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Universität Konstanz
Fachbereich Mathematik und Statistik

Übungsblatt Nr. 3

Aufgabe 1: Begleitendes Dreibein

a) $s(t) = \int_0^t |\vec{v}(T)|dT = \int_0^t |\dot{\vec{r}}(T)|dT = \int_0^t |(-v_{0,r}\sin(\omega_c T), v_{0,r}\cos(\omega_c T), v_{0,z})|\, dT = \int_0^t \sqrt{v_{0,r}^2 + v_{0,z}^2}dT = \sqrt{v_{0,r}^2 + v_{0,z}^2}t$
Also $t(s) = \frac{s}{\sqrt{v_{0,r}^2+v_{0,z}^2}}$, folglich:

$$\vec{r}(s) = \left(\frac{v_{0,r}}{\omega_c}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}s\right), \frac{v_{0,r}}{\omega_c}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}}}s\right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}s\right)$$

b)

$$\vec{T} = \frac{d\vec{r}}{ds} = \left(-\frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}s\right), \frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}}}s\right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}\right)$$

$$\begin{aligned}
\vec{N} &= \frac{\frac{d\vec{T}}{ds}}{\left|\frac{d\vec{T}}{ds}\right|} \\
&= \frac{\left(-\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}}s\right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}}s\right), 0\right)}{\sqrt{\frac{v_{0,r}^2\omega_c^2}{(v_{0,r}^2+v_{0,z}^2)^2}}} \\
&= \frac{\left(-\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}}s\right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}}s\right), 0\right)}{\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}} \\
&= \left(-\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}s\right), -\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}s\right), 0\right)
\end{aligned}$$

$$
\begin{aligned}
\vec{B} &= \vec{T} \times \vec{N} \\
&= \left(\frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \right. \\
&\quad - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \\
&\quad \left. \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin^2\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) - \left[- \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos^2\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) \right] \right) \\
&= \left(\frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 1 \right)
\end{aligned}
$$