# Übungsblatt Nr. 3

**Aufgabe 1:** Begleitendes Dreibein

a) $s(t) = \int_0^t |\vec{v}(T)| dT = \int_0^t |\dot{\vec{r}}(T)| dT = \int_0^t |(-v_{0,r}\sin(\omega_c T), v_{0,r}\cos(\omega_c T), v_{0,z})| \, dT = \int_0^t \sqrt{v_{0,r}^2 + v_{0,z}^2} \, dT = \sqrt{v_{0,r}^2 + v_{0,z}^2} \, t$

Also $t(s) = \dfrac{s}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}$, folglich:

$$\vec{r}(s) = \left( \frac{v_{0,r}}{\omega_c} \cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,r}}{\omega_c} \sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right)$$

b)

$$\vec{T} = \frac{d\vec{r}}{ds} = \left( -\frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \right)$$

$$\vec{N} = \frac{\frac{dT}{ds}}{\left| \frac{dT}{ds} \right|}$$

$$= \frac{\left( -\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 0 \right)}{\sqrt{\frac{v_{0,r}^2 \omega_c^2}{(v_{0,r}^2 + v_{0,z}^2)^2}}}$$

$$= \frac{\left( -\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2} \sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 0 \right)}{\frac{v_{0,r}\omega_c}{v_{0,r}^2 + v_{0,z}^2}}$$

$$= \left( -\cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), -\sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 0 \right)$$

$$\vec{B} = \vec{T} \times \vec{N}$$

$$= \left( \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \right.$$

$$- \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right),$$

$$\left. \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin^2\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) - \left[ - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos^2\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) \right] \right)$$

$$= \left( \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left( \frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 1 \right)$$

## Aufgabe 2: Reibungsprobleme

a) $F_{R,max} = \mu \cdot F_N$, also $F_{E,max} = \frac{5}{8} \cdot ma \cos\alpha \approx 9627.548\,\text{m/s}^2$

b) $F_{R,max} = \mu_G \cdot F_N$, also $F_G \sin\alpha = 0.3 \cdot F_G \cos\alpha$, d.h. $\tan\alpha = 0.3 \iff \alpha = \arctan 0.3 \approx 0.29$
Die Höhe des Kegels über dem Silo ist also $\frac{d}{2} \sin \arctan 0.3$ und das Volumen insgesamt ist:

$$\frac{1}{3}\pi \cdot \left( \frac{d}{2} \right)^2 \cdot \frac{d}{2} \sin \arctan 0.3 + \pi \cdot \left( \frac{d}{2} \right)^2 \cdot h = \pi \cdot \left( \frac{d}{2} \right)^2 \left( \frac{d}{6} \sin \arctan 0.3 + h \right) \approx 2393.8\,\text{m}^3$$