_get_ui(exp_mpz);
00155     mpz_clear(exp_mpz);
00156     return 1;
00157 }
00158
00159 static int parse_numeric_term_mpz(mpz_t out, const char *term)
00160 {
00161     char *trimmed = dup_trimmed(term);
00162     if (trimmed == NULL || trimmed[0] == '\0')
00163     {
00164         free(trimmed);
00165         return 0;
00166     }
00167
00168     char *pow_op = strchr(trimmed, '^');
00169     char *sci_op = strchr(trimmed, 'e');
00170     if (!sci_op)
00171         sci_op = strchr(trimmed, 'E');
00172
00173     if (pow_op && sci_op)
00174     {
00175         free(trimmed);
00176         return 0;
00177     }
00178
00179     if (pow_op != NULL)
00180     {
00181         if (strchr(pow_op + 1, '^') != NULL || strchr(pow_op + 1, 'e') != NULL || strchr(pow_op + 1, 'E') != NULL)
00182         {
00183             free(trimmed);
00184             return 0;
00185         }
00186
00187         *pow_op = '\0';
00188         const char *base_str = trimmed;
00189         const char *exp_str = pow_op + 1;
00190
00191         mpz_t base;
00192         mpz_init(base);
00193         unsigned long exp = 0;
00194         int ok = parse_integer_token_mpz(base, base_str) && parse_exponent_u64(exp_str, &exp);
00195         if (ok)
00196             mpz_pow_ui(out, base, exp);
00197
00198         mpz_clear(base);
00199         free(trimmed);
00200         return ok;
00201     }
00202
00203     if (sci_op != NULL)
00204     {
00205         if (strchr(sci_op + 1, 'e') != NULL || strchr(sci_op + 1, 'E') != NULL || strchr(sci_op + 1, '^') != NULL)
00206         {
00207             free(trimmed);

```

```

00208         return 0;
00209     }
00210
00211     *sci_op = '\0';
00212     const char *base_str = trimmed;
00213     const char *exp_str = sci_op + 1;
00214
00215     mpz_t base, pow10;
00216     mpz_inits(base, pow10, NULL);
00217     unsigned long exp = 0;
00218     int ok = parse_integer_token_mpz(base, base_str) && parse_exponent_u64(exp_str, &exp);
00219     if (ok)
00220     {
00221         mpz_ui_pow_ui(pow10, 10, exp);
00222         mpz_mul(out, base, pow10);
00223     }
00224
00225     mpz_clears(base, pow10, NULL);
00226     free(trimmed);
00227     return ok;
00228 }
00229
00230 int ok = parse_integer_token_mpz(out, trimmed);
00231 free(trimmed);
00232 return ok;
00233 }
00234
00235 static int parse_range_parts(const char *left_expr, const char *right_expr, mpz_t lower, mpz_t upper)
00236 {
00237     mpz_t left, right;
00238     mpz_inits(left, right, NULL);
00239
00240     int ok = parse_numeric_expr_mpz(left, left_expr) && parse_numeric_expr_mpz(right, right_expr) &&
    mpz_cmp(right, left) >= 0;
00241     if (ok)
00242     {
00243         mpz_set(lower, left);
00244         mpz_set(upper, right);
00245     }
00246
00247     mpz_clears(left, right, NULL);
00248     return ok;
00249 }
00250
00251 int is_numeric_str(const char *str)
00252 {
00253     if (str == NULL || *str == '\0')
00254         return 0;
00255
00256     while (*str)
00257     {
00258         if (*str < '0' || *str > '9')
00259             return 0;
00260         ++str;
00261     }
00262     return 1;
00263 }
00264
00265 int parse_numeric_expr_mpz(mpz_t out, const char *expr)
00266 {
00267     if (expr == NULL)
00268         return 0;
00269
00270     size_t expr_len = strlen(expr);
00271     char *copy = malloc(expr_len + 1);
00272     if (copy == NULL)
00273         return 0;
00274     memcpy(copy, expr, expr_len + 1);
00275
00276     mpz_set_ui(out, 0);
00277     int has_terms = 0;
00278
00279     char *cursor = copy;
00280     while (cursor != NULL)
00281     {
00282         char *plus = strchr(cursor, '+');
00283         if (plus != NULL)
00284             *plus = '\0';
00285
00286         mpz_t term_value;
00287         mpz_init(term_value);
00288         int ok = parse_numeric_term_mpz(term_value, cursor);
00289         if (!ok)
00290         {
00291             mpz_clear(term_value);
00292             free(copy);
00293             return 0;
00294         }
00295     }
00296 }

```



```

00301     }
00302
00303     mpz_add(out, out, term_value);
00304     mpz_clear(term_value);
00305     has_terms = 1;
00306
00307     cursor = (plus != NULL) ? (plus + 1) : NULL;
00308 }
00309
00310 free(copy);
00311 return has_terms;
00312 }
00313
00314 int parse_numeric_expr_u64(const char *expr, uint64_t *out)
00315 {
00316     if (out == NULL)
00317         return 0;
00318
00319     mpz_t value;
00320     mpz_init(value);
00321     if (!parse_numeric_expr_mpz(value, expr) || mpz_sgn(value) < 0 || mpz_sizeinbase(value, 2) > 64)
00322     {
00323         mpz_clear(value);
00324         return 0;
00325     }
00326
00327     uint64_t parsed = 0;
00328     size_t words = 0;
00329     mpz_export(&parsed, &words, 1, sizeof(parsed), 0, 0, value);
00330     *out = (words == 0) ? 0 : parsed;
00331     mpz_clear(value);
00332     return 1;
00333 }
00334
00335 int parse_inclusive_range_mpz(const char *range_expr, mpz_t lower, mpz_t upper)
00336 {
00337     if (range_expr == NULL)
00338         return 0;
00339
00340     char *range = dup_trimmed(range_expr);
00341     if (range == NULL || range[0] == '\\0')
00342     {
00343         free(range);
00344         return 0;
00345     }
00346
00347     size_t len = strlen(range);
00348     if (len > 7 && strncmp(range, "range[", 6) == 0 && range[len - 1] == ']')
00349     {
00350         memmove(range, range + 6, len - 7);
00351         range[len - 7] = '\\0';
00352     }
00353
00354     len = strlen(range);
00355     if (len > 2 && range[0] == '[' && range[len - 1] == ']')
00356     {
00357         memmove(range, range + 1, len - 2);
00358         range[len - 2] = '\\0';
00359     }
00360
00361     char *sep = strstr(range, "..");
00362     if (sep != NULL)
00363     {
00364         *sep = '\\0';
00365         int ok = parse_range_parts(range, sep + 2, lower, upper);
00366         free(range);
00367         return ok;
00368     }
00369
00370     sep = strchr(range, ':');
00371     if (sep != NULL)
00372     {
00373         *sep = '\\0';
00374         int ok = parse_range_parts(range, sep + 1, lower, upper);
00375         free(range);
00376         return ok;
00377     }
00378
00379     for (char *comma = range; (comma = strchr(comma, ',')) != NULL; ++comma)
00380     {
00381         char saved = *comma;
00382         *comma = '\\0';
00383         int ok = parse_range_parts(range, comma + 1, lower, upper);
00384         *comma = saved;
00385         if (ok)
00386         {
00387             free(range);

```

```

00388         return 1;
00389     }
00390 }
00391
00392 free(range);
00393 return 0;
00394 }
00395
00402 int create_dir(const char *dir)
00403 {
00404     if (iz_platform_create_dir(dir) != 0)
00405     {
00406         log_error("Failed to create directory: %s", dir ? dir : "(null)");
00407         return -1;
00408     }
00409     return 0;
00410 }
00411
00419 uint64_t gcd(uint64_t a, uint64_t b)
00420 {
00421     while (b != 0)
00422     {
00423         uint64_t temp = b;
00424         b = a % b;
00425         a = temp;
00426     }
00427     return a;
00428 }
00429
00447 uint64_t modular_inverse(uint64_t a, uint64_t m)
00448 {
00449     if (m == 1)
00450         return 0;
00451
00452     uint64_t m0 = m;
00453     int64_t x0 = 0, x1 = 1;
00454
00455     while (a > 1)
00456     {
00457         uint64_t q = a / m;
00458         uint64_t t = m;
00459
00460         m = a % m;
00461         a = t;
00462         t = x0;
00463
00464         x0 = x1 - (int64_t)q * x0;
00465         x1 = t;
00466     }
00467
00468     if (x1 < 0)
00469         x1 += (int64_t)m0;
00470     return (uint64_t)x1;
00471 }
00472
00481 void gmp_seed_randstate(gmp_randstate_t state)
00482 {
00483     unsigned long seed;
00484     if (!iz_platform_fill_random(&seed, sizeof(seed)))
00485         seed = (unsigned long)time(NULL);
00486
00487     gmp_randseed_ui(state, seed);
00488 }
00489
00495 int get_cpu_cores_count(void)
00496 {
00497     return iz_platform_cpu_cores_count();
00498 }
00499
00514 int get_cpu_L2_cache_size_bits(void)
00515 {
00516     return iz_platform_l2_cache_size_bits();
00517 }

```

Chapter 14

Examples

14.1 `/Users/hazemmounir/Workspace/iZproject/iZprime/src/toolkit/↵ bitmap.c`

Clears bits at regular intervals, typically used in sieve algorithms.

Clears bits at regular intervals, typically used in sieve algorithms. Starting from `start_idx`, this function clears every `step`-th bit up to the specified limit (inclusive). This is commonly used in prime sieve algorithms where you need to mark all multiples of a number as composite.

The function includes:

- Automatic capping of limit to bitmap size
- Integer overflow protection in the loop
- Early termination to prevent infinite loops

Parameters

| | |
|-----------|---|
| bitmap | Pointer to the bitmap to modify |
| step | Interval between bits to clear (must be > 0, typically a prime) |
| start_idx | Starting bit index (inclusive, typically first multiple) |
| limit | Upper bound for clearing (inclusive, auto-capped to size-1) |

Note

If `limit > bitmap->size`, it is automatically reduced to `bitmap->size - 1`

No operation if `bitmap` is `NULL`, `step` is 0, or `start_idx > limit`

Includes overflow prevention: stops if next index would overflow

```
// For Sieve of Eratosthenes: mark multiples of 3 starting from 9 bitmap_clear_steps(bitmap, 3, 9, 1000); // Clears 9, 12, 15, 18, ...
```

```
#include <bitmap.h>

#ifdef __aarch64__
#include <arm_neon.h>
#elif defined(__x86_64__) || defined(_M_X64)
#include <immintrin.h>
#endif

BITMAP *bitmap_init(size_t size, int set_bits)
{
    assert(size > 0 && "Bitmap size must be positive");

    BITMAP *bitmap = (BITMAP *)malloc(sizeof(BITMAP));
    if (bitmap == NULL)
    {
        log_error("Memory allocation failed for BITMAP struct");
        return NULL;
    }

    bitmap->size = size;
    bitmap->byte_size = (size + 7) / 8;
    bitmap->data = (unsigned char *)malloc(bitmap->byte_size);
    if (bitmap->data == NULL)
    {
        free(bitmap);
        log_error("bitmap_create: Memory allocation failed for BITMAP");
        return NULL;
    }

    if (set_bits)
        memset(bitmap->data, 0xFF, bitmap->byte_size);
    else
        memset(bitmap->data, 0x00, bitmap->byte_size);

    memset(bitmap->sha256, 0, SHA256_DIGEST_LENGTH); // Initialize SHA-256 to zero

    return bitmap;
}

void bitmap_free(BITMAP **bitmap)
{
    // Safe guard against double free and NULL pointer
    if (bitmap && *bitmap)
    {
        if ((*bitmap)->data)
        {
            free((*bitmap)->data);
            (*bitmap)->data = NULL;
        }

        free(*bitmap);
        *bitmap = NULL;
    }
}

void bitmap_set_bit(BITMAP *bitmap, size_t idx)
{
    bitmap->data[idx / 8] |= (1 « (idx % 8));
}

int bitmap_get_bit(BITMAP *bitmap, size_t idx)
{
    return (bitmap->data[idx / 8] & (1 « (idx % 8))) != 0;
}

void bitmap_flip_bit(BITMAP *bitmap, size_t idx)
{
    bitmap->data[idx / 8] ^= (1 « (idx % 8));
}

void bitmap_clear_bit(BITMAP *bitmap, size_t idx)
{
    // Clear the bit by ANDing with the negation of the bit mask
    bitmap->data[idx / 8] &= ~(1 « (idx % 8));
}

void bitmap_set_all(BITMAP *bitmap)
{
    memset(bitmap->data, 0xFF, (bitmap->size + 7) / 8);
}

void bitmap_clear_all(BITMAP *bitmap)
```

```

{
    memset(bitmap->data, 0x00, (bitmap->size + 7) / 8);
}

void bitmap_clear_steps(BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
{
    assert(step > 0 && "Step must be positive in bitmap_clear_steps.");
    limit = MIN(limit, bitmap->size - 1);

    for (uint64_t idx = start_idx; idx <= limit; idx += step)
    {
        bitmap->data[idx / 8] &= ~(1 « (idx % 8));
    }
}

void bitmap_clear_steps_simd(BITMAP *bitmap, uint64_t step, uint64_t start_idx, uint64_t limit)
{
    assert(step > 0 && "Step must be positive in bitmap_clear_steps_simd.");
    limit = MIN(limit, bitmap->size - 1);

#ifdef __aarch64__
    // NEON implementation for ARM64
    uint64_t idx = start_idx;

    // Process 4 steps at a time
    // Condition: idx + 3*step <= limit

    if (limit >= 3 * step && idx <= limit - 3 * step)
    {
        uint64x2_t v_step2 = vdupq_n_u64(2 * step);
        uint64x2_t v_step4 = vdupq_n_u64(4 * step);

        // Initialize indices: [idx, idx+step] and [idx+2step, idx+3step]
        uint64x2_t v_idx_01 = vdupq_n_u64(idx);
        v_idx_01 = vsetq_lane_u64(idx + step, v_idx_01, 1);

        uint64x2_t v_idx_23 = vaddq_u64(v_idx_01, v_step2);

        while (idx <= limit - 3 * step)
        {
            // Extract indices and perform operations
            // Note: NEON doesn't have efficient scatter for bytes, so we extract and do scalar stores

            uint64_t i0 = vgetq_lane_u64(v_idx_01, 0);
            uint64_t i1 = vgetq_lane_u64(v_idx_01, 1);
            uint64_t i2 = vgetq_lane_u64(v_idx_23, 0);
            uint64_t i3 = vgetq_lane_u64(v_idx_23, 1);

            bitmap->data[i0 / 8] &= ~(1 « (i0 % 8));
            bitmap->data[i1 / 8] &= ~(1 « (i1 % 8));
            bitmap->data[i2 / 8] &= ~(1 « (i2 % 8));
            bitmap->data[i3 / 8] &= ~(1 « (i3 % 8));

            // Advance indices
            v_idx_01 = vaddq_u64(v_idx_01, v_step4);
            v_idx_23 = vaddq_u64(v_idx_23, v_step4);

            idx += 4 * step;
        }
    }

    // Handle remaining steps
    for (; idx <= limit; idx += step)
    {
        bitmap->data[idx / 8] &= ~(1 « (idx % 8));
    }
}

#elif defined(__AVX2__)
    // AVX2 implementation for x86_64
    uint64_t idx = start_idx;

    // Process 4 steps at a time
    if (limit >= 3 * step && idx <= limit - 3 * step)
    {
        __m256i v_step4 = _mm256_set1_epi64x(4 * step);

        // Initialize indices: [idx, idx+step, idx+2step, idx+3step]
        // Note: _mm256_set_epi64x takes arguments in reverse order (e3, e2, e1, e0)
        __m256i v_idx = _mm256_set_epi64x(idx + 3 * step, idx + 2 * step, idx + step, idx);

        while (idx <= limit - 3 * step)
        {
            // Extract indices
            uint64_t i0 = _mm256_extract_epi64(v_idx, 0);
            uint64_t i1 = _mm256_extract_epi64(v_idx, 1);
            uint64_t i2 = _mm256_extract_epi64(v_idx, 2);

```

```

uint64_t i3 = _mm256_extract_epi64(v_idx, 3);

bitmap->data[i0 / 8] &= ~(1 « (i0 % 8));
bitmap->data[i1 / 8] &= ~(1 « (i1 % 8));
bitmap->data[i2 / 8] &= ~(1 « (i2 % 8));
bitmap->data[i3 / 8] &= ~(1 « (i3 % 8));

// Advance indices
v_idx = _mm256_add_epi64(v_idx, v_step4);
idx += 4 * step;
}
}

// Handle remaining steps
for (; idx <= limit; idx += step)
{
    bitmap->data[idx / 8] &= ~(1 « (idx % 8));
}

#elif defined(__SSE2__)
// SSE2 implementation for x86_64
uint64_t idx = start_idx;

// Process 2 steps at a time
if (limit >= step && idx <= limit - step)
{
    _mm128i v_step2 = _mm_set1_epi64x(2 * step);
    // Note: _mm_set_epi64x takes arguments in reverse order (e1, e0)
    _mm128i v_idx = _mm_set_epi64x(idx + step, idx);

    while (idx <= limit - step)
    {
        // Extract indices
        // _mm_cvtsi128_si64 extracts the lower 64 bits (element 0)
        uint64_t i0 = _mm_cvtsi128_si64(v_idx);
        // Unpack high to low to extract element 1
        uint64_t i1 = _mm_cvtsi128_si64(_mm_unpackhi_epi64(v_idx, v_idx));

        bitmap->data[i0 / 8] &= ~(1 « (i0 % 8));
        bitmap->data[i1 / 8] &= ~(1 « (i1 % 8));

        // Advance indices
        v_idx = _mm_add_epi64(v_idx, v_step2);
        idx += 2 * step;
    }
}

// Handle remaining steps
for (; idx <= limit; idx += step)
{
    bitmap->data[idx / 8] &= ~(1 « (idx % 8));
}

#else
// Fallback to scalar implementation
for (uint64_t idx = start_idx; idx <= limit; idx += step)
{
    bitmap->data[idx / 8] &= ~(1 « (idx % 8));
}
#endif
}

BITMAP *bitmap_clone(BITMAP *src)
{
    assert(src && src->data && "Invalid source bitmap passed to bitmap_clone.");

    BITMAP *dest = bitmap_init(src->size, 0);
    memcpy(dest->data, src->data, src->byte_size);
    memcpy(dest->sha256, src->sha256, SHA256_DIGEST_LENGTH);

    return dest;
}

void bitmap_compute_hash(BITMAP *bitmap)
{
    assert(bitmap && bitmap->data && "Invalid bitmap passed to bitmap_compute_hash.");

    // Generate SHA-256 hash and store it in the struct
    SHA256((unsigned char *)bitmap->data, bitmap->byte_size, bitmap->sha256);
}

int bitmap_validate_hash(BITMAP *bitmap)
{

```

```

assert(bitmap && bitmap->data && "Invalid bitmap");

unsigned char correct_hash[SHA256_DIGEST_LENGTH]; // Buffer to hold the computed hash

// Generate SHA-256 hash and store it in correct_hash
SHA256((unsigned char *)bitmap->data, bitmap->byte_size, correct_hash);

// Compare actual_hash with the stored hash in bitmap->sha256
if (memcmp(correct_hash, bitmap->sha256, SHA256_DIGEST_LENGTH) == 0)
{
    return 1; // SHA-256 match
}
else
{
    log_error("SHA-256 checksum validation failed.");
    return 0; // SHA-256 mismatch
}
}

int bitmap_fwrite(BITMAP *bitmap, FILE *file)
{
    assert(bitmap && bitmap->data && "Invalid bitmap passed to bitmap_fwrite.");
    assert(file && "File pointer is NULL in bitmap_fwrite.");

    // Write size
    if (fwrite(&bitmap->size, sizeof(size_t), 1, file) != 1)
    {
        log_error("Failed to write bitmap size to file.");
        return 0;
    }

    // Write data
    if (fwrite(bitmap->data, sizeof(unsigned char), bitmap->byte_size, file) != bitmap->byte_size)
    {
        log_error("Failed to write bitmap data to file.");
        return 0;
    }

    // Compute SHA-256 hash if not already computed
    // Check if hash is all zeros by examining all bytes
    int hash_is_zero = 1;
    for (size_t i = 0; i < SHA256_DIGEST_LENGTH; i++)
    {
        if (bitmap->sha256[i] != 0)
        {
            hash_is_zero = 0;
            break;
        }
    }

    if (hash_is_zero)
    {
        bitmap_compute_hash(bitmap);
    }

    // Write SHA-256 hash
    if (fwrite(bitmap->sha256, 1, SHA256_DIGEST_LENGTH, file) != SHA256_DIGEST_LENGTH)
    {
        log_error("Failed to write SHA-256 hash to file.");
        return 0;
    }

    return 1; // Success
}

BITMAP *bitmap_fread(FILE *file)
{
    assert(file && "File pointer is NULL in bitmap_fread.");

    // Read size
    size_t size;
    if (fread(&size, sizeof(size_t), 1, file) != 1)
    {
        log_error("Failed to read bitmap size from file.");
        return NULL;
    }

    BITMAP *bitmap = bitmap_init(size, 0);
    if (bitmap == NULL)
    {
        log_error("Failed to initialize bitmap.");
        return NULL;
    }

    // Read data directly into the already-allocated buffer
    if (fread(bitmap->data, sizeof(unsigned char), bitmap->byte_size, file) != bitmap->byte_size)
    {

```

```
    log_error("Failed to read complete bitmap data from file.");
    bitmap_free(&bitmap);
    return NULL;
}

// Read SHA-256 hash
if (fread(bitmap->sha256, 1, SHA256_DIGEST_LENGTH, file) != SHA256_DIGEST_LENGTH)
{
    log_error("Failed to read SHA-256 hash from file.");
    bitmap_free(&bitmap);
    return NULL;
}

if (!bitmap_validate_hash(bitmap))
{
    log_error("SHA-256 hash validation failed.");
    bitmap_free(&bitmap);
    return NULL;
}

return bitmap;
}
```


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