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University of Konstanz
Faculty of Science
(Computer and Information Science)

Recursion, Devide and Conquer

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Exercise 1: Heapsort I

a)
  A. It is not a Heap, because there is a hole where the fifth node should be, but there is a sixth one.

  B. It is a Heap because it has no holes and every parent is greater than it's childs.

  C. It is not a Heap, because there is a hole where the fifth node should be, but there is a sixth one.

  D. It is not a Heap, because the third node is smaller than it's child.

b)

$$[10, 5, 0, 12, 11, 7, 9, 8, 3, 4, 6]$$

$$[10, 5, 9, 12, 11, 7, 0, 8, 3, 4, 6]$$

$$[10, 5, 9, 12, 11, 7, 6, 8, 3, 4, 0]$$

$$[10, 12, 9, 5, 11, 7, 6, 8, 3, 4, 0]$$

$$[10, 12, 9, 8, 11, 7, 6, 5, 3, 4, 0]$$

$$[12, 10, 9, 8, 11, 7, 6, 5, 3, 4, 0]$$

$$[12, 11, 9, 8, 10, 7, 6, 5, 3, 4, 0]$$

Exercise 2: $\mathcal{O}$-notation

a) to be proofen: $\exists c > 0 : \exists n_0 : \forall n > n_0 : |105n + 100| \leq c \cdot n^2$
let $c := 205, n_0 := 1$ so that:
$\forall n \geq n_0 : 100n + 105 \leq 100n + 105n \leq 205n \leq 205 \cdot n^2 = c \cdot n^2$ $\blacksquare$

b) to be proofen: $\exists c > 0 : \exists n_0 : \forall n > n_0 : |0.1n^2 - 5| \geq c \cdot n$
let $c := 1, n_0 := 20$ so that:
$\forall n \geq n_0 : 0.1n^2 - 5 \geq 2n - 5 \geq n + n - 5 \geq n + 20 - 5 \geq 1 \cdot n = c \cdot n$ $\blacksquare$

c) to be proofen: $\exists c_0, c_1 > 0 : \exists n_0 : \forall n > n_0 : c_0 \cdot n^3 \leq |6n^2 + 6n + 6| \leq c_1 \cdot n^3$
let $c_0 := 1, c_1 := 20, n_0 := 1$ so that:
$c_0 \cdot n^3 = 1 \cdot n^3 \leq n^3 + n^2 + 1 \leq 6n^3 + 6n^2 + 6 \leq 6n^3 + 6n^3 + 6n^3 \leq 20 \cdot n^3 = c_1 \cdot n^3$ $\blacksquare$

Exercise 3: Analysing a new Algorithm

a)

$$[1, 5_0, 5_1, 3, 8]$$

$$[1, 5_1, 3, 5_0, 8]$$

$$[1, 3, 5_1, 5_0, 8]$$

$$[1, 3, 5_1, 5_0, 8]$$

b) The result is an ordert Array

c) One has to change the $\geq$ sign in line 4 to $>$