Justification of and Parameter Estimation for a Conceptual Functional Equivalent (CFE) Formulation of the NOAA-NWS National Water Model (version 2.1)

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Created June, 2021, DRAFT updated June, 2023

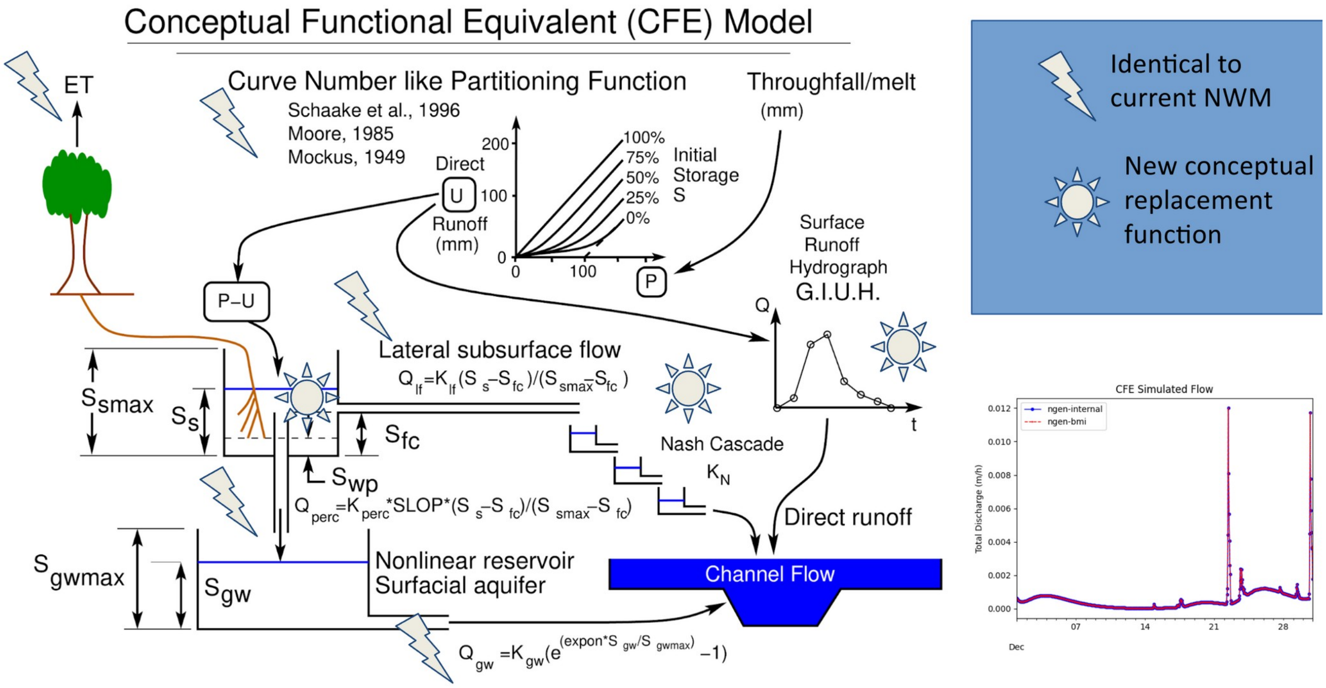

Figure 1: Conceptual Functional Equivalent (CFE) model conceptual diagram. For description of parameter calculation or estimation, see Section 4 of this document.

1. Introduction

Analysis of the National Water Model (NWM) code revealed certain aspects of that model that present opportunities for simplification. In particular, the methods employed in the NWM to perform soil moisture accounting, lateral subsurface and surface (overland) flow routing, use coarse discretizations that diminish their physical representativeness. Therefore, one might consider replacing them with conceptualizations. The Conceptual Functional Equivalent (CFE) to the National Water Model does that. The following sections describe the conceptualizations employed to represent portions of the NWM that are applied in a manner that makes the amenable to replacement.

2.0 Simulation of Soil Moisture in the Noah-MP Land Surface Model

The Noah-MP land surface model was designed to partition sensible and latent heat fluxes over large areas as a lower boundary condition for weather and climate simulation codes. Typical applications of the Noah-MP model involve model grids such as 1/8-degree. Because the Noah-MP model applications involved scales much larger than the hillslope scale, it does not resolve topography. It assumes that the earth is flat, and the land surface slope does not appear in the code. The portions of the operational National Water Model, currently version 2.1, that account for land-surface slope exist in the WRF-Hydro routing routines discussed later.

2.1 Soil Moisture Dynamics

The WRF-Hydro version of the National Water Model employs the Noah-MP scheme on a 1 km grid to simulate soil column processes. As is commonly done with gridded models, the methods and parameters applied in each 1 km grid are assumed to represent the areally-averaged response of that 1 km grid. As a land-surface model, Noah-MP serves as a lower boundary condition for weather and climate models (Niu et al., 2011). Its primary purpose in this role is partitioning of net radiation into sensible and latent heat fluxes. Noah-MP simulates soil moisture to determine the amount of potential evapotranspiration (PET) that becomes actual evapotranspiration (AET) because drier soils limit water uptake by plants. The formulation of Noah-MP was not driven by a requirement to accurately model the vertical profile of soil moisture, rather its design emphasizes bulk soil moisture content in a computationally fast and reliable fashion to limit AET as soils dry.

The Noah-MP soil moisture routine solves the moisture content form of the Richardson/Richards equation (RRE), which uses the soil moisture diffusivity. This form of RRE is strictly invalid for heterogeneous or layered soils because its derivation assumes that the soil moisture content is a continuous, smooth variable (Hillel, 1998). Soil layers with differing soil hydraulic properties naturally produce discontinuities in soil moisture content, and add considerable algorithmic complexity to RRE solvers which affect their stability and accuracy (Brunone et al., 2003). For this reason, the current operational NWM (v2.1) assumes the entire 2m column of soil is homogeneous, and neglects layering. Soil layering is ubiquitous (Phillips and Lorz, 2008) and often dominates soil moisture fluxes and distributions. This is particularly true where repeated tillage results in a nearly impervious layer of compacted soil below plow depth or “fragipan” (Dingman, 1994). It is important to note that what the Noah-MP documentation refers to as “layers” its formulation are in fact “discretizations” of a homogeneous soil. Attempts to employ dissimilar soil hydraulic variables in the different discretizations with the moisture content form of RRE are not justified by theory or the literature (Talbot et al. 2004).

The moisture content form of RRE is invalid for saturated soils, because it does not explicitly include the soil pressure head. For this reason, the NOAH-MP implementation is intentionally applied in a fashion that prevents saturation in the uppermost discretization by using a coarse discretization. The exception is when the WRF-Hydro parameter that specifies the head gradient at the bottom of the soil column is set to, or very nearly, 0.0. In that case, saturation occurs from the bottom-up. The thickness of the upper discretization immediately below the land surface in Noah-MP is 10 cm. Going downward in the 2 m soil column, the second discretization is 30 cm, the third is 60 cm, and the fourth and bottom discretization is 100 cm thick. Coarse discretization of the soil at the land surface generally prevents saturation because rainfall inputs do not cause rapid wetting, producing an artificially muted response. The literature contains many instances where appropriate RRE solutions for simulation of saturated soils require discretizations near or smaller than 1 cm for accurate solutions (Farthing and Ogden, 2017).

The fine discretization required to accurately solve the mixed or head forms of RRE to simulate saturated soil conditions is computationally expensive, and reduces solution scheme reliability (Farthing and Ogden, 2017). Computationally expensive and unreliable numerical solutions pose serious problems for operational numerical codes. The standard application of Noah-MP overcomes this problem by using four coarse discretizations of a 2 m thick soil column as shown in Fig 2.

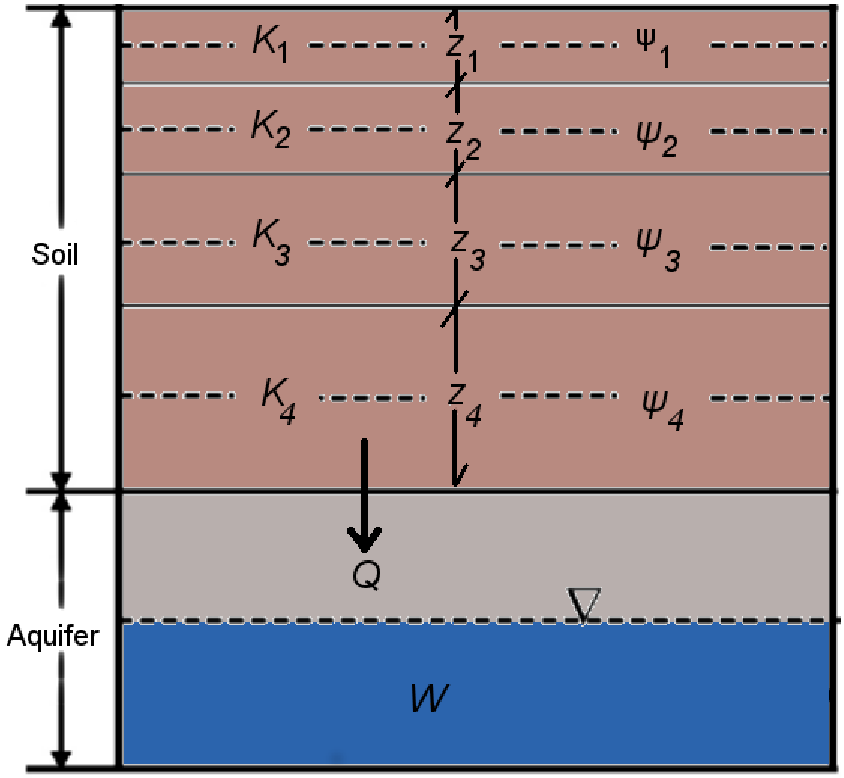

Figure 2: Soil column discretization employed in NWM.
Z_1=0.1m, Z_2=0.3m, Z_3=0.6m, Z_4=1.0m.

The Noah-MP soil moisture routine as implemented in the National Water Model applies a free-drainage boundary condition at the bottom of the soil profile, which assumes that the vertical head gradient equals -1 [L L-1] (Niu et al., 2011). A calibrated parameter that allowed to vary between zero and one may limit the downward percolation of soil water to the aquifer below. This parameter is often calibrated to a small value (near zero) in humid and semi-humid climates where the water table is often near the land surface. This serves to limit percolation and maximize water availability for plant updake. This lower boundary condition disallows the capillary rise of groundwater upward into the soil from groundwater below the soil column, an important process in humid and in certain semi-humid regions, as well as riparian areas in semi-arid and arid regions.

The formulation of the soil moisture routine in the NWM makes it incapable of simulating infiltration in layered soils, which are ubiquitous and often have a dominant effect on vertical soil moisture distributions. Because the moisture-content form of RRE does not include the pressure head, this solution is strictly invalid for saturated soils. The use of coarse discretizations in Noah-MP greatly reduces the likelihood of saturation from above because natural rainfall cannot fill all the pore space in the top-most discretization during one computational time step. The use of coarse discretization offers the benefit of reducting the number of computational elements, which increases solution speed and robustness by smoothing. Furthermore, the Noah-MP soil moisture accounting scheme design applied in the National Water Model is incomplete because it disallows capillary rise of shallow groundwater into the soil column, a known and important hydrologic flux.

2.2 Soil Moisture Performance Evaluation

The performance of the Noah-MP RRE solver was compared against the numerical solution of the advection term of the Soil Moisture Velocity Equation (Ogden et al. 2017) using a finite moisture-content discretization (Talbot and Ogden, 2008; Lai et al. 2015). The simulated domain represented a 2 m homogeneous soil column using default van Genuchten parameters (ref). forced using tropical rainfall and employing a free drainage boundary condition.

Results show that the moisture content profile in Noah-MP seldom differs from approximately hydrostatic. Transients during precipitation produce a muted response in the soil column owing to the coarse discretization. The lack of true soil layers and downward increase in wetness promotes downard movement of soil water.

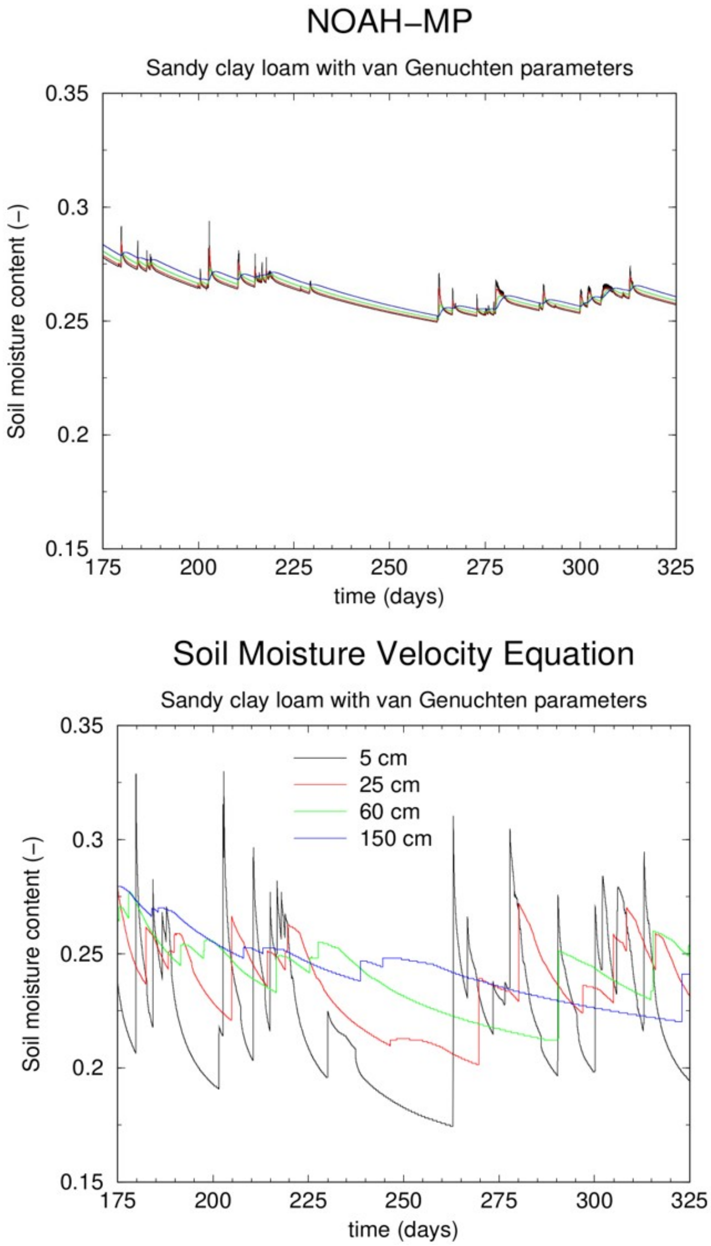

Figure 3: Comparison of Noah-MP moisture-content RRE solution against finite moisture-content domain solution of the advection term of the soil moisture velocity equation for a sand-clay soil texture. Inset legend valid for both figures.

Results also show that despite regular forcing by high intensity rainfall, the upper discretization of the soil domain does not achieve saturation. In this regard the coarse discretization serves its purpose and maintains method validity for strictly unsaturated conditions.

For these reasons, the Noah-MP soil moisture routine as applied in the National Water Model is incapable of accurate soil moisture profile predictions except in rare situations where soils are well approximated as homogeneous, with high permeability with limited influence of near-surface groundwater. Errors due to these formulation features are likely most significant in the near-surface upper discretizations, particularly in tilled agricultural lands where compaction just below plow depth creates a fragipan that dominates the near-surface soil moisture, in disturbed lands where compaction can produce saturation from above, and in situations where near-surface groundwater provides water for use by plants as in humid catchments and riparian areas in arid and semi-arid catchments.

2.3 Implications of Soil Moisture Performance Evaluation

The strict assumptions required by the moisture-content solution of the RRE of a uniform, unsaturated soil domain and the observation that the Noah-MP soil moisture code seldom deviates from hydrostatic conditions supports the notion that the soil domain might be equivalently simulated as a hydrologic reservoir or storage element that stores water as a hydrostatic moisture profile of equivalent volume.

3. Stormflow Generation in the WRF-Hydro based National Water Model

The current operational National Water Model, version 2.1, conceptualizes three sources of stormflow. These include: direct runoff, lateral subsurface flow through the 2 m soil, and a near-surface aquifer that is well described as a non-linear exponential hydrologic reservoir. This section describes the partitioning of rainfall into direct runoff and soil moisture, and the routing of overland and lateral subsurface flow.

3.1 Direct Runoff Generation in the WRF-Hydro based National Water Model

Most land-surface schemes employed in Earth system models posit a direct runoff mechanism, wherein some fraction of liquid precipitation or snow melt is partitioned into surface flow without entering the soil column. The Noah-MP scheme employs the conceptualization published by Schaake et al (1995), which is very similar to the Probability Distributed Principle (Moore, 1985), which resembles the Curve Number method developed by the U.S. Soil Conservation Service (Mockus, 1949). Similar schemes include the VIC (ref) model which exhibits many similarities with Xinanjiang (ref) method.

Each of these methods have common features. First, they were each developed for simulating time-averaged behavior over time periods such as daily or storm-total, meaning they are responsive to rainfall amount not rainfall rate. Secondly, these methods all produce some runoff for any rainfall amount after filling some initial storage threshold. Thirdly, once the entire soil storage is filled, all applied rainfall becomes direct runoff. These features limit the strict applicability to catchments without rainfall-rate sensitive thesholds, with some degree of imperviousness owing to either exposed bedrock or pavement at the land surface, and limited percolation that minimizes the role of groundwater contributions to stormflow. For these reasons, these methods are not valid for simulation of rainrate-dependent runoff generation mechanisms that are common in many arid and semi-arid catchments.

3.2 Occurrence of Lateral Subsurface Flow in the WRF-Hydro based National Water Model

The WRF-Hydro model configured as the current operational National Water Model (version 2.1) uses the field capacity soil moisture as a threshold for activation of lateral flow paths. When any of the soil discretizations in the Noah-MP soil column exceed the field capacity soil moisture content, the WRF-Hydro model activates a lateral flow path in that discretization.

This threshold represents a significant deviation from established studies of unsaturated lateral flow. Using a rigorous first-principles analysis, Or et al. (2015) found that lateral motion in the unsaturated zone seldom exceeds a few meters during interstorm periods, and that therefore lateral flow occurs predominantly when some strata within the soil reaches saturated conditions.

As a conceptual model, the plausible use of the field capacity moisture content as a lateral flow threshold cannot be automatically discounted. After all, that assumption is employed in the current operational National Water Model. The question becomes one of the advantages of applying this threshold within the Noah-MP soil discretizations, or if a single threshold based on the total storage of soil moisture within the entire 2 m soil column. This question is addressed in the parameter equivalence section.

3.3 Routing of direct runoff and lateral subsurface flow in the WRF-Hydro based National Water Model

The current operational National Water Model routes direct runoff as surface overland flow and lateral subsurface flow on a 250m grid. Therefore, each 1 km x 1 km Noah-MP grid includes 16 of these routing grids. Digital Elevation Model (DEM) resolutions have continuously increased over the geospatial data era that began in the 1980's. What is considered a high-resolution DEM has changed from 90 m to 1 over that period. The this changing definition has resulted from changes in data acquisition from digitized contour maps to LiDAR, vast increases in digital storage and computational processing power, increased network bandwidth, and a realization in the literature that certain process require high-fidelity representations of the Earth's surface.

The 250 m grid used in the NWM does not accurately represent land surface slope in most regions. This is shown in Figure 4, which shows analysis of slope prediction errors over five different topographically diverse 1-degree x 1-degree regions. Each of the five plots shows the CDF of slope derived from 30-m (black) and 250-m (red) grids. Also shown are the error in percent as a function of slope (blue), and the percent difference in slope percentile (magenta). The absolute errors are largest in flatter regions, where hillslopes are not well described using a 250 m grid, and smallest in mountainous regions where hillslopes are better defined using a 250 m grid. Even though the errors due to grid size are smaller in mountainous areas, they are still significant.

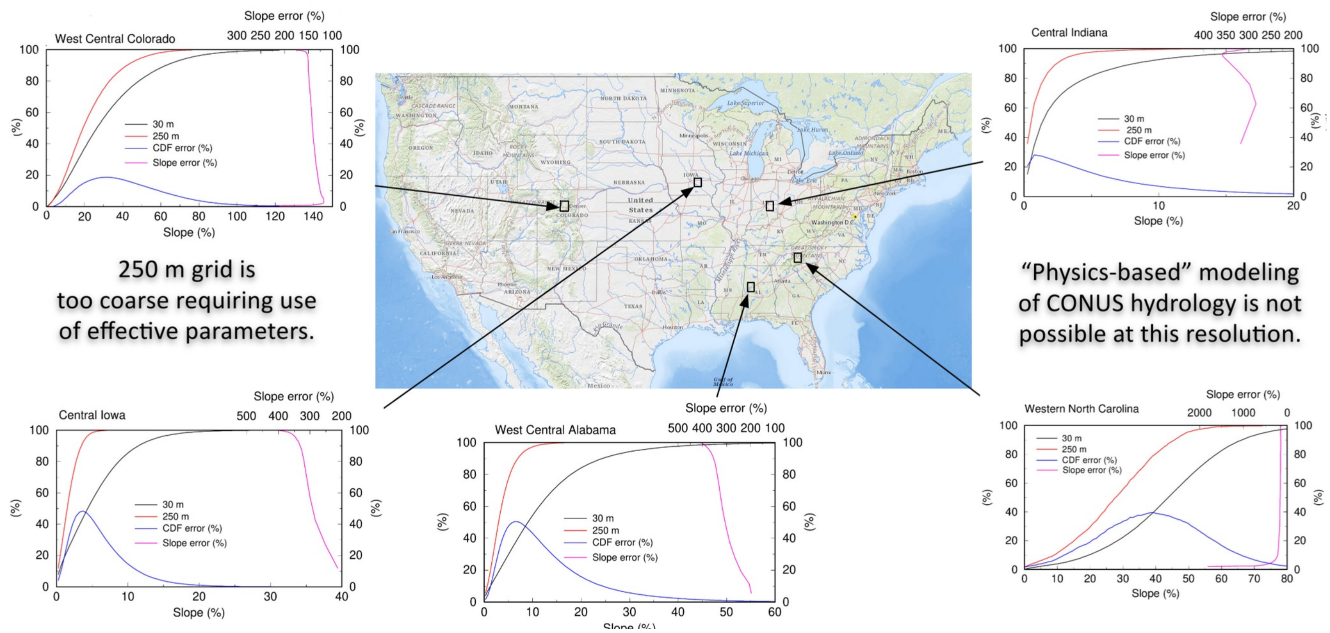

Figure 4: CDFs of topographical slope and their relative difference, expressed as a percent error for five topographically distinct 1-degree x 1-degree blocks in the contiguous U.S.

Since land-surface slope *S* provides the driving force for both overland flow using Manning’s equation as $\sqrt{S}$ and lateral flow linearly assuming Boussinesque behavior, systematic errors in slope require the use of effective parameter values. When overland flow occurs, it persists on the land-surface for relatively short distances (e.g. < 10 m) before being intercepted by a rill, then a gully, which delivers water to the channel. Once overland flow concentration occurs, the relative influence of grid-scale processes greatly diminishes. Similarly, lateral subsurface flow, an extremely difficult process to observe and/or quantify, cannot persist at scales longer than the hillslope scale. Based on results shown in Figure 3, the 250 m grid scale most accurately represents hillslopes in the Rocky Mountains, and even then with slope errors in the range of 100-150% over the entire CDF.

The application of a 250 m routing grid significantly distorts description of the land surface and introduces bias into the CDF of land surface slope. The amount of this bias is a function of both the slope, and the local geomorphology given the significant differences between the plots shown in Fig. 4 from different regions of the contiguous U.S. The us of a 250 m grid generally reduces land surface slope by 20-40% for the range of slopes represented by a 30m DEM.

3.4 Simulation of Run-On Infiltration in the WRF-Hydro based National Water Model

When overland flow is routed from one 250 m grid to another that flow is exposed to another opportunity to be partitioned into soil moisture or continue as overland flow. While the validity of stable overland flow at the scale of 250 m is not supported by the literature or observations, the WRF-Hydro based National Water Model applies the Schaake function to partition overland flow. The intent and design of the Schaake function is partitioning of precipitation, which experiences infiltration opportunity over the entire surface of a model grid. Except in the most extreme case where overland flow depth is sufficient to overwhelm microtopography and completely cover a grid, the infiltration opportunity seen in overland flow occurs over a small fraction of the land surface area. This recursive application of the Schaake function represents a significant misapplication of the method.

4.0 Parameter Equivalence between NWM and CFE Formulations

Given soil parameters from the NWM, with definitions from Noah-MP SOILPARM.TBL:

`maxsmc`	saturated soil moisture content [-]
`wltsmc`	wilting point soil moisture content [-]
`satdk`	saturated hydraulic conductivity [m s^{-1}]
`satpsi`	saturated capillary head [m]
`bb`	'b' exponent on Clapp-Hornberger (1978) soil water relations [-]

`mult`, the multiplier applied to satdk to route water rapidly downslope in subsurface aka `LKSATFAC`
Four fixed soil discretizations Δz of thickness from top to bottom of: 0.1 m, 0.3 m, 0.6 m, and 1.0 m. Total soil column depth D=2.0 m.

4.1 Direct Runoff Partitioning using the Schaake et al. (1996) Scheme

The inputs for the Schaake (Schaake et al. 1996) partitioning scheme are:

```
void Schaake_partitioning_scheme
     (double timestep_s,
      double Schaake_adjusted_magic_constant_by_soil_type,
      double column_total_soil_moisture_deficit_m,
      double water_input_depth_m
```

The value of the Schaake_adjusted_magic_constant_by_soil_type, abbreviated as $C_{schaake}$ is:

$C_{schaake}$=`REFKDT * satdk/2.0E-06.`

The total soil water storage is Ss_{max} = `maxsmc` * D, where

D = the total thickness of the soil column [2.0 m]

Therefore, if the current storage in the soil column nonlinear reservoir is S_s, with $0<=S<=S_{max}$, then the column_total_soil_moisture_deficit = $S_{max} - S$.

4.2 Lateral Subsurface Flow

In the NWM implementation of the WRF-Hydro model lateral flow is assumed to occur any time the water content in any soil discretization (incorrectly referred to as a "layer" in WRF-Hydro) exceeds the water content associated with field capacity, which is defined as the water content where free drainage stops. In finer textured soils, the field capacity water content coincides with a soil suction pressure of approximately 1/3 Atm. Here we will assume that this represents a suction head of 3.44 m. In general, we can assume that for a given soil there is a constant α<1 that describes the field capacity so that the suction head above the water table is given by $H_{wt} = \alpha\, H_{atm}$. Here $H_{atm} = P_{atm} / \gamma$, where P_{atm}=101,300 [Pa] and γ = being the weight of water per unit volume which is very nearly 9,810 [N m^{-3}].

The WRF-Hydro code uses the Clapp-Hornberger (1978) relation to describe soil water retention. This is the relationship between soil water content θ and soil suction head ψ. Clapp and Hornberger neglected the residual water content by assuming that it is equal to zero. Other parameters in this relation are the water content at effective saturation θ_e , which is called `SMCMAX` in WRF-Hydro, the saturated soil suction head parameter ψ_s , which is called `SATPSI` in WRF-Hydro, and the Clapp-Hornberger exponent parameter b, which is called `bexp` in WRF-Hydro. That soil water retention function is:

$$\psi = \psi_s \left(\frac{\theta}{\theta_e} \right)^{-b} \tag{1}$$

With reference to the geometry shown in Figure 2, the parameters needed to simulate the lateral flow response include four storage thresholds, and four linear reservoir constants. Because we assume that groundwater flow is dominated by Darcy's law, linear reservoirs are appropriate for calculation of these fluxes, because under the Boussinesq assumptions, the flux varies approximately linearly with saturated thickness.

Assuming that the soil is in hydrostatic equilibrium, the activation moisture content at the center of each soil discretization coincides with value that results in a soil suction head equal to some fraction of atmospheric pressure head as calculated using $H_{wt} = \alpha\, H_{atm}$, we can calculate the total storage of moisture in the soil in that condition.

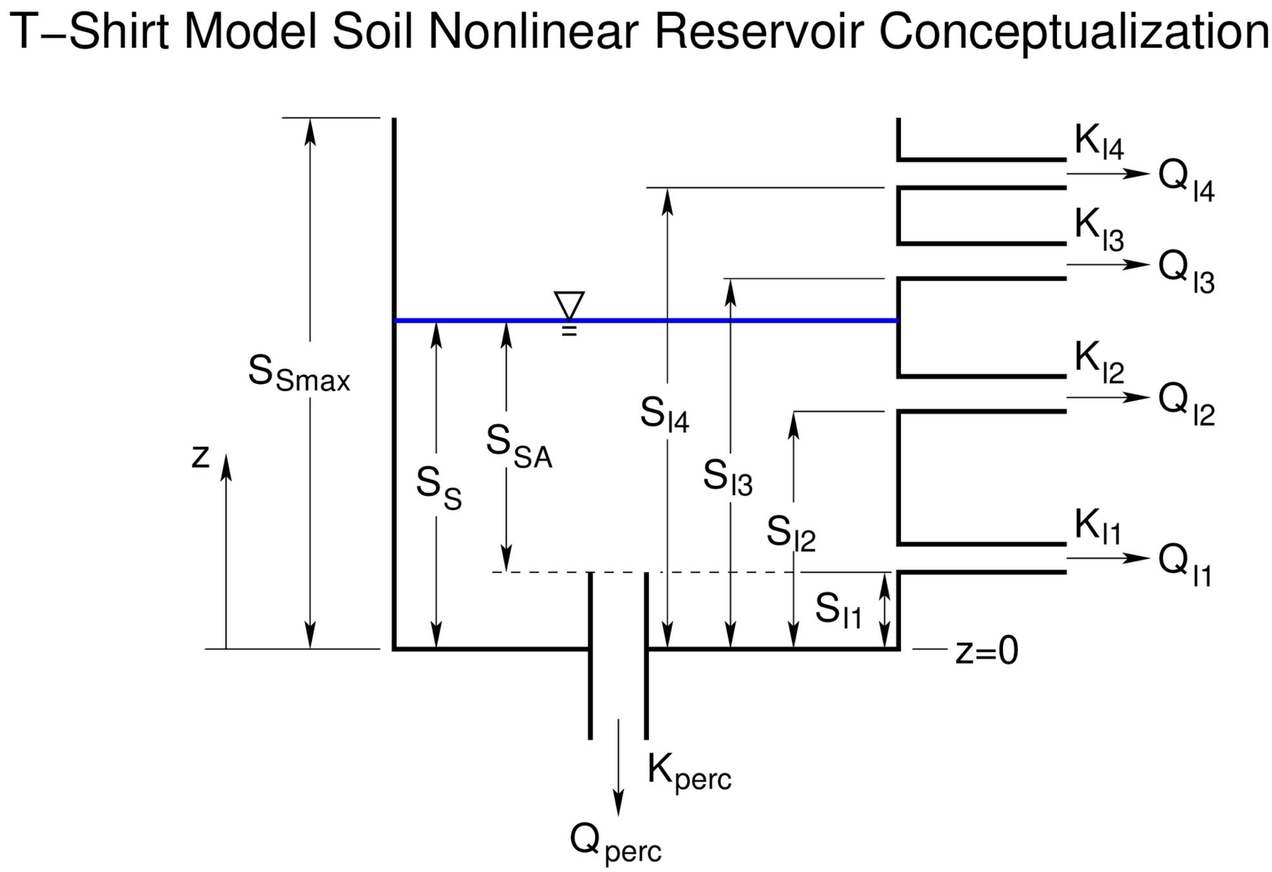

Figure 5: Definition sketch of variables and fluxes for calculation from the soil moisture reservoir, assuming that the flux in each soil discretization is calculated separately. It turns out that this level of detail is not necessary as described below.

If the soil is in hydrostatic equilibrium, then the suction head at the center of each soil discretization relative to the bottom of the soil column will equal the vertical distance z upward from the water table to the center of each soil discretization. Solving Eqn. 1 for water content gives:

$$\theta = \theta_e \left(\frac{z}{\psi_s} \right)^{-1/b} \tag{2}$$

The means to estimate a field capacity water content for a particular soil is uncertain. Its value depends on soil structure, heterogeneity, and other difficult to measure/observe factors. In the absence of data, approximate field capacity moisture contents are used. In lieu of data on field capacity, the common approach is to assume that the field capacity occurs at a water content that results in a soil suction head equal to some fraction $\alpha < 1$ of the atmospheric pressure head H_{atm}. In equation form, $H_{\mathrm{wt}} = \alpha\, H_{\mathrm{atm}}$. In the case of fine textured soils, α is taken as 1/3, while in sandy soils it is nearer to 1/10. The wilting point water content can be solved using:

$$\theta_{fc} = \theta_e \left(\frac{H_{wt}}{\psi_s} \right)^{-1/b} = \theta_e \left(\frac{\alpha P_{atm}}{\gamma_w \psi_s} \right)^{-1/b} \tag{3}$$

In the case of the four discretizations used in the NWM, numbered from bottom up, the distances are z_i={0.5, 1.3, 1.75, and 1.95 m}.

The linear reservoir requires that we convert this water table into an equivalent storage of water in the control volume. When the water content is equal to the value given by Eqn. 2, then the water stored in the soil is given by the following equation for the ith discretization z_i :

$$S_{fci} = \theta_e \left(\frac{1}{\psi_s}\right)^{-1/b} \int_{z_i=\Omega_i}^{z=\Omega_i+2} \zeta^{-1/b} d\zeta \tag{4}$$

where ζ is the variable of integration, and Ω is a vertical offset [m] that depends on the sought after activation storage for a particular discretization {1-4}. The offset Ω varies by soil discretization or outlet number, which are the same in Figure 2. Assuming that α = 1/3, then the suction head at the wilting point water content, H_{wt} = 3.44 m, then for the first outlet, Ω_i= (H_{wt}- z_i) = (3.44-0.5) m, where z_i = 0.5 m is the height from the bottom of the 2.0 m thick soil column up to the center of the bottom (first) discretization.

Integration of Eqn. 4 yields the following solution:

$$S_{fci} = \theta_e \left(\frac{1}{\psi_s}\right)^{-1/b} \frac{z^{1-1/b}}{1-1/b} \Big|_{\Omega_i}^{\Omega_i+2} \tag{5}$$

S_i calculated using Eqn. 5 is the storage threshold for activation of the ith lateral flowpath. They are close together because the field capacity definition of lateral flow path activation occurs on a portion of the water retention curve where there is not a lot of change of storage with in response to changes in pressure that coincide with changes in elevation for hydrostatic equilibrium. The use of the field capacity as a threshold is dubious because macropores that convey large quanitities of lateral subsurface flow do not activate when the soil is under suction.

For example, given a loam soil with the following parameters: θ_e=0.439, ψ_s = 0.355 m, b = 4.05, we calculate the following lateral flow activation threshold storage values for S_i given in Table 1.

Table 1. Example calculated storage threshold values for preferential flow path activation S_i

i	z_i	Ω_i [m]	Ω_i+2 [m]	Upper limit	Lower limit	S_i [m]
1	0.5	2.942	4.942	1.504	1.018	0.487
2	1.3	2.142	4.142	1.316	0.801	0.516
3	1.75	1.692	3.692	1.207	0.671	0.536
4	1.95	1.492	3.492	1.158	0.610	0.547

Note that the maximum water storage in a 2 m thick soil column with the given effective porosity is 0.878 m, so these storage activation thresholds are only a bit more than about half of the available storage at most. This further supports the notion that the field capacity is not a particularly realistic lateral flow activation threshold from a soil physics standpoint.

In the case of a sand-dominated soil where α =0.1, the hypothetical water table can appear in the soil column when calculating the storage at field capacity for the reservoir outlets that are higher in the soil column. When this occurs, the lower limit used to evaluate the integral in Eqn. 5, Ω_i becomes invalid because it is negative. When Ω_i is negative, then this indicates that the water table appears in the soil column. In this case use 0.0 for the lower limit of evaluation and the upper limit of integration equal to (2.0- z_i +H_{wt}), and the water stored in the saturated soil below the water table must be added to the integral. This situation is mathematically represented in Eqn. 6.

$$S_{fc} = \theta_e \left(\frac{1}{\psi_s}\right)^{-1/b} \left[\frac{z_i^{1-1/b}}{1-1/b} \Big|_{0}^{2-z_i+H_{wt}}\right] + \theta_e * \left(z_i - H_{wt}\right) \tag{6}$$

The fact that the activation thresholds in Table 1 are nearly the same, ranging from 55-62% of the total storage suggests that they could be replaced with a single value that is either 55% or perhaps the mid-point of this range, 58.5%. Also not that S_1 is the activation threshold for percolation to deep groundwater.

Table 2. Relative Soil Moisture Storage Threshold for Lateral Flow Activation [m]

				RELATIVE STORAGE [-] Distance to center of discretization			
Soil Texture	Assumed field capacity suction head fraction α	Distance to w.t. H_{wt} (m)	Eqn. 4 constant $\theta_s(1/\psi_s)^{-1/b}$	z_1 0.5	z_2 1.3	z_3 1.75	z_4 1.95
Sand	0.1	1.03	0.130	0.158	0.257	0.326	0.355
Loamy sand	0.15	1.55	0.193	0.278	0.321	0.404	0.446
Sandy loam	0.18	1.86	0.287	0.419	0.463	0.514	0.560
Silt loam	0.22	2.27	0.452	0.714	0.766	0.811	0.839
Silt	0.28	2.89	0.452	0.687	0.724	0.753	0.768
Loam	0.33	3.41	0.360	0.489	0.512	0.528	0.537
Sandy clay loam	0.33	3.41	0.299	0.395	0.409	0.419	0.424
Silty clay loam	0.33	3.41	0.439	0.698	0.717	0.731	0.738
Clay loam	0.33	3.41	0.395	0.623	0.641	0.654	0.661
Sandy clay	0.33	3.41	0.327	0.468	0.479	0.486	0.490
Silty clay	0.33	3.41	0.420	0.690	0.706	0.717	0.723
Clay	0.33	3.41	0.438	0.730	0.745	0.756	0.761

Notes on Table 2: These relative storage values are dimensionless and calculated by dividing the calculated threshold storage using Eqns 3 and 4 by the maximum storage in the 2.0 m deep soil column thickness, which is equal to 2.0*θ_e. Soil parameters taken from Noah-MP SOILPARM.TBL file.

While the spread between the heights of the centers of the z_1 and z_4 is 73% of the total soil column thickness, the spread between the activation storage contents for the four soil discretizations is less than 20% for the four coarsest soil textures and less than 10% for the remainder. This observation supports the notion that using multiple linear reservoir outlets to calculate the lateral subsurface flow does not provide benefit. Therefore, the relative storage for activation of lateral flow and the percolation flow path should occur when the pressure at the center of the first discretization, z_1, equals the field capacity.

Darcy's law together with some assumptions about watershed geometry (slope, flow area) allow calculation of the lateral flow constants K_{li} in Figure 2. In the NWM, the saturated hydraulic conductivity values associated with saturated flow in are taken as a scalar multiple of the soil vertical saturated hydraulic conductivity, `SATDK` [m s^{-1}], which is more commonly described using the notation K_s. This multiplier is a calibration parameter called `LKSATFAC`. Calibrated values of this parameter are bounded between 10 and 10,000 during the calibration process. This means that in the case of the loam soil used in the example calculations for Table 1, with `SATDK`=3.4E-06 m s^{-1} the lateral hydraulic conductivity in the calibrated NWM can vary from 3.4E-05 to 3.4E-02 m s^{-1}, or 3.4E-03 to 3.4 cm s^{-1}. The upper limit of this range is 122 m h^{-1}!

The maximum transmissivity determining the flux of lateral subsurface flow through the i^{th} discretization, $T_i = K_s$ * `LKSATFAC`* Δz_i, where Δz_i is the thickness of the i^{th} soil discretization [m]. The flux through each soil discretization in the NWM relies on the Prickett-Lonnquist Aquifer Simulation Model (PLASM) (Prickett and Lonnquist, 1971) to route water once it is placed into the lateral flow routine. The NWM implementation neglects capillary effects and assumes a constant specific yield, which is generally a poor assumption for groundwater tables near the land surface.

Darcy's Law in the context of one-dimensional saturated groundwater flow in an unconfined aquifer in the principal flow direction, q_s is:

$$q_s = -T \frac{dH}{ds} \tag{6}$$

In the Boussinesq approximation assumes that vertical velocity components are negligible and that groundwater flow is driven by topography in the case of catchment models. Because the CFE model does not include an explicit representation of topography which is necessary to calculate the land surface slope, we cannot use Darcy's law to calculate the nonlinear reservoir K_l.

The findings of this analysis into the 250 m routing employed the lateral zone flow code in the NWM and design of its conceptual equivalent suggest two findings. First, the relatively small range of values of the activation storage for the proposed four lateral flow linear reservoir outlets suggests that they can be replaced with a single reservoir outlet. Secondly, the lack of topographical data and linearity of the Darcy's law (Eqn. 6) suggests that the linear reservoir K factor for this single outlet be treated like a calibration factor, just like the NWM treats the `LKSATFAC` parameter.

4.3 Deep Groundwater Reservoir

Given the following nonlinear reservoir parameters as applied in the NWM using an exponential function, with the same parameters as defined in the NWM code `C`, `expon`, and $\mathtt{z}_{\mathtt{max}}$:

$$Q = C\left[\exp\left(expon * z / z_{max}\right) - 1\right] \tag{7}$$

writing this in the notation used in Figure 1:

$$Q = K_{gw}\left[\exp\left(expon_{gw} S_{gw}/S_{gwmax}\right)-1\right] \quad (8)$$

from this we can see that there is a one-to-one equivalence between the CFE model and NWM parameters: K_{gw} = C, $Expo_{gw}$ = *expon*, and $Sgw_{max} = z_{max}$. The K_{gw} should have units of [m h^{-1}] if the model is formulated with a 1 h time step.

4.4 Summary- Estimating CFE Model Parameters

The conceptual model structure shown in Fig 1. is proposed as a functional equivalent of the National Water Model implementation of NOAH-MP with the routing functionality of the WRF-Hydro code. This document goes through the steps of estimating equivalent parameters for the so-called T-Shirt model.

Considering the differences between the representation of the soil reservoir in Figs 1 and 2, and example calculations of activation storage levels for the standard Noah-MP soil parameter values, the differences between the activation storage levels for the soil discretizations are small. Given that the lateral flow calculation in the NWM is entirely conceptual due to the use of the LKSATFAC parameter which is allowed to vary over three orders of magnitude in calibration, an equivalent conceptual model can use one lateral flow outlet. Its activation storage should be the same as that for the percolation flux, based on field capacity.

Here is a summary of how to calculate the parameters for the CFE model from NWM parameters. With reference to the parameters in Figure 1:

Schaake Partitioning Function

This function is identical to the one used in the NWM. Three parameters are required. Note `satdk` is the calibrated saturated hydraulic conductivity value from the NWM, using variable names ftom Fig. 1) where applicable. NWM parameter names are listed in courier font:

1. S_{smax} = 2.0 m * `smcmax`, where `smcmax` is the soil porosity used in the NWM which is either calibrated or from SOILPARM.TBL. Note that 2.0 m is the assumed thickness of the soil in the National Water Model, applied uniformly over CONUS.

2. Schaake_adjusted_magic_constant_by_soil_type = $C_{schaake}$= `REFKDT * satdk/2.0E-06`, with `satdk` being the soil saturated hydraulic conductivity in [m s^{-1}]. The default value of `REFKDT` = 3.0 in NOAH-MP (see GENPARM.TBL), but is calibrated in WRF-Hydro.

3. `column_total_soil_moisture_deficit_m` = $S_{smax} - S_s$ (using variable names from Fig. 1)

Lateral Flow Function

This function requires four parameters, S_{smax}, S_{fc}, K_{lf}, and K_n as defined in Figure 1. As before, NWM parameters are listed in courier font. S_{fc} is calculated using the following steps:

1. Calculate the field capacity water content using Eq. 3. Assume α =0.1 for Sand soil texture, increasing linearly to α =0.33 for a Loam soil texture. For soil textures from Loam to Clay, use α =0.33.

2. Calculate the height above a water table $H_{wt} = \alpha\ H_{atm}$.

3. H_{wt} should be greater than 0.5 m for all soils. Calculate the soil water storage in the soil column when the lowest soil discretization is equal to field capacity S_{fc} using Eqn. 5, with the limits of integration being: Ω_1= (H_{wt} -0.5) m, and $\Omega_2=\Omega_1$+2 m.

4. If a different discretization or height in the soil column is used to trigger the lateral flow function, then it is possible that Eqn. 5 will have a negative lower limit of integration. In this case follow the procedure outlined to use Eqn. 6.

5. The third lateral flow function parameter, K_{lf}, is pretty much a calibration parameter in the NWM with no strictly definable relationship to the NWM parameters. One could assume that the contour length in the subsurface through which shallow subsurface low passes depends on the drainage density, d_d which for CONUS averages about 3.5 km of stream channel per sq. km of land surface area, `A`. The area that lateral flow would pass through might equal the soil thickness `D` (2 m) time twice the length of channel because the channel has two sides. Then the maximum flux into the streams from the subsurface assuming a water table slope of 0.01 and lateral flow conductivity of `LKSATFAC`*`DKSAT`:

$$Q_{maxlf} = 0.01 * LKSATFAC * DKSAT * D * 2 * (A * d_d) \quad [\mathrm{m^3\ s^{-1}}] \qquad (9)$$

With `A` in km^2, and d_d expressed in units of m of channel per m^2 of catchment area (note d_d= 3.5 km/km^2 = 0.0035 m/m^2). The linear reservoir constant is calculated using, with `LKSATFAC` as a calibration parameter that is allowed to vary from 10 to 10,000 as in the NWM, with :

$$K_{lf} = Q_{maxlf} / A = 0.02 * LKSATFAC * DKSAT * D * d_d \quad [\mathrm{m\ s^{-1}}] \qquad (10)$$

6. The fourth lateral flow function parameter, K_n is the Nash cascade discharge linear reservoir coefficient. Adjust to improve hydrograph timing and magnitude late direct runoff component on falling limb of the hydrograph.

Percolation Function

This function becomes active whenever the storage in the soil S_s becomes greater than the threshold given by S_{fc}. The flow rate per unit area is given by the following equation (this equation is very similar to the calculation used in NWM 2.0):

$$Q_{perc} = K_{perc} * SLOP * (S_s - S_{fc}) / (S_{smax} - S_{fc}) \quad [\text{m s}^{-1}] \tag{11}$$

where:
K_{perc}= the calibrated saturated hydraulic conductivity of the soil, `satdk`
$SLOP$ = calibrated NWM `SLOPE` parameter

Deep Groundwater Response Nonlinear Reservoir

The deep groundwater response nonlinear reservoir is identical to that used in the NWM v2.0. It has two parameters, C_{gw}, and *expon*.

$$Q_{gw} = C_{gw} \left(e^{\left(expon * S_{gw} / S_{gwmax} \right)} - 1 \right) \quad [\text{m s}^{-1}] \tag{12}$$

Use calibrated NWM values of C_{gw} and *expon* parameters, with S_{gw} and S_{gwmax} defined as in Figure 1.

5.0 Discussion and Conclusions

The WRF-Hydro based National Water Model, current operational version 2.1, applies the Noah-MP land surface model on a 1 sq. km grid to simulate point column processes. Noah-MP does not consider land surface topography, and uses the soil moisture content solution of Richardson/Richards Equation (RRE) which is limited to a uniform, unsaturated soil. Because this soil moisture transport scheme is invalid for saturated conditions, it cannot simulate infiltration from above. For this reason, a conceptual precipitation partitioning scheme is required to estimate direct runoff and increase in soil moisture due to precipitation. A coarse discretization of the soil smooths the RRE solution, promotes computational stability, reduces computational effort, and effectively moves water down into the soil with a correct tendency to produce a hydrostatic water content profile between storms. The coarse discretization greatly damps solution dynamics compared to a solution of the advection term of the soil moisture velocity equation using a finite moisture content discretization. Because of these limitations the Conceptual Functional Equivalent of the National Water Model replaces the RRE solver with a hydrologic reservoir.

The WRF-Hydro based National Water Model includes different sources three stormflow production: direct runoff, lateral subsurface flow through a uniform two meter thick soil, and base flow simulated by a nonlinear exponential hydrologic reservoir. Overland flow is routed on a 250 m grid using a quasi-2D steepest descent kinematic or diffusive wave option using Manning's equation and a finite-volume routing solution similar to the CASC2D model (Julien et al. 1995). In the WRF-Hydro based operational National Water Model, runon infiltration is simulated by recursive application of the Schaake et al. (1996) partitioning function, which violates its fundamental original assumption as a rainfall partitioning method. Because of slope reductions introduced by the use of the 250 m grid, and the fact that overland flow seldom persists as sheet flow for distances larger than 10 m much less 250 m, this overland flow routing function is replaced in CFE using a generic Geomorphic Instantaneous Unit Hydrograph of the user's preference.

The WRF-Hydro based National Water Model routes lateral subsurface flow on the same 250 m grid used in overland flow routing using a 2D Boussinesq solution. Lateral subsurface flow is triggered using a field capacity threshold. The lateral flow transmissivity is very loosely tied to the vertical saturated hydraulic conductivity of the soil multiplied by a calibration factor that is allowed to vary between 10 and 10,000. This calibration factor overwhelms the magnitude of the original parameter, rendering the approach largely conceptual in nature. For this reason, in the CFE model the 250 m grid Boussinesq lateral flow function is replaced by a cascade of Nash linear reservoirs.

The CFE model retains signficant portions of the current operational National Water Model. For instance, the Schaake et al. (1996) rainfall partitioning method is retained. The National Weather Service recently added a Xinanjiang partitioning scheme as well that will serve as an option in NWM version 3.0 The conceptual exponential shallow aquifer model was retained, as were the field capacity soil moisture threshold for activation of vertical percolation and lateral subsurface flow paths.

The purpose of the CFE model is to apply appropriate conceptualizations where warranted in situations where deviations from physics due to model structural biases, grid-size distortion of descriptive inputs, use of methods beyond required conditions, or inappropriate application of methods. The net gain provided by the CFE model is to greatly reduce computational effort with very little loss in capability. It provides a sound basis as a null hypothesis in evaluating other alternative model formulations for hydrologic predictions in National Water Model Version 4.0, which will become operational in fiscal year 2025 configured using the Next Generation Water Resources Modeling Framework (Ogden et al. 2021).

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