

Exercise Sheet 7

Issue Date: December 5th, 2023

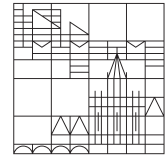
Due Date: December 11th, 2023 – 10:00 a.m.

Σ 10 Points

Konzepte der Informatik INF-11700

Winter 2023/2024

Universität
Konstanz



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Complexity, Verification & Correctness

Exercise 1: Analyzing an Algorithm (6 points)

Given the following algorithm working on an array $A[1, \dots, n]$:

```
1 f(A[1, ..., n]) begin
2   for  $i \leftarrow n, n-1, \dots, 2$  do
3     for  $j \leftarrow 2, 3, \dots, i$  do
4       if  $A[j-1] \geq A[j]$  then
5         swap value in  $A[j-1]$  and  $A[j]$ 
```

- a) (2 points) Give a suitable loop invariant $\mathcal{I}\mathcal{N}\mathcal{V}$ that can be used in Floyd's Verification Method.
- b) (2 points) Prove the loop-invariant by induction.
- c) (2 points) Prove the correctness of algorithm by using Floyd's verification method.

Exercise 2: Hoar-Logic – Analysis (2 points)

Give the weakest precondition for the following assignments and conditions:

- a) $\text{WP}[x = x - 2;](x < 0)$
- b) $\text{WP}[x = z - 5;](x > 5)$

Exercise 3: Dynamic Programming – Knapsack (2 points)

You have backpacks B_0, \dots, B_6 which can be filled up to a maximum weight j for a backpack B_j . You have items I_1, \dots, I_n with values $v(I)$ and weights $w(I)$. Four of these are:

$$I_1 : v(1) = 3, w(1) = 2$$

$$I_2 : v(2) = 2, w(2) = 1$$

$$I_3 : v(3) = 1, w(3) = 2$$

$$I_4 : v(4) = 5, w(4) = 3$$

About the *other items* you don't know anything other than the maximal values using only these items (given in the last row). Use the *Knapsack* algorithm from the lecture on Dynamic

Programming to update the table below. In there, cell $[i, j]$ records the maximum value that a backpack B_j with weight-limit j can carry when being allowed to pack items $I_i \dots I_n$.

B	0	1	2	3	4	5	6
I_1							
I_2							
I_3							
I_4							
<i>using only other items</i>	0	1	4	5	7	8	9