3/23/2025

AI Coursework

Artificial Intelligence and Data Mining

Module CRN: 34123

Roll No:

Tasks

Task 4: 4.1. A Contradiction and a Tautology

⇒ 1: In this task, we need to show that $(p \Rightarrow q) \wedge (p \wedge \neg q)$ is a contradiction.

Step 1: Constructing the Truth Table

We first break the expression into its components: $p \Rightarrow q$ (Implication Law: $p \Rightarrow q \equiv \neg p \vee q$), $p \wedge \neg q$, $(p \Rightarrow q) \wedge (p \wedge \neg q)$

Step 2: Row by Row Results For Verification:

Let's see row by row of how we are getting different results.

Row 1 (p = T, q = T):

p	q	¬q	p ⇒ q	p ∧ ¬q	(p ⇒ q) ∧ (p ∧ ¬q)
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

- $\neg q = F \Rightarrow p \wedge \neg q = T \wedge F = F \Rightarrow p \Rightarrow q = T \Rightarrow T = T \Rightarrow (p \Rightarrow q) \wedge (p \wedge \neg q) = T \wedge F = F$

Row 2 (p = T, q = F): $\neg q = T$ =? $p \wedge \neg q = T \wedge T = T \Rightarrow p \Rightarrow q = T \Rightarrow F = F \Rightarrow (p \Rightarrow q) \wedge (p \wedge \neg q) = F \wedge T = F$

Row 3 (p = F, q = T): $\neg q = F \Rightarrow p \wedge \neg q = F \wedge F = F \Rightarrow p \Rightarrow q = F \Rightarrow T = T \Rightarrow (p \Rightarrow q) \wedge (p \wedge \neg q) = T \wedge F = F$

Row 4 (p = F, q = F): $\neg q = T \Rightarrow p \wedge \neg q = F \wedge T = F \Rightarrow p \Rightarrow q = F \Rightarrow F = T \Rightarrow (p \Rightarrow q) \wedge (p \wedge \neg q) = T \wedge F = F$.

o, we can conclude that as the final column contains only F values, the compound proposition $(p \Rightarrow q) \wedge (p \wedge \neg q)$ is false under all possible truth value assignments. Therefore, it is a contradiction.

2. We need to show that $(p \wedge \neg q) \Rightarrow \neg(\neg p \wedge q)$ is a tautology using a formal proof. We will transform the given expression step by step, applying one law of logic at each step:

Original expression: $(p \wedge \neg q) \Rightarrow \neg(\neg p \wedge q)$

Material implication: $(A \Rightarrow B) \equiv (\neg A \vee B = \neg(p \wedge \neg q) \vee \neg(\neg p \wedge q)$

De Morgan's law: $\neg(A \wedge B) \equiv (\neg A \vee \neg B = (\neg p \vee \neg\neg q) \vee \neg(\neg p \wedge q))$

Double negation: $\neg\neg A \equiv A$ = $(\neg p \lor q) \lor \neg(\neg p \land q)]$

De Morgan's law: $\neg(A \land B) \equiv (\neg A \lor \neg B)$ = $(\neg p \lor q) \lor (\neg\neg p \lor \neg q)$

Double negation: $\neg\neg A \equiv A$ = $(\neg p \lor q) \lor (p \lor \neg q)$

Associativity of disjunction: $(A \lor B) \lor C \equiv A \lor (B \lor C)$ = $\neg p \lor q \lor p \lor \neg q$

Commutativity and associativity of disjunction: $(\neg p \lor p) \lor (q \lor \neg q)$

Law of excluded middle: $p \lor \neg p \equiv T$ = $T \lor (q \lor \neg q)$

Law of excluded middle: $q \lor \neg q \equiv T$ = $T \lor T$

Idempotent law of disjunction:

Now finally, apply the Idempotent law of disjunction. $T \lor T \equiv T = T$

Since the expression evaluates to true in all cases, it is a tautology.

4.2 Solar-powered Smart-car

Solution: We are given the following axioms (premises) in propositional logic:

1. If a misalignment of the solar panels implies a shortage of power, then there will be a delay to the journey. $(M \Rightarrow S) \Rightarrow D$
2. A misalignment of the solar panels prevents the electronics in the car from functioning and delays the journey. $M \Rightarrow (\neg E \land D)$
3. If the car battery is flat (fully drained) or the journey is delayed, then the car will suffer a power shortage. $(B \lor D) \Rightarrow S$

We need to prove the conclusion, D =====> (There is a delay to the journey?????)

Step-by-Step Proof

Let's define our variables:

M: There is a misalignment of the solar panels , S: The car suffers a power shortage, D: There is a delay to the journey, E: The electronics in the car are functioning,B: The car battery is flat (fully drained)

Step 1: Extract from Premise (2) :*From the second premise: $M \Rightarrow (\neg E \land D)$ & Using the Implication Expansion Rule: $M \Rightarrow \neg E$,$M \Rightarrow D$.(Since conjunction elimination allows us to split $M \Rightarrow (\neg E \land D)$) into two separate implications.)*

Step 2: Conclusion from Step 1 *Since we have: $M \Rightarrow D$ .This means that if the solar panels are misaligned, then there is definitely a delay to the journey.*

Step 3: Apply Premise (3) :The third premise states: $(B \lor D) \Rightarrow S$ & Using the Contrapositive Rule: $\neg S \Rightarrow \neg(B \lor D)$, By De Morgan's Law: $\neg S \Rightarrow (\neg B \land \neg D)$, From this, we extract: $\neg S \Rightarrow \neg D$,which means: $D \Rightarrow S$

Step 4: Apply Premise (1): Premise (1) states: $(M \Rightarrow S) \Rightarrow D$, Since we derived $D \Rightarrow S$ from Step 3, we substitute this into Premise (1): $(M \Rightarrow S) \Rightarrow D$, Since we already established that $M \Rightarrow D$ from Step 2, it follows that: D which confirms the conclusion.So,We have successfully deduced that, D (There is a delay to the journey.)

4.3 Binary Logical Connectives

Binary logical connectives are fundamental operations in logic that combine two boolean values (true or false) to produce a single boolean result. These connectives play a crucial role in mathematical logic, computer science, and digital circuit design.

Commonly Used Binary Logical Connectives

The most commonly used binary logical connectives include:

AND (∧): True only if both operands are true.

OR (∨): True if at least one operand is true.

XOR (⊕):True if exactly one operand is true.

IMPLICATION (⇒): False only if the first operand is true and the second is false.

BICONDITIONAL (⇔): True if both operands are the same.

NAND (Not AND): True unless both operands are true.

NOR (Not OR): True only if both operands are false.

How Many Binary Logical Connectives Exist?

Since each binary connective takes two boolean inputs (A and B) and outputs a single boolean value, there are 2^4 = 16 possible binary logical connectives. These include the trivial ones, such as:Always True (T), Always False (F), Identity functions (A or B alone), Negation of A or B

The 16 Binary Logical Connectives

A set of logical connectives determines truth values through different conditions. Contradiction (⊥) functions as an operation that yields false value while tautology (⊤) produces true value. The conjunction (∧) results in truth only when both values maintain their truth status and disjunction (∨) becomes verified by a single true value. The logical forms ⊕ produce True outcomes only during value dissimilarity whereas the logic form ↔ yields True results based on value equivalence. Material implication (→) as well as its converse form (←) produce falsehood when certain conditions apply. Assessment theory requires the fundamental digital logic components NAND (↑) and NOR (↓). The value negation operator (¬p, ¬q) performs inversion and the projection operation returns source inputs directly. Logic and computing use these basic connectives in their operations.

Are Other Connectives Useful? Yes, several of the 16 possible connectives are useful in different contexts:*The NAND (↑) and NOR (↓) gates work as essential digital circuit elements and computer architecture elements because they achieve functional completeness which allows building any logical function. The prohibition or verification of relationships between conditions is possible through the concepts of Material nonimplication (p ⊄ q) and Converse nonimplication (p ⊅ q). User data validation and encryption as well as parity checking and binary addition processes depend heavily on the cryptographic function of Exclusive OR (⊕).*

Why Are Some Connectives Not Very Useful? NAND (↑) & NOR (↓): Essential in digital circuits and computer architecture due to their functional completeness. The two trivial connectives ⊤ and ⊥ lack significance because they yield output continuously. Some connectives demonstrate redundancy since they can be substituted with primary logical operations (for example biconditional (↔) is equivalent to ((p→q) ∧ (q→p)). The human understanding of logic finds better alignment with the natural language through AND & OR connectives than the more complex NAND & NOR. Infinitesimal utility exists in unary operators such as ¬p and ¬q and projections since these operators disregard half of the input variables. The digital system utilizes six fundamental logical connectives (AND, OR, XOR, NAND, NOR, IMPLICATION) from a total inventory of sixteen possible connective components. The analysis deals with non-pestering cases using Material and Converse Nonimplication methods. The exclusive OR operation (⊕) serves as a vital element for checking parity and binary arithmetic and encryption methods.

Reflections on Propositional Logic Assessment:

My understanding of formal reasoning as well as problem-solving and logical transformations increased through propositional logic analysis. Truth tables together with transformation proofs created a verification system while giving insight into logical reasoning. The method of logical representation taught me to translate realistic situations into organized expressions that made me better at systematic deductive application. The study of binary connectives provided me with valuable insights about their functional completeness features. The learning process helped me develop abilities in systematic problem-solving as well as logical translation skills together with critical analysis competencies and structured communication methods. Propositional logic serves computational science through NAND and NOR gates and it supports applications in artificial intelligence and mathematical collection as well as algorithm development. When refining proofs and supplying logical step-by-step logic we encountered precision needs in addition to precise reasoning requirements. My future goals include developing improved proof-writing skills with an emphasis on how logical rules affect Artificial Intelligence development and formal verification systems. My number-based evaluation has enhanced my abilities in organized problem-solving while improving my detective-style logical analysis which will support my academic and professional career.