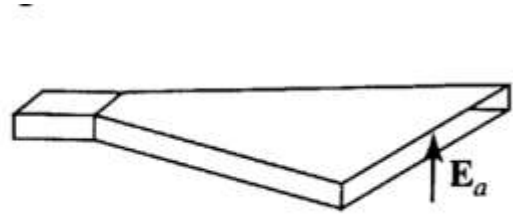


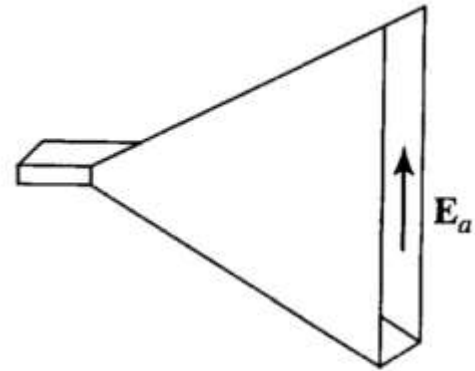
# HORN Antenna Design

- The rectangular horns are ideally suited for rectangular waveguide feeds. The horn acts as a gradual transition from a waveguide mode to a free-space mode of the EM wave. When the feed is a cylindrical waveguide, the antenna is usually a conical horn.
- Horn antennas are popular in the microwave bands (above 1 GHz). Horns provide high gain, low VSWR (with waveguide feeds), relatively wide bandwidth, and they are not difficult to make.

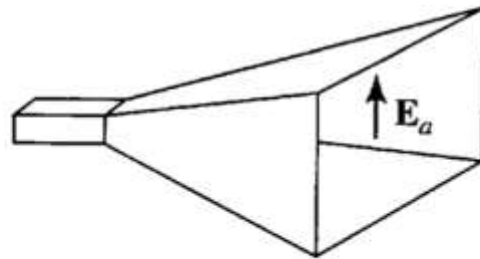
# 3 Types..



(a) *H*-plane sectoral horn.

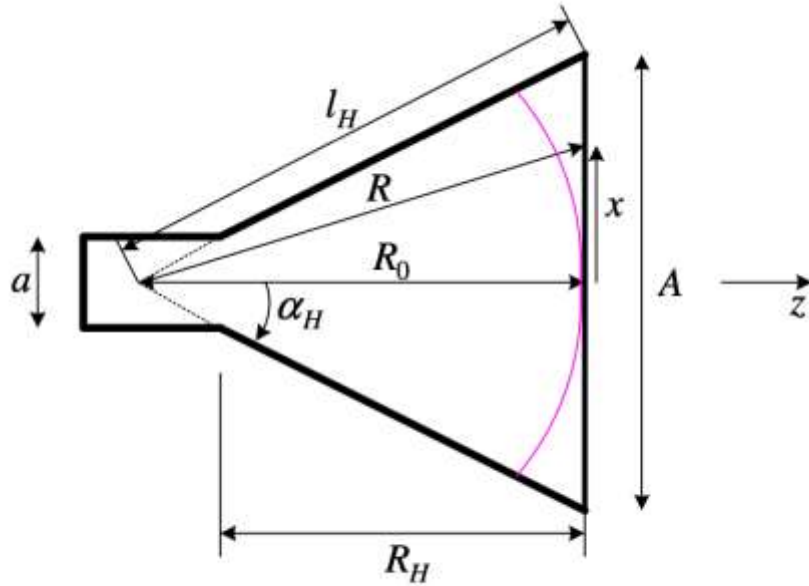


(b) *E*-plane sectoral horn.



(c) Pyramidal horn.

# H-plane (x-z)



H-PLANE (X-Z) CUT OF AN *H*-PLANE SECTORAL HORN

$$l_H^2 = R_0^2 + \left(\frac{A}{2}\right)^2,$$

$$\alpha_H = \arctan\left(\frac{A}{2R_0}\right),$$

$$R_H = (A - a) \sqrt{\left(\frac{l_H}{A}\right)^2 - \frac{1}{4}}.$$

The amplitude pattern of the *H*-plane sectoral horn is obtained as

$$\bar{E}(\theta, \varphi) = \left(\frac{1 + \cos \theta}{2}\right) \cdot \left[ \frac{\sin(0.5\beta b \cdot \sin \theta \cdot \sin \varphi)}{(0.5\beta b \cdot \sin \theta \cdot \sin \varphi)} \right] \cdot I(\theta, \varphi).$$

$$D_H = \frac{b}{\lambda} \frac{32}{\pi} \left( \frac{A}{\lambda} \right) \varepsilon_{ph}^H = \frac{4\pi}{\lambda^2} \varepsilon_t \varepsilon_{ph}^H (Ab)$$

where

$$\varepsilon_t = \frac{8}{\pi^2};$$

$$\varepsilon_{ph}^H = \frac{\pi^2}{64t} \left\{ [C(p_1) - C(p_2)]^2 + [S(p_1) - S(p_2)]^2 \right\};$$

$$p_1 = 2\sqrt{t} \left[ 1 + \frac{1}{8t} \right], \quad p_2 = 2\sqrt{t} \left[ -1 + \frac{1}{8t} \right];$$

$$t = \frac{1}{8} \left( \frac{A}{\lambda} \right)^2 \frac{1}{R_0 / \lambda}.$$

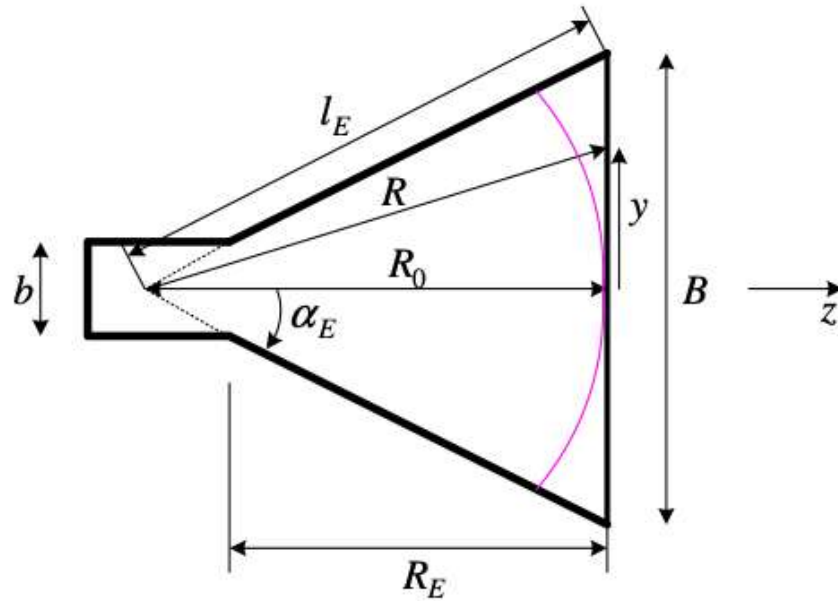
Epsilon ph is the phase aperture efficiency factor.

Epsilon t is the taper factor

From the family of directivity curves it is clear that for a given axial length  $R_0$  and at a given wavelength, there is an optimal aperture width  $A$  corresponding to the maximum directivity.

# E-plane (y-z)

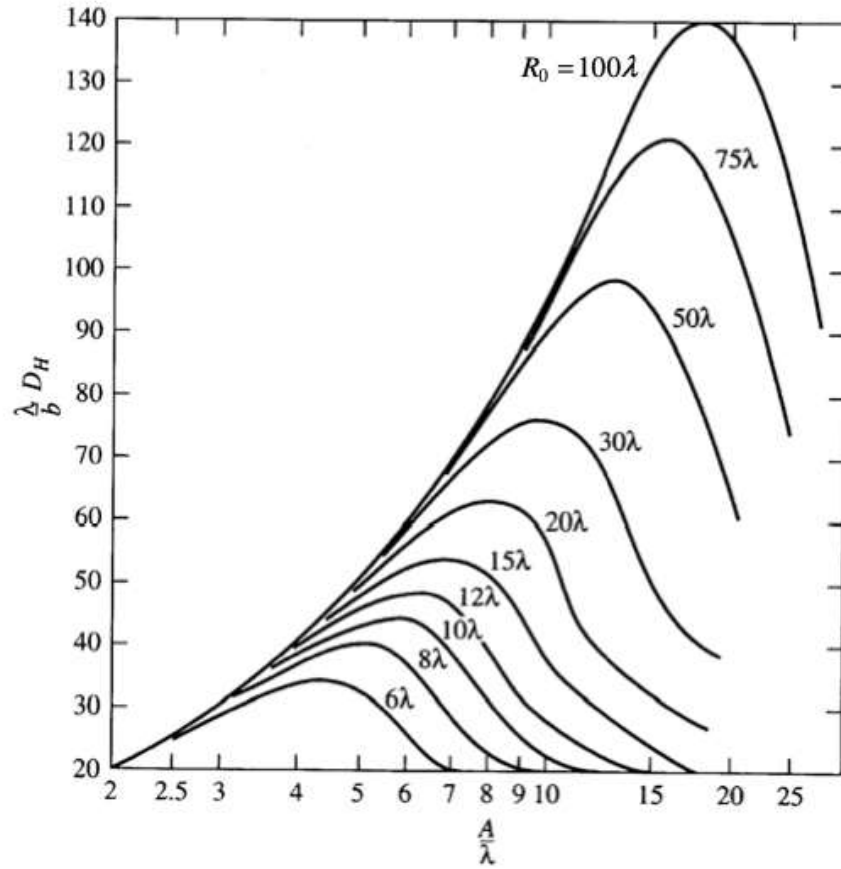
## 1.2 The *E*-plane sectoral horn



*E*-plane (y-z) cut of an *E*-plane sectoral horn

$$D_E = \frac{a}{\lambda} \frac{32}{\pi} \frac{B}{\lambda} \epsilon_{ph}^E = \frac{4\pi}{\lambda^2} \epsilon_t \epsilon_{ph}^E aB,$$

$$\epsilon_t = \frac{8}{\pi^2}, \quad \epsilon_{ph}^E = \frac{C^2(q) + S^2(q)}{q^2}, \quad q = \frac{B}{\sqrt{2\lambda R_0}}.$$



[Stutzman&Thiele, Antenna Theory and Design]

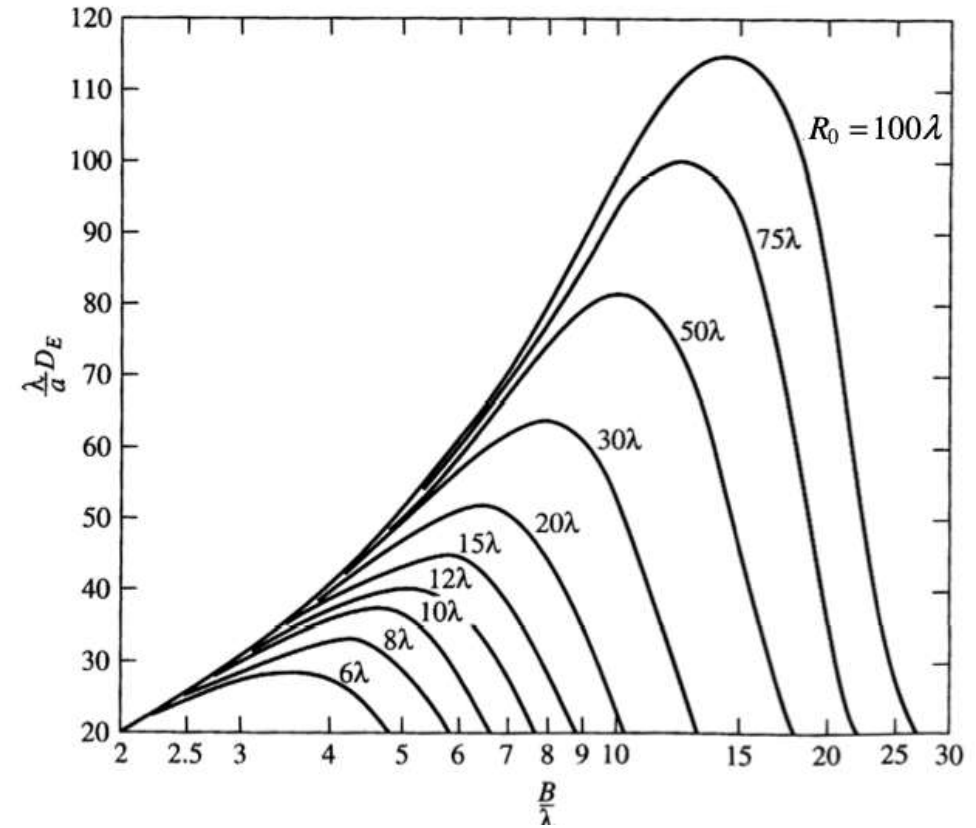
It can be shown that the optimal directivity is obtained if the relation between  $A$  and  $R_0$  is

$$A = \sqrt{3\lambda R_0}, \quad (18.24)$$

or

$$\frac{A}{\lambda} = \sqrt{3 \frac{R_0}{\lambda}}. \quad (18.25)$$

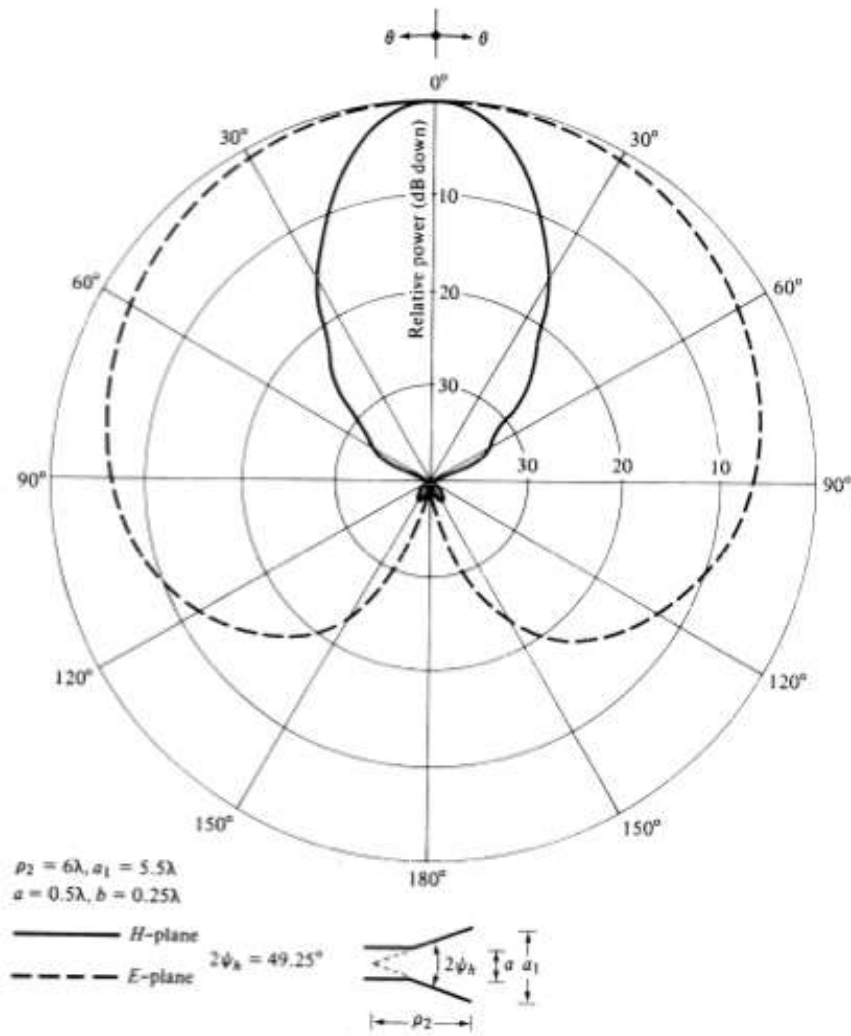
A family of universal directivity curves  $\lambda D_E / a$  vs.  $B / \lambda$  with  $R_0$  being a parameter is given below.



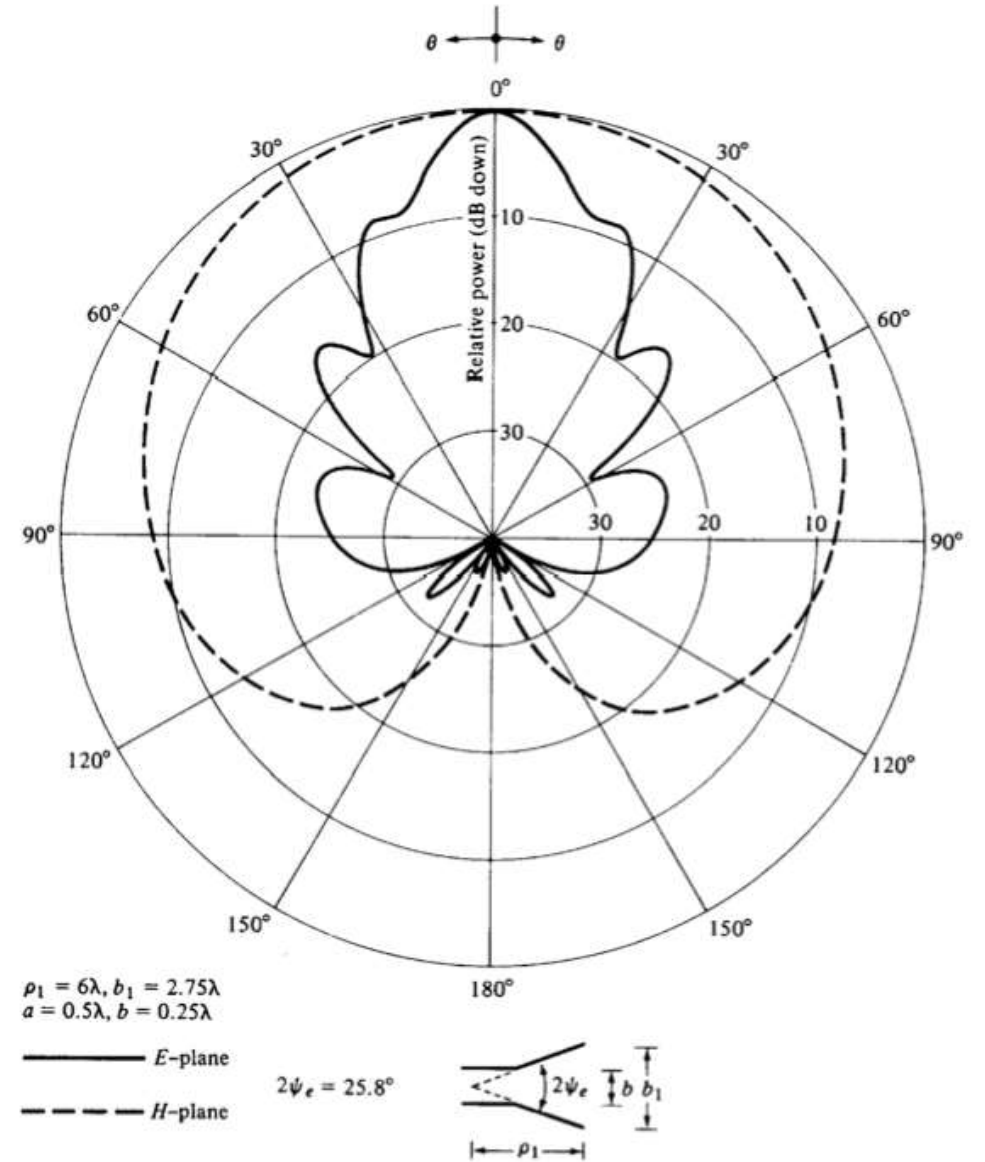
The optimal relation between the flared height  $B$  and the horn apex length  $R_0$  that produces the maximum possible directivity is

$$B = \sqrt{2\lambda R_0}. \quad (18.37)$$

# *E*- AND *H*-PLANE PATTERN OF *H*-PLANE SECTORAL HORN



# *E*- AND *H*-PLANE PATTERN OF *E*-PLANE SECTORAL HORN



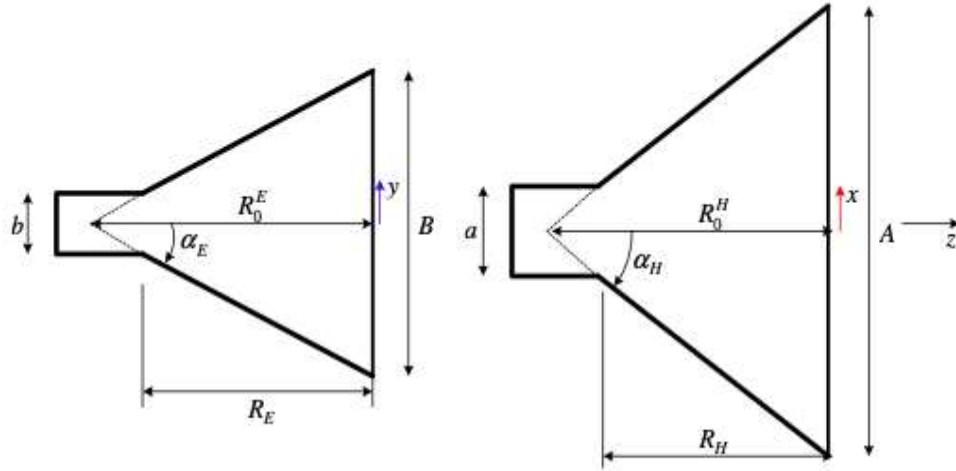


# Pyramidal horn antenna

- The pyramidal horn is a popular antenna in the microwave frequency ranges (from  $\approx 1$  GHz to 30 GHz). The feeding waveguide is flared in both directions, the  $E$ -plane and the  $H$ -plane.
- All results are combinations of the  $E$ -plane and  $H$ -plane sectoral horn analyses.
- The directivity of the pyramidal horn can be found by introducing the phase efficiency factors of both planes and the taper efficiency factor of the  $H$ -plane.
- The gain of a horn is usually very close to its directivity because the radiation efficiency is very good (low losses). The directivity, as calculated, is very close to measurements.

- The optimal directivity of an  $E$ -plane horn is achieved at  $q=1$
- $\epsilon_{ph} E = 0.8$ .
- The optimal directivity of an  $H$ -plane horn is achieved at  $t = 3/8$ ,  $\epsilon_{ph} H = 0.79$
- $\epsilon_{ph} P = 0.632 * \epsilon_t = 0.51$
- Therefore, the best achievable directivity for a rectangular waveguide horn is about half that of a uniform rectangular aperture.
- We reiterate that the best accuracy is achieved if  $\epsilon H$  and  $\epsilon E$  are calculated numerically without using the second-order phase approximation.
- For a horn antenna to be realizable, the following must be true :

$$RE = RH = RP.$$



It can be shown that

$$\frac{R_0^H}{R_H} = \frac{A/2}{A/2 - a/2} = \frac{A}{A-a}, \quad (18.43)$$

$$\frac{R_0^E}{R_E} = \frac{B/2}{B/2 - b/2} = \frac{B}{B-b}. \quad (18.44)$$

The optimum-gain condition in the  $E$ -plane (18.37) is substituted in (18.44) to produce

$$B^2 - bB - 2\lambda R_E = 0. \quad (18.45)$$

There is only one physically meaningful solution to (18.45):

$$B = \frac{1}{2} \left( b + \sqrt{b^2 + 8\lambda R_E} \right). \quad (18.46)$$

Similarly, the maximum-gain condition for the  $H$ -plane of (18.24) together with (18.43) yields

$$R_H = \frac{A-a}{A} \left( \frac{A^2}{3\lambda} \right) = A \frac{(A-a)}{3\lambda}. \quad (18.47)$$

$$D_P = \frac{4\pi}{\lambda^2} \epsilon_t \epsilon_{ph}^E \epsilon_{ph}^H AB,$$

where

$$\epsilon_t = \frac{8}{\pi^2};$$

$$\epsilon_{ph}^H = \frac{\pi^2}{64t} \left\{ [C(p_1) - C(p_2)]^2 + [S(p_1) - S(p_2)]^2 \right\};$$

$$p_1 = 2\sqrt{t} \left[ 1 + \frac{1}{8t} \right], \quad p_2 = 2\sqrt{t} \left[ -1 + \frac{1}{8t} \right], \quad t = \frac{1}{8} \left( \frac{A}{\lambda} \right)^2 \frac{1}{R_0^H / \lambda};$$

$$\epsilon_{ph}^E = \frac{C^2(q) + S^2(q)}{q^2}, \quad q = \frac{B}{\sqrt{2\lambda R_0^E}}.$$

Substituting in the expression for the horn's gain,

$$G = \frac{4\pi}{\lambda^2} \epsilon_{ap} AB,$$

gives the relation between  $A$ , the gain  $G$ , and the aperture efficiency  $\epsilon_{ap}$

$$G = \frac{4\pi}{\lambda^2} \epsilon_{ap} A \frac{1}{2} \left( b + \sqrt{b^2 + \frac{8A(a-a)}{3}} \right),$$

$$\Rightarrow A^4 - aA^3 + \frac{3bG\lambda^2}{8\pi\epsilon_{ap}} A - \frac{3G^2\lambda^4}{32\pi^2\epsilon_{ap}^2} = 0.$$

- The optimum gain value of  $\epsilon_{ap} = 0.51$  is usually used, which makes the equation a fourth-order polynomial equation in  $A$ . Its roots can be found analytically and numerically.
- In a numerical solution, the first guess is usually set at  $A(0) = 0.45\lambda(G)^{-0.5}$ . Once  $A$  is found,  $B$ ,  $RE = RH$  can be computed.
- Horn antennas operate well over a bandwidth of 50%. However, gain performance is optimal only at a given frequency.

# Design Parameters

- **S11 (Return Loss):** Reflection coefficient at the antenna's input port. Indicates how much power is reflected from the antenna instead of being radiated, a lower value (e.g., -10 dB or less) signifies better impedance matching between the antenna and the transmission line. **Ideal Value:**  $S_{11} < -10$  dB over the operating frequency range (less than 10% of power is reflected).
- The radiation pattern corresponds to the energy distribution of the radiated power by the antenna. The antenna converts received signal/electrical power into an electromagnetic wave. The AZ pattern is a radiated wave of power density/gain/directivity along the azimuthal plane at fixed elevation.
- Taper angle: Basically the cone angle. A wider taper angle typically produces a broader radiation pattern, while a narrower one produces a more focused beam. **Gain and Directivity**-Narrow taper angles generally increase antenna gain and directivity by focusing the energy into a smaller area. A smoother taper angle ensures better impedance matching, reducing reflected power.
- Aperture loss – Results from spillover of radiated power, unmatched radiation pattern, poor feed design, edge diffraction.

- It shows how the antenna radiates energy horizontally around its axis, indicating whether the coverage is uniform or directional. A parabolic dish antenna will show a **highly directional main lobe** with minimal side lobes in the AZ plane, while an omnidirectional antenna will have a circular pattern.
- **Beamwidth:** The **3-dB beamwidth** (angular width of the main lobe at half its maximum power) can be determined. This indicates the area of strong signal coverage in the azimuth plane.
- **Directionality:** The shape of the pattern helps identify the directionality. Directional antennas (e.g., parabolic dishes) will have a single, narrow lobe.
- **Interference Analysis:** The pattern helps identify **side lobes** or **back lobes**, which can lead to unwanted radiation or interference.
- The 11dB gain is the maximum value to which the antenna gain can fall within the taper angle.