Interim Presentation: Analysis and Conclusions - Battle of Bunker Hill

Group 36:
Abdullah Alasfar
Sudhahar Jayapalan
Alanna Mozzetti
Jason Hironimus
Zubair Lakhia

Fig. 1. Chandler, Winthrop. *The Battle of Bunker Hill*. about 1776–77. *Museum of Fine Arts, Boston*, https://collections.mfa.org/objects/34496/the-battle-of-bunker-hill.

Description of the Battle

Outcome: The Battle of Bunker Hill ended in a British victory, but the British suffered over 1,000 casualties, more than twice the American losses (History.com; National Park Service).

Significance: The colonial forces demonstrated resilience, shaking British confidence in achieving a quick victory (History.com).

Debate: Historians debate whether British tactics of advancing in tight formations across open terrain unnecessarily increased their casualties, or if the high losses were primarily due to the strategic American defensive fortification and optimal shooting range, which exposed British soldiers to sustained colonial fire (National Park Service).

Research Design & Analysis

Research Question: How does colonial engagement distance and British formational tactics impact the casualties during the Battle of Bunker Hill?

Hypothesis: The hypothesis posits that British reliance on close-order formations led to higher British casualties and lower Colonist casualties. Additionally, Colonial short engagement distances led to higher British casualties and higher Colonial casualties.

Variables:

- Independent Variables:
  - Colonial engagement distance
  - British formational tactics
    i. the dimensions of (rows/columns) British troop formations
- Dependent Variable: British and American casualties

History and Context: Battle of Bunker Hill

- One of the first battles of the American Revolutionary War

- It took place in the Charlestown peninsula of Boston between British troops and the American Continental Army (Battle of Bunker Hill [British Battles])

- British won the battle
  - The American forces were very effective in defending and making offensive move
  - Despite of loss, the battle gave a significant morale boost to the American Army

- Timeline:
  - June 17th 1775, the battle lasted for around 2 hours (Battle of Bunker Hill [British Battles])

- Technology Used:
  - Bayonets, Muskets, Cannons, and Mortars (Battle of Bunker Hill [British Battles])

Fig. 2. Mayer, Frank. *The Continentals*. 1875. *Smithsonian American Art Museum*, https://americanart.si.edu/artwork/continentals-16617.

Agents (Turtles)

- British
- Colonists

British and Colonists

- Soldiers
- Colonists: 1,500–4,000 (Frothingham 190)
- British: 2,000–4,000 (Frothingham 191)
- Non Constant Parameters:
  - Movement Speed
  - Firing Speed
  - Ammunition Count
  - Can Shoot
  - Hit Rate
  - Closest Soldier
  - Soldier Distance
  - Formation Number
  - Formation Row
  - Retreat

Fig. 3. Trumbull, John. *The Death of General Warren at the Battle of Bunker's Hill, 17 June, 1775*. After 1815–Before 1831. *Museum of Fine Arts, Boston*, https://collections.mfa.org/objects/34260/the-death-of-general-warren-at-the-battle-of-bunkers-hill.

Movement Speed

"$Z_i = x_i + \boldsymbol{\alpha}\, y_i$" (Scarf 722)

Scarf tests and adds on to Naismith's rule, which is "based on trigonometry and simple assumptions about rate of ascent" (Scarf 719). Scarf proves that regardless of speed, there is a equivalence ratio of 1:8 for men (Scarf 719). Flat movement speed rate is 1, historically slow speed (Armies March).*

Firing Speed

Colonist: <= 4 times a minute (Hopkins)

British: <= 3.75 a minute (Ketchum 137)

Adjusted by depth of formation and tick ratio*.

Ammunition Count

Colonist: <= 24 (Hopkins)

- Using sensitivity 2 + (random 0 - 2)

British: 60 (Harrington)

It will decrement with each shot fired.

Fig. 4. Avery, Amos. Powder horn. 1775. *Museum of Fine Arts, Boston*, https://collections.mfa.org/objects/45025/powder-horn.

*Tick-Ratio: 5 seconds per tick.

Can Shoot

Fig. 5. Warren, Henry. *Red Coat Shooting a Colonial*. After 1825–50. *Museum of Fine Arts, Boston*, https://collections.mfa.org/objects/473295/red-coat-shooting-a-colonial.

A boolean on if the soldier can shoot. It is determined by three factors.

1. Formation:
    - In formations with multiple rows, only one row fires at a time.
2. Distance to shoot:
    - Colonists: At one point Colonel Prescott "commanded a cessation till the enemy advanced within thirty yards" (Frothingham 396). 30 yards = 90 feet.
    - British: Max effective distance of their weaponry– 100 yards (Reid 59). 100 yards = 300 feet.
3. Visibility:
    - Test intersection of coordinates (SW, NW, SE, NE). If two parts of the hill are in-between the enemy soldier is not visible.

Can Shoot is true if the soldier is in the firing row and the target is both in range and visible; otherwise it is false.

Hit Rate

Brown Besses and Charlesvilles, as flintlock muskets had a "maximum effective range… [of] 100yd" (Reid 59).

- For uphill or downhill shots, we will use TBR. "With the angle measured, you can reference the corresponding cosine, multiply it by the distance in yards, and use the resulting number in yards as your actual shoot-to range" (Benedikt).
- "At 75m (82yd), 60 per cent hits were recorded; at 150m (164 yd), 40 per cent hit the target; at 225m (246 yd) the hit rate fell to only 25 per cent; and at 300m (328yd), 20 per cent. At even closer ranges the performance improved dramatically….it was actually found difficult to miss at 25yd" (Reid 59).
- Yards converted to feet to match environment.
- Redoubt and fence both reduced accuracy as well (sensitivity analysis and percentage of covered person)
    - Rail Fence 80%
    - Redoubt 95%
    - Hill Cover 85%

Fig. 6. *Call to arms - American Revolution*. 1909. *Museum of Fine Arts, Boston*, https://collections.mfa.org/objects/583236/call-to-arms--american-revolution.

Closest Soldier

This is determined using NetLogo's distance and min-one-of.

Closest Distance

This is determined using NetLogo's distance and the closestSoldier. Multiplied by 15ft (the distance of each patch).

Formation Number

The number of the formation that the soldier is in.

Formation Row

The number of rows in this soldier's formation.

Retreat

This is a boolean on if the soldier is retreating.

Fig. 7. *Battle of Bunker Hill, June 17, 1775*. 1909. *Museum of Fine Arts, Boston*, https://collections.mfa.org/objects/583233/battle-of-bunker-hill-june-17-1775.

Environment

- Battlefield size is approximately 4800 feet x 2100 feet
- Each patch is 15 x 15 feet
- There is roughly 44800 total patches
- Each patch is uniform in terms of ground type as battle was open field
- Each patch is characterized by dry, traversable grass

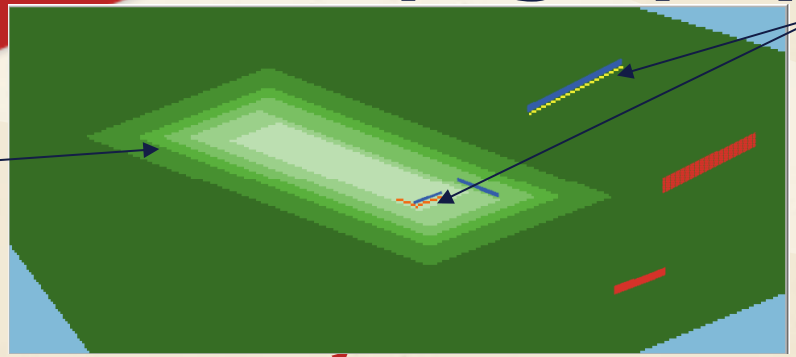
Fig. 8. Fawkes, John. *Battle of Bunker Hill*. British Battles, https://www.britishbattles.com/war-of-the-revolution-1775-to-1783/battle-of-bunker-hill/. Accessed 3 Oct. 2024.

Topography

- Patches will adjust for the elevation of Breed's Hill
- Other patches will be flat
- American fortifications will be present on top of Breed's Hill and Eastern flank:
  - 150 feet x 150 feet x 6 feet tall redoubt western and southern portion of Breed's Hill
  - 200 yard rail fence East side (Frothingham 135)

Fig.9. Frye, Charlies E. *Array of American Forces on the Field at the Battle of Breeds Hill*. 13 Aug. 2011. *Wikipedia*. Accessed 15 Oct. 2024.

Topography Continued



- Changes in green represent changes in elevation, as seen in our NetLogo model
- Each level up(color change) represents 15 ft. in elevation

NetLogo Model

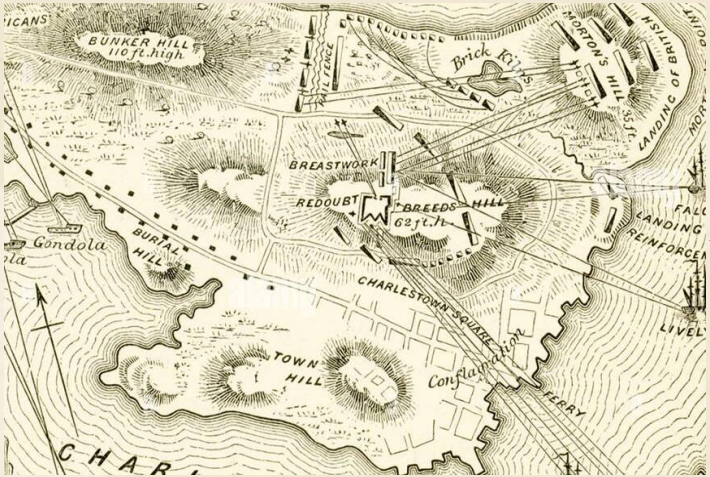
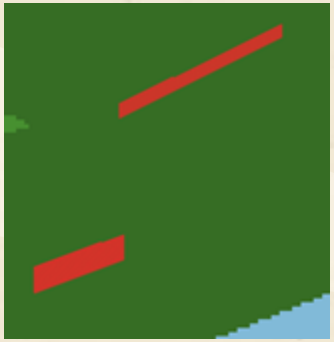


Fig. 10. *Plan of the Battle of Bunker Hill*. 23 Feb. 2020, www.alamy.com/plan-of-battle-of-bunker-hill-on-june-17-1775-early-in-the-revolutionary-war-1775-83-the-british-defeated-the-americans-at-the-battle-of-bunker-hill-in-massachusetts-despite-their-loss-the-inexperienced-colonial-forces-inflicted-significant-casualties-against-the-enemy-and-the-battle-provided-them-with-an-important-confidence-boost-during-the-siege-of-boston-april-1775-march-1776-although-commonly-referred-to-as-the-battle-of-bunker-hill-most-of-the-fighting-occurred-on-nearby-breeds-hill-image383196047.html?imageid=A60C9E56-06B0-4617-9379-BCDC38C3F1E8&p=32409&pn=1&searchId=a9cd353e0fce8bfc032272c6c4ed729a&searchtype=0. Accessed 15 Nov. 2024.

Historical Information – British Formations

- While in formation, soldiers could march in two main styles:
  - Open Order: Six paces between the ranks, allowing for greater maneuverability
  - Closed Order: Ranks only one pace apart, creating a compact and unified line
- Because of the tighter formations on the Charlestown Peninsula, the British were forced to use something resembling a closed order formation
- These were replicated in our NetLogo models
- For the Battle of Bunker Hill, the first wave that the British sent up on the right was a flank, but the subsequent waves(2nd and 3rd waves) were just a diversion



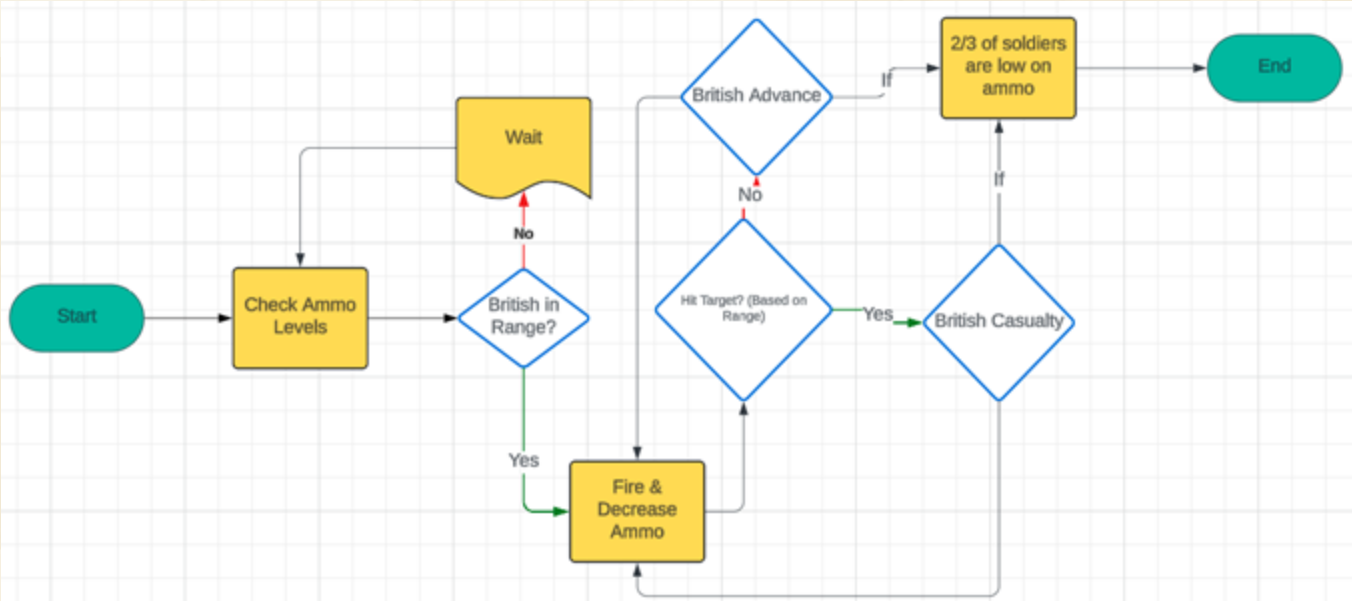
Line formation:

- 160 columns

NetLogo British Soldier Formation

Fig. 11. Conflicting British Strategies Executing the American Revolution. American Battlefield Trust, www.battlefields.org/learn/articles/conflicting-british-strategies-executing-american-revolution

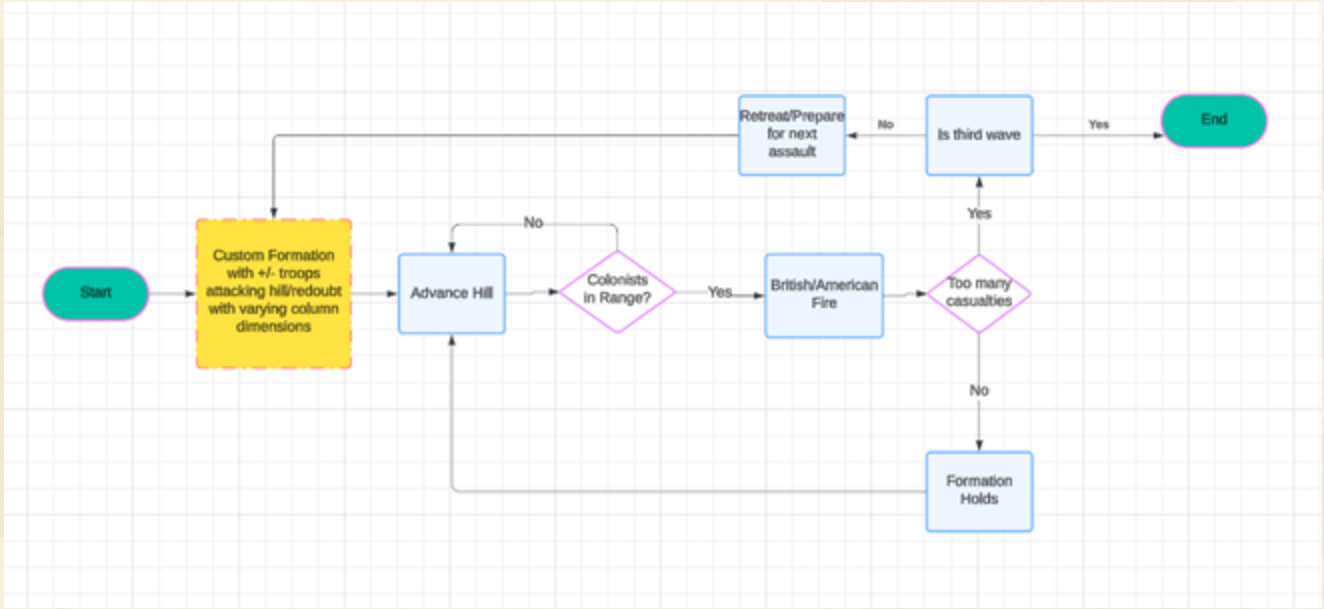
American Militia Combat Diagram



Key Logic:

- Fire only when British troops are close, maximizing accuracy.
- Ammo depletion influences when they switch to retreat behavior.

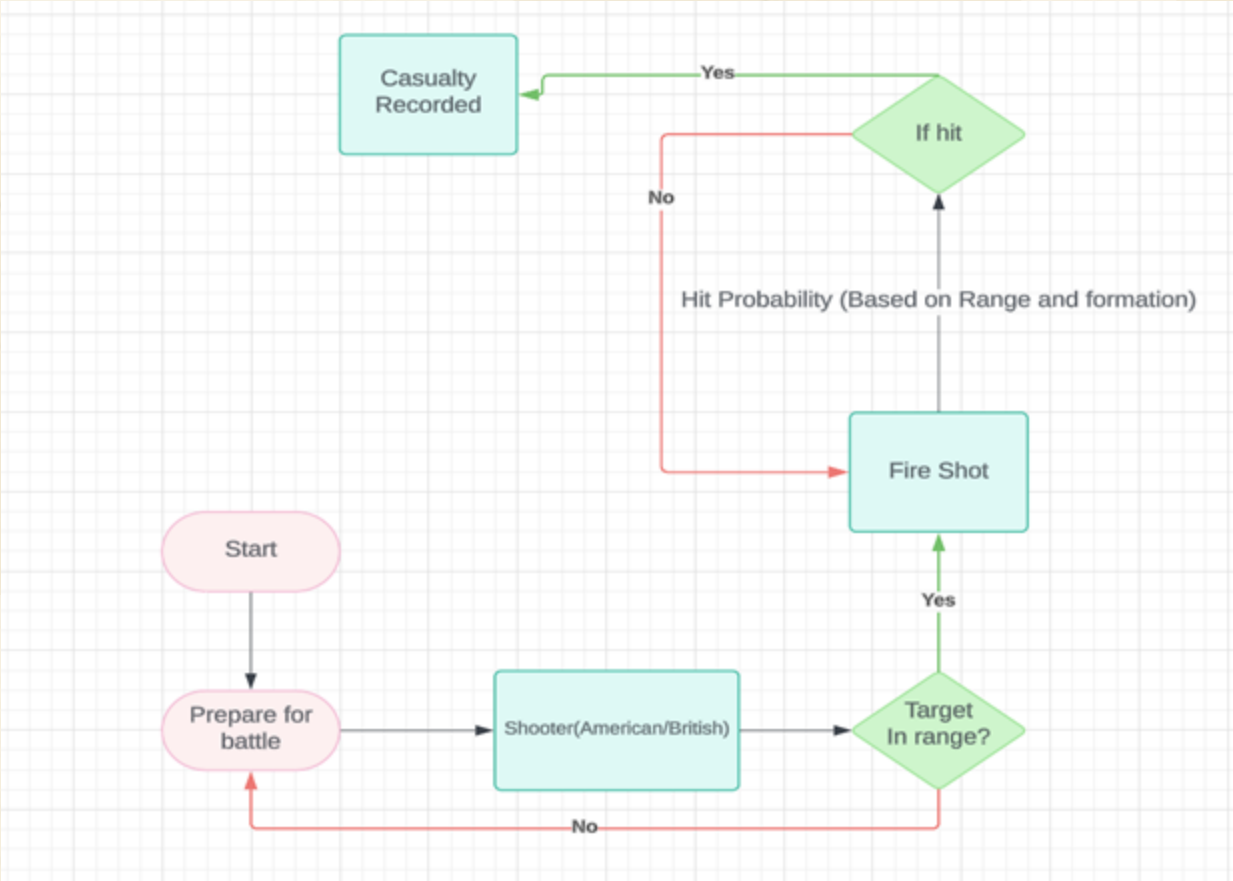
British Soldier Combat Diagram



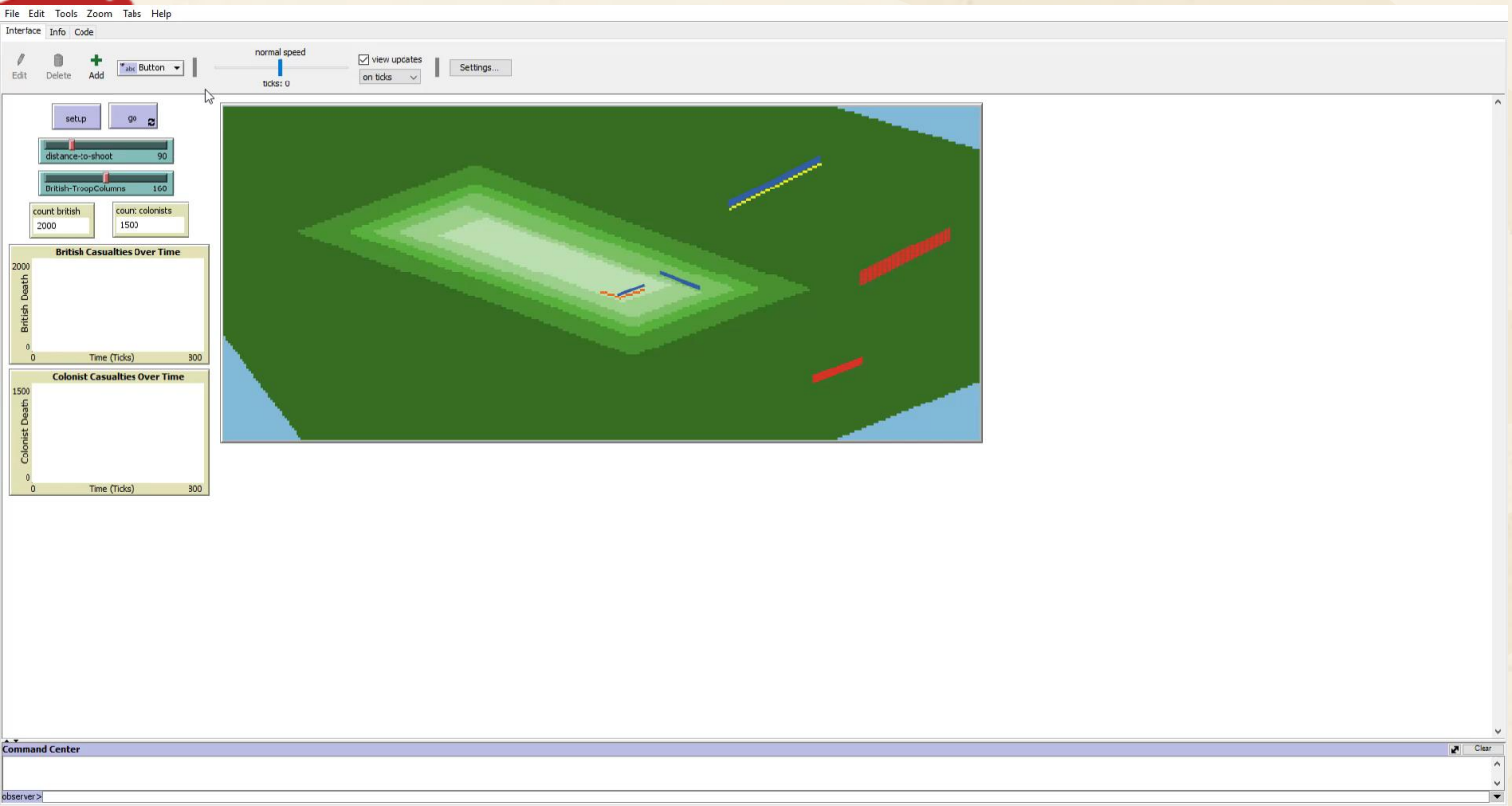
Key Logic:

- Move toward Colonist positions in formations.
- Fire when within range.
- Retreat if casualties are too high.
- Track total casualties and formation stability.

General Combat Diagram



NetLogo Simulation



Redoubt

Rail Fence

Water

Elevation

Flat

Higher

Top

Colonists

British

Analysis

Research Question

How does colonial engagement distance and British formational tactics impact the casualties during the Battle of Bunker Hill?

Independent Variables

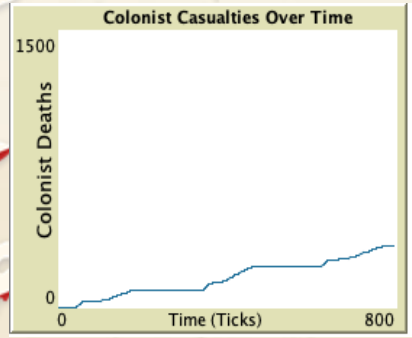
Colonial Distance to shoot- spans from 30 feet to 300 feet

Width of British formation in number of columns between both formations- 80, 160, 240 columns observed

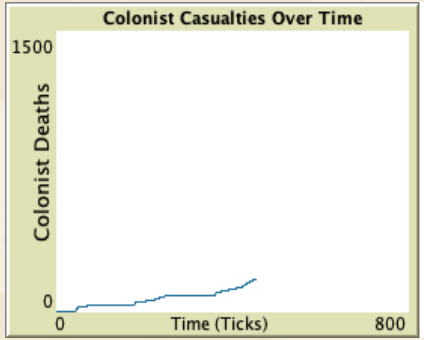
Dependent Variables

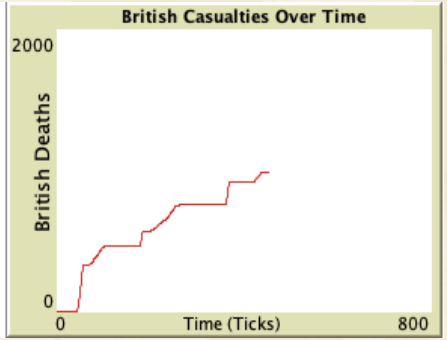
British and Colonial deaths out of 1500 initial Colonists and 2000 initial British

Modeling Casualties: British vs. Colonial

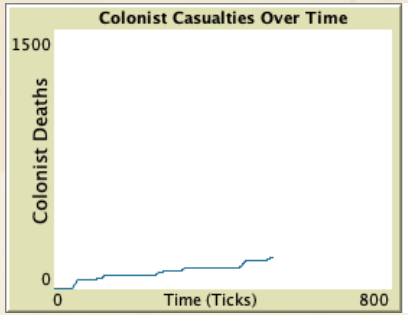


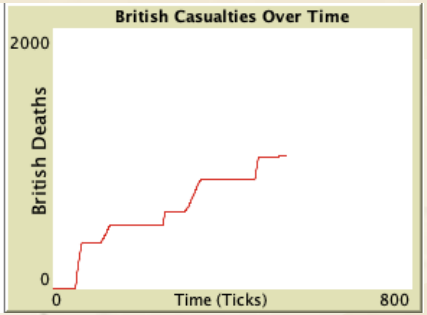
British-TroopColumns - 80
Distance-to-shoot – 90



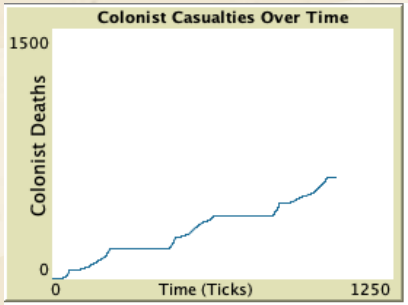


British-TroopColumns - 240
Distance-to-shoot – 270





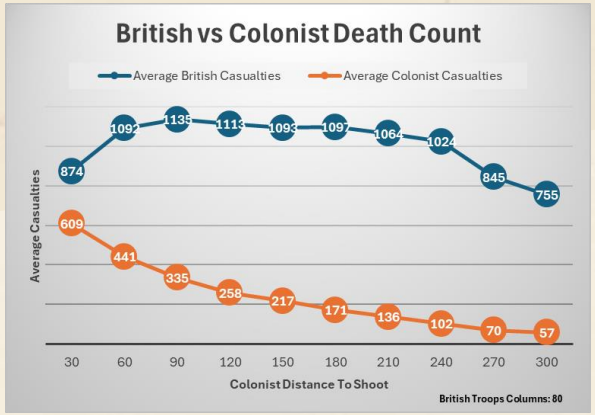
British-TroopColumns - 160
Distance-to-shoot – 180



British-TroopColumns - 80
Distance-to-shoot – 30

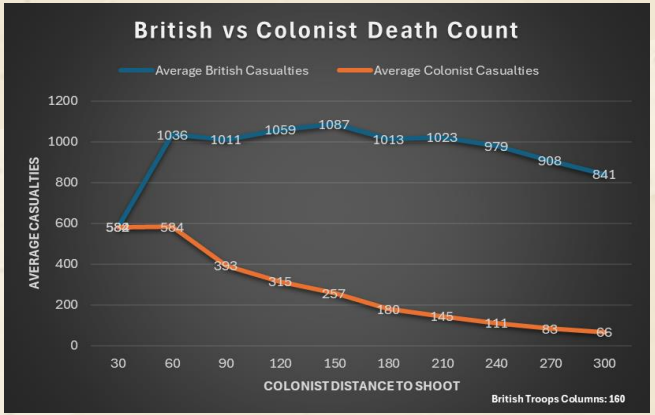
Modeling Simulation: Distance To Shoot vs Casualties

- The longer shooting distances tend to result in fewer casualties for both sides, particularly for the Colonists.
- The critical distances where British vulnerability was highest in terms of casualties are:
  - **90 units** for a formation of **80 columns**
  - **150 units** for a formation of **160 columns**
  - **240 units** for a formation of **240 columns**
- The chart indicates specific points at which British forces were most susceptible to higher casualty rates based on their formation size and shooting distance.



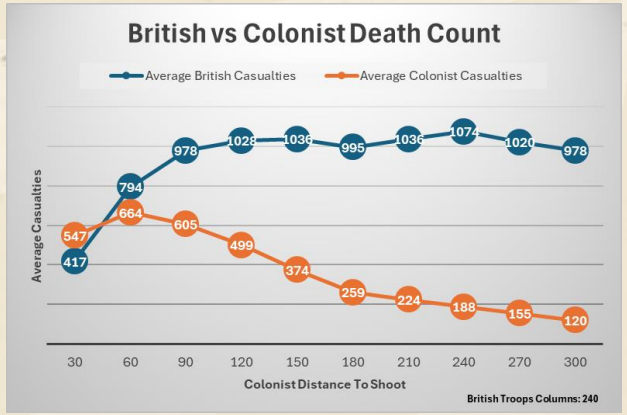
Standard Deviation British: 134
Mean British: 1009

Standard Deviation Colonist: 178
Mean Colonist: 240



Standard Deviation British: 149
Mean British: 954

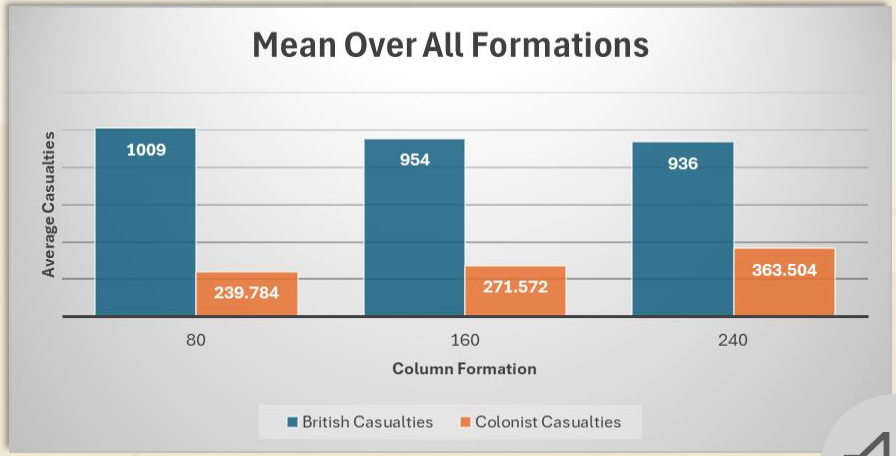
Standard Deviation Colonist: 194
Mean Colonist: 272

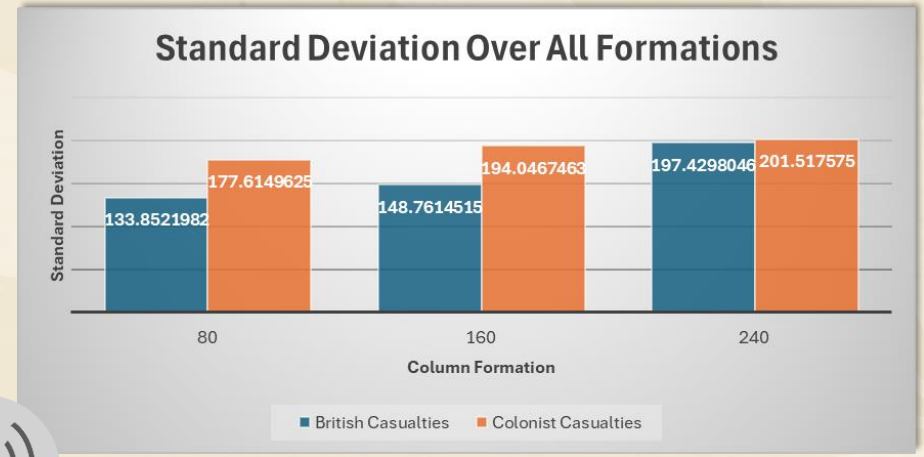


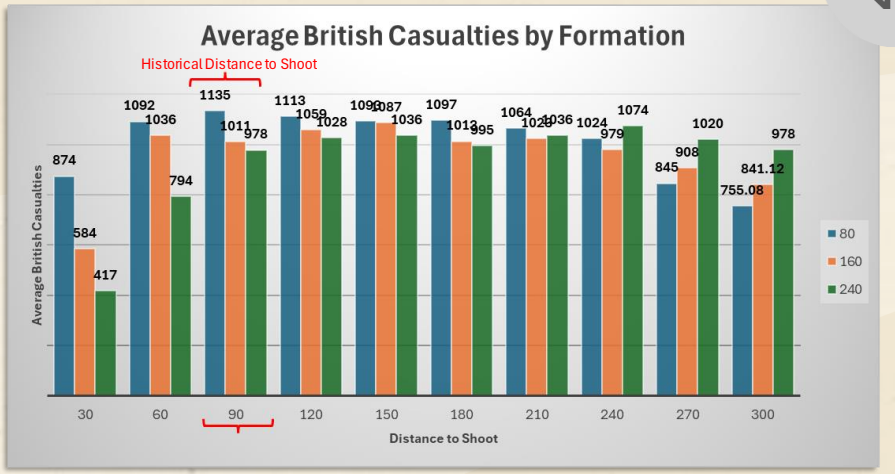
Standard Deviation British: 197
Mean British: 936

Standard Deviation Colonist: 202
Mean Colonist: 364

Modeling Simulation: Average Casualties by Formation







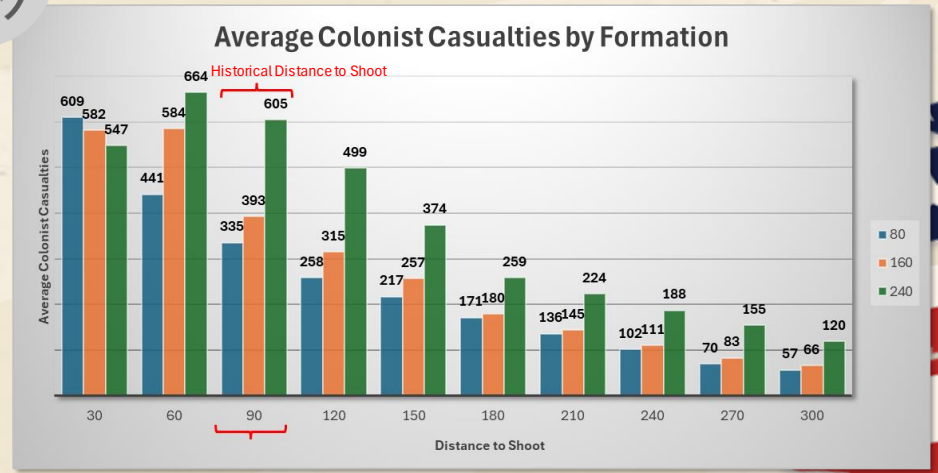


Fig. 12. Homer, Winslow. *The Battle of Bunker-Hill - Watching the Fight from Copp's Hill, in Boston*. 1875. *National Gallery of Art*, https://www.nga.gov/collection/art-object-page.45626.html.

Conclusions

●**Hypothesis:** The hypothesis posits that British reliance on close-order formations led to higher British casualties and lower Colonist casualties. Additionally, Colonial short engagement distances led to higher British casualties and higher Colonial casualties.

Our hypothesis was mostly correct!

- British reliance on close-order formations led to higher British casualties and lower colonist casualties.
- Colonial short engagement distance did not lead to higher British casualties but did lead to higher Colonial casualties.

Conclusions on Engagement Distance

- **Conclusion:**
  - Colonist engagement distance directly impacted both British and Colonist casualties during the Battle of Bunker Hill.
  - Up to a point, increasing the max engagement distance led to increased British casualties and decreased Colonist casualties.
  - When the Colonist engagement distance begins approaching the max effective distance of the flintlock muskets, British casualties begin to decline again.
- **Why:**
  - Increasing the effective shooting range for the Colonists led to more opportunities for British casualties; despite the low ammunition count this was still fairly effective.
  - However, when the Colonist shooting range begins reaching the apex of it's max effective range, shots become less effective. Due to the limited ammunition, at this point in time we see British casualties begin to decline again due to the decreased accuracy.
  - Colonist deaths also decrease due to the limited ammunition; Colonists end up retreating earlier.

Conclusions on British Formation

- **Conclusion:**
  - British formational tactics directly impact both British and Colonist casualties.
  - On average the wider and less column dense the British formation, the less British casualties.
    - The exceptions to this occur when the Colonist distance to shoot is greater than 210 feet.
  - On average, the wider and less column dense the British formation, the more Colonist casualties.
    - The one exception to this is at a Colonist distance to shoot of 30 feet.
- **Why:**
  - A more spread out troop formation was more effective for the British when they had a wider firing range compared to the Colonists. More British could remain out of the Colonist's shooting range for longer, while still being within their own max effective shooting range. Thus the ability to fire without being fired at in return both reduced their casualties and increased Colonist casualties.

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