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Übungsblatt Nr. 3

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Aufgabe 1: Begleitendes Dreibein

a) $s(t) = \int_0^t |\vec{v}(T)| dT = \int_0^t |\dot{\vec{r}}(T)| dT = \int_0^t |(-v_{0,r}\sin(\omega_c T), v_{0,r}\cos(\omega_c T), v_{0,z})|\, dT = \int_0^t \sqrt{v_{0,r}^2 + v_{0,z}^2} dT = \sqrt{v_{0,r}^2 + v_{0,z}^2}\, t$

Also $t(s) = \frac{s}{\sqrt{v_{0,r}^2+v_{0,z}^2}}$, folglich:

$$\vec{r}(s) = \left(\frac{v_{0,r}}{\omega_c}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s\right), \frac{v_{0,r}}{\omega_c}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}}} s\right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s\right)$$

b)

$$\vec{T} = \frac{d\vec{r}}{ds} = \left(-\frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s\right), \frac{v_{0,r}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}}} s\right), \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}}\right)$$

$$\begin{aligned}
\vec{N} &= \frac{\frac{dT}{ds}}{\left|\frac{dT}{ds}\right|} \\
&= \frac{\left(-\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}} s\right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}} s\right), 0\right)}{\sqrt{\frac{v_{0,r}^2\omega_c^2}{(v_{0,r}^2+v_{0,z}^2)^2}}} \\
&= \frac{\left(-\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}} s\right), -\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2+v_{0,z}^2}} s\right), 0\right)}{\frac{v_{0,r}\omega_c}{v_{0,r}^2+v_{0,z}^2}} \\
&= \left(-\cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s\right), -\sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s\right), 0\right)
\end{aligned}$$

$$
\begin{aligned}
\vec{B} &= \vec{T} \times \vec{N} \\
&= \left(\frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \right. \\
&\quad - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), \\
&\quad \left. \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin^2\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) - \left[- \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos^2\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right) \right] \right) \\
&= \left(\frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \sin\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), - \frac{v_{0,z}}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} \cos\left(\frac{\omega_c}{\sqrt{v_{0,r}^2 + v_{0,z}^2}} s \right), 1 \right)
\end{aligned}
$$

Aufgabe 2: Reibungsprobleme

a) $F_{R,max} = \mu \cdot F_N$, also

$$
\begin{aligned}
(F_H + F_{E,max}) &= \mu \cdot F_N \\
mg \sin\alpha + F_{E,max} &= \frac{5}{8} \cdot mg \cos\alpha \\
F_{E,max} &= \frac{5}{8} \cdot mg \cos\alpha - mg \sin\alpha \\
F_{E,max} &\approx 3.64 \cdot 10^7 \, \frac{\mathrm{kg\,m}}{\mathrm{m}^2}
\end{aligned}
$$

b) $F_{R,max} = \mu_G \cdot F_N$, also $F_G \sin\alpha = 0.3 \cdot F_G \cos\alpha$, d.h. $\tan\alpha = 0.3 \iff \alpha = \arctan 0.3 \approx 2.9 \cdot 10^{-1}$
Die Höhe des Kegels über dem Silo ist also $\frac{d}{2} \sin \arctan 0.3$ und das Volumen insgesamt ist:

$$
\frac{1}{3}\pi \cdot \left(\frac{d}{2}\right)^2 \cdot \frac{d}{2} \sin \arctan 0.3 + \pi \cdot \left(\frac{d}{2}\right)^2 \cdot h = \pi \cdot \left(\frac{d}{2}\right)^2 \left(\frac{d}{6} \sin \arctan 0.3 + h \right) \approx 2.3938 \cdot 10^3 \,\mathrm{m}^3
$$