Qualitative analysis of a bulk ferromagnetic hysteresis model

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Qualitative Analysis of a Bulk Ferromagnetic Hysteresis Model

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Abstract

In this paper, we present a set of equations for a *bulk ferromagnetic hysteresis model*. We establish the connection between this set of equations and classical energy loss expressions for ferromagnetic hysteresis. We show analytically that for periodic inputs the system of equations has a limit set that is a periodic orbit in the $H - M$ plane, under certain constraints on the parameters.

The results on the Ω limit set proved in this paper is used in extensions of our work to a bulk magnetostriction model in [1].

1 Introduction

In 1983, D.C. Jiles and D.L. Atherton [2, 3] proposed a model for ferromagnetic hysteresis from phenomenological and thermodynamic considerations. This model is very appealing from the control point of view as it describes hysteresis as the solution of four differential equations. The verification of the periodic nature of the hysteresis curves with sinusoidal inputs, has been achieved by computer simulation[2, 3]. V. Rizzoli et al.[4] use the harmonic balance method to show this fact. But by the very nature of the harmonic balance/describing function method, it is only a means to find the approximate resonant frequency in some systems. It cannot answer the following question for instance: Does the solution trajectory of the Jiles-Atherton model have an orbitally asymptotically stable periodic orbit for sinusoidal inputs?

In this paper, we present a modification of the Jiles - Atherton model which we call the *bulk ferromagnetic hysteresis model*. The mathematical properties of the two are quite different. We show that there is a periodic orbit which is the Ω limit set of a trajectory starting at the origin (see Theorem 1 in Section 3).

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2 Bulk Ferromagnetic hysteresis model

We begin with a statement of the problem. Consider a thin ferromagnetic rod or a toroid. The magnetic field inside such a ferromagnet is uniform with a scalar value H and directed along the axis of the rod or toroid. The average magnetic moment per unit volume has a scalar value M and is also directed along the axis. H is assumed to be equal to the magnetic field of external origin in the material, as the demagnetizing field is approximately equal to zero for the shapes under consideration. The relationship between H and M is hysteretic as has been verified by experiments dating back atleast to J.A. Ewing [5]. We are interested in obtaining a state space model of low dimension that is physics based and simulates the behaviour of a bulk ferromagnetic rod.

We equate the work done by an external source (a battery) δW_{bat}, with the change in the internal energy of the material δW_{mag} and losses in the magnetization process δL_{mag}.

$$\delta W_{bat} = \delta W_{mag} + \delta L_{mag} \tag{1}$$

W.F. Brown in his remarkable monograph[6] derives the work done by the battery in changing the magnetization per unit volume, in one cycle, to be given by

$$\delta W_{bat} = \oint \mu_0 H \, dM \tag{2}$$

The relationship between the above energy expression and the usual expression of hysteresis energy loss per unit volume can be derived easily. As $\oint \mu_0 H \, dH$ and $\oint \mu_0 M \, dM$ are loop integrals of exact differentials and hence equal to zero, we have

$$
\begin{aligned}
\oint H\,dB &= \oint \mu_0\,H\,dH + \oint \mu_0\,H\,dM \\
&= \oint \mu_0\,H\,dM \\
&= -\oint \mu_0\,M\,dH \\
&= -\oint \mu_0\,M\,dH - \alpha \oint \mu_0\,M\,dM \\
&= -\oint \mu_0\,M\,d(H + \alpha\,M) \\
&= -\oint M\,dB_e \qquad (3)
\end{aligned}
$$

where the constant α can take any value. The last equation is of interest because in Weiss's molecular field theory for ideal ferromagnetic rods (no losses), $M_{an} = M$ is a function of B_e and $\alpha > 0$ is the molecular field parameter. For an ideal ferromagnetic rod, M_{an} is given by the Langevin function [7, 8] – $M_{an} = M_s\,\mathcal{L}(z) = M_s\,(\coth z - \frac{1}{z})$ where M_s is the saturation magnetization. $z = \frac{H+\alpha M}{a}$ and a is a parameter that depends on the temperature of the specimen. Thus for an ideal ferromagnet, $\oint H\,dB$ and $\oint H\,dM$ are equal to zero as we expect them to be. Hence for periodic inputs, the same curve is traced for both the increasing and decreasing branches for periodic H (Figure 1). This curve is called the *anhysteretic* curve.

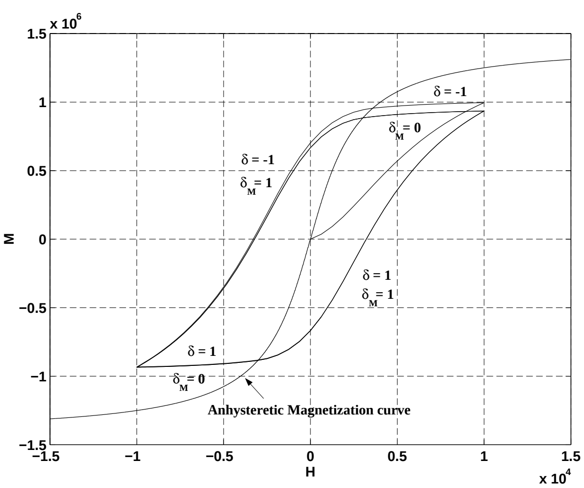

Figure 1: M vs H relationship for an ideal and a lossy ferromagnet.

Using Equation (3), we obtain the expression for δW_{mag} from the ideal case. $\delta W_{mag} = \delta W_{bat} = -\oint M_{an}\,dB_e$ per unit volume, per cycle. For a lossy ferromagnet, the expression for the magnetic hysteresis losses δL_{mag} is due to Jiles and Atherton. The motivation for this term (see Equation (6) below) is the observation that the hysteresis losses are due to irreversible domain wall motions in a ferromagnetic solid. They arise from various defects in the solids and are discussed in detail by Jiles and Atherton [3]. Here we provide a gist of their results. They consider the average magnetic moment per unit volume M to be comprised of an irreversible component M_{irr} and a reversible component M_{rev}. Furthermore, they claim M_{rev} to be related to the anhysteretic or ideal magnetization by the following expression.

$$
\begin{aligned}
M &= M_{rev} + M_{irr}. \qquad (4) \\
M_{rev} &= c\,(M_{an} - M_{irr}). \qquad (5)
\end{aligned}
$$

where $0 < c < 1$ is a parameter that depends on the material. The energy loss due to the magnetization is only due to M_{irr},

$$
\delta L_{mag} = \oint k\,\delta\,(1-c)\,dM_{irr} \qquad (6)
$$

In the above equation, k is a nonnegative parameter, and δ is defined as follows

$$
\delta = sign(\dot{H}). \qquad (7)
$$

Furthermore, Jiles and Atherton make the assumption that if the actual magnetization is less than the anhysteretic value and the magnetic field strength H is lowered, then until the value of M becomes equal to the anhysteretic value M_{an}, the change in magnetization is reversible. That is,

$$
\frac{dM_{irr}}{dH} = 0\,if \left\{ \begin{array}{l} \dot{H} < 0\,and\,M_{an}(H_e) - M(H) > 0 \\ \dot{H} > 0\,and\,M_{an}(H_e) - M(H) < 0 \end{array} \right. \qquad (8)
$$

At this point we pause and take stock of all the assumptions given by Equations (4 - 8). The reasoning behind Equation (5) is provided by Jiles and Atherton [3]. They use phenomenology based arguments the correctness of which is unclear. It is clear that the motivation for the assumptions is mathematical. Without Equation (8), the incremental susceptibility at the reversal points $\frac{dM}{dH}$ can become negative. This can be checked by numerical simulations. Experimental observations suggest that the quasi-static incremental susceptibility is a non-negative quantity. Therefore we adopt the same assumptions as (a) they do not violate the laws of thermodynamics, (b) they make the quasi-static incremental susceptibility a non-negative quantity and (c) the extra structure makes the model numerically well conditioned as will be seen later. With these qualifying comments we proceed with the derivation of the state equations.

By Equations (4) and (5) we get,

$$M = (1-c)\, M_{irr} + c\, M_{an} \tag{9}$$

Let

$$\delta_M = \begin{cases} 0 & : \quad \dot{H} < 0 \, and \, M_{an}(H_e) - M(H) > 0 \\ 0 & : \quad \dot{H} > 0 \, and \, M_{an}(H_e) - M(H) < 0 \\ 1 & : \quad otherwise. \end{cases} \tag{10}$$

Then by Equations (8) and (9),

$$\frac{dM}{dH} = \delta_M \,(1 - c)\, \frac{dM_{irr}}{dH} + c\, \frac{dM_{an}}{dH} \tag{11}$$

From equations (1), (2), (3) and (6) and the expression for W_{mag} we get,

$$\oint (M_{an} - M - k\,\delta\,(1-c)\,\frac{dM_{irr}}{dB_e})\, dB_e = 0$$

Note that the above equation is valid only if the orbits of $M(t)$ and $H(t)$ are periodic in the $M-H$ plane. In other words, the trajectory is a periodic orbit. We now make the *hypothesis* that the following equation is valid when we go from *any point on this periodic orbit to another point on the periodic orbit.*

$$\int (M_{an} - M - k\,\delta\,(1-c)\,\frac{dM_{irr}}{dB_e})\, dB_e = 0 \tag{12}$$

The above equation holds only on the periodic orbit. Therefore on the periodic orbit, the integrand must be equal to zero.

$$M_{an} - M - k\,\delta\,(1-c)\,\frac{dM_{irr}}{dB_e} = 0 \tag{13}$$

Using Equations (11) and (13) we can show after some manipulations that

$$\frac{dM}{dH} = \frac{\frac{k\,\delta}{\mu_0}\, c\, \frac{dM_{an}}{dH} + \delta_M\,(M_{an} - M)}{\frac{k\,\delta}{\mu_0} - \delta_M\,(M_{an} - M)\,\alpha} \tag{14}$$

Setting $k = 0$ gives us $\delta_M\,(M_{an} - M)\,\frac{dM}{dH} = -\frac{\delta_M\,(M_{an} - M)}{\alpha}$. As mentioned before, compatibility with the physical phenomenon demands that $\frac{dM}{dH} \geq 0$. α is a non–negative parameter and so for the above equation to make sense we must have $M_{an} - M = 0$. Thus $k = 0$ represents the lossless case.

Rewriting Equation (14) so that we have $\frac{dM_{an}}{dH_e}$ in the numerator on the right hand side we get

$$\frac{dM}{dH} = \frac{\frac{k\,\delta}{\mu_0}\, c\, \frac{dM_{an}}{dH_e} + \delta_M\,(M_{an} - M)}{\frac{k\,\delta}{\mu_0} - \delta_M\,(M_{an} - M)\,\alpha - \frac{k\,\delta}{\mu_0}\,\alpha\, c\, \frac{dM_{an}}{dH_e}} \tag{15}$$

This equation is different from the one obtained by Jiles and Atherton [2, 3]. We henceforth refer to it as the *bulk ferromagnetic hysteresis model* so as not to confuse it with the model in [3] that is popularly known as Jiles-Atherton model. For the sake of completeness we write down the other equations satisfied by the system.

$$M_{an}(H_e) = M_s \left(\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e} \right). \tag{16}$$

$$H_e = H + \alpha\, M. \tag{17}$$

$$\delta = sign(\dot{H}). \tag{18}$$

$$\delta_M = \begin{cases} 0 : \dot{H} < 0 \, and \, M_{an}(H_e) - M(H) > 0 \\ 0 : \dot{H} > 0 \, and \, M_{an}(H_e) - M(H) < 0 \\ 1 : otherwise. \end{cases} \tag{19}$$

Equations (15 - 19) describe the bulk ferromagnetic hysteresis model. There are 5 non–negative parameters in this model namely a, α, M_s, c, k. Also $0 < c < 1$. Figure 1 shows the values taken by the discrete variables δ, δ_M at different sections of the hysteresis curve in the $M-H$ plane.

Remarks : The starting point for Jiles and Atherton's theory is the "energy" function $E = \int M\, dB_e$. The reasoning behind the choice of the above energy function is never explained in their papers. The relationship between the chosen energy function and the classical $\oint H\, dB$ hysteresis losses is also unclear. The clarification is of paramount importance for obvious reasons of validity of the theory but also for providing further insights into extending the model to cover the phenomenon of magnetostriction. Hence we first bridged this gap by starting with W.F. Brown's expression for the work done by a battery in changing the magnetization of a rigid ferromagnetic body.

3 Qualitative analysis of the model

The model equations are only valid when the variables M and H are on a periodic orbit. In other words, the solution of the equations represent the physics of the system under these conditions. But, usually in practice we do not know apriori what state the system is in. Then can we use the above model? The answer in the affirmative is provided in this section. We show analytically that even if we start at the origin in the $M - H$ plane (which is usually not on the hysteresis loop), and apply a periodic input $\dot{H}$, we *tend* asymptotically towards this periodic solution.

First we prove an important property.

Claim 1 *Consider the system described by Equations (15 - 19). Let the initial condition (M_0, H_0) be on the anhysteretic curve.*

$$M_0 = M_{an}(H_0 + \alpha M_0). \tag{20}$$

Let the parameters satisfy the following conditions.

$$\frac{c\,\alpha\,M_s}{3\,a} < 1 \tag{21}$$
$$0 < c < 1 \tag{22}$$

If $H(t)$ is a monotonically increasing function of time for $t \in [a, b]$ where $b > a \geq 0$, then $(M_{an} - M) \geq 0 \;\; \forall t \in [a, b]$. Else if $H(t)$ is a monotonically decreasing function of time for $t \in [c, d]$ where $d > c \geq 0$, then $(M_{an} - M) \leq 0 \;\; \forall t \in [c, d]$.

Proof:

Later it is shown that the solutions of the system exist and they are unique. We first prove the first statement in the claim. Suppose $H(t)$ is a monotonically increasing function of time for $t \in [a, b]$. Let $y = M_{an} - M$. At $t = a$, $y(a) = 0$. We want to show $y \geq 0 \;\forall\; t \in [a, b]$. We will prove this by contradiction. Suppose $y < 0$ for some $t^* \in [a, b]$. Since $\dot{H} \geq 0$ at $t = t^*$, by the condition given by Equation (10), $\delta_M = 0$. By Equation (15) $\frac{dM}{dH} = \frac{c\,\frac{dM_{an}}{dH_e}}{1 - c\,\alpha\,\frac{dM_{an}}{dH_e}}$.

Therefore

$$\begin{aligned}\frac{dy}{dH} &= \frac{d(M_{an} - M)}{dH} \\ &= \frac{dM_{an}}{dH_e}\left(1 + \alpha\,\frac{dM}{dH}\right) - \frac{dM}{dH} \\ &= \frac{dM_{an}}{dH_e} - \left(1 - \alpha\,\frac{dM_{an}}{dH_e}\right)\frac{dM}{dH}\end{aligned}$$

It is easily shown (using series expansions) that $\max_{H,M} \frac{dM_{an}}{dH_e} = \frac{M_s}{3\,a}$. By Inequality (21), the denominator of $\frac{dM}{dH}$ is always positive. We now have two cases to consider. If $\frac{\alpha\,M_s}{3\,a} \geq 1$, then obviously $\frac{dy}{dH} > 0$. Else if $\frac{\alpha\,M_s}{3\,a} < 1$ then $\frac{1 - \alpha\,\frac{dM_{an}}{dH_e}}{1 - c\,\alpha\,\frac{dM_{an}}{dH_e}} < 1$.

Therefore,

$$\begin{aligned}\frac{dy}{dH} &= \frac{dM_{an}}{dH_e} - \left(1 - \alpha\,\frac{dM_{an}}{dH_e}\right)\frac{dM}{dH} \\ &> (1 - c)\,\frac{dM_{an}}{dH_e}\end{aligned}$$

By Inequality (22), $\frac{dy}{dH} > 0$. Hence $y \not< 0 \;\forall\; t \in [a, b]$. The second statement in the claim is similarly proved by considering the case $\delta = -1$.

■

We remark here that it can be easily shown that as $H \to \infty$, $M \to M_s$. We omit the proof of this property.

Define state variables, $x_1 = H$, $x_2 = M$. Let, $z = \frac{x_1 + \alpha\,x_2}{a}$. Then the state equations are:

$$\dot{x_1} = u \tag{23}$$
$$\dot{x_2} = g(x_1, x_2, x_3, x_4)\,u. \tag{24}$$
$$x_3 = sign(u). \tag{25}$$
$$x_4 = \begin{cases} 0 & : \; x_3 < 0 \; and \; \coth(z) - \frac{1}{z} - \frac{x_2}{M_s} > 0 \\ 0 & : \; x_3 > 0 \; and \; \coth(z) - \frac{1}{z} - \frac{x_2}{M_s} < 0 \\ 1 & : \; otherwise. \end{cases} \tag{26}$$

In this paper, we study the system described by Equations (23 - 26), together with the input given by,

$$u(t) = b\,\cos(\omega\,t). \tag{27}$$

The periodicity of the limiting trajectory produced by the system of Equations (23 - 27) is proved in 4 steps. To make the analysis easier, we consider the system separately during the times when $sign(u(t))$ has the value +1 or -1. The key fact that is exploited was proved in Claim 1.

1. Starting from $(x_1, x_2) = (0, 0)$, $x_2(t)$ increases for $t \in [0, \frac{\pi}{2\,\omega}]$, but lies below the anhysteretic magnetization curve.

2. For $t \in [\frac{\pi}{2\,\omega}, \frac{3\,\pi}{2\,\omega}]$, $x_2(t)$ first intersects the anhysteretic curve, then lies above it.

3. For $t \in [\frac{3\,\pi}{2\,\omega}, \frac{5\,\pi}{2\,\omega}]$, $x_2(t)$ first intersects the anhysteretic curve, then lies below it.

By repeating the analysis in Steps 2, 3, we can conclude that the solution trajectory to the ferromagnetism model lie within the compact set $[-\frac{b}{\omega}, \frac{b}{\omega}] \times [-M_s, M_s]$.

4. We then look at $\{x_2\left(\frac{2n\pi}{\omega}\right)\};\ n = 0, 1, 2, \ldots$. This sequence of points lie on the x_2 axis ($x_1 = 0$ line). We then show that the sequence has a unique accumulation point. This shows the existence of an asymptotically stable periodic orbit in the $x_1 - x_2$ plane. Since x_3 and x_4 depend on x_1, x_2, we conclude that the system of Equations (23 - 27) produce asymptotically stable periodic solutions.

3.1 Analysis of the Model for $t \in [0, \frac{\pi}{2\omega}]$

For sake of brevity, we give the salient points of the proofs without going into too much detail.

Claim 2 *Consider the system described by Equations (23 - 27), and* $(x_1(0), x_2(0)) = (0, 0)$. *Suppose the parameters satisfy conditions (21, 22) and*

$$\frac{k}{\mu_0}\left(1 - \frac{\alpha c M_s}{3a}\right) - 2\alpha M_s = A > 0. \qquad (28)$$

In the time interval $[0, \frac{\pi}{2\omega}]$, *there exists a unique solution and it satisfies the condition* $|x_2(t)| \leq M_s$.

Proof:

The existence of the solution at $t = 0$ is first established by noting that the system equation $\dot{x} = f(t, x)$ is a *Carathéory* equation [9, 10]. This solution can be extended to a maximal interval of existence in time. From the proof of this extension theorem [9], we see that the solution can be continued upto $[0, \frac{\pi}{2\omega}]$. The uniqueness of this solution is easily proved by showing that $\|f(t, x) - f(t, y)\| \leq L\|x - y\|$ where L is a constant, and using the uniqueness theorem in Hale [9].

To prove the second part, consider the solution of the system with initial condition $(x_1(0), x_2(0)) = (0, 0)$. The anhysteretic curve passes through this point. Hence by Claim 1, $x_2(t)$ is bounded above by the anhysteretic curve $\forall\ t \in [0, \frac{\pi}{2\omega}]$. It is also a monotonically increasing function because $g(t, x) \geq 0$. Hence, $|x_2(t)| \leq M_s \ \ \forall\ t \in [0, \frac{\pi}{2\omega}]$. ■

3.2 Analysis of the Model for $t \in [\frac{\pi}{2\omega}, \frac{5\pi}{2\omega}]$

Claim 3 *Consider the system described by Equations (23 - 27), and* $(x_1(0), x_2(0)) = (0, 0)$. *Suppose the parameters satisfy conditions (21, 22 and 28). In the time interval* $[\frac{\pi}{2\omega}, \frac{3\pi}{2\omega}]$, *there exists a unique solution and it satisfies the condition* $|x_2(t)| \leq M_s$

Proof: The proof is very much similar to the one in the previous subsection, except for one detail. We claim that the trajectory first intersects the anhysteretic curve for some time $t^* \in (\frac{\pi}{2\omega}, \frac{\pi}{\omega})$. Then by Claim 1 the trajectory $x_2(t)$ is bounded below by the anhysteretic curve for $t \in [t^*, \frac{3\pi}{2\omega}]$. It is also a monotonically decreasing function and hence, $|x_2(t)| \leq M_s \ \ \forall\ t \in [\frac{\pi}{2\omega}, \frac{3\pi}{2\omega}]$. As $f(t, x)$ is continuous for $t \in (\frac{\pi}{2\omega}, t^*)$ and $t \in (t^*, \frac{3\pi}{2\omega})$, it follows that $x(t)$ is also continuously differentiable in the both intervals. ■

Claim 4 *Consider the system described by Equations (23 - 27), and* $(x_1(0), x_2(0)) = (0, 0)$. *Suppose the parameters satisfy Equations (21, 22, and 21). the time interval* $[\frac{3\pi}{2\omega}, \frac{5\pi}{2\omega}]$, *there exists a unique solution and it satisfies the condition* $|x_2(t)| \leq M_s$.

The proof is similar to that of Claim 3.

3.3 Proof of Periodic behaviour of the Model for Sinusoidal Inputs

Let us first collect together some important properties of the bulk ferromagnetic hysteresis model. We don't provide the proofs as they are of a very straight forward nature.

Property 1 *Consider the system described by Equations (23–27). If the conditions expressed by Inequalities (21), (22) and (28) are satisfied and* $x = (x_1, x_2) = (0, 0)$, *then there exists a unique solution to the system.*

Property 2 *If Inequalities (21), (22) and (28) are satisfied, then*

$$\frac{dx_2}{dx_1} \geq 0 \ \ \forall\ x \in \sum. \qquad (29)$$

Property 3 (Anti-symmetry) *Consider Equations (23–26), with* $u(t) \geq 0$. *Let* $y_u(t) = (y_1, y_2)(t, u)$ *and* $x_u(t) = (x_1, x_2)(t, u)$, *where we have made the dependence on the input explicit. If* $y(0) = -x(0)$, *then* $y_u(t) = -x_{-u}(t)$.

Property 4 *If* $u(t)$ *does not change its sign* $\forall t \in [a, b]$ *and if* $\tilde{x}(a), \breve{x}(a)$ *are two initial states of the system with* $\tilde{x}_2(a) \geq \breve{x}_2(a)$, *then* $\tilde{x}_2(t) \geq \breve{x}_2(t)\ \forall t \in [a, b]$.

Property 5 *Consider the system given by Equations (23–26), with input given by Equation (27). If*

$(x_1, x_2)(0) = (0,0)$, *then* $|x_2(t)| \leq M_s \; \forall t \geq 0$. *Thus the trajectory lies in the compact region* $[-\frac{b}{\omega}, \frac{b}{\omega}] \times [-M_s, M_s]$ *in the* $x_1 - x_2$ *plane.*

Theorem 1 *Consider the system given by Equations (23–26), with input given by Equation (27).*

If $(x_1, x_2)(0) = (0,0)$, *then the* Ω*-limit set of the system is a periodic orbit.*

Proof:

Let $\theta = \omega t$. Then the non-autonomous system given by Equations (23–27), can be transformed into an autonomous one with the auxiliary equation, $\dot{\theta} = \omega$. Because of Properties 2 and 3, we only need to look at the trajectories in the $x_1 - x_2$ plane for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. By Equation (23), the trajectory in the $x_1 - x_2$ plane intersects transversally with the x_2 axis. This is because $\frac{dx_2}{dx_1}$ is bounded. Thus there exists a sequence of intersections $\Pi_1 = p_k$. This sequence has a convergent subsequence $?_1 = p_{n_k}$ because by Property(5), it lies in the compact set $[-M_s, M_s]$ on the x_2 axis. Let, $p_{n_k} \rightarrow p^*$. Let $\Pi_2 = p_k \setminus p_{n_k}$. If this sequence is finite, then we have nothing to prove. If however, this sequence is infinite, then it has a convergent subsequence, $?_2 = p_{m_k}$. If the limit point of this subsequence is also p^*, then again we have nothing to prove. In this case, we proceed further by extracting subsequences until we find one with the limit point $q^* \neq p^*$. Both the points $(0, q^*), (0, p^*)$ on the $x_1 - x_2$ plane belong to the Ω limit set. Consider the trajectories with $\tilde{x_2}(0) = q^*$, and $\breve{x_2}(0) = p^*$, and $q^* > p^*$. By Property 4, for $0 \leq t \leq \frac{\pi}{2\omega}$, $\tilde{x}_2(t) \geq \breve{x}_2(t)$. Also for $\frac{\pi}{2\omega} \leq t \leq \frac{3\pi}{2\omega}$, $\tilde{x}_2(t) \geq \breve{x}_2(t)$. and for $\frac{3\pi}{2\omega} \leq t \leq \frac{2\pi}{\omega}$, $\tilde{x}_2(t) \geq \breve{x}_2(t)$. Hence for one period of the input sinusoid $\tilde{x}_2(t) \geq \breve{x}_2(t)$. This is true for any period of the sinusoid and so we conclude that atleast one of the following statements must be true.

$$\begin{aligned} p^* &\notin \tilde{x}_2(t) \; \forall t \geq 0 \\ q^* &\notin \breve{x}_2(t) \; \forall t \geq 0 \end{aligned}$$

That is, atleast one of the points q^*, p^* do not belong to the Ω limit set. Hence it is not possible that $p^* \neq q^*$. (In a longer version of this paper, we argue how this orbit is asymptotically stable.)

■

Remarks:

1. Instead of condition (28), if $\frac{k\,(1-c)}{\mu 0} - 2\,\alpha\, M_s > \tilde{A} > 0$, then we obtain existence and uniqueness for the Jiles – Atherton model [3].
2. If Claim (1) is reproved for their set of equations, then by using the same method, we can show that the Ω limit set is asymptotically stable for the Jiles – Atherton model.
3. The important difference between the bulk ferromagnetic hysteresis model and the Jiles – Atherton model is that $k = 0$ *does not* represent the lossless case for the latter.

4 Conclusion

In this paper, a bulk ferromagnetic hysteresis model is shown to have a limit set that is a periodic orbit in the $M - H$ plane. Three conditions given by Inequalities (21), (22) and (28), must be satisfied by the parameters of the system, so that the result is true. This analytical proof shows that the model is numerically well conditioned.

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