2024-05-05
Winter Semester 2023/2024

University of Konstanz
Faculty of Science
(Physics)

# Übungsblatt 04

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## **Aufgabe 1:** Satz von Stokes

$$\begin{aligned}
\int_S \vec{\nabla} \times \vec{F} \mathrm{d}\vec{f} &= \int_S \begin{pmatrix} \frac{\partial}{\partial y} y^2 z - \frac{\partial}{\partial z} y z^2 \\ \frac{\partial}{\partial z} y - \frac{\partial}{\partial x} y^2 z \\ \frac{\partial}{\partial x} y z^2 - \frac{\partial}{\partial y} y \end{pmatrix} \mathrm{d}\vec{f} \\
&= \int_S \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \cdot \frac{1}{R} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mathrm{d}f \\
&= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} -\frac{1}{R} R \cos(\theta) R^2 \sin(\theta) \mathrm{d}\theta \mathrm{d}\varphi \\
&= R^2 \int_0^{2\pi} \left[ \frac{1}{4} \cos 2\theta \right]_0^{\frac{\pi}{2}} \mathrm{d}\varphi \\
&= R^2 \int_0^{2\pi} -\frac{1}{2} \mathrm{d}\varphi \\
&= -\frac{1}{2} [\varphi]_0^{2\pi} R^2 \\
&= -\pi R^2
\end{aligned}$$

$$\begin{aligned}
\oint_C \vec{F} \cdot \mathrm{d}\vec{r} &= \oint_C \begin{pmatrix} y \\ y z^2 \\ y^2 z \end{pmatrix} \cdot \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \mathrm{d}\varphi \\
&= -R^2 \int_0^{2\pi} \sin(\varphi) \sin(\varphi) \mathrm{d}\varphi \\
&= -R^2 \left[ \frac{1}{2} \varphi - \sin(2\varphi) \right]_0^{2\pi} \\
&= -\pi R^2
\end{aligned}$$

## Aufgabe 2: Satz von Gauß

$$
\begin{aligned}
\oint_S \vec{A}(\vec{r}) \cdot \mathrm{d}\vec{f} = & \int_0^a \int_0^b \begin{pmatrix} 2x^2 \\ y \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} dxdy \quad + \int_0^a \int_0^b \begin{pmatrix} 2x^2 \\ y \\ c^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} dxdy \\
& + \int_0^a \int_0^c \begin{pmatrix} 2x^2 \\ 0 \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} dxzy + \int_0^a \int_0^c \begin{pmatrix} 2x^2 \\ b \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} dxdz \\
& + \int_0^b \int_0^c \begin{pmatrix} 0 \\ y \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} dyzy \quad + \int_0^b \int_0^c \begin{pmatrix} 2a^2 \\ y \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} dydz \\
= & \, abc^2 + abc + 2a^2bc
\end{aligned}
$$

$$
\begin{aligned}
\int_V \operatorname{div} \vec{A}(\vec{r}) \mathrm{d}V &= \int_V 4x + 1 + 2z \mathrm{d}V \\
&= \int_0^c \int_0^b \int_0^a 4x + 1 + 2z \mathrm{d}x \mathrm{d}y \mathrm{d}z \\
&= \int_0^c \int_0^b [2x^2 + x + 2xz]_0^a \mathrm{d}y \mathrm{d}z \\
&= \int_0^c \int_0^b 2a^2 + a + 2az \mathrm{d}y \mathrm{d}z \\
&= \int_0^c [2a^2y + ay + 2azy]_0^b \\
&= \int_0^c 2a^2b + ab + 2abz \mathrm{d}z \\
&= [2a^2bz + abz + abz^2]_0^c \\
&= 2a^2bc + abc + abc^2
\end{aligned}
$$