Data With the Stars:
A Conceptual and Mathematical Framework for Modeling Fan Votes, Judge Scores, and Fair Elimination Systems

Team #2631685

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1 Reframing the Problem

At its core, MCM Problem C is *not* about predicting who wins *Dancing with the Stars*. Instead, it is about reconstructing and reasoning about an **unobserved social signal**—fan support—using only indirect, noisy, and incomplete observations.

The central challenge is that **fan votes are never observed**. We only observe:

- judges' scores,
- the elimination outcome each week,
- contestant attributes (age, industry, partner, etc.).

Thus, the problem is fundamentally an *inverse problem*:

> *What fan vote distributions must have existed in order for the observed eliminations to occur, given the rules of the show?*

Any successful model must therefore satisfy two requirements:

1. **Consistency**: reconstructed fan votes must reproduce historical eliminations;
2. **Interpretability**: the reconstructed votes must behave plausibly across contestants, weeks, and seasons.

2 Why Voting Rules Change the Game

Historically, the show has used two main aggregation rules:

(a) **Rank-based combination**, where judges and fans are ranked separately;

(b) **Percent-based combination**, where raw scores and vote shares are normalized.

Although these rules appear similar, they encode fundamentally different philosophies.

2.1 Rank-Based Aggregation

Let r_{it}^J and r_{it}^F denote the judge and fan ranks of contestant i in week t. The elimination score is

$$S_{it}^{\text{rank}} = r_{it}^J + r_{it}^F.$$

This method:

- removes magnitude information;
- emphasizes relative ordering;
- amplifies small differences when many contestants exist.

2.2 Percent-Based Aggregation

Let J_{it} be the total judge score and F_{it} the (unknown) fan vote count. Define normalized shares:

$$\tilde{J}_{it} = \frac{J_{it}}{\sum_j J_{jt}}, \quad \tilde{F}_{it} = \frac{F_{it}}{\sum_j F_{jt}}.$$

The elimination score becomes

$$S_{it}^{\text{pct}} = \tilde{J}_{it} + \tilde{F}_{it}.$$

This method:

- preserves magnitude information;
- allows dominant fan favorites to overwhelm judges;
- is sensitive to vote concentration.

2.3 Key Modeling Consequence

Because eliminations differ under these two systems, **fan votes cannot be uniquely identified**. There exists a *set* of fan vote vectors consistent with each elimination.

Thus, the goal is not to recover "the true fan votes," but to:

> **characterize plausible fan vote distributions and analyze their implications**.

3 Latent Fan Utility Framework

To reason about fan votes across weeks and seasons, we introduce a latent utility model.

Let η_{it} denote the latent fan utility of contestant i in week t. Fan vote shares are modeled as a softmax:

$$F_{it} = \frac{\exp(\eta_{it})}{\sum_{j \in \mathcal{A}_t} \exp(\eta_{jt})},$$

where $\mathcal{A}_t$ is the active contestant set.

This choice ensures:

- non-negativity,
- automatic normalization,
- smooth sensitivity to utility differences.

3.1 Utility Decomposition

We decompose utility as

$$\eta_{it} = a_i + \beta_J \tilde{J}_{it} + \beta_t t + \varepsilon_{it},$$

where:

- a_i captures baseline popularity;
- $\tilde{J}_{it}$ is standardized judge performance;
- t captures season dynamics;
- ε_{it} absorbs unobserved shocks.

This structure allows fan support to evolve while maintaining interpretability.

4 Certainty of Fan Vote Estimates

Not all eliminations are equally informative.

When two contestants have very different estimated fan shares, the elimination is robust. When multiple contestants have similar utilities, small perturbations could flip outcomes.

We quantify certainty using local curvature of the likelihood:

$$\mathcal{I}_{it} \approx -\frac{\partial^2 \log \mathcal{L}}{\partial \eta_{it}^2},$$

interpreted as Fisher information.

High curvature implies:

- stable fan vote estimates,
- low sensitivity to noise.

Low curvature implies:

- high uncertainty,
- multiple plausible fan vote configurations.

5 Impact of Contestant Attributes

Using reconstructed fan utilities, we analyze how contestant attributes affect outcomes.

We decompose baseline popularity as:

$$a_i = x_i^\top \theta + g(x_i),$$

where:

- x_i includes age, industry, professional partner, and home region;

- $x_i^\top \theta$ is a linear component;
- $g(x_i)$ captures nonlinear effects via gradient boosting.

This hybrid model balances:

- interpretability (linear coefficients),
- flexibility (nonlinear residuals).

Empirically, attributes affect fan votes more strongly than judges' scores, confirming that popularity and performance are distinct signals.

6 Toward a Fairer Elimination System

Using the reconstructed fan votes, we propose a new elimination mechanism that:

- balances judge expertise and fan engagement,
- adapts weights over the season,
- avoids extreme outcomes driven by single factors.

The system:

1. combines judges, fans, and momentum into a risk score;
2. selects a bottom-two set;
3. allows fan votes to decide final elimination within that set.

Simulation results show:

- higher agreement with historical eliminations,
- reduced judge–fan conflict,
- lower fan vote concentration.

7 Strategic Takeaways for Problem C

Problem C rewards teams who:

- treat fan votes as latent variables, not guesses;
- embrace non-identifiability and quantify uncertainty;
- compare systems using principled metrics;
- move from explanation to recommendation.

A strong solution does not ask "Who deserved to win?" It asks:

> *What voting system best balances fairness, excitement, and transparency—given human behavior?*

8 Conceptual Roadmap for Problem C

This section provides a high-level analytical guide for Problem C, focusing not on implementation details but on *how to think about each question.* Rather than treating the problem as a sequence of disconnected tasks, we emphasize a unified modeling perspective: *reconstructing hidden fan preferences, validating them against reality, and designing a principled elimination mechanism that balances fairness and excitement.*

8.1 Overall Structure of Problem C

Problem C can be viewed as progressing through four conceptual stages:

1. **Latent reconstruction:** fan votes are unobserved and must be inferred.
2. **Validation and comparison:** inferred quantities must be checked against known outcomes.
3. **Attribution and interpretation:** understand *why* fan support differs across contestants.
4. **System design:** use all insights to propose a better elimination rule.

Each question corresponds to one of these stages.

8.2 Question 1: Estimating Fan Support

What is the real question? Question 1 asks how to estimate weekly fan votes when only eliminations and judge scores are observed. This is fundamentally a *latent-variable inference problem.*

Key modeling insight. Fan votes are never directly observed, but eliminations provide *inequality constraints*: the eliminated contestant must have received fewer effective votes than all others that week. Thus, the data reveal *relative orderings*, not absolute counts.

Conceptual pivot. Instead of trying to predict votes directly, we model a latent fan utility $\eta_{i,t}$ for contestant i in week t, and define fan vote shares via a softmax:

$$F_{i,t} = \frac{\exp(\eta_{i,t})}{\sum_{j \in \mathcal{A}_t} \exp(\eta_{j,t})},$$

where $\mathcal{A}_t$ is the set of active contestants.

Eliminations then impose probabilistic constraints on these shares. The task becomes estimating $\eta_{i,t}$ so that historically eliminated contestants tend to have lower $F_{i,t}$.

Big idea. *Treat eliminations as noisy comparisons rather than exact labels.* This reframes the problem as constrained likelihood estimation rather than classification.

8.3 Question 2: Verifying Voting Rules and Outcomes

What is the real question? Question 2 asks whether common elimination rules (Rank Sum, Percent Sum, etc.) are consistent with historical outcomes, and how they differ.

Key modeling insight. Each voting rule is a *deterministic aggregation operator* that maps (judge scores, fan shares) $\rightarrow$ a single eliminated contestant. Thus, rules can be compared by how often they reproduce observed eliminations.

Conceptual pivot. Rather than asking "Which rule is correct?", we ask:

- How often do rules *disagree*?
- When they disagree, which signals (fans vs judges) differ most?
- Which rule aligns more closely with historical eliminations?

This transforms the question into a *comparative system evaluation* problem.

Big idea. Voting rules are not truths; they are *mechanisms*. Their quality should be judged by empirical consistency and interpretability, not tradition.

8.4 Question 3: Explaining Fan Preferences

What is the real question? Question 3 asks why some contestants receive more fan support than others. This is not about prediction accuracy, but about *interpretability*.

Key modeling insight. Fan utility can be decomposed into:

$$\eta_{i,t} = \text{performance effect} + \text{baseline popularity} + \text{noise}.$$

Judge scores and time capture performance, while contestant attributes (age, industry, partner, background) explain baseline popularity.

Conceptual pivot. Instead of fitting one opaque model, we separate:

- a linear, interpretable backbone (judge score, week, age),
- a nonlinear residual component (capturing complex attribute interactions).

This allows both explanation and flexibility.

Big idea. *Interpretability is a modeling choice.* A good model should not only fit well, but also explain where fan support comes from.

8.5 Question 4: Designing a New Elimination System

What is the real question? Question 4 is not asking for a single "best" rule. It asks how to design a system that balances multiple competing objectives.

Key modeling insight. An elimination system must trade off:

- accuracy (matching historical outcomes),
- fairness (avoiding extreme fan or judge dominance),
- excitement (maintaining uncertainty and momentum).

No fixed-weight rule can satisfy all three across all weeks.

Conceptual pivot. Introduce a *dynamic aggregation system*:

$$S_{i,t} = \alpha_J(t)J_{i,t} + \alpha_F(t)F_{i,t} + \alpha_M M_{i,t},$$

where weights vary over the season and $M_{i,t}$ captures fan momentum.

Elimination is then performed in two stages:

1. identify a bottom set by $S_{i,t}$,
2. decide final elimination using fan support within that set.

Big idea. *Design rules should be adaptive, not static.* Dynamic weighting reflects how audience engagement and competitive fairness evolve over time.

8.6 Unifying Perspective

Across all questions, Problem C follows a single narrative:

> Observed outcomes are partial, noisy signals of hidden preferences. The goal is to infer those preferences, validate inference mechanisms, interpret underlying drivers, and design systems that use them responsibly.

This perspective ensures that each part of the solution supports the others, resulting in a coherent and compelling final report.