University of Konstanz
Faculty of Science (Physics)

Übungsblatt Nr. 3

Jörg und Elias

Aufgabe 1: Newtonsche Reibung

a) $F_{ges} = F_g + \kappa v^2$

b) für die Endgeschwindigkeit gilt $a = 0\,\mathrm{m/s^2}$, also

$$\begin{aligned}
F_{ges} &= 0\,\frac{\mathrm{kg\,m}}{\mathrm{s}^2} \\
F_g + \kappa v_\infty^2 &= 0\,\frac{\mathrm{kg\,m}}{\mathrm{s}^2} \\
\kappa v_\infty^2 &= -F_g \\
v^2 &= -\frac{F_g}{\kappa} \\
|v_\infty| &= \sqrt{-\frac{F_g}{\kappa}}
\end{aligned}$$

c) $u := \sqrt{-\frac{\kappa}{F_g}}v, \xi := \sqrt{-\frac{m}{4\kappa a}}, \tau := u - 1, \vartheta := u + 1$, daraus folgt:
$v = \sqrt{-\frac{F_g}{\kappa}}, dv = \sqrt{-\frac{\kappa}{F_g}}du, du = d\tau, du = d\vartheta$ Es gilt:

$$\begin{aligned}
F_{ges} &= F_g + \kappa v^2 \\
m\frac{dv}{dt} &= F_g + \kappa v^2 \\
dt &= \frac{m}{F_g + \kappa v^2}dv && \mid \text{Substitution} \\
dt &= \frac{m}{F_g(1-u^2)}\sqrt{-\frac{F_g}{\kappa}}du \\
dt &= \sqrt{-\frac{m}{4\kappa a}} \cdot \frac{2}{1-u^2}du \\
dt &= \xi \cdot \frac{2}{u^2-1}du \\
\frac{1}{x}dt &= \cdot\frac{2}{u^2-1}du
\end{aligned}$$

Da

$$\begin{aligned}
\frac{1}{\tau}d\tau - \frac{1}{\vartheta}d\vartheta &= \frac{1}{u-1}du - \frac{1}{u+1}du \\
&= \left(\frac{1}{u-1} - \frac{1}{u+1}\right)du \\
&= \left(\frac{u+1-(u-1)}{(u-1)(u+1)}\right)du \\
&= \frac{2}{u^2-1}du
\end{aligned}$$

gilt:

$$\begin{aligned}
\frac{1}{\xi}dt &= \frac{2}{u^2-1}du \\
\frac{1}{\xi}dt &= \frac{1}{\tau}d\tau - \frac{1}{\vartheta}d\vartheta \\
\int_{t_0}^{t_1}\frac{1}{\xi}dt &= \int_{\tau_0}^{\tau_1}\frac{1}{\tau}d\tau - \int_{\vartheta_0}^{\vartheta_1}\frac{1}{\vartheta}d\vartheta \\
\frac{1}{\xi}t &= \ln|\tau_1| - \ln|\tau_0| - (\ln|\vartheta_1| - \ln|\vartheta_0|)
\end{aligned}$$

Durch Rücksubstituieren erhält man:

$$\begin{aligned}
\frac{1}{\xi}t &= \ln|u_1-1| - \ln|u_0-1| - (\ln|u_1+1| - \ln|u_0+1|) \\
\frac{1}{\xi}t &= \ln|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1| - \ln|\underbrace{\sqrt{-\frac{\kappa}{F_g}}v_0}_{=0} - 1| - \left(\ln|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1| - \ln|\underbrace{\sqrt{-\frac{\kappa}{F_g}}v_0}_{=0} + 1|\right) \\
\frac{1}{\xi}t &= \ln|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1| - \ln|-1| - \left(\ln|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1| - \ln|1|\right) \\
\frac{1}{\xi}t &= \ln|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1| - 0 - \left(\ln|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1| - 0\right) \\
\frac{1}{\xi}t &= \ln|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1| - \ln|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1| \\
\frac{1}{\xi}t &= \ln\frac{|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1|}{|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1|} \\
-\frac{1}{\xi}t &= \ln\frac{|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1|}{|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1|} \\
\exp(-\frac{1}{\xi}t) &= \frac{|\sqrt{-\frac{\kappa}{F_g}}v_1 + 1|}{|\sqrt{-\frac{\kappa}{F_g}}v_1 - 1|} \\
\exp(-\frac{1}{\xi}t) &= \frac{|v_1 + \sqrt{-\frac{F_g}{\kappa}}|}{|v_1 - \sqrt{-\frac{F_g}{\kappa}}|}
\end{aligned}$$

Nach der b) ist $|v_1| <= \sqrt{-\frac{F_g}{\kappa}}$ und da $a <= 0$, also $-v_1 <= \sqrt{-\frac{F_g}{\kappa}}$ und $v_1 - \sqrt{-\frac{F_g}{\kappa}} <= 0$:

$$\exp\left(-\frac{1}{\xi}t\right) = \frac{|v_1 + \sqrt{-\frac{F_g}{\kappa}}|}{|v_1 - \sqrt{-\frac{F_g}{\kappa}}|}$$

$$\exp\left(-\frac{1}{\xi}t\right) = \frac{v_1 + \sqrt{-\frac{F_g}{\kappa}}}{-v_1 + \sqrt{-\frac{F_g}{\kappa}}|}$$

$$-v_1 \exp\left(-\frac{1}{\xi}t\right) + sqrt-\frac{F_g}{\kappa}\exp\left(-\frac{1}{\xi}t\right) = v_1 + \sqrt{-\frac{F_g}{\kappa}}$$

$$v_1\left(\exp\left(-\frac{1}{\xi}t\right) + 1\right) = \sqrt{-\frac{F_g}{\kappa}}\left(\exp\left(-\frac{1}{\xi}t\right) - 1\right)$$

$$v_1 = \sqrt{-\frac{F_g}{\kappa}} \cdot \frac{\exp\left(-\frac{1}{\xi}t\right) - 1}{\exp\left(-\frac{1}{\xi}t\right) + 1}$$

$$v_1 = \sqrt{-\frac{F_g}{\kappa}}\tanh -\frac{1}{2\xi}t$$