1. In Fig. 1, the continuous-time controller and the (continuous-time) plant are

$$K(s) = \frac{s+7}{s(s+10)} \qquad G(s) = \frac{6}{s+4}.$$

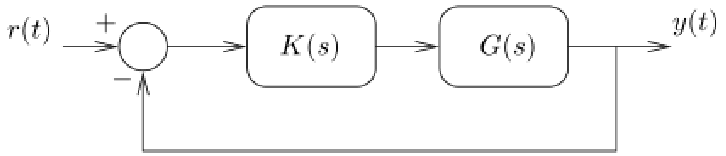


Figure 1: Problem 1.

We want to implement the controller using a discrete–time approximation $K(z)$ (Fig. 2).

Figure 2: Problem 1.

(a) Verify that the bandwidth of the closed-loop system in Fig. 1 is 1.3 rad/s.

(b) Choose a suitable sampling period and use Euler's forward rectangular rule to obtain an approximating discrete–time controller $K(z)$. Write both the difference equation and the transfer function of the controller.

(c) Write a pseudocode for a procedure to implement the above control algorithm. Assume that procedures `ADC` and `DAC` are available to read the A/D and to send values to the D/A and that the procedure is going to be called at sampling instants. Ignore computational delay.

a) First we find the transfer function

$$K(s) = \frac{s+7}{s(s+10)} \qquad G(s) = \frac{6}{s+4}.$$

$$T(s) = \frac{K(s)G(s)}{1+K(s)G(s)} \qquad \therefore K(s)G(s) = \frac{6(s+7)}{s(s+10)(s+4)}$$

$$T(s) = \frac{\frac{6(s+7)}{s(s+10)(s+4)}}{1+\frac{6(s+7)}{s(s+10)(s+4)}} = \frac{6(s+7)}{s(s+10)(s+4)+6(s+7)} = \frac{6s+42}{(s^2+10s)(s+4)+6(s+7)}$$

$$T(s) = \frac{6s+42}{s^3+4s^2+10s^2+40s+6s+42} = \frac{6s+42}{s^3+14s^2+46s+42}$$

• using MATLAB

EDITOR:

```
clear all
clc

%Find the transfer function
syms s
k=(s+7)/((s)*(s+10)) ;
g=6/(s+4) ;
v=k*g ;
T=v/(1+v); % closd loop TF
[n,d]=numden(sym(T))

%Calculate the Bandwidth of transfer function
sys= tf([6,42],[1,14,46,42])
Wbw= bandwidth(sys)
```

Command Window:

```
n =

6*s + 42


d =

s^3 + 14*s^2 + 46*s + 42



sys =

        6 s + 42
  ------------------------
  s^3 + 14 s^2 + 46 s + 42

Continuous-time transfer function.


Wbw =

   1.2958
```

$W_{bw} \approx 1.3$ rad/s

(or) CL: closed loop

$$|G_{CL}(j\omega_{BW})| = 0.707\,|G_{CL}(j0)|$$

So

$$\frac{|G_{CL}(j1.3)|}{|G_{CL}(j0)|} = 0.707$$

$$T(s) = \frac{6s+42}{s^3+14s^2+46s+42}$$

$$G_{CL}(j\omega) = \frac{6j\omega+42}{(j\omega)^3+14(j\omega)^2+46(j\omega)+42}$$

$$G_{CL}(j0) = \frac{42}{42} = 1$$

$$G_{CL}(j1.3) = \frac{7.8j+42}{-2.197j-23.66+59.8j+42} = \frac{7.8j+42}{57.603j+18.34}$$

or

$$G_{cl}(s) = \frac{K(s)G(s)}{1+K(s)G(s)} = \frac{(s+7)6}{s(s+10)(s+4) + 6(s+7)}$$

$$G_{cl}(s) = \frac{6s+42}{s^3+14s^2+46s+42}$$

$$G_{cl}(j\omega) = \frac{6j\omega + 42}{(j\omega)^3 + 14(j\omega)^2 + 46(j\omega) + 42}$$

$$= \frac{6j\omega + 42}{-j\omega^3 - 14\omega^2 + 46j\omega + 42} = \frac{6j\omega + 42}{j(46\omega - \omega^3) - 14\omega^2 + 42}$$

$$|G_{CL}(j\omega)| = \frac{\left(42^2 + (6\omega)^2\right)^{1/2}}{\left((46\omega - \omega^3)^2 + (-14\omega^2 + 42)^2\right)^{1/2}}$$

$\omega_B = 1.3$

$$|G_{CL}(j\omega_B)| = 0.707$$

$$G_{CL}(j1.3) = \frac{7.8j + 42}{-2.197j - 23.66 + 59.8j + 42} = \frac{7.8j + 42}{57.603j + 18.34}$$

$$|G_{CL}(j1.3)| = 0.707$$

$$\therefore \frac{|G_{CL}(j1.3)|}{|G_{CL}(j0)|} = 0.707$$

b) The transfer function was already computed in part a)

EDITOR:

```
ws=30*Wbw          % sampling frequency (rad/s)
Ts=2*pi/ws         % sampling period (s)
```

Command Window:

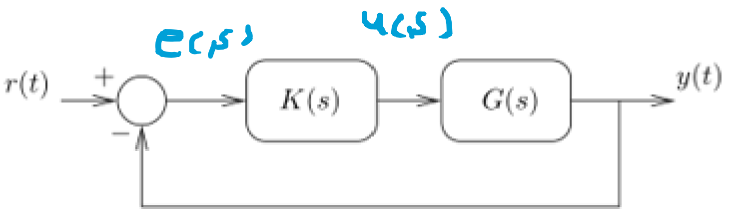
ws = 38.8735

Ts = 0.1616 (suitable sampling period $T_s = 0.1616$ sec)

$$K(s) = \frac{s+7}{s(s+10)}$$

Eulers Forward Rectangular Rule

$$\dot{x}(k) \cong \frac{x(k+1) - x(k)}{T}$$



$$K(s) = \frac{U(s)}{E(s)} = \frac{s+7}{s(s+10)}$$

$$K(s) = \frac{U(s)}{e(s)} = \frac{s+7}{s(s+10)}$$

$$\dot{x}(k) \cong \frac{x(k+1) - x(k)}{T}$$

$$U(s)\,(s^2+10s) = e(s)\,(s+7)$$

$$\ddot{u} + 10\dot{u} = \dot{e} + 7e$$

$$\dot{u}(k) = \frac{u(k+1) - u(k)}{T}$$

$$\dot{e}(k) = \frac{e(k+1) - e(k)}{T}$$

$$\ddot{u}(k) = \frac{\dot{u}(k+1) - \dot{u}(k)}{T}$$

$$\dot{u}(k+1) = \frac{u(k+2) - u(k+1)}{T}$$

$$\ddot{u} + 10\dot{u} = \dot{e} + 7e$$

$$\frac{\dot{u}(k+1) - \dot{u}(k)}{T} + 10\left(\frac{u(k+1) - u(k)}{T}\right) = \frac{e(k+1) - e(k)}{T} + 7e(k)$$

$$\frac{1}{T}\left(\frac{u(k+2) - u(k+1)}{T} - \left(\frac{u(k+1) - u(k)}{T}\right)\right) + 10\left(\frac{u(k+1) - u(k)}{T}\right)$$

$$= \frac{e(k+1) - e(k)}{T} + 7e(k)$$

$$\frac{1}{T^2}\left(u(k+2) - 2u(k+1) + u(k)\right) + \frac{10}{T}\left(u(k+1) - u(k)\right)$$

$$= \frac{1}{T}\left(e(k+1) - e(k)\right) + 7e(k)$$

$$\frac{1}{T^2}\left(u(k+2)-2u(k+1)+u(k)\right)+\frac{10}{T}\left(u(k+1)-u(k)\right)$$
$$=\frac{1}{T}\left(e(k+1)-e(k)\right)+7e(k)$$

$$u(k+2)-2u(k+1)+u(k)+10T\left(u(k+1)-u(k)\right)$$
$$=T\left(e(k+1)-e(k)\right)+7T^2e(k)$$

$$\boxed{\begin{aligned}u[k+2]&=u(k+1)(2-10T)+u(k)(10T-1)\\&\quad+e(k+1)T+e(k)(7T^2-T)\end{aligned}}$$

if $T=0.1616$, Difference equation is

$$u[k+2]=0.384\,u[k+1]+0.616\,u[k]+0.1616\,e[k+1]+0.0212\,e[k]$$

or

$\ddot{u}[k] + 10\dot{u}[k] = \dot{e}[k] + 7e[k]$

$\dot{u}[k] = \dfrac{u(k+1) - u(k)}{T}$

$\ddot{u}[k] = \dfrac{1}{T^2}\{u[k+2] - 2u[k+1] + u[k]\}$

$\dot{e}[k] = \dfrac{e(k+1) - e(k)}{T}$

$T = 0.1616$

$$\frac{1}{T^2}\{u[k+2] - 2u[k+1] + u[k]\} + \frac{10}{T}[u(k+1) - u(k)] = \frac{e(k+1) - e(k)}{T} + 7e[k]$$

$$\frac{1}{T^2}u[k+2] + u[k+1]\left(\frac{1}{T^2}(-2) + \frac{10}{T}\right) + u[k]\left(\frac{1}{T^2} - \frac{10}{T}\right)$$

$$= e(k+1)\left(\frac{1}{T}\right) + e(k)\left(7 - \frac{1}{T}\right)$$

-0.384 0.616

$$u[k+2] + u[k+1]\,(T^2)\left(\frac{1}{T^2}(-2) + \frac{10}{T}\right) + u[k]\,T^2\left(\frac{1}{T^2} - \frac{10}{T}\right)$$

$$= e(k+1)\,T + e(k)\,T^2\left(7 - \frac{1}{T}\right)$$

0.1616 0.0212

1. In Fig. 1, the continuous-time controller and the (continuous-time) plant are

$$K(s) = \frac{s+7}{s(s+10)} \qquad G(s) = \frac{6}{s+4}.$$

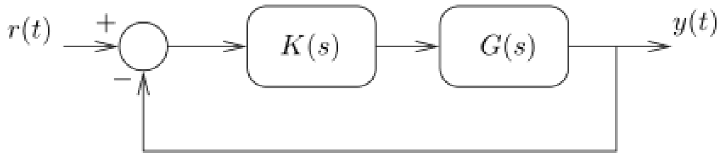


Figure 1: Problem 1.

We want to implement the controller using a discrete–time approximation $K(z)$ (Fig. 2).

Figure 2: Problem 1.

(a) Verify that the bandwidth of the closed-loop system in Fig. 1 is 1.3 rad/s.

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(c) Write a pseudocode for a procedure to implement the above control algorithm. Assume that procedures `ADC` and `DAC` are available to read the A/D and to send values to the D/A and that the procedure is going to be called at sampling instants. Ignore computational delay.

The transfer function K[z] is obtained via z-transform (Time advance/delay property) assume no initial condition

- $\mathcal{Z}\{x[n+1]\} = z\,X(z) - z\,x[0]$
- $\mathcal{Z}\{x[n+2]\} = z^2X(z) - z^2x[0] - z\,x[1]$

? why no initial condition?

what happens if there is initial condition

$$u(k+2) = u(k+1)(2-10T) + u(k)(10T-1) + e(k+1)T + e(k)(7T^2-T)$$

$$z^2U(z) = zU(z)(2-10T) + U(z)(10T-1) + zE(z)T + E[z](7T^2-T)$$

$$U(z)(z^2 - (2-10T)z - (10T-1)) = E[z](zT + (7T^2-T))$$

$$\boxed{\frac{U(z)}{E[z]} = K[z] = \frac{zT + 7T^2 - T}{z^2 + (10T-2)z + (1-10T)}}$$

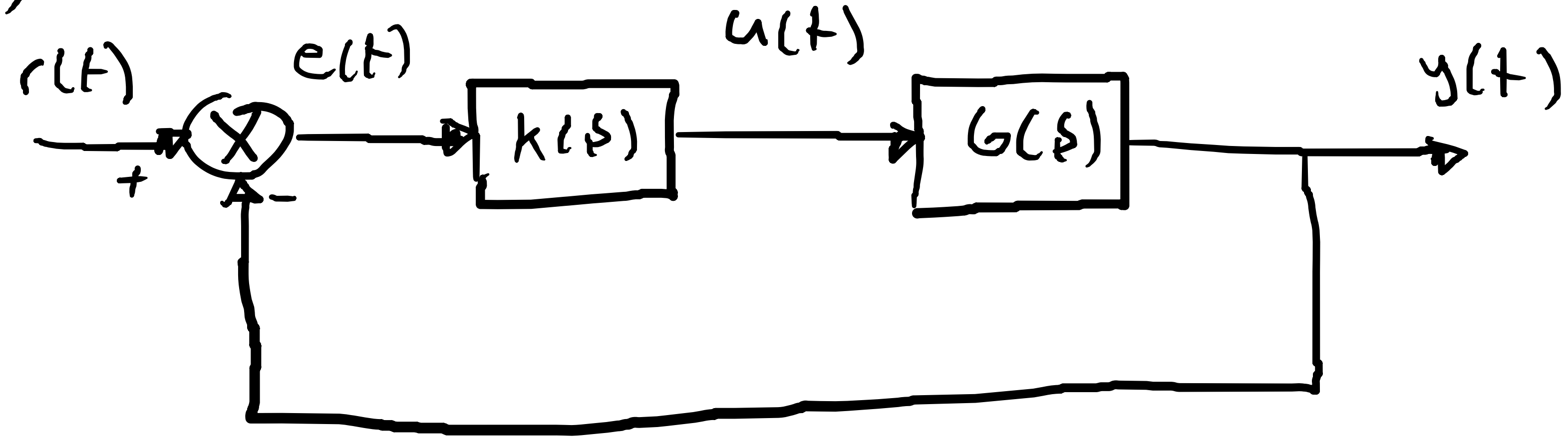
✓ Definition of transfer function ratio of output to input with 0 initial condition

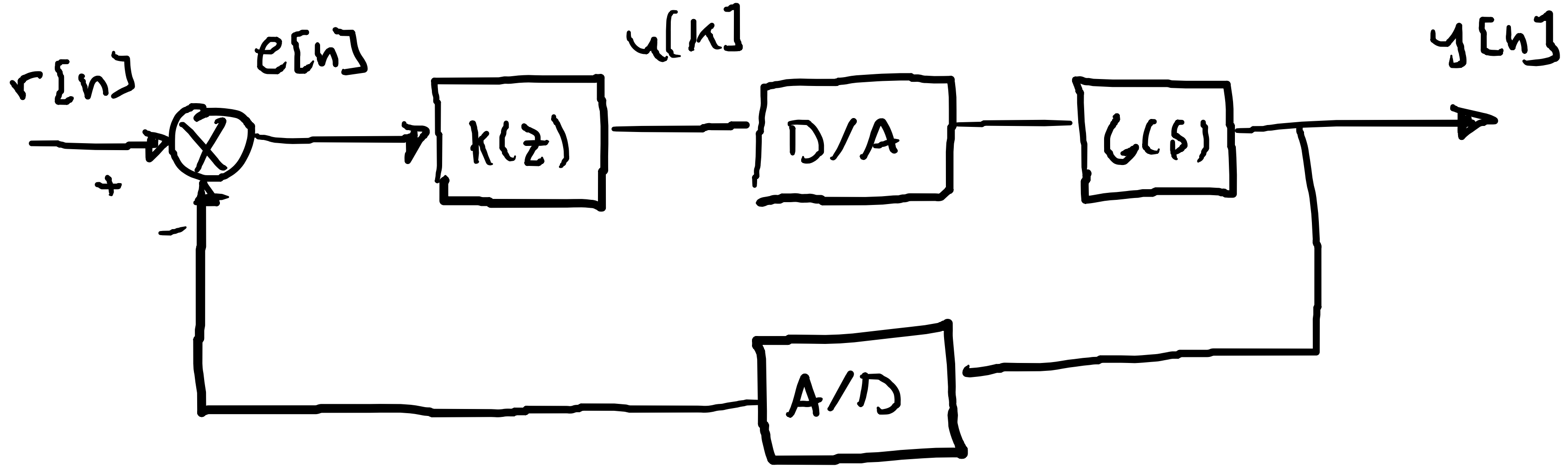
✓ if you assume initial condition it will only give you contributions of input + initial conditions you have 0 state and 0 input response

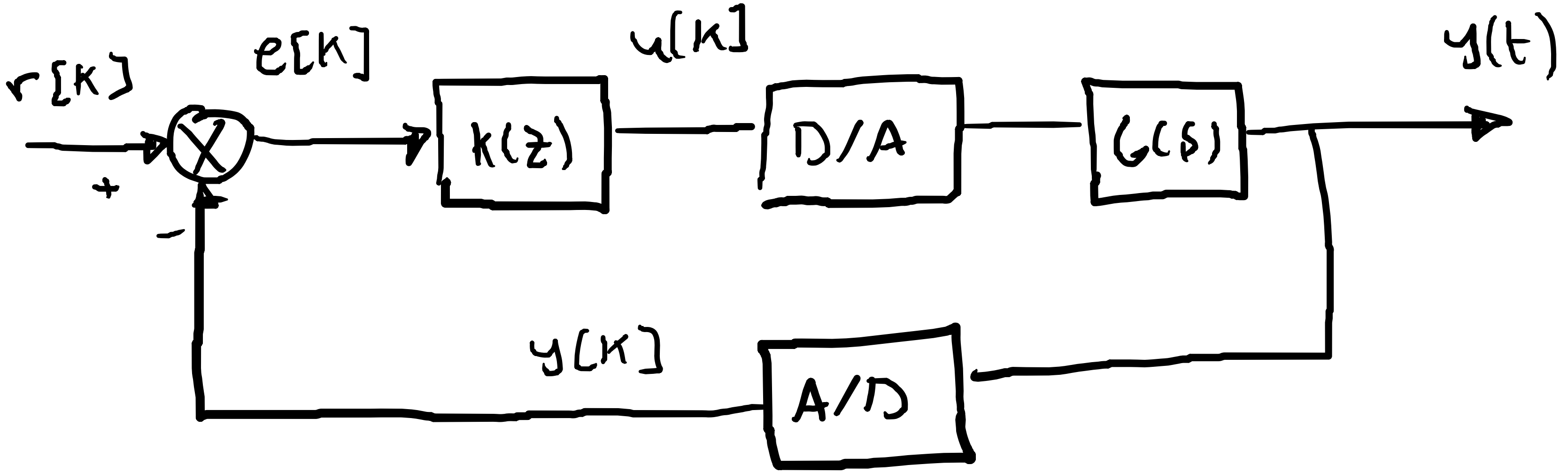
0 state → what you get output from input

0 input response → initial conditions only

c)







$$u[k+2] = 0.384\, u[k+1] + 0.616\, u[k] + 0.1616\, e[k+1] + 0.0212\, e[k]$$

UK2 = 0.384UK1 + 0.616UK + 0.1616 EK1 + 0.0212 EK

```
Subroutine Control (EK, EK1, UK, UK1)
  CALL ADC (EK2) % call current error from ADC

  UK2 = 0.384UK1 + 0.616UK + 0.1616EK1 + 0.0212EK
   % Define UK2

  DAC (UK2)  % convert UK2 to analog

   EK = EK1     % error updated
   EK1 = EK2

   UK = UK1     % control signal updated
   UK1 = UK2
Return   % return to subroutine and start all over...
```

a) $T(s) = \dfrac{6s+42}{s^3+14s^2+46s+42}$

USING MATLAB:

```
sys= tf([6,42],[1,14,46,42])
Wbw= bandwidth(sys)
```

b)

$u(k+2) = u(k+1)(1-10T) + u(k)(10T) + e(k+1)T + e(k)(7T^2-T)$

$T = 0.1616$

$u(k+2) = -0.616\,u(k+1) + 1.616\,u(k) + 0.1616\,e(k+1) + 0.0212\,e(k)$

c)

```
Subroutine Control(ek, uk)
  CALL ADC(ek1)
 uk2 = -0.616 uk1 + 1.616 uk + 0.1616 ek1 % Define uk2
  CALL DAC(uk2) %convert uk2 to analog
   uk = uk2
   ek = ek1
   Return
```

2. The transfer function of a dc motor is

Correction in future slides

$$G(s) = \frac{0.8}{s+1}$$

The input is armature voltage and the output is angular velocity.

(a) Design a continuous PI controller

$$K(s) = k\,\frac{s+a}{s}$$

so that the closed–loop system (Fig. 3) has a rise time $t_r < 1$ sec and an overshoot $M_p < 20\%$.

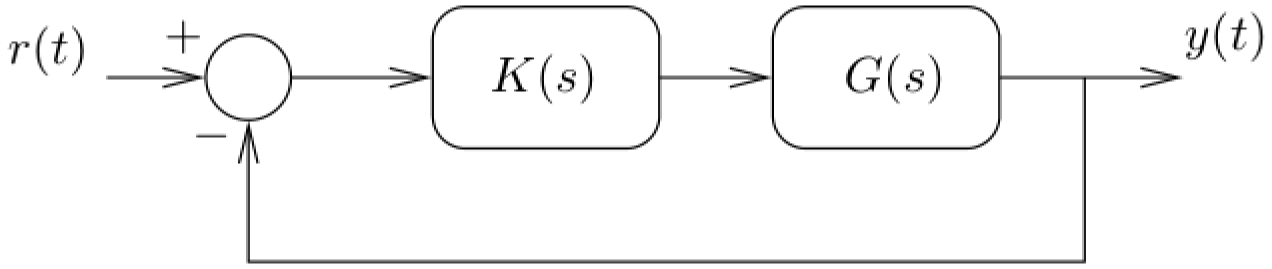


Figure 3: Problem 2.

- Find closed loop TF
- Solve for K & a

$$T(s) = \frac{K(s)\,G(s)}{1+K(s)G(s)} = \frac{\frac{0.8}{s+1} * K\frac{(s+a)}{s}}{1+\frac{0.8}{s+1} * K\frac{(s+a)}{s}} = \frac{0.8 * K(s+a)}{(s+1)s + 0.8K(s+a)}$$

$$= \frac{0.8Ks + 0.8aK}{s^2 + s + 0.8Ks + 0.8aK} = \frac{0.8Ks + 0.8aK}{s^2 + s(1+0.8K) + 0.8aK}$$

$$\boxed{\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}}$$

$$\boxed{\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100}$$

For $M_p < 20\%$

$\zeta > 0.456$

[?] why not pick $\zeta = 0.9$

let $\zeta = 0.6$

$t_r = \frac{1.8}{\omega_n}$ $\quad t_r < 1$

$\frac{1.8}{\omega_n} < 1$ $\quad \omega_n > 1.8$

[?] why not pick $\omega_n = 10K$

let $\omega_n = 1.8$

$\zeta = 0.6$ gives you 10% but not good, we design for 15% to cover the entire 20%, so if 20% we aim for 15%

Why? if you make it high you put too much effort on system, the rise time is minimum and the control signal is huge or system will not work (big)

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

checked by MATLAB that $\zeta = 0.6$ gives overshoot less than 20%

$$2\zeta\omega_n = 1 + 0.8K$$

$$\therefore K = \frac{2\zeta\omega_n - 1}{0.8} = 1.45$$

$$0.8aK = \omega_n^2$$

$$a = \frac{\omega_n^2}{0.8K} = 2.79$$

$$\boxed{K(s) = \frac{1.45(s + 2.79)}{s}}$$

(b) Use Euler's forward rectangular rule to obtain an approximating discrete–time controller $K(z)$ (Fig. 4).

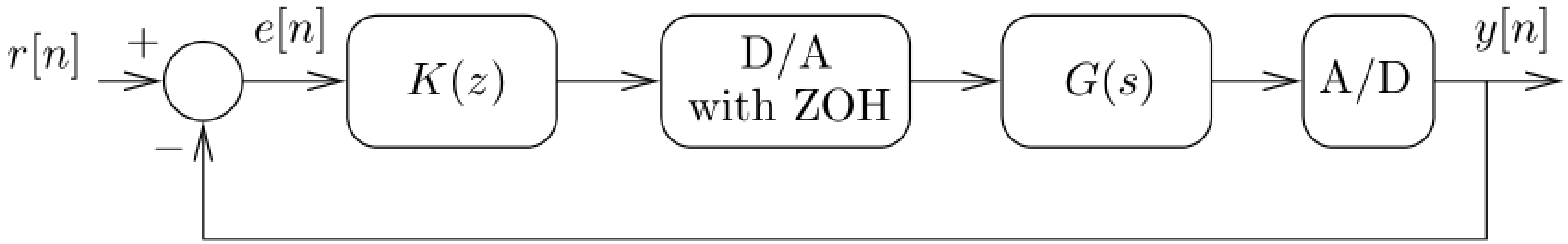


Figure 4: Problem 2.

$$\dot{x}(k) \cong \frac{x(k+1) - x(k)}{T}$$

$$\dot{u}(k) = \frac{u(k+1) - u(k)}{T}$$

$$\dot{e}(k) = \frac{e(k+1) - e(k)}{T}$$

$$K(s) = \frac{U(s)}{E(s)} = \frac{k(s+a)}{s}$$

$$\dot{u} = k\dot{e} + ake$$

$\dot{u} = k\dot{e} + ake$, $k = 1.45$, $a = 2.79$

$$\frac{1}{T}(u(k+1) - u(k)) = \frac{k}{T}(e(k+1) - e(k)) + ake(k)$$

$$u(k+1) - u(k) = k(e(k+1) - e(k)) + akTe(k)$$

$$u(k+1) = u(k) + e(k+1)k + e(k)(akT - k)$$

$$u(k+1) = u(k) + 0.802\,e(k+1) + (4.05T - 0.802)e(k)$$

Find $k[z]$ using z-transform

$$zU(z) = U(z) + zkE(z) + (akT - k)E(z)$$

$$U(z)(z-1) = E(z)(zk + akT - k)$$

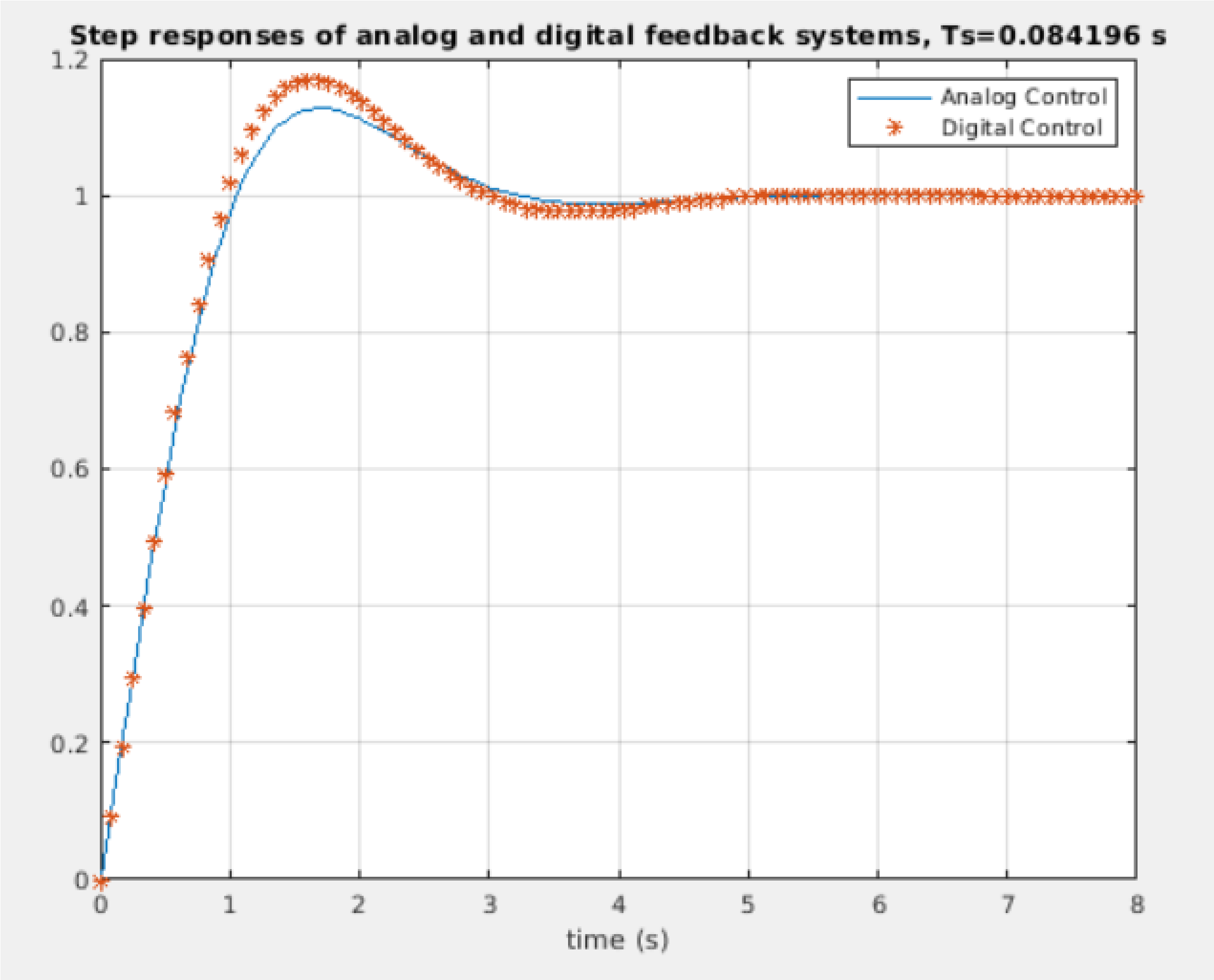
$$k(z) = \frac{U(z)}{E(z)} = \frac{(zk + akT - k)}{(z-1)} = \frac{1.45z + 4.05T - 1.45}{z-1}$$

(c) Consider a range of sampling frequencies and for each sampling frequency plot the step responses of the analog and digital systems. To obtain the step responses, you can modify and use the MATLAB file `asn1.m` which is posted on the course Moodle web page. Find the slowest sampling frequency for which the overshoot does not exceed 25%. Compare the sampling frequencies with the bandwidth of the analog closed–loop system.

c) For $\omega_s = 30\,\omega_B$

$M_p \approx 17\%$

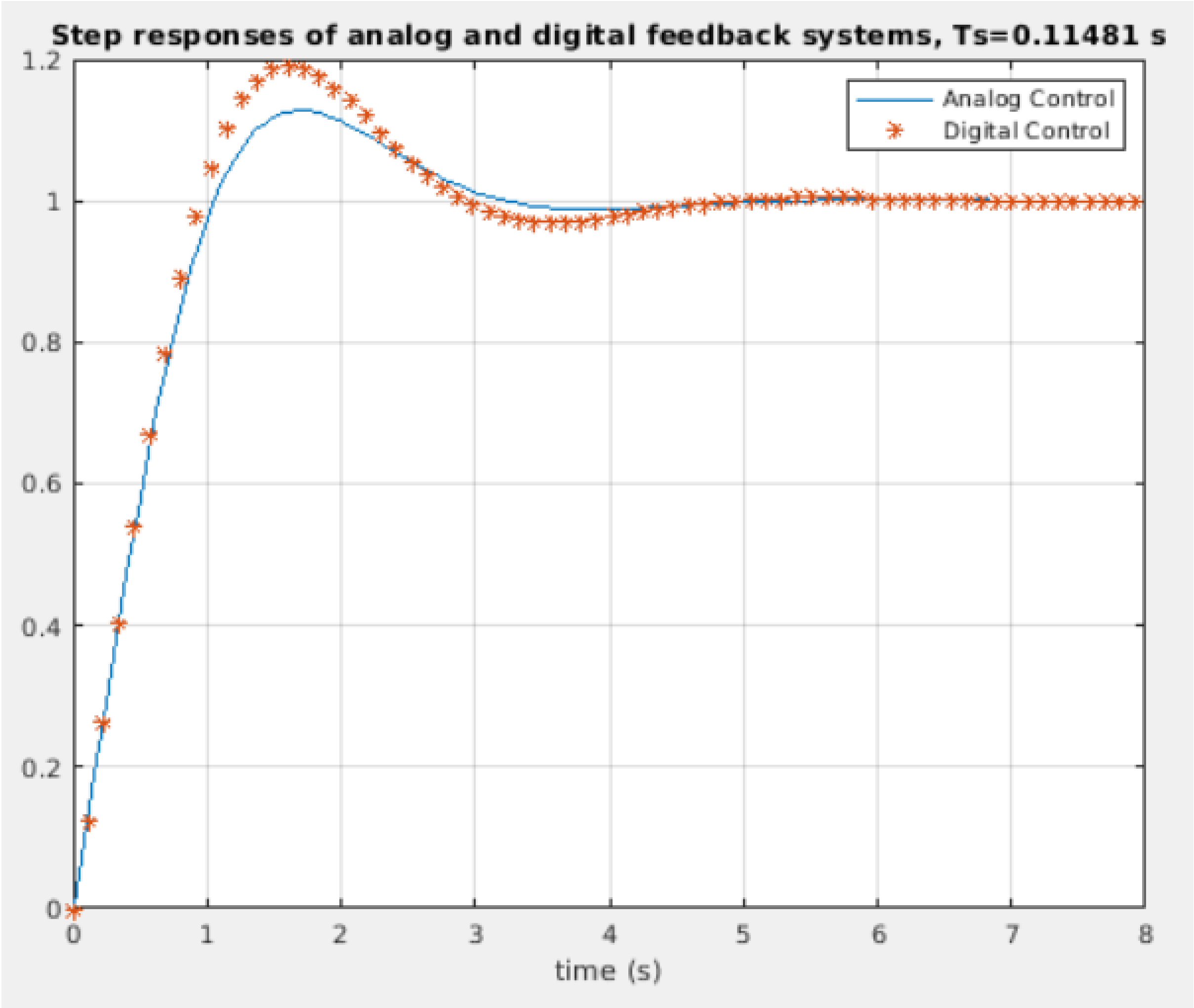
$T_s = 0.084$ sec



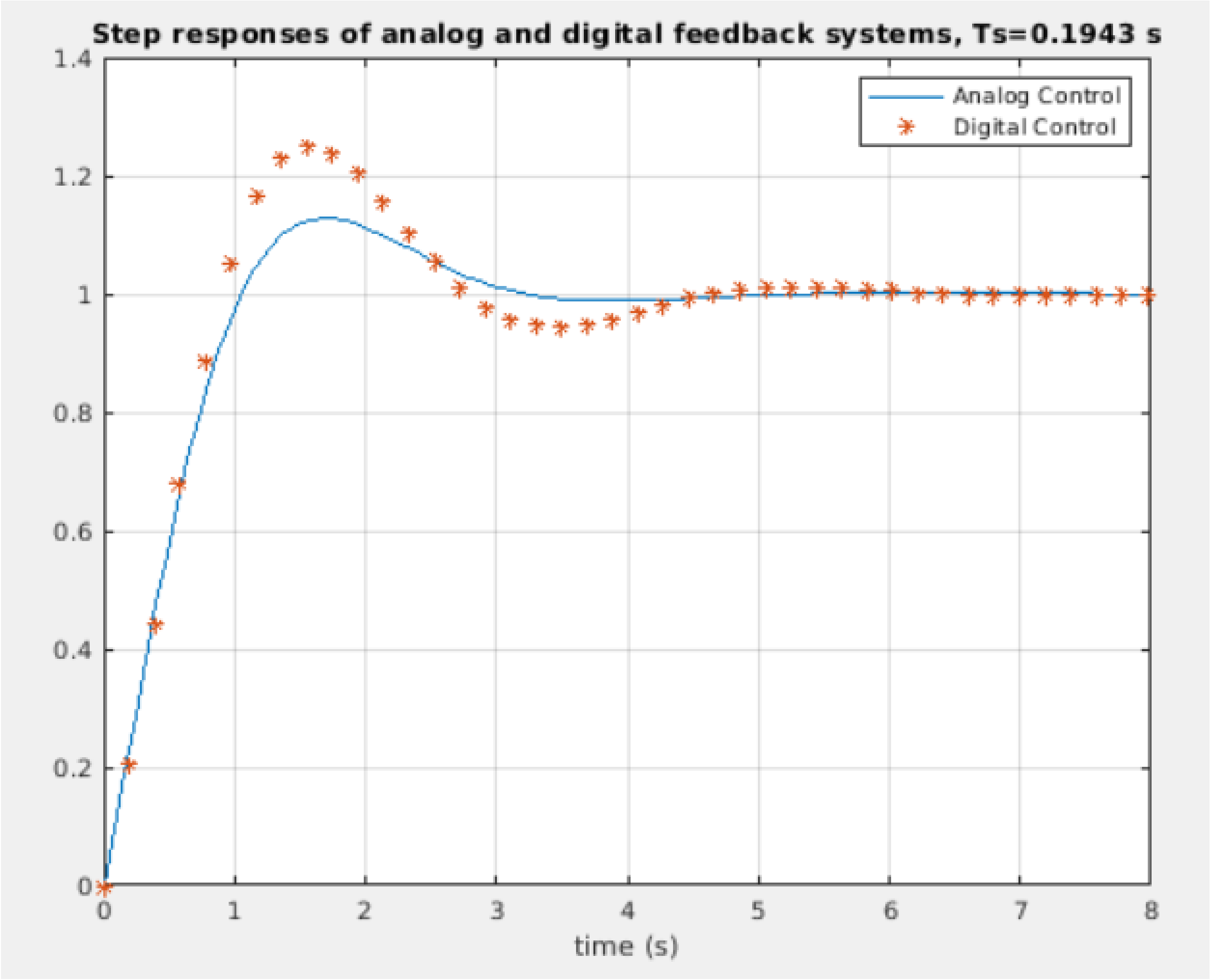
c) For $w_s = 22\, w_B$

$M_p \approx 19\%$

$T_s = 0.1148$ sec



For $\omega_s = 13\omega_B$
$M_p > 25\%$
$T_s = 0.1943$ sec



So for $\omega_s > 13\omega_B$
we get $M_p \leq 25\%$

? Digital control has worse performance?

? will there be coding questions in exam?

- [x] depends on sampling time so follow the 3σ b.w rule
- [x] only pseudocodes

(d) Repeat (b) and (c) for Euler's backward rectangular rule. Compare the results of the forward and backward rules.

$$\dot{x}[k] = \frac{x[k]-x[k-1]}{T}$$

$$\dot{u}[k] = \frac{u[k]-u[k-1]}{T}$$

$$\dot{u} = K\dot{e} + aKe \quad , \quad K = 1.45 \quad , \quad a = 2.79$$

$$\dot{e}[k] = \frac{e[k]-e[k-1]}{T}$$

$$\frac{1}{T}\left(u[k]-u[k-1]\right) = \frac{K}{T}\left(e[k]-e[k-1]\right) + aKe[k]$$

$$u[k]-u[k-1] = K\left(e[k]-e[k-1]\right) + aKTe[k]$$

$$u[k] = u[k-1] + e[k](K+aKT) - e[k-1]K$$

$$U[z] = z^{-1}U[z] + E[z]K(1+aT) - z^{-1}KE[z]$$

$$U[z](1-z^{-1}) = E[z]\left(K(1+aT) - z^{-1}K\right)$$

$$U(z)(1 - z^{-1}) = E[z]\left(K(1+aT) - z^{-1}K\right)$$

$$K(z) = \frac{U(z)}{E[z]} = \frac{K\left((1+aT) - z^{-1}\right)}{(1 - z^{-1})} = \frac{1.45\left((1+2.79T) - z^{-1}\right)}{(1 - z^{-1})}$$

$$= \frac{K\left(z(1+aT) - 1\right)}{z-1} = \frac{1.45\left(z(1+2.79T) - 1\right)}{z-1}$$

$k = 1.45$, $a = 2.79$,

Forward Euler:

$$K(z) = \frac{V(z)}{E(z)} = \frac{(zk + akT - k)}{(z-1)} = \frac{K(z + aT - 1)}{z-1}$$

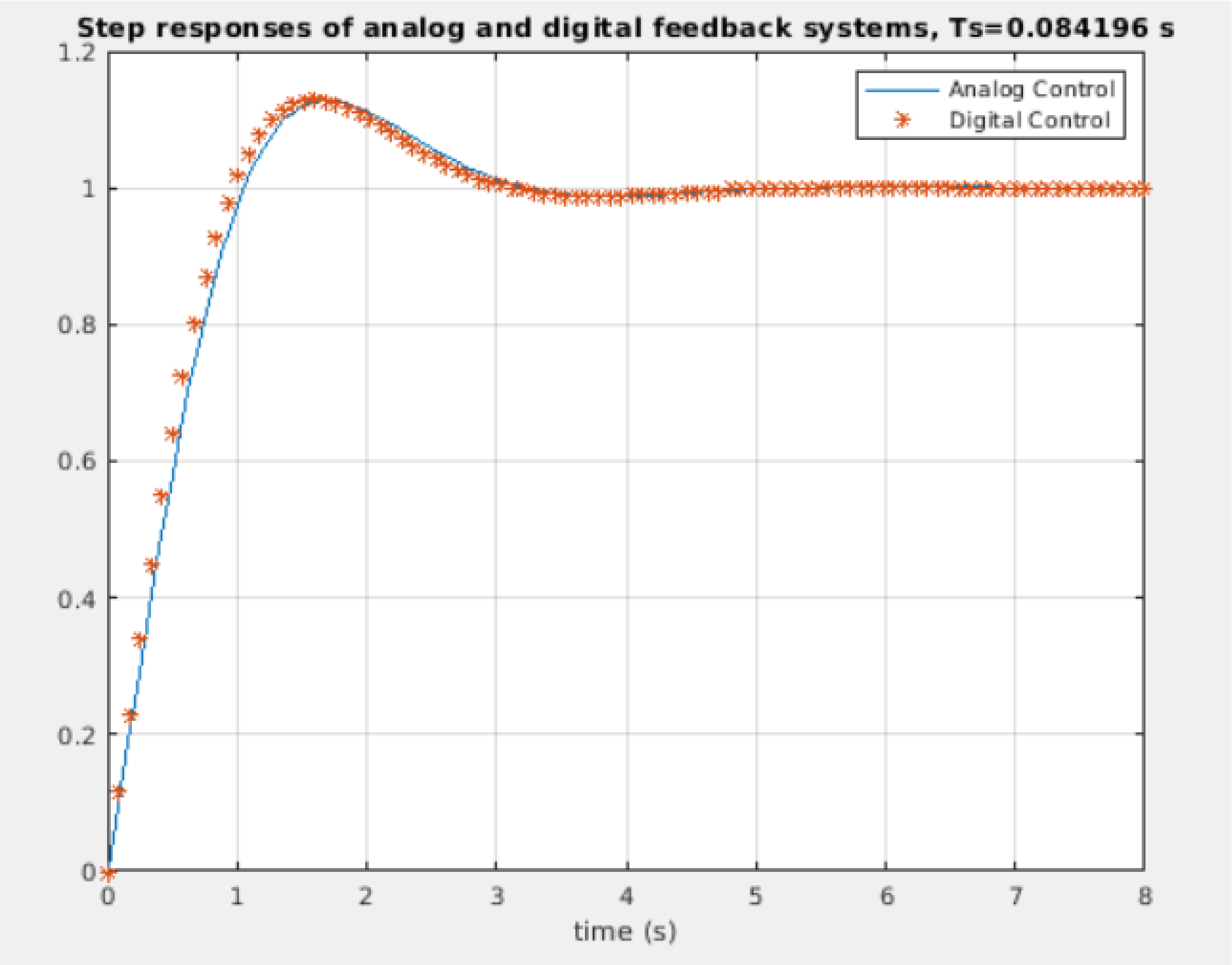
Backward Euler:

$$K(z) = \frac{V(z)}{E[z]} = \frac{K((1+aT) - z^{-1})}{(1 - z^{-1})} = \frac{K(z(1+aT) - 1)}{z-1}$$

For $\omega_s = 30\omega_B$

$M_p \approx 13\%$

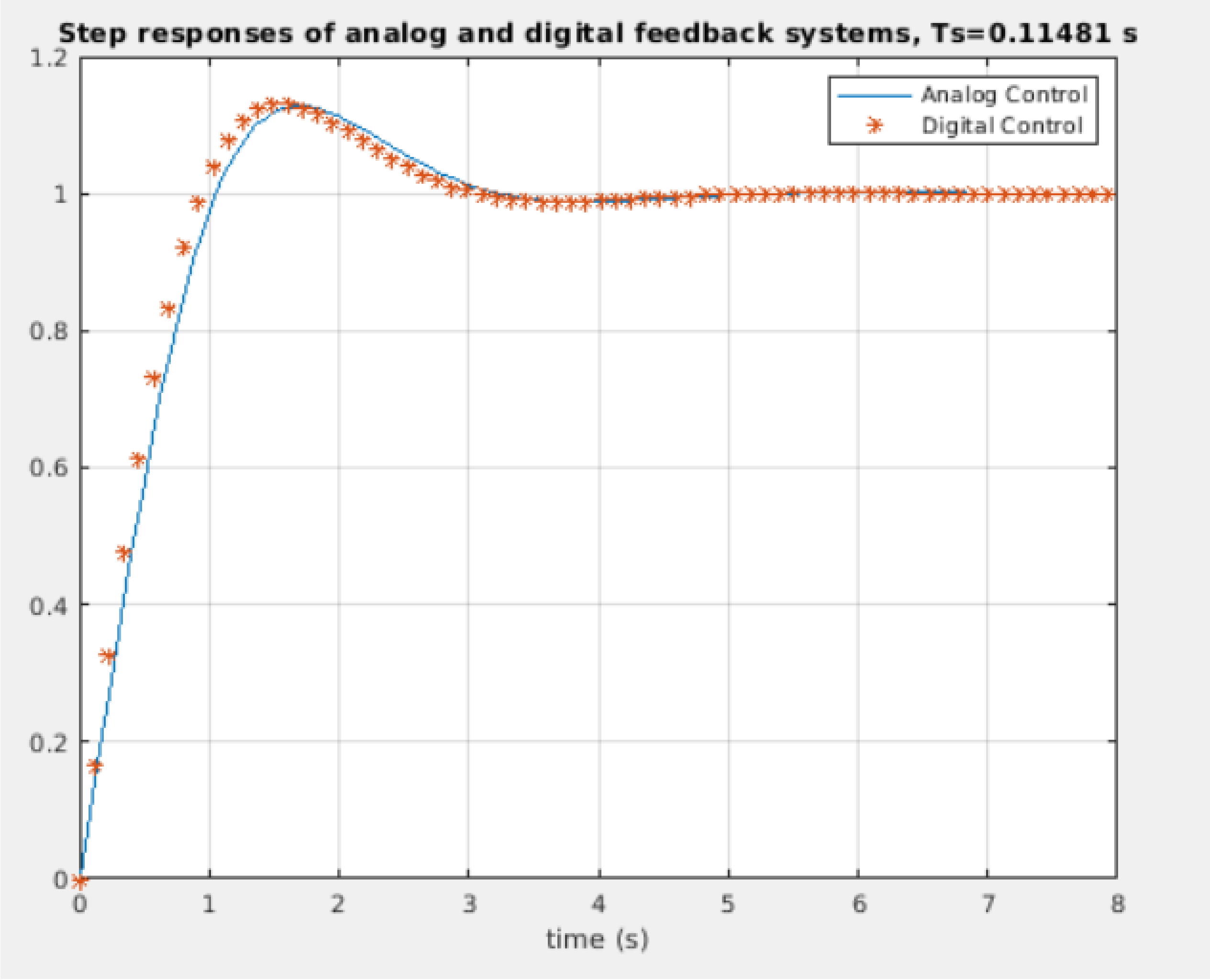
$T_s = 0.084196$



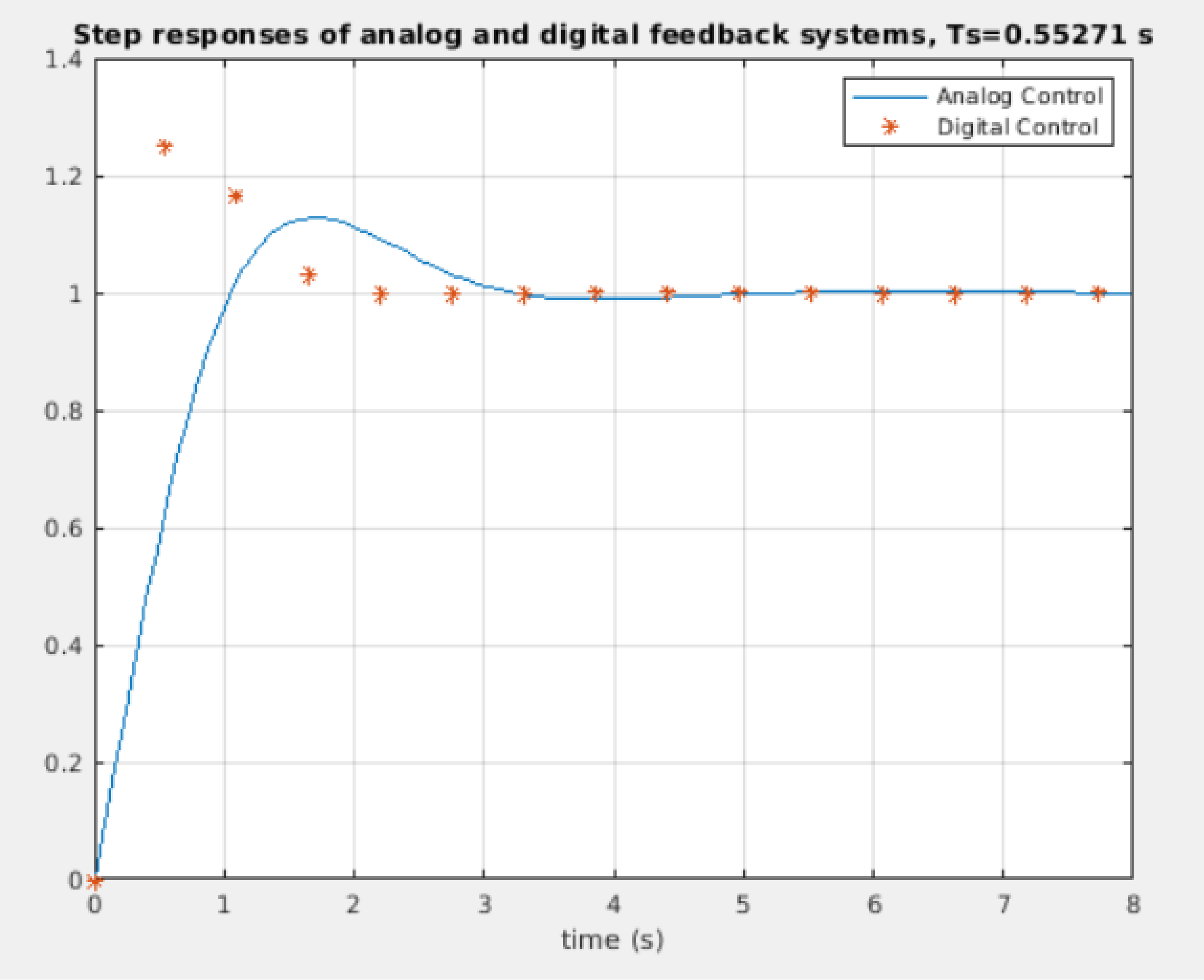
For $\omega_s = 22\omega_B$

$M_p \approx 13\%$

$T_s = 0.11481\,sec$



For $w_s = 4.57 w_B$
$M_p > 25\%$
$T_s = 0.552\,sec$



we notice that Euler backward gives lower overshoot for same sample rate and can function at lower sample rate
(less constraint on hardware)

2. The transfer function of a dc motor is

$$G(s) = \frac{0.8}{s+1}$$

The input is armature voltage and the output is angular velocity.

(a) Design a continuous PI controller

$$K(s) = k\,\frac{s+a}{s}$$

so that the closed–loop system (Fig. 3) has a rise time $t_r < 1$ sec and an overshoot $M_p < 20\%$. (choose something that covers specs up to Mp 15% atleast)

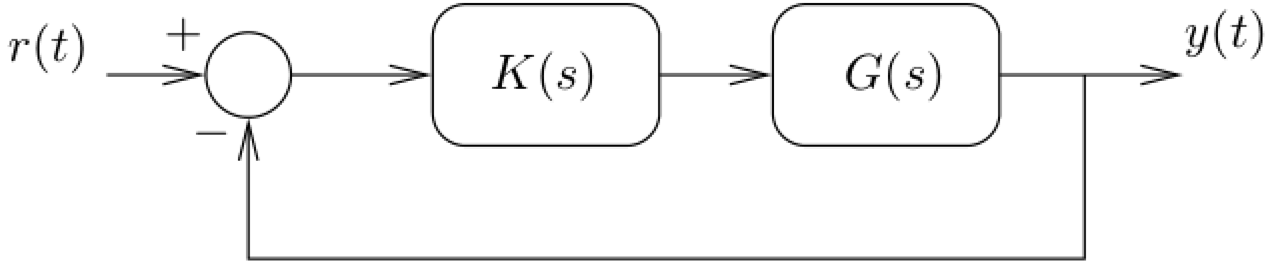


Figure 3: Problem 2.

For 2nd-order system with no zero

$$G_{CL}(s) = T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

$$\omega_{BW} \approx (-1.196\zeta + 1.85)\omega_n \quad \text{For} \quad (0.3 \leq \zeta \leq 0.8)$$

$\omega_{BW} \approx \omega_n$ if $\zeta = 0.7$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 = \%OS = \frac{y_{max} - F.V}{F.V}$$

$$t_s = \frac{4.6}{\zeta\omega_n}$$

$$t_r = \frac{1.8}{\omega_n}$$

$$\boxed{\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}}$$

$$\boxed{\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100}$$

$$G_{CL}(s) = \frac{K(s)G(s)}{1+K(s)G(s)}$$

$$G(s) = \frac{0.8}{s+1} \qquad K(s) = k\,\frac{s+a}{s}$$

$$= \frac{\frac{0.8k(s+a)}{(s+1)s}}{1+\frac{0.8k(s+a)}{(s+1)s}} = \frac{0.8k(s+a)}{(s+1)s+0.8k(s+a)} = \frac{0.8ks+0.8ka}{s^2+s+0.8ks+0.8ka}$$

$$= \frac{0.8ks+0.8ka}{s^2+s(1+0.8k)+0.8ka}$$

$t_r < 1$ $\qquad \frac{1.8}{\omega_n} < 1$ $\qquad \omega_n > 1.8$

$\omega_n = 1.8$

$$G_{CL}(s) = T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\boxed{\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}}$$

- $\zeta > 0.456$ for $\%OS < 20\%$
- we chose ζ that gives $\%OS$ not less than 15% to satisfy specs
- $\zeta = 0.5$ gives $k=1$ so good choice and gives $\%OS$ of 16%

$(1+0.8K) = 2\zeta\omega_n$, $0.8K\alpha = \omega_n^2$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

- $\zeta = 0.5$ gives $K=1$ so good choice and gives %OS of 16%

$$K = \frac{2\zeta\omega_n - 1}{0.8} = 1 \qquad \therefore \alpha = \frac{\omega_n^2}{0.8K} = \frac{(1.8)^2}{0.8 \cdot 1} = 4.05$$

$$K(s) = \frac{s+4.05}{s}$$

(b) Use Euler's forward rectangular rule to obtain an approximating discrete–time controller $K(z)$ (Fig. 4).

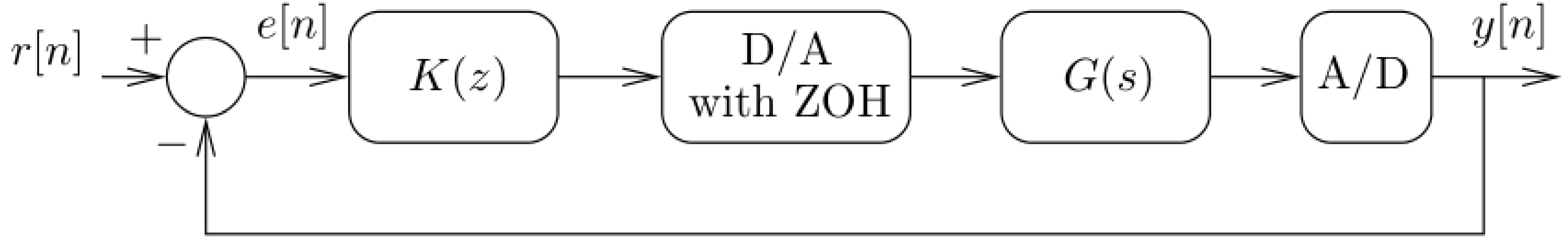


Figure 4: Problem 2.

$$K(s) = \frac{s+4{,}05}{s}$$

$$K(s) = k\,\frac{s+a}{s}$$

(b) Use Euler's forward rectangular rule to obtain an approximating discrete–time controller $K(z)$ (Fig. 4).

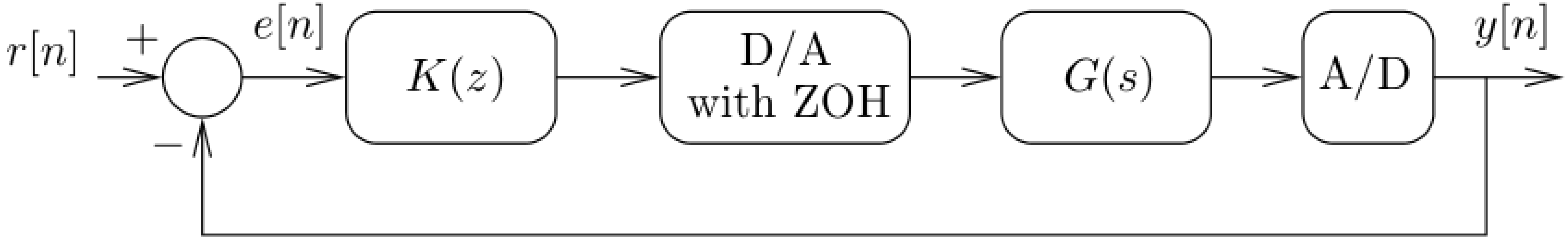


Figure 4: Problem 2.

$$\dot{x}(k) \cong \frac{x(k+1) - x(k)}{T}$$

$$K(s) = k\,\frac{s+a}{s}$$

$$\frac{U(s)}{E(s)} = k\,\frac{s+a}{s}$$

$$sU(s) = E(s)\,k\,(s+a)$$

$$\frac{U(s)}{E(s)} = k\frac{s+a}{s} \qquad sU(s) = E(s)k(s+a)$$

$$\dot{x}(k) \cong \frac{x(k+1) - x(k)}{T}$$

$$\dot{u}(k) = \dot{e}(k)k + e(k)ka$$

$a = 4.05$
$k = 1$

$$\frac{u(k+1) - u(k)}{T} = \left(\frac{e(k+1) - e(k)}{T}\right)k + e(k)ka$$

$$u(k+1) - u(k) = \left(e(k+1) - e(k)\right)k + Tka\,e(k)$$

$$u(k+1) = u(k) + e(k+1)k + e(k)k(Ta-1)$$

$$zU(z) = U(z) + zkE(z) + E(z)k(Ta-1)$$

$$U(z)(z-1) = E(z)(zk + k(Ta-1))$$

$$\frac{U(z)}{E(z)} = k(z) = \frac{k(z+Ta-1)}{z-1} = \frac{k(z+4.05T-1)}{z-1}$$

(c) Consider a range of sampling frequencies and for each sampling frequency plot the step responses of the analog and digital systems. To obtain the step responses, you can modify and use the MATLAB file `asn1.m` which is posted on the course Moodle web page. Find the slowest sampling frequency for which the overshoot does not exceed 25%. Compare the sampling frequencies with the bandwidth of the analog closed–loop system.

(d) Repeat (b) and (c) for Euler's backward rectangular rule. Compare the results of the forward and backward rules.

(b) Use Euler's forward rectangular rule to obtain an approximating discrete–time controller $K(z)$ (Fig. 4).

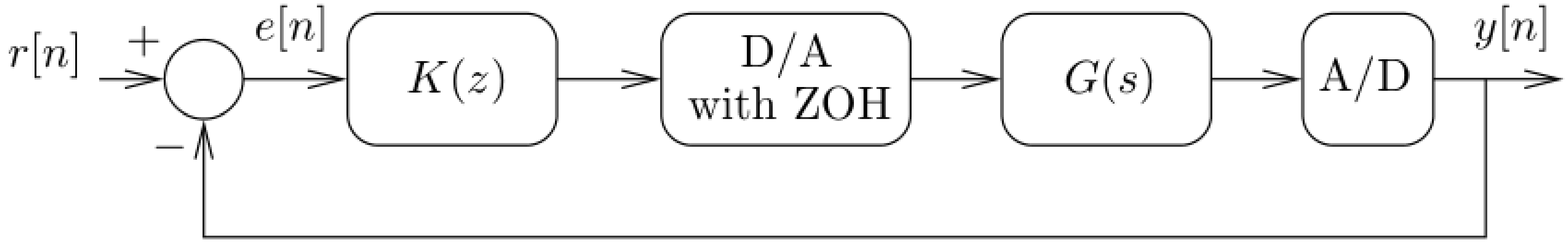


Figure 4: Problem 2.

$$K(s) = k\,\frac{s+a}{s}$$

(d) Repeat (b) and (c) for Euler's backward rectangular rule. Compare the results of the forward and backward rules.

$$\dot{x}[k] = \frac{x[k]-x[k-1]}{T}$$

$$\dot{u}(k) = \dot{e}(k)K + e(k)Ka$$

$$\frac{u(k)-u(k-1)}{T} = \frac{K}{T}\left(e(k)-e(k-1)\right) + e(k)Ka$$

$$u(k)-u(k-1) = K\left(e(k)-e(k-1)\right) + TKa\,e(k)$$

$$u(k) = u(k-1) - Ke(k-1) + e(k)K(1+Ta)$$

$$U(z) = z^{-1}U(z) - Kz^{-1}E(z) + E[z]K(1+Ta)$$

$$U(z)(1-z^{-1}) = E[z]K\left[(1+Ta) - z^{-1}\right]$$

$$\frac{U(z)}{E(z)} = K\frac{(1+Ta-z^{-1})}{(1-z^{-1})} = K\frac{(z(1+Ta)-1)}{(z-1)} = \frac{(z(1+4.05T)-1)}{(z-1)}$$