Exercise Sheet 3

Issue Date: November 7th, 2023
Due Date: November 13th, 2023 – 10:00 a.m.
$\sum$ 10 Points

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Algorithms & data structures: lists, arrays

Exercise 1: Lists and Arrays (*2 points*)

Given an array A and a singly-linked list L of uneven length (fixed and known for the array, unknown for the list). Assume you can only use constant space. How many elements do you have to access in order to find the middle element in

a) array A? Chose one option:
   i) $\frac{length(A)+1}{2}$ elements
   ii) $length(A) + 1$ elements
   iii) direct access to element $A[\frac{length(A)+1}{2}]$
   iv) $length(A) + \frac{length(A)+1}{2}$ elements

b) list L? Chose one option:
   i) length(L) elements
   ii) $length(L) + \frac{length(L)+1}{2}$ elements
   iii) $\frac{length(L)+1}{2}$ elements
   iv) direct access to element $L[\frac{length(L)+1}{2}]$

Exercise 2: Stacks (*3 points*)

In the lecture we introduced the following algorithm for checking the correctness of sequences of parentheses:

Input: sequence of parentheses stored in an array
Data: stack S
Output: boolean for correctness

```
begin
    for i = 1 ... K.length do
        if K[i] is an opening parenthesis then
            S.push(K[i])
        if K[i] is a closing parenthesis then
            if S.isEmpty() then
                return false
            x = S.pop()
    return true
```

a) (1 point) Give two small counter-examples to prove the algorithm wrong.

b) (2 points) The algorithm can be corrected, how?

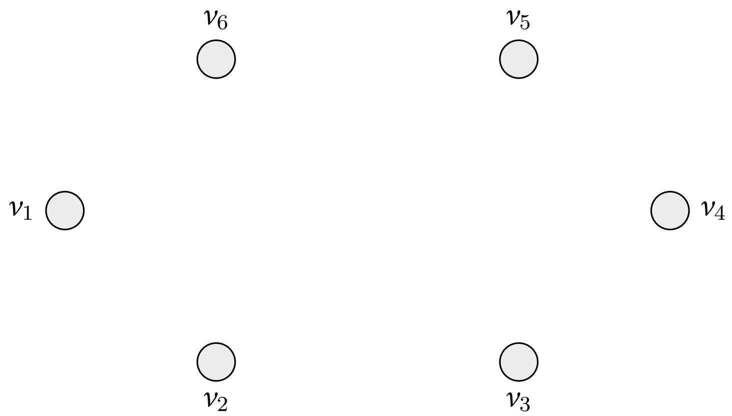
Exercise 3: Graphs (*5 points*)

Notation. The adjacency matrix of a graph $G = (V, E)$ is a *square matrix*, more precisely, an $n \times n$ matrix, where $n = |V|$. The entry $a_{v,w}$ (where the first index indicates the *row* and the second index indicates the *column* in the matrix) is one if $(v, w) \in E$ and it is zero else. For an *undirected* graph G the equality $A(G)_{i,j} = A(G)_{j,i}$ holds for all i, j.

Given the **directed** graph G_1 specified in the following adjacency matrix.

$$A(G_1) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

a) (3 points) Add the edges for G_1 to the drawing below. Please leave the vertices arranged like this, to help making it possible to check your solution quickly!



b) (2 points) Give an adjacency list representation for G_1.