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## KDI exercise sheet no 1

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For the record, I'm using . 's as decimal separator

### Exercise 1: Number conversions

a)  $1011\ 1010\ 1110\ 1001\ 0101\ 1100_2 = \text{bae95c}_{16} = 12\ 249\ 436_{10}$

b)  $1011.001_2 = 11.125_{10}$

c)  $13.25_{10} = 1101.01_2$

### Exercise 2: Arithmetics

a)  $13_{10} = 1101_2$

$17_{10} = 1\ 0001_2$

Ones' Complement:

$$\begin{aligned} 0000\ 1101_2 - 0001\ 0001_2 &= 0000\ 1101_2 + (-0001\ 0001_2) \\ &= 0000\ 1101_2 + 1110\ 1110_2 \\ &= 1111\ 1011_2 \\ &= -0000\ 0100_2 \\ &= -4_{10} \end{aligned}$$

Two's Complement:

$$\begin{aligned} 0000\ 1101_2 - 0001\ 0001_2 &= 0000\ 1101_2 + (-0001\ 0001_2) \\ &= 0000\ 1101_2 + (1110\ 1110_2 + 1_2) \\ &= 0000\ 1101_2 + (1110\ 1111_2) \\ &= 1111\ 1100_2 \\ &= -(0000\ 0011_2 + 1_2) \\ &= -0000\ 0100_2 \\ &= -4_{10} \end{aligned}$$

b)  $19_{10} = 1\ 0011_2$

Ones' Complement:

$$\begin{aligned}
 0001\,0011_2 - 0001\,0011_2 &= 0001\,0011_2 + (-0001\,0011_2) \\
 &= 0001\,0011_2 + 1110\,1100_2 \\
 &= 1111\,1111_2 \\
 &= 0_{10}
 \end{aligned}$$

Two's Complement:

$$\begin{aligned}
 0001\,0011_2 - 0001\,0011_2 &= 0001\,0011_2 + (-0001\,0011_2) \\
 &= 0001\,0011_2 + (1110\,1100_2 + 1_2) \\
 &= 0001\,0011_2 + (1110\,1101_2) \\
 &= 0000\,0000_2 \\
 &= 0_{10}
 \end{aligned}$$

c)  $14_{10} = 1110_2$

$13_{10} = 1101_2$

Ones' Complement:

$$\begin{aligned}
 0000\,1110_2 - 0000\,1101_2 &= 0000\,1110_2 + (-0000\,1101_2) \\
 &= 0000\,1110_2 + (1111\,0010_2) \\
 &= 0_2 \\
 &= 0_{10}
 \end{aligned}$$

Two's Complement:

$$\begin{aligned}
 0000\,1110_2 - 0000\,1101_2 &= 0000\,1110_2 + (-0000\,1101_2) \\
 &= 0000\,1110_2 + (1111\,0010_2 + 1_2) \\
 &= 0000\,1110_2 + (1111\,0011_2) \\
 &= 1_2 \\
 &= 1_{10}
 \end{aligned}$$

d)  $25_{10} = 1\,1001_2$

$21_{10} = 1\,0101_2$

Ones' Complement:

$$\begin{aligned}
 0001\,1001_2 - 0001\,0101_2 &= 0001\,1001_2 + (-0001\,0101_2) \\
 &= 0001\,1001_2 + (1110\,1010_2) \\
 &= 11_2 \\
 &= 3_{10}
 \end{aligned}$$

Two's Complement:

$$\begin{aligned}
 0001\,1001_2 - 0001\,0101_2 &= 0001\,1001_2 + (-0001\,0101_2) \\
 &= 0001\,1001_2 + (1110\,1010_2 + 1_2) \\
 &= 0001\,1001_2 + (1110\,1011_2) \\
 &= 100_2 \\
 &= 4_{10}
 \end{aligned}$$

### Exercise 3: Algorithms

- a) One can adapt the algorithm by replacing the 2 with a 5, so that the algorithm looks like:

#### Algorithm 0.1

let  $x_k$  be the  $k$ th digit of the 5-adic number  
 Step 1:  $k := 0$   
 Step 2:  $x_k := n \bmod 5; n := \lfloor \frac{n}{5} \rfloor; k := k + 1$   
 Step 3: If  $n > 0$  go to Step 2  
 Step 4: output  $x_{k-1} \dots x_0$

- b) One can adapt the algorithm to convert decimal places from decimal to binary, by replacing all 2s with 5s, the algorithm will look like:

#### Algorithm 0.2

let  $x_k$  be the  $k$ th digit of the 5-adic number  
 Step 1:  $k := -1$   
 Step 2:  $x_k := \lfloor n \times 5 \rfloor; n := n \times 5 - x_k; k := k + 1$   
 Step 3: if not  $n = 0$  go to Step 2  
 Step 4: output  $0.x_{-1}x_{-2} \dots x_{k+1}$

- c) If one wants to convert  $11.4_{10}$  to base 5 with the algorithms 0.1 and 0.2 one needs to add the algorithm 0.1 of  $\lfloor 11.4_{10} \rfloor$  and the algorithm 0.2 of  $11.4_{10} - \lfloor 11.4_{10} \rfloor$ .

The algorithm 0.1 of  $\lfloor 11.4 \rfloor$  does the following:

Step 1:  $k := 0$   
 Step 2:  $x_0 := 11 \bmod 5 = 1; n := \lfloor \frac{11}{5} \rfloor = 2; k := 0 + 1;$   
 Step 3: if  $2 > 0$  go to Step 2  
 Step 2:  $x_1 := 2 \bmod 5 = 2; n := \lfloor \frac{2}{5} \rfloor = 0; k := 1 + 1 = 2;$   
 Step 3: if  $0 > 0$  go to Step 2  
 Step 4: output  $x_{k-1} \dots x_0 = 21$

The algorithm 0.2 of  $11.4_{10} - \lfloor 11.4_{10} \rfloor$  does the following:

Step 1:  $k := -1$   
 Step 2:  $x_{-1} := \lfloor 0.4 \times 5 \rfloor = 2; n := 0.4 \times 5 - 2 = 0; k := k + 1$   
 Step 3: if not  $0 = 0$  go to Step 2  
 Step 4: output  $0.x_{-1}x_{-2} \dots x_{k+1} = 0.2$

That means  $11.4_{10} = 21_5 + 0.2_5 = 21.2_5$