Distributed Systems

Pearson Correlation Algorithm

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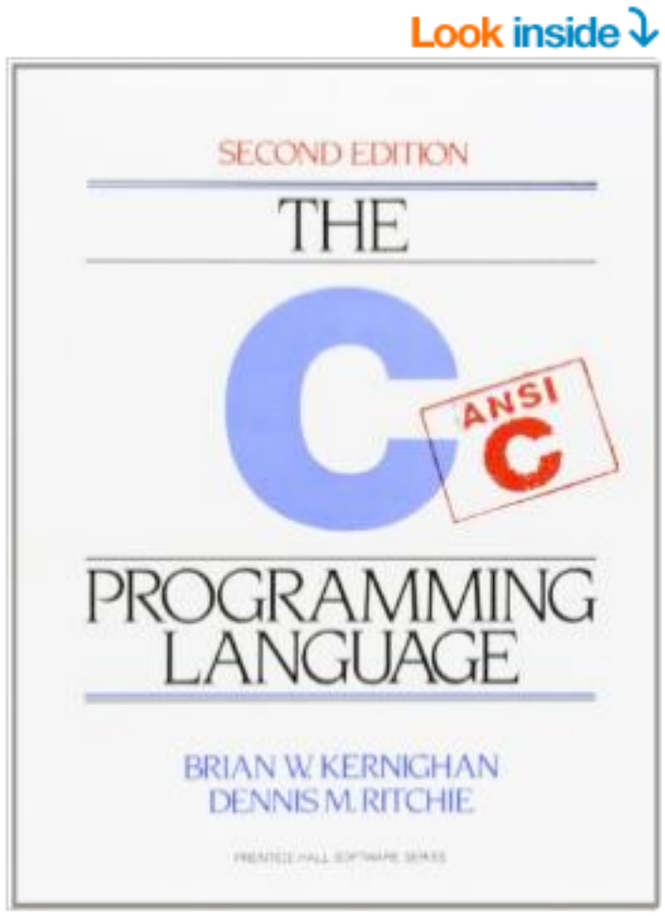
Context



A ‘Recommendation System’ is a functionality of an Information System that allows to make recommendations of products, contents, objects, or many other things to users within that system (depending of the type of application or system). The pioneer in this approach is Amazon. When someone browses Amazon, the system will recommend or make suggestions of products based on the users’ previous purchases, the history of other users with similar behavior, preferences, etc.

The C Programming Language - 2nd Edition
http://www.amazon.com/dp/0131103628/

Context

There are many techniques or methods to implement ‘Recommendation Systems’, specially in movies, but the one we’re going to use its based on the historical behavior of all registered users in a movie website, finding potential similar users (and movies to watch) by comparing the ratings they’ve given to the same movies.

	Movie 1	**Movie** 2	**Movie** 3	**Movie** 4	**Movie** 5	**Movie** 6	**Movie** 7	**Movie** j	**Movie** M
User 1	0	1	2	1	0	1	4	2	2
User 2	4	0	4	2	4	3	0	4	0
User 3	3	2	5	0	2	0	5	4	0
User 4	5	3	2	1	1	0	3	0	1
User 5	2	0	1	2	0	4	4	4	5
User 6	5	1	2	1	4	0	0	3	1
User 7	2	2	1	4	4	0	5	2	4
User i	5	0	1	3	4	2	5	5	2
User U	1	1	2	1	0	4	4	4	4

(5) : Rating that **User** i has given to **Movie** j

M: Total Amount of Available **Movies**
U: Total Amount of Registered **Users**
B: Total Amount of **most correlated Users** we want to find for each given User.

0 : Haven’t watched Movie

1 **Worst**
2
3 **Rating Scale**
4
5 **Best**

Ideally, what those ‘Recommendation Systems’ do, is once the most suitable ~ compatible users are found, then suggestions are made to the current user, where such suggestions are the things the user haven’t watched, but that the compatible user(s) did and liked the most.

Correlation

How can we find such Compatibility between users? We're going to find the **Correlation** between them. In Statistics, Correlation refers to statistical relationships in random variables, measuring how similar or non deviated are them. In this case, our random variables will be the ratings each user give to a set of movies.

	Movie 1	Movie 2	Movie 3	Movie 4	Movie 5	Movie 6	Movie 7	Movie j	Movie M
User 1	0	1	2	1	0	1	4	2	2
User 2	4	0	4	2	4	3	0	4	0
User 3	3	2	5	0	2	0	5	4	0
User 4	5	3	2	1	1	0	3	0	1
User 5	2	0	1	2	0	4	4	4	5
User 6	5	1	2	1	4	0	0	3	1
User 7	2	2	1	4	4	0	5	2	4
User i	5	0	1	3	4	2	5	5	2
User U	1	1	2	1	0	4	4	4	4

Finding the Correlation between **3's & 7's** ratings.

$$\rho_{X,Y} = \operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

← Covariance

← Standard Deviations

$$cov(x,y) = \frac{\sum_{i-1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)}$$

(n - 1) cancels out

X_i / Y_i each element

$\overline{X}$ / $\overline{Y}$ ratings' mean

Δ_i - $\overline{\Delta}$ distance to mean

for x $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$ for y $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

This aims to find similar ratings in movies and similar means.

Resulting Formula: Pearson Correlation

Standard statistical measures are often used to calculate the similarity between users in step 1 (e.g. Spearman correlation, Pearson correlation, etc.).[32] In this work, similar users are found using the Pearson correlation coefficient formula (7.1):

$$\operatorname{corr}_{a,u} = \frac{\sum_{i=1}^{m}(r_{a,i} - \bar{r_a}) \times (r_{u,i} - \bar{r_u})}{\sqrt{\sum_{i=1}^{m}(r_{a,i} - \bar{r_a})^2} \times \sqrt{\sum_{i=1}^{m}(r_{u,i} - \bar{r_u})^2}} \qquad (7.1)$$

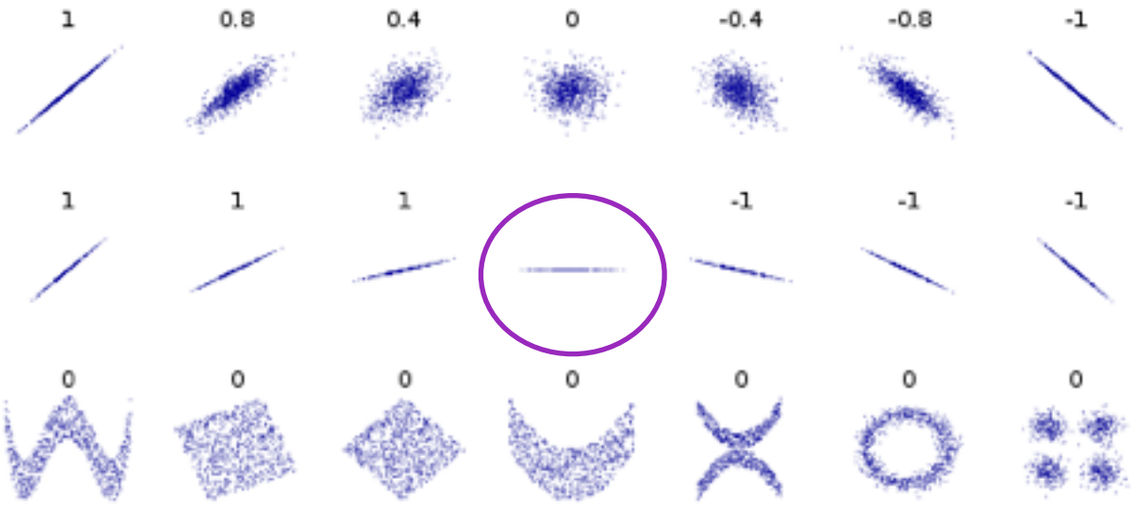
Personalization Techniques and Recommender Systems
By G. Uchyigit, Matthew, M. Y. Ma

http://books.google.com.co/books?id=tKWJArCo7msC&pg=PA172

Result ~ Issues

The result will be a value between -1 and 1. **-1 or 1 means a very strong correlation, and closer to 0 means that there's a very weak correlation, or nothing can be inferred from the current set.** The sign shows if its a positive or a negative correlation, being 1 a direct relationship, and -1 an inverse relationship (anti-correlation).

https://en.wikipedia.org/wiki/Correlation_and_dependence



⊖ Edge case:

The variance of a user's ratings is 0. Nothing can be inferred.
How is it possible? When a user has rated **ALL** the available movies with the same value, so distance to mean is **ALWAYS** 0.
Or a user hasn't watched **ANY** single movie, so all ratings are 0, therefore implying what was said, mean is 0, distances are 0, then **Standard Deviation** is 0, so **corr(X,Y)** divides by 0.
If we find such a case, we'll simply return a correlation of 0.

$$\rho_{X,Y} = \operatorname{corr}(X,Y) = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

← Covariance
← Standard Deviations

$$cov(x,y) = \frac{\sum_{i-1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)}$$

(n - 1) cancels out
X_i / Y_i each element
$\bar{X}$ / $\bar{Y}$ ratings' mean
Δ_i - $\bar{\Delta}$ distance to mean

for x $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$ for y $s_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

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Multiprocessing

*Using **OpenMPI**, design with **PCAM***

*Given a **Ratings Matrix** with the Ratings **U** Users have given to **M** Movies, we want to obtain a **Recommendation Matrix** with the **B** Users all **U** users are the most correlated with.*

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M i	M U
U 1	0	1	2	1	0	1	4	2	2
U 2	4	0	4	2	4	3	0	4	0
U 3	3	2	5	0	2	0	5	4	0
U 4	5	3	2	1	1	0	3	0	1
U 5	2	0	1	2	0	4	4	4	5
U 6	5	1	2	1	4	0	0	3	1
U 7	2	2	1	4	4	0	5	2	4
U i	5	0	1	3	4	2	5	5	2
U U	1	1	2	1	0	4	4	4	4

Values: Movie Ratings

0 : Haven't watched Movie

1 **Worst**
2
3 **Rating Scale**
4
5 **Best**

Ratings Matrix

	B 1	B 2	B 3	B B
U 1	5	4	2	3
U 2	1	3	8	5
U 3	4	1	5	9
U 4	2	3	6	5
U 5	7	3	2	1
U 6	8	4	2	3
U 7	9	5	1	3
U i	2	1	4	3
U U	3	2	5	9

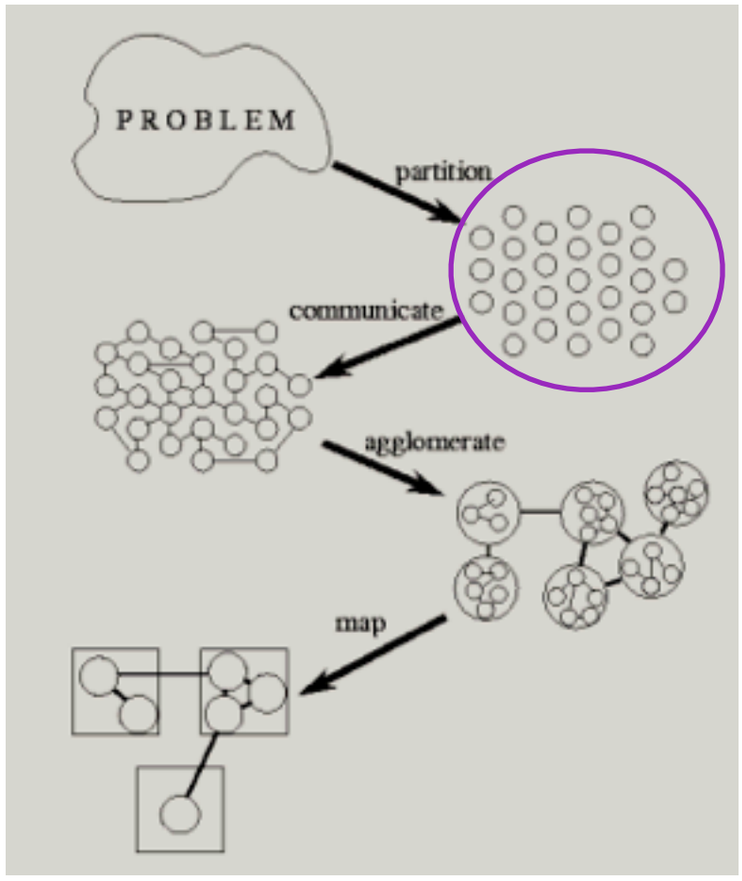
Values: User Indices

B
3
2
1 **Correlation Order**

Recommendation Matrix

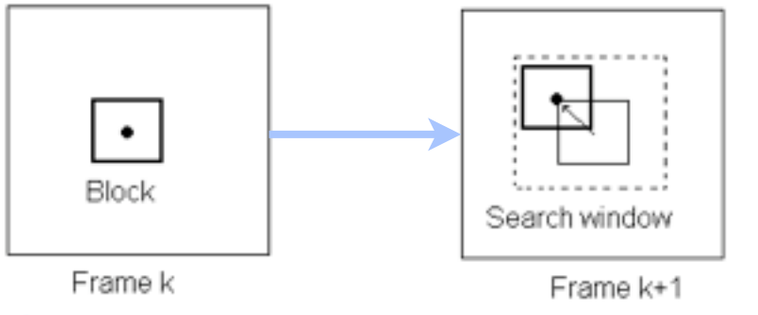
PCAM - Partition

Data



We've devised some strategies to '**partition'** the program to be distributed across several processing nodes. First and foremost, remembering a previous project, the **Motion Vector Finding Project**, there were two particular things given to each node: First, a **single, big, invariant** search area *(the **target frame**)*, and second, a **lot of small** bits of data to find in that search area *(macro-blocks from the **origin frame**)*. The search area was given as a whole because it was **invariant**, and a complete search was to be done on it to find the displacements of macro-blocks against the previous frame *(being the benefits of multiprocessing a significant reduction in time by having several nodes perform separately such a computationally intensive complete search for each macro-block)*. The previous frame wasn't sent in its entirety, because each processing node only had to process some fixed certain parts of it, so its useless to send all the data from the origin frame *(So such a data partitioning into small bits instead of sending it all would be helpful with time and space, but mostly space, and transmission)*.

Motion Vector Finding Project



Only the blocks the current node cares about.

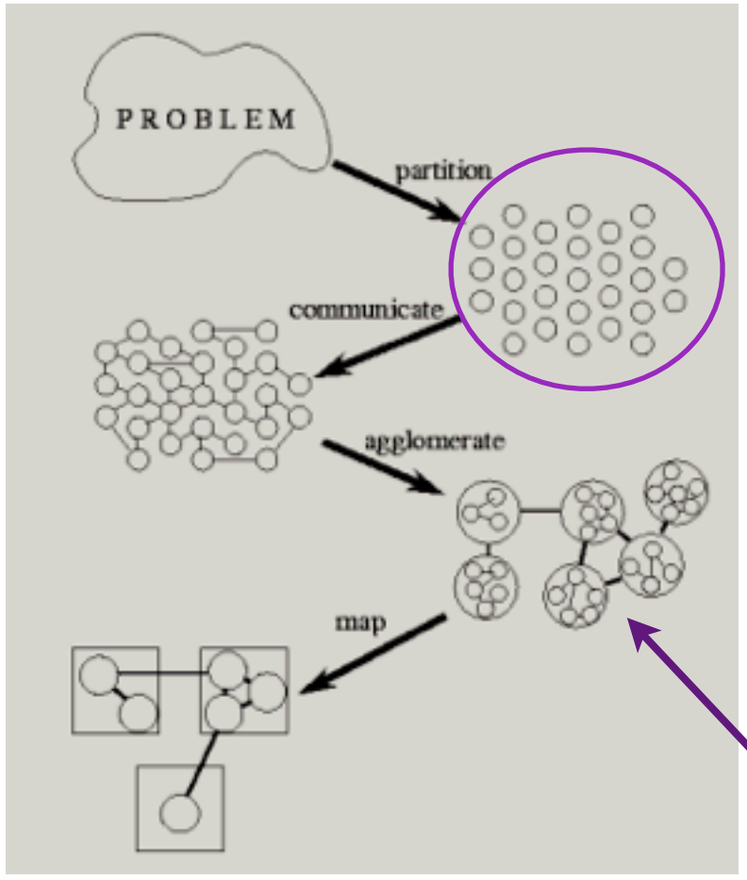
The full target frame for performing a complete search over it.

Now, back to the **Pearson Correlation Algorithm Project**, the search area will be the matrix that holds the ratings each user has given to each movie, then a complete search must be done over it because we have to compare **every** user against **every other** user to find the most correlated users. **But then**, the curious thing is that the data to compare from *(i.e. the current user)* **is ALSO CONTAINED** in this search area *(after all, its a matrix for all possible users)*, **so it seems that only sending the original matrix will suffice and no division of data is required.**

So, we'll simply send to each node the matrix with the ratings, no divisions, no subdata, there will be only a functional partitioning. The opportunity found in multiprocessing is having separate nodes compute the best correlated users for a certain users assigned with a deterministic algorithm (in a round robin sequence).

PCAM - Partition

Data



A division of data **CAN** be done, though it would be very complex, as in:

1. The **Master** has to load the whole data into its memory
2. The **Master** sends to each **Node** the rows *(users)* they care about
3. The **Master** sends to each **Node** just a part *(half?)* of the dataset to perform the correlation algorithm *(full search)* in with the given rows *(users)*.
4. The **Nodes** process that part *(half?)* of the dataset against the rows *(users)* that were sent to them.
5. The **Master** sends to each **Node** another part *(remaining half?)* of data.
6. The **Nodes** process this new part *(half?)* with the rows (users) they already were assigned with previously, and then **merge** internally in each block the results with the results of the previous part *(half?)* retaining only the best.

Such a division is only helpful in case the nodes have **very** limited memory, but being the case plain text files, we won't worry about such a case. Also would be slower because an increased number of connections to be opened across the network. **Also we are assuming multiprocessing on Machine level, if we were to go a level deeper (In each Machine's Processors) this could be applied, each Machine has the whole dataset, only a certain Users to calculate, and then, give each available Processor a part of the Dataset, to find independently within the Machine the best correlated Users, and then perform a Merge at the end. Such a Merge would be the Agglomeration part of PCAM.**

Node A Users/Rows

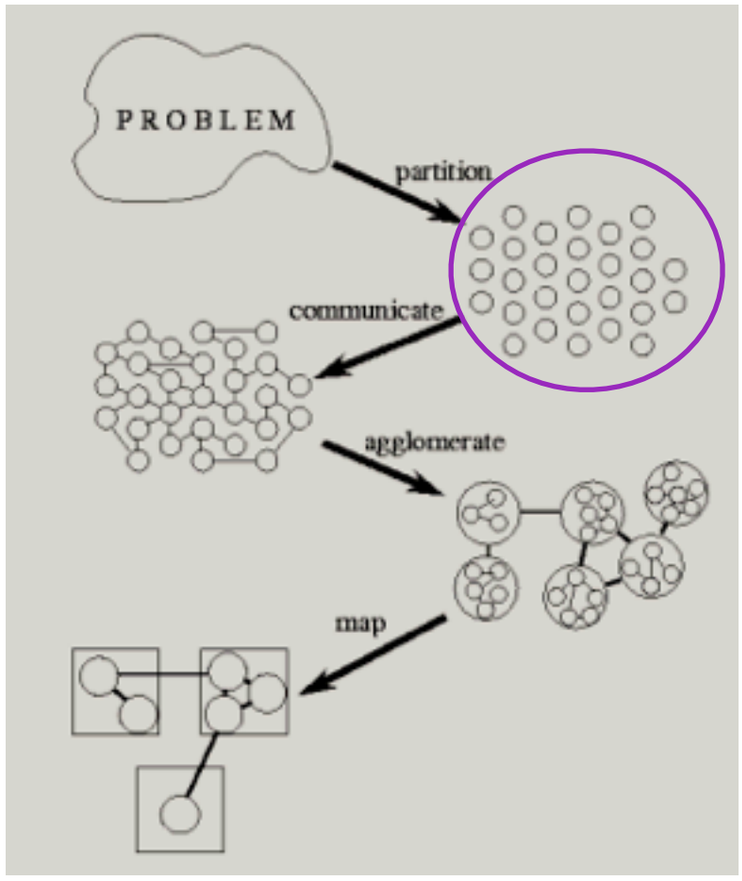
	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M j	M U
U 1	0	1	2	1	0	1	4	2	2
U 2									
U 3	3	2	5	0	2	0	5	4	0
U 4									
U 5	2	0	1	2	0	4	4	4	5
U 6									
U 7	2	2	1	4	4	0	5	2	4
U i									
U U	1	1	2	1	0	4	4	4	4

Node B Users/Rows

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M j	M U
U 1									
U 2	4	0	4	2	4	3	0	4	0
U 3									
U 4	5	3	2	1	1	0	3	0	1
U 5									
U 6	5	1	2	1	4	0	0	3	1
U 7									
U i	5	0	1	3	4	2	5	5	2
U U									

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M j	M U	
U 1	0	1	2	1	0	1	4	2	2	
U 2	4	0	4	2	4	3	0	4	0	**First Part**
U 3	3	2	5	0	2	0	5	4	0	
U 4	5	3	2	1	1	0	3	0	1	
	... (n/2 size of data) ...									
U 5	2	0	1	2	0	4	4	4	5	
U 6	5	1	2	1	4	0	0	3	1	**Last Part**
U 7	2	2	1	4	4	0	5	2	4	
U i	5	0	1	3	4	2	5	5	2	
U U	1	1	2	1	0	4	4	4	4	

PCAM - Partition Functionality



In terms of functionality the program consists of 3 parts:

An initial conditional block where the **Master** loads the file and starts sending it to all other **Nodes** (Master already counts as a **Node**), and where **Workers** will listen to such incoming data.

Then, a non-conditional block of code, which all **Nodes,** be it the **Master** or a **Worker,** will run, and its calculating the correlation of a ***given user*** against ***all other users,*** and storing just the best **B** users.
To determine the current user a node will process, a deterministic algorithm will be applied in a loop. Then compare such an user with everyone, of course, excluding itself ;-)

```
for(int U1 = TASK_ID; U1 < NUM_USERS; U1 += NUM_PROCS) {
```

Equivalent

```
    int *recList;
    recList = findBests(NUM_USERS, NUM_MOVIES, NUM_BEST, U1, matrixUI);
    // Will get the indices of the top B Users the U1 User correlates
    // the most with depending on the ratings the pairs of Users have.
    results.push_back(recList);
    // Store the result for later merging them all.
```

U1 ← **TASK_ID** + **(i * NUM_PROCS)**

- **U1**: User to process
- **TASK_ID**: ID of the current task, acting like an offset.
- **i**: increasing iterator
- **NUM_PROCS**: Total number of processes given MPI was run with.

Do this until **U1** is a value that exceeds the total amount of users.

function each Node calls for all users it deals with:

```
int* findBests(int NUM_USERS, int NUM_MOVIES, int NUM_BEST,
               int U1, int *matrixUI) {
// Will return the NUM_BEST best matches for a user U1
// It has a loop that will call findCorrelation between U1 and
// every other user, designed as U2.
```

Now, for all other users, as said, designed as U2:

```
findCorrelation(U1, U2, NUM_MOVIES, matrixUI)
```

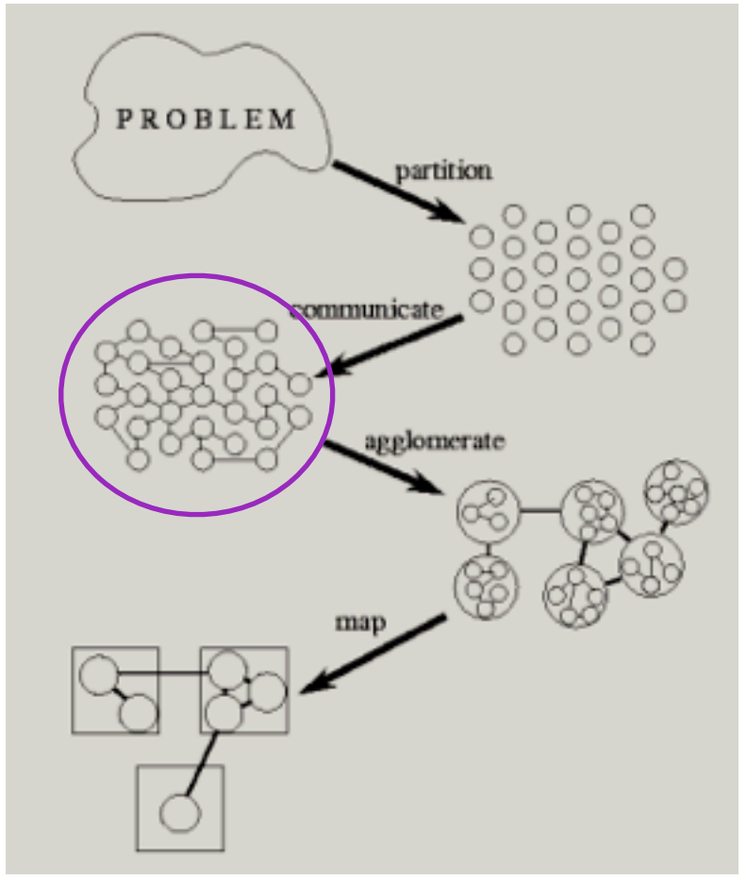
Returning only the NUM_BEST most correlated.

```
double findCorrelation(int U1, int U2, int NUM_MOVIES, int *matrixUI) {
```

findCorrelation is the application of the Pearson Correlation Algorithm that was handed to us.

Then comes the third block of the program, which is again a conditional block, where the **Workers** will send back to the **Master** the **Recommendation Lists** they made for their respective Users. **The Master, will merge all these Recommendation Lists from the Workers (and the Recommendations Lists it calculated itself) into a final Recommendation Matrix.** The **Recommendation Matrix** is the final output of the program, and its a **U * B matrix**, where the first dimension is the number of **Users (U)** and the second dimension is the amount of **Best Matches** we wanted to find **(B)**. **For each User then, there will be B values, which are the indices of the B Users this User is the most correlated with.**

PCAM - Communication



The communication we've established will be to distribute the data among **Nodes** in a **Round Robin** fashion. The Nodes will simply receive the **Ratings Matrix** once *(we are assuming we don't have an Storage distributed across a Network, so instead of simply sending the filename to open it in each Node, we're sending all the contents)*.

From there on each **Node** will calculate several **Recommendation Lists** for Users its concerned with. At the end, the **Nodes** will broadcast back to the **Master** all these **Recommendation Lists**, the Blocking Operations **RECV and SEND** are to be used, but it doesn't really matter because the **Master** will be listening to **MPI_ANY_SOURCE** *(meaning its not blocked waiting for a message from a certain specific Node)*, instead the first message that arrives will be grabbed immediately by the **Master**. Then, how can we know which **Node** sent it? Using the status of the request we can know the ID of such **Node**. Knowing exactly where a result request comes from, helps **MAPPING** it in the overall result, but waiting for a specific ID will make the use of blocking operations go very slow and crazy waiting, so, listen for any incoming request **(U times)**, and then check where did it come from.

Node Incoming Data: *all the Ratings Matrix*

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M i	M U
U 1	0	1	2	1	0	1	4	2	2
U 2	4	0	4	2	4	3	0	4	0
U 3	3	2	5	0	2	0	5	4	0
U 4	5	3	2	1	1	0	3	0	1
U 5	2	0	1	2	0	4	4	4	5
U 6	5	1	2	1	4	0	0	3	1
U 7	2	2	1	4	4	0	5	2	4
U i	5	0	1	3	4	2	5	5	2
U U	1	1	2	1	0	4	4	4	4

Apply the Algorithm →

U1, U4, U7, the users this Node will deal with.

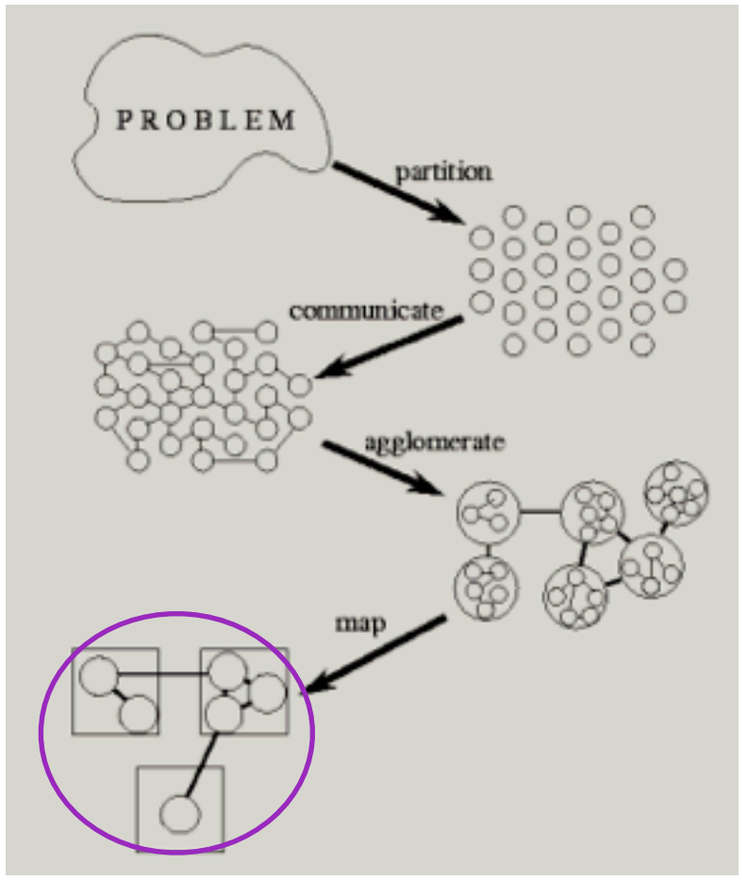
Node Outgoing Data: *the Recommendation List for the Users it processed.*

	B 1	B 2	B 3	B B
U 1	5	4	2	3
U 4	2	3	6	5
U 7	9	5	2	3

The Recommendation List of a User is a list with the B most recommended Users for the current User.

The Master will later merge all these Recommendation Lists into the resulting Recommendation Matrix.

PCAM - Mapping



The same algorithm that was designed to establish the order of communications is the one to map the work of **Nodes**, both into the **Nodes** and back from them. Each **Node** knows which **User** to process using the formula based on its own ID:

U1 ⟵ **TASK_ID** **+** **(i * NUM_PROCS)**

- **U1**: User to process
- **TASK_ID**: ID of the current task, acting like an offset.
- **i**: increasing iterator
- **NUM_PROCS**: Total number of processes given MPI was run with.

And then the **Node** will process the **Users** its concerned with in the **order they appear in the Dataset.** Preserving this order is a useful property, because its the order the **Recommendation Lists** will be stored in, and the order they will be sent back to the **Master**. The **Master** is listening for data, but not to an specific **Node**, then the **ID** of the Node is checked, and with the same formula we'll determine to which **User** does the currently returned **Recommendation List** belongs to. The iterator in this case will be the amount of already received results from a given **Node**, starting from 0. This way, we don't need to pass around the ID of a **User**, we can know to which **User** a **Recommendation List** belongs to, just knowing from which **Node** it comes from, and how many other results have been previously received from it. This way, all the **Users** are mapped into different Machines, and all **Recommendation Lists** are mapped back into the final **Recommendation Matrix**

Mapping Users To Node With Formula

	M 1	M 2	M 3	M 4	M 5	M 6	M 7	M i	M U
U 1	0	1	2	1	0	1	4	2	2
U 2	4	0	4	2	4	3	0	4	0
U 3	3	2	5	0	2	0	5	4	0
U 4	5	3	2	1	1	0	3	0	1
U 5	2	0	1	2	0	4	4	4	5
U 6	5	1	2	1	4	0	0	3	1
U 7	2	2	1	4	4	0	5	2	4
U i	5	0	1	3	4	2	5	5	2
U U	1	1	2	1	0	4	4	4	4

U1, U4, U7, the users this Node will deal with.

U1	⟵	**TASK_ID**	**+**	**(i * NUM_PROCS)**	
1		1		0	3
4		1		1	3
7		1		2	3

Mapping back works the same!
The master is only concerned with receiving a Recommendation List, and whom does it comes from.

Sending the User it belongs to is a waste of MPI connections, we can infer it with the same formula!

Finding the *B* Best Correlated Users

To find the **B** Best Correlated Users with a given User, we first need to find out the correlation between the given User and **ALL** other **U** Users. After having found all these correlations, a couple strategies appear to find in this list the **B** Best Correlated Users:

Multiple Linear Searches: *On the list containing the correlations the current User has with all other Users, lets iterate B times, each time storing the index of the User with the highest correlation, and then 'removing it' from the list (marking it as a impossible value so it isn't caught in the next iteration).*

Complexity: O(B * U)

For small values of **B** its negligible, but for large values, specially close to the number of Users **U**, it will become **O(U^2),** which is a very huge asymptotic running time. The specification tells us that **B** will be a small value, and in practice it should be, because we don't want to know the the correlation with all other registered **U** Users in order.

Sort And Constant Access: *Sort the list containing the correlations the current User has with all other Users. This sorted list will allow us to access the top B correlated Users in constant time! but...*

Complexity: O(U log U) + O(B)

The problem with this approach is that for large amounts of Users **U** and small **B,** it will unnecessarily sort all the elements while we only want the order of the top **B,** not the remaining ones. It is useful for large **B** because in asymptotic complexity, **O(U log U) + O(U)** is faster than **O(U^2)**

So we chose the first strategy for finding the B Best Correlated Users in respect of a given User for practical purposes.

Building and Running

Building:

$ chmod +x ./build.sh
$ ./build.sh

Running:

$ ./gendata [U] [M] [B]

Generate a data file with **U** users, **M** movies (each position with random ratings between 0 and 5) and put a **B** value for the Best Correlated Users to match later.

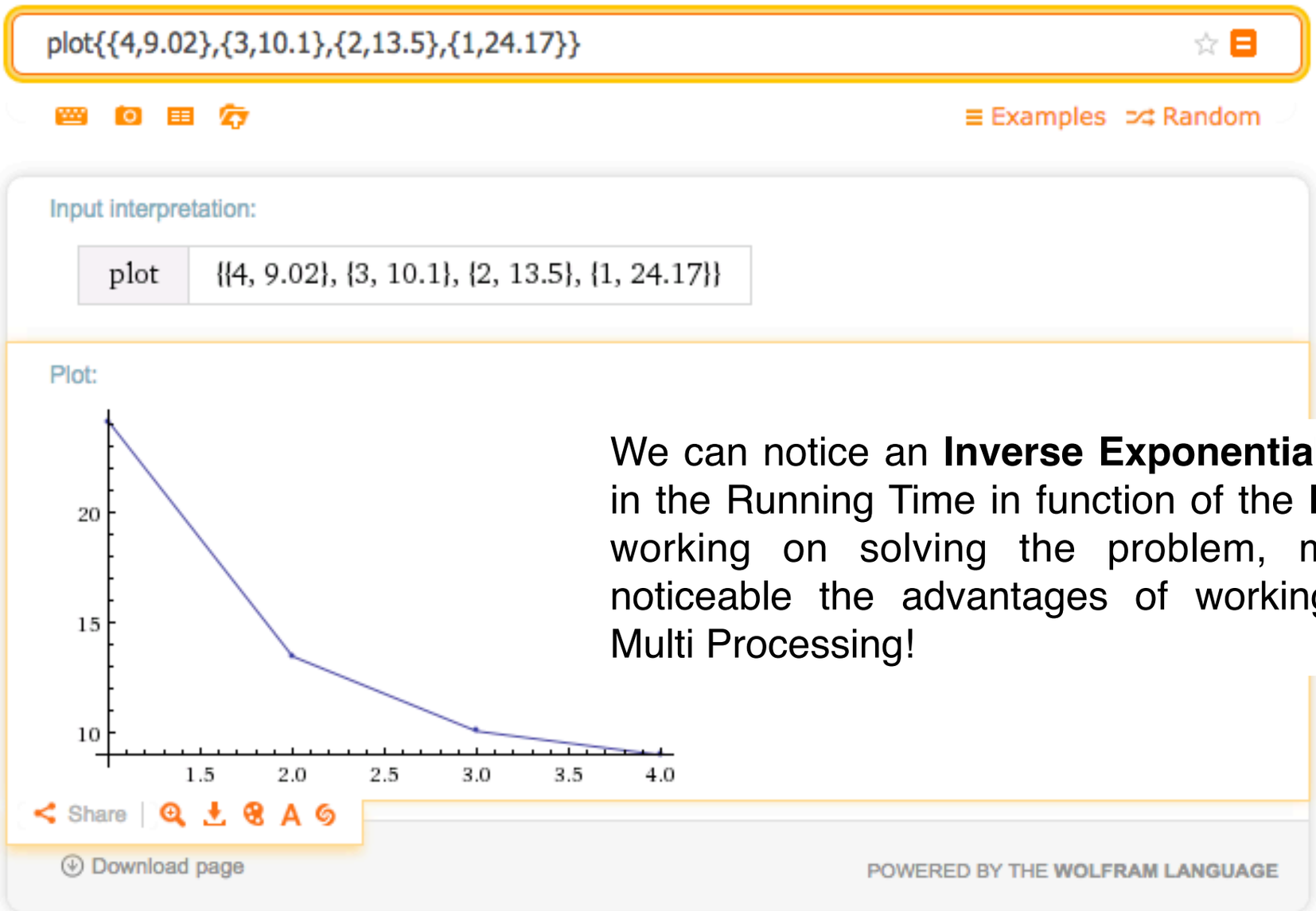
$ time ./spearson

Run the Serial version of the Pearson Correlation Algorithm Program.

$ mpirun -np [...] mpiearson

Run the MPI version of the Pearson Correlation Algorithm Program.

Time	-np 4	-np 3	-np 2	serial
1000 **U** 1000 **M** 10 **B**	*9.02s*	*10.1s*	*13.5s*	*24.17s*



We can notice an **Inverse Exponential** trend in the Running Time in function of the **Nodes** working on solving the problem, making noticeable the advantages of working with Multi Processing!

Have a nice day!