Hysteresis model for magnetic materials using the Jiles-Atherton model

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Abstract

This paper presents a hysteresis model of magnetic materials, which is based on the theory of ferromagnetic hysteresis. Results obtained by numerical simulation of hysteresis curves of the ferromagnetic, hard magnetic core and anisotropic materials have been given. The mathematical model consists of only nine parameters. The hysteresis curve obtained is in good agreement with the curve defined for these materials in the PSpice program packet. The proposed method for parameter determination enables simulation of any magnetic material, which has a characteristic R, Z or F-shaped hysteresis curve.

1. Introduction

The **PSpice** program packet for the simulation of electronic circuits is widely used in the design of electronic devices. The Jiles-Atherton model [1] for defining the hysteresis curves of ferromagnetic materials is built into this packet. However, most users of this program packet are not familiar with modeling hysteresis curves and they have problems when defining parameters of the model. A description of possibilities for the analysis of components with a ferromagnetic core is included in the instruction manual [2], but it only gives recommendations to the user on how to determine the five parameters related to magnetic properties of the material:

The **PSpice** program packet includes complete data on only one magnetic soft material: the Mn-Zn ferrite-3C8 (with the so-called R-round shaped hysteresis curve) and for a series of common core shapes. Standard catalogues of magnetic materials contain data on only 5 parameters (we need 9). The remaining four parameters have to be additionally determined. Three parameters available in catalogues can be used: M_r - residual magnetization, H_C - coercive field and χ_i - starting magnetic susceptibility. It

can be concluded that the procedure for determining these parameters is not singular, as a system of three equations with four unknowns need to be solved. The result obtained exclusively depends on the user skill in attaining satisfactory agreement with the real hysteresis curve using multiple parameter fitting. The error that can be made using this method can seriously influence validity of the **PSpice** analysis, especially when analyzing hard magnetic materials.

A satisfactory solution of this problem requires knowledge of the Jiles-Atherton model, which itself demands isotropy of the analyzed material and an R-shaped hysteresis curve. However, magnetic materials with Z or F-shaped hysteresis curves are often used, which requires certain modification of the model presented here [3].

Input data for the Mn-Zn ferrite-3C8 has been used for evaluation of the numerical simulation developed here [4], while magnetic characteristics obtained have been compared with corresponding characteristics read from the hysteresis curve of this material obtained using parameters given in the PSpice program packet.

2. Theoretical model

The basis of the theoretical analysis of ferromagnetic materials is founded on the fact that the effective magnetic field (H_C) in each individual domain is obtained as the sum of the external magnetic field (H) and the influence of surrounding magnetic domains:

$$H_e = H + \alpha \cdot M \qquad (1)$$

Where: M - magnetization of the material and $\alpha \equiv$ *ALPHA*. The anhysteresis curve represents the dependence of magnetization on the magnetic field in the case when the pinning effect of the domain wall is disregarded; i.e. it represents the minimum domain energy. It can be determined using Lengevin's function

for completely isotropic materials $L(H_C)$ [5] in the form of:

$$M_{an}(H) = M_s \cdot L(H_e) = M_s \cdot \left[coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e} \right] \quad (2)$$

Where $a \equiv A$. However, in a real material, the movement of domain walls is prevented (grain boundaries, grain inhomogeneneities, dislocations, non-magnetic inclusions, regions of inhomogeneous strain), which is defined as the pinning effect of a domain wall. This effect is active until the magnetic potential becomes high enough to cause wall movement, until the next pinning center is formed. Movement of the domain wall can be reversible and irreversible. Irreversible movement of the domain wall is described by the following equation:

$$\frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{\dfrac{\delta \cdot k}{\mu_o} - \alpha \cdot (M_{an} - M_{irr})} \quad (3)$$

Where: M_{irr} - irreversible magnetization component, k - a coefficient which depends on the energy required for domain wall movement, μ_0 - magnetic permeability of vacuum, $\delta = \pm 1$ (+ denotes the direction of the increase of the magnetic field and - denotes the decrease of the magnetic field) and finally, parameter K is defined as: $K \equiv k/\mu_0$.

If bending of the domain wall is assumed, the reversible component of magnetization M_{rev} can be described by the equation:

$$M_{rev} = c \cdot (M_{an} - M_{irr}) \quad (4)$$

Where $c \equiv C$. The total magnetization M is obtained as the sum of:

$$M = M_{irr} + M_{rev} \quad (5)$$

So, using eqs. (3), (4) and (5) the expression for differential magnetic susceptibility χ' is obtained as:

$$\chi' = \frac{dM}{dH} = \frac{(1-c) \cdot M_{an} - M_{irr}}{K \cdot \delta - \alpha \cdot (M_{an} - M_{irr})} + c \cdot \frac{dM_{an}}{dH} \quad (6)$$

Variables which can be read from the hysteresis curve and starting magnetization curve (M_r, H_C, M_m, H_m, χ_i...) do not appear in the given equations, so further model development is required for their solution.

The mathematical model for hard magnetic devices makes use of the domain coupling parameter α of the theory of ferromagnetic hysteresis. The strong coupling of the domain walls is considered as the main reason of the hysteresis of hard magnetic materials. Keeping the model as simple as possible eq. (4) is neglected. Therefore, (1) is replaced by another anhysteresis using power functions. Furthermore, the feedback loop in the differential equation is drooped too. The mathematical

model consist only of one nonlinear algebraic and one linear ordinary differential equation.

$$\frac{H}{K_c} + \alpha \frac{M}{M_s} = \frac{M_{an}/M_s}{R_\mu} + f(M_{an}/M_s) \quad (7)$$

where

$$f(M_{an}/M_s) = \begin{cases} g(M_{an}/M_s), & M_{an} \geq 0 \\ -g(-M_{an}/M_s) & M_{an} < 0 \end{cases} \quad (8)$$

$$g(M_{an}/M_s) = \hat{H}_0 \left(\frac{M_{an}}{M_s}\right)^{\beta_0} + \hat{H}_1 \left(\frac{M_{an}}{M_s}\right)^{\beta_1}$$

and

$$\frac{dM}{dH} = \begin{cases} 0, & \text{if } sign\left(\dfrac{dH}{dt}\right) = 1 \text{ and } M_{an} - M \leq 0 \\ 0, & \text{if } sign\left(\dfrac{dH}{dt}\right) = -1 \text{ and } M_{an} - \mu \geq 0 \\ \dfrac{M_{an} - M}{sign\left(\dfrac{dH}{dt}\right) K}, & elsewere \end{cases} \quad (9)$$

The parameters of the model are K_c, α, M_s, R_μ, H_0, H_1, β_0, β_1, and K. H_0, β_0 are obtained by a curve fit of the second and H_1, β_1, of the first quadrant. The additional parameter K_c is redundant. It was included in the model to make the parameter determination easier.

The existing isotropic model equations of hysteresis in magnetic materials have been extended to include the effects of anisotropy and texture. This has proved particularly important as the model is been used to describe an increasing range of magnetic materials in which anisotropy plays a significant role. Anisotropy and texture in polycrystalline magnetic materials can be adequately described by modifying the equation for the anhysteretic curve to account for these effects. To include anisotropy effects into the model, the anisotropy energy must be incorporated into the total energy of the moments. Following the development of the generalized anhysteretic function described previously [6]:

$$M_{aniso} = M_s \frac{\sum\limits_{all\ moments} e^{-E/k_B T} cos\theta}{\sum\limits_{all\ moments} e^{-E/k_B T}} \quad (10)$$

Where θ is the angle between the direction of the magnetic moment and the direction of the applied field, and

$$E = \mu_0 \langle m \rangle (H + \alpha M) + E_{aniso} \quad (11)$$

And E_{aniso} is the anisotropy energy, which depends on the anisotropy structure of material. In the case of cubic anisotropy,

$$E_{aniso} = K_1 \sum_{i \neq j}^{3} cos^2\, \theta_i\, cos^2\, \theta_j \qquad (12)$$

With the normal convention on symbols. In the present description, we only used the first anisotropy coefficient, because this approximation is in most case sufficient to provide an accurate description of the different magnetization curves along different directions. To incorporate the texture effect into the model, we introduce texture coefficient t, which is a statistical evaluation of the fraction of the textured portion of the material. Than the anhysteretic magnetization can be given as:

$$M_{an} = t \cdot M_{aniso} + (1-t) \cdot M_{iso} \qquad (13)$$

Where M_{aniso} is given by (10) and M_{an} is the isotropic anhysteretic magnetization. The general equation of hysteresis that has been described previously can now be solved with the incorporation of the anisotropy and textured anhysteretic magnetization M_{an} given above to obtain the magnetization curves along particular directions.

3. Numerical model

First, the equation for determining the parameter of reversible movement of the domain wall - c, is defined. This parameter can be determined using the starting differential normal susceptibility χ'_{in}:

$$\chi'_{in} = \left(\frac{dM}{dH} \right)_{\substack{H=0 \\ M=0}} = \frac{(1-c) \cdot M_{an}}{K \cdot \delta - \alpha \cdot M_{an}} + c \cdot \frac{dM_{an}}{dH} \quad (14)$$

Where the fact that magnetization is completely reversible for the starting magnetization curve $M = M_{rev}$ (i.e. $dM_{irr}/dH \rightarrow 0$) is used. If eq. (2) and its derivative (for $M = 0$) are introduced in eq. (14), and the development of the $coth(H/a)$ function for $H \rightarrow 0$ is used, the following is obtained:

$$\chi'_{in} = \frac{c \cdot M_s}{3 \cdot a}, \text{E.g. } c = \frac{3 \cdot a \cdot \chi'_{in}}{M_s}. \qquad (15)$$

The starting differential anhysteresis susceptibility χ'_{ian} defines a connection between parameter a and α:

$$\chi'_{ian} = lim_{\substack{H \rightarrow 0 \\ M \rightarrow 0}} \left(\frac{dM_{an}}{dH} \right) = \frac{M_s}{3 \cdot a - \alpha \cdot M_s} \qquad (16)$$

So that:

$$a = \frac{M_s}{3} \cdot \left(\frac{1}{\chi'_{ian}} + \alpha \right) \qquad (17)$$

This equation is used as the connection between parameters a and α.

Differential susceptibility in the coercive point χ'_{Hc}, representing maximal differential susceptibility on the whole magnetization curve, i.e. $\chi'_{Hc} = \chi'_{max}$ is used for determining parameter K. If eq. (6) is used to define the coercive point $H = +H_C$ and $M=0$, and it is transformed, parameter K is obtained in the following form:

$$K = \frac{M_{an}(H_C) - M_{irr}(H_C)}{\chi'_{max} - c \cdot \left[\frac{dM_{an}(H_C)}{dH} - \left(\frac{dM_{irr}(H_C)}{dH} \right) \right]} \quad (18)$$
$$+ \alpha \cdot [M_{an}(H_C) - M_{irr}(H_C)]$$

Eqs. (4) and (5) are used to determine variables M_{irr} and dM_{irr}/dH in the coercive point as:

$$M_{irr}(H_C) = -\left(\frac{c}{1-c} \right) \cdot M_{an}(H_C), \qquad (19)$$

$$\frac{dM_{irr}(H_C)}{dH} = \left(\frac{1}{1-c} \right) \cdot \chi'_{max} - \left(\frac{c}{1-c} \right) \frac{dM_{an}(H_C)}{dH}$$

If these expressions are introduced into eq.(18), the final expression used for the calculation of the pinning parameter of the domain wall, K is obtained:

$$K = \frac{M_{an}(H_C)}{1-c} \left[\alpha + \frac{1}{\frac{\chi'_{max}}{1-c} - \left(\frac{c}{1-c} \right) \cdot \frac{dM_{an}(H_C)}{dH}} \right] (20)$$

A similar procedure is repeated for the residual point ($M=+M_r$ and $H = 0$) using eqs. in the following order: (6) to define the so-called differential residual susceptibility χ'_r, then (4) and (5), then expressions for M_{irr} and dM_{irr}/dH are defined in this point, and the following is obtained:

$$M_r = M_{an}(M_r) + \frac{K}{\left(\frac{\alpha}{1-c} \right) + \frac{1}{\chi'_r - c \cdot \frac{dM_{an}(M_r)}{dH}}} \qquad (21)$$

The maximal excitation point ($M = +M_m$ and $H = H_m$) can also be used. If this point is close to the saturation point, the normal and anhysteresis differential susceptibility are almost the same ($\chi'_n \approx \chi'_{an} = \chi'_m$) and magnetization is practically equal to the irreversible value ($M = M_{irr} \approx M_m$). Differential susceptibility in the point of maximal excitation can be defined using eq.(6) as:

$$\chi'_m = \frac{M_{an}(H_m) - M_m}{K - \alpha \cdot (M_{an}(H_m) - M_m)}$$
$$+ c \cdot \left[\frac{dM_{an}(H_m)}{dH} - \frac{dM_{irr}(H_m)}{dH} \right] \qquad (22)$$

The approximations made in this point enable:

$$\frac{dM_{irr}(H_m)}{dH} = \frac{dM(H_m)}{dH} = \frac{dM_{an}(H_m)}{dH} \qquad (23)$$

And after exchanging into (22) and transformations:

$$M_m = M_{an}(H_m) - \frac{K \cdot \chi'_m}{\alpha \cdot \chi'_m + 1} \qquad (24).$$

4. The procedure used to solve the numerical model

Having in mind that the equations used for defining parameters of the Jiles-Atherton model can only be implicitly expressed as a function of the parameter being calculated and the remaining model parameters, the procedure used for determining these parameters must be iterative. Comparisons of values obtained in successive iterations are made, and if there is no significant change (for example: the deviation is smaller than $\xi=0.001$) the procedure is completed, and the solutions obtained are treated as final values. As input values we use 10 parameters (table 1).

The program was written using the *Mathematica* environment, as it requires the numerical solution of parameter differential equations, which are supported by this program packet. The parameter of reversible movement of the domain wall - *c* can be directly calculated using the starting normal susceptibility (eq. (15)). The starting value of the averaging parameter of the magnetic field is defined as $\alpha=0.001$, as this value is most often found for all isotropic materials, though another value (of the same order of magnitude) can be defined. The shape parameter of the anhysteresis curve - *a* can be initially calculated using eq. (17), while *K* is defined as 0.

Calculation of the pinning parameter of the domain wall *K* using eq. (20), followed by determination of parameter α using eq. (21) is performed in a program loop (which is exited when the difference between calculated parameter values in two successive loops is less than ξ). Parameter *a* is obtained in the same loop using eq. (24). In this case it is necessary to make a correct approximation of hyperbolic functions. In this concrete case developing this function using Bernoulli numbers approximates the coth function. This was later proved to be a correct move, as this function easily diverges, thus disrupting the complete calculation. By normalizing its argument, we have retained the argument value close to zero. The cubic equation obtained in this way was later solved using the *Mathematica 3.1* program packet. At the same time, investigation of the nature of solutions obtained and selection of real ones were essential.

The procedure for calculating parameters *K*, *a* and α is repeated the necessary number of times until the defined

accuracy is attained, though the developed program showed that results obtained converge to the final result in a small number of steps (2-5). Negative values were also obtained as final parameter values, which is normal, having in mind the nature of the hysteresis curve and the movement direction on the curve, when the sign of coefficients used in the calculation is changed. This is why the absolute value of the calculated parameter was taken in some cases. This did not have any influence of further accuracy of the procedure.

Fig. 1 shows the hysteresis curve for the Mn-Zn ferrite - 3C8 obtained by introducing model parameters (calculated using our program packet) into the *PSpice* program packet. Fig. 2 shows the hysteresis curve obtained using model parameters defined in the *PSpice* program packet for the same material.
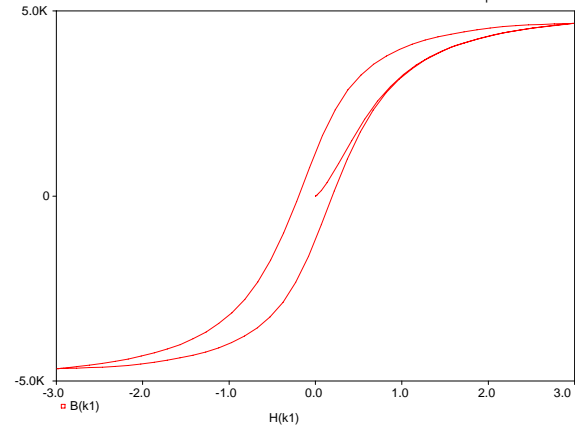
Fig. 1 Starting magnetization curve and hysteresis curve for the Mn-Zn ferrite - 3C8 obtained using model parameter values (calculated by our program) in the *PSpice* packet.
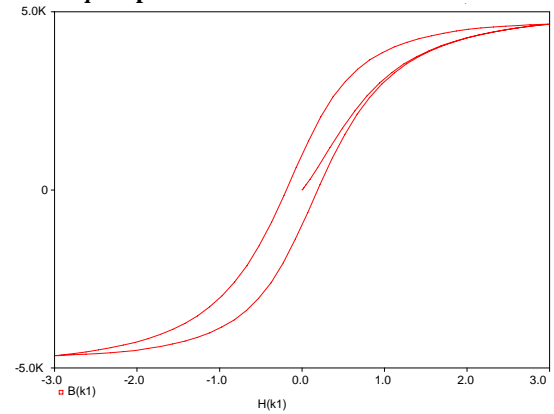
Fig. 2 Starting magnetization curve and hysteresis curve for the Mn-Zn ferrite 3C8 obtained using model parameter values defined in the *PSpice* program packet

Good agreement of both curves can be seen. Magnetic characteristics read from these curves are presented and compared with input values in table 1. We should

mention that the **PSpice** program also gives the dependence of magnetic induction B on the magnetic field strength H, which is the case when an experimental hysteresis curve is obtained. As these values do not figure in presented model equations (1-24), their connection with model variables is as follows:

$$M(H) = \frac{B(H)}{\mu_o} - H \qquad (25)$$

$$\chi = \mu_r - 1 \quad ; \quad \chi' \approx \mu_r' \qquad (26)$$

Where μ_r - the relative magnetic permeability (for soft magnetic materials $\chi \approx \mu_r$). The unit for magnetic induction in the **PSpice** program packet is Gauss - G (1G $= 10^{-4}$T), while Oersted - Oe (1Oe$\approx$79.58 A/m) is used as the magnetic field strength unit.

Table 1 Input values of magnetic characteristics for the Mn-Zn ferrite - 3C8, their values read from the curve obtained using calculated model parameter (Fig. 1) and values obtained using model parameters defined in the *PSpice* program packet

	input values	wrote from Fig.2	wrote from Fig.2
B_S (T)	0.5	0.5	0.5
χ'_{ian}	6500	6944	6006
χ'_{in}	2700	2028	2892
H_m (A/m)	240	238	238
B_m (T)	0.46	0.46	0.46
χ'_m	190	289	252
B_r (T)	0.1	0.15	0.98
χ'_r	4250	5088	5050
H_c (A/m)	16	22.4	15.1
χ'_{max}	6250	5388	5206

For hard magnetic core, due to the fact that one parameter is redundant, we chose for the feedback parameter a value α=1000 without any loss of generality. R_μ is obtained by a curve fit of the initial magnetization curve. The parameters H_1, β_1 and H_0, β_0 are determined such that they approximate the first and second quadrant, respectively. Fitting the second quadrant, initial guess for Kc, K, and H1 starts the computation. Typical start values are K_c=100...1000, K=1, and H_1=0. For a mathematical treatment M_{an} is hence replaced by M in (7). We demand that the second term of $g(.)$ is neglectible compared to the first in the second quadrant. Collects the linear part of (7), the curve fit is effected by selecting two points on demagnetization curve of the second quadrant, e.g. $(H_1$=0, M_1=$M_r)$, where M_r is the remanence magnetization and $(H_2$=-H_c/2, M_2=$M(H_2))$. The parameters β_0, H_0 are approximately given by

$$\beta_0 \approx \frac{\ln\left(c(H_1, M_1)\right) - \ln\left(c(H_2, M_2)\right)}{\ln(M_1) - \ln(M_2)} \qquad (27)$$

$$\hat{H}_0 \approx \frac{c(H_1, M_1)}{\left(\frac{M_1}{M_s}\right)^{\beta_0}}$$

$$c(H, M) = \frac{H}{K_c} + \alpha \frac{M}{M_s} - \frac{M/M_s}{R_\mu} \approx \hat{H}_0 \left(\frac{M}{M_s}\right)^{\beta_0}$$

If the slope of (7) becomes singular M_{an} changes abruptly from the upper to the lower branch and vice versa. The magnetization for which this singularity occurs is denoted as M_{Hc}. The field strength at M_{Hc} is therefore approximately H_c. The coercivity is nearly given by

$$M_{H_c}/M_s \approx {}^{(\beta_0-1)}\sqrt{\frac{\alpha - 1/R_\mu}{\beta_0 \cdot \hat{H}_0}} \qquad (28)$$

$$H_c \approx -K_c \left[\hat{H}_0 \left(\frac{M_{H_c}}{M_s}\right)^{\beta_0} - \left(\alpha - 1/R_\mu\right) \frac{M_{H_c}}{M_s} \right]$$

After choosing a better guess for K_c from (28), the procedure is repeated making use of (27). K is determined by approximating the slope of the magnetization curve near the corcivety making use of (9). Now β_1 and H_1 are computed by fitting the magnetization curve in the first quadrant. The parameters must be chosen such that they do not disturb the curve fit of the second quadrant, e.g., the second term of $g(M_{an}/M_s)$ in (7) must be negligible for $M_{an}<M_r$. We require that for $(H_1$=0, M_1=$M_r)$

$$\frac{\hat{H}_1 \left(\frac{M_r}{M_s}\right)^{\beta_1}}{\hat{H}_0 \left(\frac{M_r}{M_s}\right)^{\beta_0}} = k << 1 \qquad (29)$$

e.g., k=10^{-2}. The parameters β_1, H_1 are calculated by the same reasoning as before by (27) replacing β_0 by β_1 and H_0 by H_1.

The extended model has been used to fit measured curves of $Nd_2Fe_{14}B$ materials, which have uniaxial anisotropy. The modeled and measured curves are shown in Fig.3 and 4. The modeled curves show good agreement with the experimental results. For the anisotropic sample, the field was applied along its hard axis. The modeled texture level is t=0.32, which means statistically 32 per cent of the sample was oriented along a specific direction and the rest was randomly oriented.
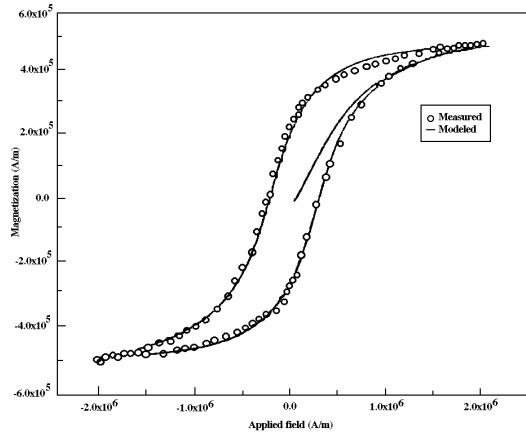
Figure 3: Modeled and measured curves of isotropic Nd$_2$Fe$_{14}$B material (Ms=610kA/m; k=750kA/m; a=420kA/m; alpha=1.5; c=0.6)
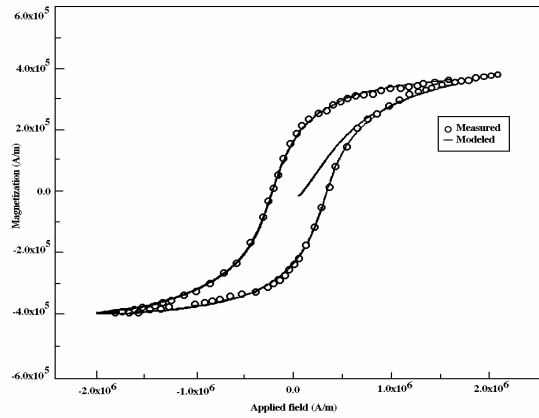
Figure 4: Modeled and measured curves of textured Nd$_2$Fe$_{14}$B material (Ms=610kA/m; k=750kA/m; a=420kA/m; alpha=2.3; c=0.6; Ka=4266J/cum; t=0.32; easy axis =001; field axis=100)

Fig.5 and 6 compares measured and simulated hysteresis curve of Alnico 2 and Alnico 5 alloys. The parameters were computed using (27) and (28). The deviation of the measured to the simulated values is not significant for all two core materials.
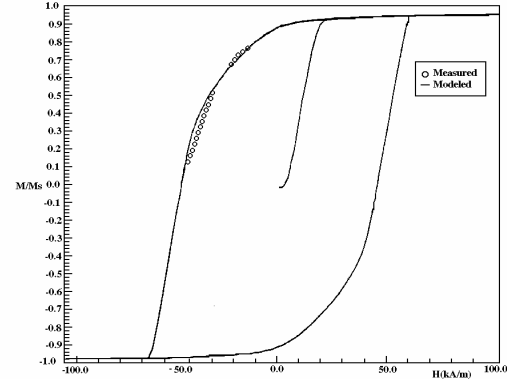
Figure 5: Simulation curve of an Alnico 2 core (β_0=1.2; H$_0$=1020; K=800; K$_c$=450; R$_\mu$=1; α=10^3)
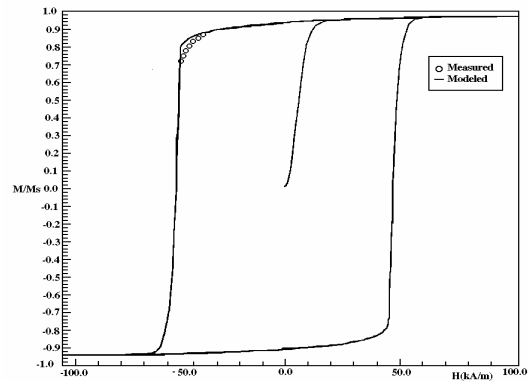
Figure 6: Simulation curve of an Alnico 5 core (β_0=16; H$_0$=2160; K=2000; K$_c$=60; R$_\mu$=1; α=10^3)

It can be noted that this algorithm required a significant number of input data, not given by any producers of magnetic materials, which necessitated the development of a procedure for their calculation. We are able to experimentally record the hysteresis curve and starting magnetization curve for any magnetic material. These curves are input into the *Matlab* program packet, which is used to determine values of input data required by the iterative procedure. The required slope coefficients are obtained using slope values for two near points. The value of saturation induction B_s can be calculated (predicted) using the maximal induction B_m value obtained from the experimental curve, as the converging point.

5. Conclusion

The problem of determining parameters of the Jiles-Atherton model for R, Z, F-shaped hysteresis curves of magnetic materials was analyzed in this paper. The algorithm was tested at the simulation level, using the *PSpice* program packet. Results obtained showed good agreement with expected values. The presented algorithm is of an adaptive type in accordance with type of magnetic materials. Expansion of this program to classes

of anisotropy (i.e. for Z and F-shaped hysteresis curves) and hard magnetic materials is made, as corresponding physical-mathematical models have been developed [3,6]. This enable the calculation of parameters required in the analysis of complex electronic circuits using the **PSpice** program packet for a wide class of magnetic materials. The proposed algorithm suitable for on-line measurements. Finally, we obtained results which much better then known results.

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