University of Konstanz
Faculty of Science
(Mathematics and Statistics)

Übungsblatt 02

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Aufgabe 2.1:

Proof

" $\Longrightarrow$ " Sei A invertierbar, zu zeigen A^t ist invertierbar mit $(A^t)^{-1} = \left(A^{-1}\right)^t$. Dazu gilt zu zeigen, dass

$$\left(A^t\left(A^{-1}\right)^t\right)_{ij} = (I_n)_{ij} = \begin{cases} 1, & i = j \\ 0, & \text{sonst} \end{cases}$$

$$\begin{aligned}
\left(A^t\left(A^{-1}\right)^t\right)_{ij} &= \sum_{k=1}^{n} A^t_{ik}\left(A^{-1}\right)^t_{kj} \\
&= \sum_{k=1}^{n} A_{ki}A^{-1}_{jk} \\
&= \sum_{k=1}^{n} A^{-1}_{jk}A_{ki} \\
&= \left(A^{-1}A\right)_{ji} \\
&= (I_n)_{ji} \\
&= \begin{cases} 1, & i = j \\ 0, & \text{sonst} \end{cases}
\end{aligned}$$

Was zu zeigen war

" $\Longleftarrow$ " Sei A^t invertierbar, zu zeigen A ist invertierbar mit $A^{-1} = \left(\left(A^t\right)^{-1}\right)^t$. Wir betrachten dazu $(\left(A^t\right)^t)_{ij} = A^t_{ji} = A_{ij}$, also $\left(A^t\right)^t = A$. Es gilt aus der Hinrichtung, dass wenn (A^t) invertierbar ist, dann auch $(A^t)^t$ mit $A^{-1} = \left(\left(A^t\right)^t\right)^{-1} = \left(\left(A^t\right)^{-1}\right)^t$ ■

Aufgabe 2.2:

(a) $g(x_1, x_2) = T^t(f)(x_1, x_2) = (f \circ T)(x_1, x_2) = f(T(x_1, x_2)) = f(x_1, 0) = ax_1 + 0 = ax_1$

(b) $g(x_1, x_2) = T^t(f)(x_1, x_2) = (f \circ T)(x_1, x_2) = f(T(x_1, x_2)) = f(-x_2, x_1) = -ax_2 + bx_1 = bx_1 - ax_2$

(c) $g(x_1, x_2) = T^t(f)(x_1, x_2) = (f \circ T)(x_1, x_2) = f(T(x_1, x_2)) = f(x_1 - x_2, x_1 + x_2) = a(x_1 - x_2) + b(x_1 + x_2) = (a + b)x_1 - (a - b)x_2$

Aufgabe 2.3:

Zu zeigen die folgenden Axiome für alle $a, b \in K, \alpha, \beta, \gamma \in V$:

(V1) Zu zeigen $\left(\overline{\alpha} + \overline{\beta}\right) + \overline{\gamma} = \overline{\alpha} + \left(\overline{\beta} + \overline{\gamma}\right)$:

$$\begin{aligned} \left(\overline{\alpha} + \overline{\beta}\right) + \overline{\gamma} &= \overline{\alpha + \beta} + \overline{\gamma} \\ &= \overline{\alpha + \beta + \gamma} \\ &= \overline{\alpha} + \overline{\beta + \gamma} \\ &= \overline{\alpha} + \left(\overline{\beta} + \overline{\gamma}\right) \end{aligned}$$

(V2) Zu zeigen $\overline{0} + \overline{\alpha} = \overline{\alpha} + \overline{0} = \overline{\alpha}$ mit:
$\overline{0} + \overline{\alpha} = \overline{0 + \alpha} = \overline{\alpha} = \overline{\alpha + 0} = \overline{\alpha} + \overline{0}$

(V3) Zu zeigen $\overline{\alpha} + \overline{-\alpha} = \overline{-\alpha} + \overline{\alpha} = 0$:
$\overline{\alpha} + \overline{-\alpha} = \overline{\alpha - \alpha} = \overline{-\alpha + \alpha} = \overline{-\alpha} + \overline{\alpha} = \overline{-\alpha + \alpha} = \overline{0}$

(V4) Zu zeigen $\overline{\alpha} + \overline{\beta} = \overline{\beta} + \overline{\alpha}$:
$\overline{\alpha} + \overline{\beta} = \overline{\alpha + \beta} = \overline{\beta + \alpha} = \overline{\beta} + \overline{\alpha}$

und

(S1) Zu zeigen $a\left(\overline{\alpha} + \overline{\beta}\right) = a\overline{\alpha} + a\overline{\beta}$:
$a\left(\overline{\alpha} + \overline{\beta}\right) = a\overline{\alpha + \beta} = \overline{a(\alpha + \beta)} = \overline{a\alpha + a\beta} = \overline{a\alpha} + \overline{a\beta} = a\overline{\alpha} + a\overline{\beta}$

(S2) Zu zeigen $(a + b)\overline{\alpha} = a\overline{\alpha} + b\overline{\alpha}$:
$(a + b)\overline{\alpha} = \overline{(a + b)\alpha} = \overline{a\alpha + b\alpha)} = a\overline{\alpha} + a\overline{\alpha}$

(S3) Zu zeigen $(a \cdot b)\overline{\alpha} = a\,(b\overline{\alpha})$:
$(a \cdot b)\overline{\alpha} = \overline{(a \cdot b)\alpha} = \overline{a\,(b\alpha))} = a\overline{b\alpha} = a\,(b\overline{\alpha})$

(S4) Zu zeigen $1 \cdot \overline{\alpha} = \overline{\alpha}$:
$1 \cdot \overline{\alpha} = \overline{1 \cdot \alpha} = \overline{\alpha}$ ∎