IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2016

BEng Honours Degree in Computing Part II
MEng Honours Degrees in Computing Part II
BEng Honours Degree in Mathematics and Computer Science Part II
MEng Honours Degree in Mathematics and Computer Science Part II
MEng Honours Degree in Mathematics and Computer Science Part III
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C240=MC240

MODELS OF COMPUTATION

Wednesday 4 May 2016, 10:00
Duration: 90 minutes

Answer ALL TWO questions

Paper contains 2 questions
Calculators not required

1 Assume the big-step semantics of the *Repeat* language given by the rules:

$$\frac{}{\langle \texttt{skip}, s\rangle \Downarrow_c s} \qquad \frac{\langle E, s\rangle \Downarrow_e n}{\langle x := E, s\rangle \Downarrow_c s[x \mapsto n]}$$

$$\frac{\langle C_1, s\rangle \Downarrow_c s' \quad \langle C_2, s'\rangle \Downarrow_c s''}{\langle C_1; C_2, s\rangle \Downarrow_c s''} \qquad \frac{\langle B, s\rangle \Downarrow_b \mathsf{True} \quad \langle C_1, s\rangle \Downarrow_c s'}{\langle \texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2, s\rangle \Downarrow_c s'}$$

$$\frac{\langle C, s\rangle \Downarrow_c s' \quad \langle B, s\rangle \Downarrow_b \mathsf{True}}{\langle \texttt{repeat}\ C\ \texttt{until}\ B, s\rangle \Downarrow_c s'} \qquad \frac{\langle B, s\rangle \Downarrow_b \mathsf{False} \quad \langle C_2, s\rangle \Downarrow_c s'}{\langle \texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2, s\rangle \Downarrow_c s'}$$

$$\frac{\langle C, s\rangle \Downarrow_c s' \quad \langle B, s'\rangle \Downarrow_c \mathsf{False} \quad \langle \texttt{repeat}\ C\ \texttt{until}\ B, s'\rangle \Downarrow_c s''}{\langle \texttt{repeat}\ C\ \texttt{until}\ B, s\rangle \Downarrow_c s''}$$

where expressions and Booleans have the same behaviour as in the standard *While* language. [For simplicity, we assume that expression and Boolean evaluation does not change the state.]

a Give an English description of the behaviour of $\texttt{repeat}\ C\ \texttt{until}\ B$.

b Give the derivation tree for $\langle x := 0; \texttt{repeat}\ x := x + 2\ \texttt{until}\ x > 3, s\rangle$. [You may simplify the derivation tree for expressions and Booleans.]

c Consider the translation f from *Repeat* commands to *While* commands defined inductively by:

$$\begin{aligned}
f(\texttt{skip}) &= \texttt{skip} \\
f(x := E) &= x := E \\
f(C_1; C_2) &= f(C_1); f(C_2) \\
f(\texttt{if}\ B\ \texttt{then}\ C_1\ \texttt{else}\ C_2) &= \texttt{if}\ B\ \texttt{then}\ f(C_1)\ \texttt{else}\ f(C_2) \\
f(\texttt{repeat}\ C\ \texttt{until}\ B) &= f(C); \texttt{while}\ \neg B\ \texttt{do}\ f(C)
\end{aligned}$$

This translation has the property that, for all C, s and s', if $\langle C, s\rangle \Downarrow_c s'$ then $\langle f(C), s\rangle \Downarrow_c s'$. Using this fact, prove that the *Repeat* language is deterministic.

d i) Give the small-step semantic rules for the $\texttt{repeat}$ command.

ii) Give the small-step evaluation path for

$$\langle x := 0; \texttt{repeat}\ x := x + 2\ \texttt{until}\ x > 3, s\rangle$$

[You may simplify the steps that evaluate the expressions and Booleans.]

The four parts carry, respectively, 10%, 30%, 20%, and 40% of the marks.

2a i) Give the graphical representation of the following register machine:

$$\begin{array}{ll} L_0: & R_1^- \to L_1, L_5 \\ L_1: & R_1^+ \to L_2 \\ L_2: & R_2^- \to L_3, L_5 \\ L_3: & R_2^+ \to L_4 \\ L_4: & R_0^+ \to L_5 \\ L_5: & HALT \end{array}$$

Given input values $R_0 = 0$, $R_1 = x$ and $R_2 = y$ for $x, y \in \mathbb{N}$, describe the resulting output values for R_0, R_1 and R_2.

ii) Give the register machine of the function $f(x,y)$, which gives the answer 1 if either x or y is non-zero and the answer 0 if both x and y are zero. The register machine should have the property that the output values of R_1 and R_2 are the same as the input values. Give the graphical representation of the register machine.

b The Church Booleans, used to represent the Boolean truth values, are given by:

$$\begin{array}{rcl} \texttt{TRUE} & \stackrel{\text{def}}{=} & \lambda x.\lambda y.x \\ \texttt{FALSE} & \stackrel{\text{def}}{=} & \lambda x.\lambda y.y \end{array}$$

An IF predicate, used to represent the conditional if statement, is given by:

$$\texttt{IF} \stackrel{\text{def}}{=} \lambda b.\lambda t.\lambda f.b\,t\,f$$

i) Show that $(\texttt{IF}\,\texttt{TRUE}\,t\,f)$ reduces to t and $(\texttt{IF}\,\texttt{FALSE}\,t\,f)$ reduces to f, giving all the steps in the reductions.

ii) Consider the NOT predicate defined by

$$\texttt{NOT} \stackrel{\text{def}}{=} \lambda b.\texttt{IF}\,b\,\texttt{FALSE}\,\texttt{TRUE}$$

Using part 2bi, show that $\texttt{NOT}(\texttt{NOT}\,b)$ reduces to b when b is TRUE and when b is FALSE.

iii) Give λ-terms corresponding to the predicates AND and OR, and explain your answer.

The two parts carry, respectively, 40% and 60% of the marks.