

Übungsblatt Nr. 3

Aufgabe 1: Begleitendes Dreibein

a) $s(t) = \int_0^t |\vec{v}(T)| dT = \int_0^t |\dot{\vec{r}}(T)| dT = \int_0^t |(-v_{0,r} \sin(\omega_c T), v_{0,r} \cos(\omega_c T), v_{0,z})| dT = \int_0^t \sqrt{v_{0,r}^2 + v_{0,z}^2} dT = \sqrt{v_{0,r}^2 + v_{0,z}^2} t$
Also $t(s) = \frac{s}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}$, folglich:

$$\vec{r}(s) = \left(\frac{v_{0,r}}{\omega_c} \cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,r}}{\omega_c} \sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right)$$

b)

$$\vec{T} = \frac{d\vec{r}}{ds} = \left(-\frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \right)$$

$$\begin{aligned} \vec{N} &= \frac{\frac{d\vec{T}}{ds}}{\left| \frac{d\vec{T}}{ds} \right|} \\ &= \frac{\left(-\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 0 \right)}{\sqrt{\frac{v_{0,r}^2 \omega_c^2}{(v_{0,r}^2 + v_{0,z}^2)^2}}} \\ &= \frac{\left(-\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 0 \right)}{\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2}} \\ &= \left(-\cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), -\sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 0 \right) \end{aligned}$$

$$\begin{aligned}
\vec{B} &= \vec{T} \times \vec{N} \\
&= \left(\frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \right. \\
&\quad \left. - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \right. \\
&\quad \left. \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin^2 \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) - \left[- \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos^2 \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) \right] \right) \\
&= \left(\frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos \left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 1 \right)
\end{aligned}$$

Aufgabe 2: Reibungsprobleme

a) $F_{R,max} = \mu \cdot F_N$, also $F_{E,max} = \frac{5}{8} \cdot ma \cos \alpha \approx 9627.548 \text{ m/s}^2$

b) $F_{R,max} = \mu_G \cdot F_N$, also $F_G \sin \alpha = 0.3 \cdot F_G \cos \alpha$, d.h. $\tan \alpha = 0.3 \iff \alpha = \arctan 0.3 \approx 0.29$