October 26, 2023
Wintersemester 2023/2024

Universität Konstanz
Fachbereich Mathematik und Statistik

KDI exercise sheet no 1

For the record, I'm using .'s as decimal separator

Exercise 1: Number conversions

a) $1011\,1010\,1110\,1001\,0101\,1100_2 = \text{bae95c}_{16} = 12\,249\,436_{10}$

b) $1011.001_2 = 11.125_{10}$

c) $13.25_{10} = 1101.01_2$

Exercise 2: Arithmetics

a) $13_{10} = 1101_2$
$17_{10} = 1\,0001_2$
Ones' Complement:

$$
\begin{aligned}
0000\,1101_2 - 0001\,0001_2 &= 0000\,1101_2 + (-0001\,0001_2) \\
&= 0000\,1101_2 + 1110\,1110_2 \\
&= 1111\,1011_2 \\
&= -0000\,0100_2 \\
&= -4_{10}
\end{aligned}
$$

Two's Complement:

$$
\begin{aligned}
0000\,1101_2 - 0001\,0001_2 &= 0000\,1101_2 + (-0001\,0001_2) \\
&= 0000\,1101_2 + (1110\,1110_2 + 1_2) \\
&= 0000\,1101_2 + (1110\,1111_2) \\
&= 1111\,1100_2 \\
&= -(0000\,0011_2 + 1_2) \\
&= -0000\,0100_2 \\
&= -4_{10}
\end{aligned}
$$

b) $19_{10} = 1\,0011_2$

Ones' Complement:

$$\begin{aligned} 0001\,0011_2 - 0001\,0011_2 &= 0001\,0011_2 + (-0001\,0011_2) \\ &= 0001\,0011_2 + 1110\,1100_2 \\ &= 1111\,1111_2 \\ &= 0_{10} \end{aligned}$$

Two's Complement:

$$\begin{aligned} 0001\,0011_2 - 0001\,0011_2 &= 0001\,0011_2 + (-0001\,0011_2) \\ &= 0001\,0011_2 + (1110\,1100_2 + 1_2) \\ &= 0001\,0011_2 + (1110\,1101_2) \\ &= 0000\,0000_2 \\ &= 0_{10} \end{aligned}$$

c) $14_{10} = 1110_2$
$13_{10} = 1101_2$
Ones' Complement:

$$\begin{aligned} 0000\,1110_2 - 0000\,1101_2 &= 0000\,1110_2 + (-0000\,1101_2) \\ &= 0000\,1110_2 + (1111\,0010_2) \\ &= 0_2 \\ &= 0_{10} \end{aligned}$$

Two's Complement:

$$\begin{aligned} 0000\,1110_2 - 0000\,1101_2 &= 0000\,1110_2 + (-0000\,1101_2) \\ &= 0000\,1110_2 + (1111\,0010_2 + 1_2) \\ &= 0000\,1110_2 + (1111\,0011_2) \\ &= 1_2 \\ &= 1_{10} \end{aligned}$$

d) $25_{10} = 1\,1001_2$
$21_{10} = 1\,0101_2$
Ones' Complement:

$$\begin{aligned} 0001\,1001_2 - 0001\,0101_2 &= 0001\,1001_2 + (-0001\,0101_2) \\ &= 0001\,1001_2 + (1110\,1010_2) \\ &= 11_2 \\ &= 3_{10} \end{aligned}$$

Two's Complement:

$$\begin{aligned} 0001\,1001_2 - 0001\,0101_2 &= 0001\,1001_2 + (-0001\,0101_2) \\ &= 0001\,1001_2 + (1110\,1010_2 + 1_2) \\ &= 0001\,1001_2 + (1110\,1011_2) \\ &= 100_2 \\ &= 4_{10} \end{aligned}$$

Exercise 3: Algorithms

a) One can adapt the algorithm by replacing the 2 with a 5, so that the algorithm looks like:

Algorithm 0.1

let x_k be the kth digit of the 5-adic number
Step 1: $k := 0$
Step 2: $x_k := n \bmod 5; n := \lfloor \frac{n}{5} \rfloor; k := k + 1$
Step 3: If $n > 0$ go to Step 2
Step 4: output $x_{k-1} \dots x_0$

b) One can adapt the algorithm to convert decimal places from decimal to binary, by replacing all 2s with 5s, the algorithm will look like:

Algorithm 0.2

let x_k be the kth digit of the 5-adic number
Step 1: $k := -1$
Step 2: $x_k := \lfloor n \times 5 \rfloor; n := n \times 5 - x_k; k := k - 1$
Step 3: if not $n = 0$ go to Step 2
Step 4: output $0.x_{-1}x_{-2} \dots x_{k+1}$

c) If one wants to convert 11.4_{10} to base 5 with the algorithms 0.1 and 0.2 one needs to add the algorithm 0.1 of $\lfloor 11.4_{10} \rfloor$ and the algorithm 0.2 of $11.4_{10} - \lfloor 11.4_{10} \rfloor$.
The algorithm 0.1 of $\lfloor 11.4 \rfloor$ does the following:
Step 1: $k := 0$
Step 2: $x_0 := 11 \text{ mod } 5 = 1; n := \lfloor \frac{11}{5} \rfloor = 2; k := 0 + 1;$
Step 3: if $2 > 0$ go to Step 2
Step 2: $x_1 := 2 \text{ mod } 5 = 2; n := \lfloor \frac{2}{5} \rfloor = 0; k := 1 + 1 = 2;$
Step 3: if $0 > 0$ go to Step 2
Step 4: output $x_{k-1} \dots x_0 = 21$

The algorithm 0.2 of $11.4_{10} - \lfloor 11.4_{10} \rfloor$ does the follwing:
Step 1: $k := -1$
Step 2: $x_{-1} := \lfloor 0.4 \times 5 \rfloor = 2; n := 0.4 \times 5 - 2 = 0; k := k - 1$
Step 3: if not $0 = 0$ go to Step 2
Step 4: output $0.x_{-1}x_{-2} \dots x_{k+1} = 0.2$

That means $11.4_{10} = 21_5 + 0.2_5$