

# The Merging of Water Droplets Base-on Metaball

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**Abstract**—Generating realistic world containing water is among the most challenging aspects in virtual reality. One important area in computer graphics simulations for water is the animation of droplets, which is vital for driver simulators. The method for rendering water droplets which is proposed in this paper originates from the method that is published by Yu et al [11]. For speed and practicability purposes, we use GPU-based approaches to generate realistic images of water droplets. Modified metaball technique are used to approximate the merging of water droplets and this model can also be apply to the real-time systems completely due to our optimization for the 3D metaball.

**Keywords**—water droplets; metaball; merging; modeling; real-time rendering

## I. INTRODUCTION

The ever-changing nature of liquids and its inherent complexity present an incredible challenge and have captured the attention of lots of computer researchers in recent decades. In this situation, several computer graphics professionals strive to find successfully model for displaying the realistic characters of liquids. However, liquids with its many different forms of appearance need different approaches for simulation, several different methods addressing modeling and rendering have been developed.

The great majority of researchers have mainly focus on representing a large mass of the fluids such as ocean [15], wave [16] and connected fluids, much of the work has been devoted to body motion and regularity for change of water surface, but very little concentrated on small mass of the fluids as the form of water droplet. Water droplet moving on a given surface has its own features. Its shape and motion are influenced by many indiscoverable factors including gravity, interfacial tension and air resistance and so on.

In this paper, we propose a new droplet model animation using modified meatballs technique in real-time. Yu [11] selects Wyvill's the degree six polynomial for the field function and deforms the shape of metaball owing to the gravity force, however at the cost of a high computational complexity. He also uses 12 control points to fix the outline of the water droplets which is inflexible and inaccurate. We

propose to overcome these limitations by considering the surface as a bump map and rendering using an approach on GPU that the refraction and reflection of light are in our mind. We also point out the modified meatballs to flexibly simulate water droplets merging in real-time applications.

## II. PREVIOUS WORK

Visualizing water droplets has been mainly addressed in three ways: empirical models, image-based approaches and physically based methods. The advantage and disadvantage exist in these ideas.

In empirical models, most methods provide various ways to model water droplets efficiently and realistically but remain incapable of taking some of the physical motions into considerations. The first method [10] introduced by Kaneda et al. for realistic animation of water droplets and their streams on a glass plate. In this method, a discrete model of a glass plate divided into small meshes is developed to simulate the streams from the water droplets, and there is high-speed rendering approaches taking into account reflection and refraction of light. However, it can not simulate a water droplet on a curved surface, which is very indispensability for drive simulators. Thus an extended method [9] moving on curved surfaces was proposed by Kaneda et al. In this method, affinity is the main factor to the meander of the streams and to the wetting phenomenon and two different rendering versions are pointed out, the faster one uses spheres, the more sophisticated uses metaball. In 1999, Kaneda et al proposed the method [8] once again involving extensions of previously discussed above methods. The main improvement is modeling of obstacles that influence water drop motions, like the windshield. Fournier et al. presented a model by mass-spring system with surface tension and volume conservation constraints to determine the shape of a water droplet. The result was realistic but not efficient in handling droplets merging and separating. Yu et al. [11] successfully modeled static droplet shapes merging fluently. They select metaball technique to control interaction of two droplets. Later this concept was extended by Tong et al. who supported a volume-preserving approach to model water flows using metaballs [6].

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Recently, Hajjar [1] et al. presented a real-time empirical method which pursues the animation of the droplets lying on a flat surface, they also show how to simply incorporate absorption and ink transport effects in simulation scheme. All in all, these methods do not think interaction between the fluid dynamics internal to the water drops and the surface tension at the liquid interfaces, so it is hard to make a wide range of droplets deformation and motion accurately and realistically.

In image-based approaches, the representation of these methods is the combination of bump maps technique with water droplet motion. The approaches was found by Jonsson [5] in SIGRAD2002 that used bump maps technique in order to simulate the flow of a droplet on structured surfaces. In this method, the normals of the bump map, that describes the geometry of the micro structured surface, are used in the flow computation of the droplets. It can produce a physically plausible real-time animation. However, this method is incapable of present a physically correct simulation of water droplets flow on a structured surface.

In physically based methods, researchers have performed numerous experiments to determine the liquid-solid interfacial tension and produced lots of techniques for simulating the liquid-solid interactions. Gennes [13] in 1985 represents an attempt towards a unified picture with special emphasis on certain features of “dry spreading” which can reveal the wetting of solids in Physical Description. In 2005, Wang et al. [2] presented an algorithm to solve the capillary solid coupling problem by modeling surface tensions between liquid-solid surface and solid object. The results are realism but time cost is about 5-8 days for water droplets animations.

### III. MODELING FOR WATER DROPLET

The metaball technology [14] is the development of algorithms for drawing curved surfaces directly from their mathematical definitions rather than by dividing them into large numbers of polygons. It can generate a more general solution to the imaging problem for implicit surfaces and apply to a class of ever-changing objects surfaces. The surface of water droplet created by metaball is defined by the points satisfying the following (1), where  $T_0$  is a threshold,  $q_i$  is the maximum density, and  $f_i$  is the density function of metaball.

$$f(x, y, z) = \sum_{i=0}^n q_i f_i - T_0 = 0 \quad (1)$$

There are many existing forms of metaball density functions that can be used in Equation. The following are four typical forms proposed by Blinn (2), Murakami (3), Wyvill (4), and Nishimura (5) respectively.

$$f_i(x, y, z) = \exp(-ar^2) \quad (2)$$

$$f_i(x, y, z) = \begin{cases} 1 - 3\left(\frac{r}{R_i}\right)^2, & 0 \leq r \leq \frac{R_i}{3} \\ \frac{3}{2} \left[ 1 - \left(\frac{r}{R_i}\right) \right]^2, & 0 \leq r \leq R_i \end{cases} \quad (3)$$

$$f_i(x, y, z) = \begin{cases} \left[ 1 - \left(\frac{r}{R_i}\right)^2 \right]^2, & 0 \leq r \leq R_i \\ 0, & r > R_i \end{cases} \quad (4)$$

$$f_i(x, y, z) = \begin{cases} -\frac{4}{9} \left(\frac{r}{R_i}\right)^6 + \frac{17}{9} \left(\frac{r}{R_i}\right)^2 + 1, & 0 \leq r \leq R_i \\ 0, & r > R_i \end{cases} \quad (5)$$

Where value  $r$  is the distance from point  $(x, y, z)$  to the center of metaball,  $R_i$  is the influence radius of metaball. By adjusting  $T_0$ ,  $q_i$  or  $f_i$  parameters, different effects can be achieved for the same arrangement. In order to make the approach suitable for water droplet, we explored the features of merging shape and extended the basic metaball model.

Individual water droplets are often modeled as spheres defined by Gaussian distributions and all values below an arbitrary threshold are regarded as outside the sphere. Because the surface is completely determined according to this threshold, the areas of overlapping spheres can be added to create new values which form a new curve surface. Figure 1 shows the smooth curve which results from the addition of two Gaussian curves.

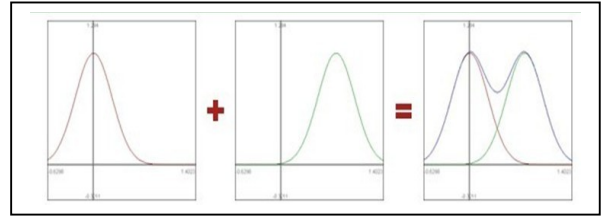


Figure 1. the addition of two Gaussian curves.

When many water droplets merge together, there is visible problem in overlapping spheres areas if simply overlapping every water droplet using above method. In this case, in order to achieve the realistic merging effect, particular blend technique must be used instead of simply overlapping. For simplicity, firstly we consider two water droplets case. The Linear Weighted Average Method is set as follows Equation (6):

$$f = t_1 q_1 \exp(-ar_1^2) + t_2 q_2 \exp(-ar_2^2) \quad (6)$$

$$t_1 = (R_1 - r_1) / \Delta t \quad (7)$$

$$t_2 = (R_2 - r_2) / \Delta t \quad (8)$$

$$t_1 + t_2 = 1 \quad (9)$$

$$\Delta t = (R_1 - r_1) + (R_2 - r_2) \quad (10)$$

Where  $f(r_1)$  and  $f(r_2)$  are the density function of metaball respectively. The parameters  $q_1$  and  $q_2$  are two water droplets maximum density respectively. The value  $r_1$

and  $r_2$  are the distance from this overlapping point to the center of metaball.

#### IV. MOTION ON THE BUMP MAP [22]

This section displays two simple efforts to test our particles shadow algorithm. In order to obtain better results from the particle system, it requires that real nature be shown in scene. We achieve this by adding complete particles attributes [7] and setting the initial attributes.

A water droplet contains a position vector  $P$ , a velocity vector  $V$ , an acceleration vector  $A$ , and a mass  $M$ . These motion properties are capable of describing the statement of every droplet in 3D scene. To be able to create an accurate physical simulation of the phenomenon of water droplets, a large amount of factors including external forces such as gravity and wind and internal forces such as surface tension and inter-facial tension would have to be taken into account.

The internal forces are complication factor. Wang et al. [2] presented water droplets simulation absolutely base-on physics model that describes surface tensions between liquid-solid surface and solid object. However, the results are proved not applying to the real-time rendering because time cost is about 5-8 days. Thus, in this paper, the affinity ( $\alpha$ ) is assumed to be experimentally constant value all over the surface for simplicity. The direction  $d_0$  of water droplets is opposite to the force direction (12).

$$F_{int} = v_d * \beta \quad (11)$$

$$d_0 = -\frac{1}{\alpha} F_{int} \quad (12)$$

The direction of movement affected by external forces can be computed as (14). In this equation,  $N$  is the unit length normal which is retrieved from the bump map. It will determine the meander of the water droplets.

$$F_{ext} = M_0 * \sin \theta. \quad (13)$$

$$d_0 = F_{ext} - (N \cdot F_{ext})N \quad (14)$$

Realistic interface details affecting the motion of water droplets need to consider macro-geometry that is too complex to efficiently render by individual polygons. Using bump mapping, we can alleviate this serious problems. These adjust the computing parameters at the pixel level in such a way that the viewer perceives small perturbations away from the base geometry.

The key idea of calculating direction is to estimate the acceleration of the water droplet, the forces and the mass are used, as shown in (15).

$$a_0 = \frac{(F_{ext} + F_{int})}{m} \cdot d_0 \quad (15)$$

By adding the acceleration to the velocity for each step, we can get the value of velocity. This method gives us the new position and the animation of the water droplets during one frame.

#### V. RENDERING

Commonly used method for rendering accurate water droplets is Ray tracing [18]. This technique can perfectly show the result of reflection and refraction effect. But this method requires a lot of calculation time. To solve this problem, lighting is performed with the Fresnel model combining reflections and refractions. The final rendering effect depends on background textures using environment mapping [17] technique and interface textures using a dynamic 2D environment map technique.

The refraction effect depends on the original view ray and on the surface normal at the point where the view ray reaches the surface. In the terms of incident ray ( $I$ ) and surface normal ( $N$ ), we can compute the reflected vector ( $R$ ) with Equation (16).

$$R = I - 2N \cdot (N \cdot I). \quad (16)$$

If light passes through a boundary between two materials of different density such as air and water, light's direction changes. The refracted angle ( $\theta_r$ ) can be computed using Snell's Law which is expressed mathematically by Equation (17).

$$N_i * \sin \theta_i = N_r * \sin \theta_r \quad (17)$$

Where  $\theta_i$  is the incident angle and an index of refraction for air ( $N_i$ ) and water ( $N_r$ ).

When light reaches an interface between air and water droplets, some light reflects off the surface at the interface, and some refracts through the surface. For this phenomenon, the Fresnel Model can be used to describe how much light is reflected and how much is refracted. The Fresnel equations are complicated. Instead of using the equations, the rendering will use the empirical approximation in Equation (18), which gives realistic results with less complication.

$$reCoeff = \max(0, \min(1, bias + scale \times (1 + I \cdot N)^{power})) \quad (18)$$

$$Fin = reCoeff * refracted + (1 - reCoeff) * refraced \quad (19)$$

#### VI. RESULT

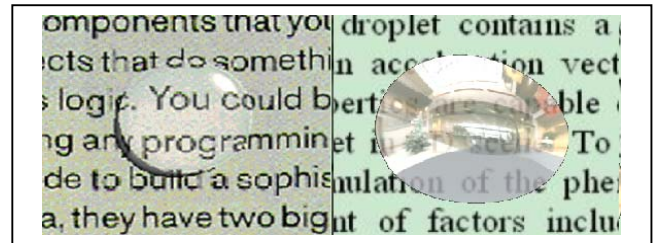


Figure 2. Yu's method of rendering a water droplet (left); new modeling for a water droplet in our method (right).

Figure 2 (left) shows a Ray-traced droplets with gravity effect on plane metaplate using [11] method. In comparison, Figure 2 (right) shows several frames of an animation

rendered by using the proposed method described in this paper. The application renders a background made of paper. On the surface, the water droplets solely influenced by gravity force are moving down on bump map. The metaball texture is used for optimization in order to avoid the complexity of 3D metaball in real-time.

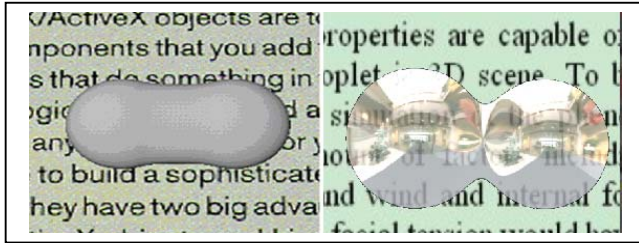


Figure 3. Yu's water droplets merging (left); new merging effect in our method (right).

Figure 3 (left) shows a scene of the two water droplets merging using [11] method. Obviously, the overlapping area is not nature. Turning attention to our method, as shown in Figure 3 (right), the merging expresses better effect.

## VII. CONCLUSION

A method applying to real-time simulation for animation of water droplets based-on metaball technique was proposed and the motion of droplets was determined by the underlying bump mapped surface in order to save rendering time. The great difference from the previous works is using modified metaball model which has more realistic merging effect. The key point that makes water droplets simulation fast in real-time is 3D metaball optimization and the introduction of normal mapping technique. For rendering water droplets, Fresnel-model is implemented taking into account refraction and refraction of light.

However, there are several improvements that can be done to this animation. We believe that a method taking surface tensions effects into consideration and coherent shapes of the droplets could greatly achieve higher quality. Even though it is the cost of a high computational complexity, GPU approaches can help us a lot. Another improvement would be to use affinity of the surface depending on the bump map, because the meandering of the water droplet is determined mainly by it, if we add it to our account, it probably gives a much more realistic and natural look.

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