Designing an MPC Controller for a Simplified Rocket Landing Problem

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Index Terms—MPC, Rocket Landing, Model Predictive Control, Control Theory, Control Engineering

I. INTRODUCTION

The objective of the assignment is to implement a model-predictive control (MPC) scheme to land a simplified model of a rocket. The problem is fundamentally one of 'controllability', ie $x(0) = x_s \rightarrow x(t_f) = 0$. The assignment gives an abstracted model, we start by rewriting the model as follows:

$$
\begin{bmatrix} r_x(k+1) \\ r_y(k+1) \\ r_z(k+1) \\ v_x(k+1) \\ v_y(k+1) \\ v_z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 & T_s & 0 \\ 0 & 0 & 1 & 0 & 0 & T_s \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x(k) \\ r_y(k) \\ r_z(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \end{bmatrix} +
$$

$$
\begin{bmatrix} \frac{T_s^2}{2m} & 0 & 0 \\ 0 & \frac{T_s^2}{2m} & 0 \\ 0 & 0 & \frac{T_s^2}{2m} \\ T_s & 0 & 0 \\ 0 & T_s & 0 \\ 0 & 0 & T_s \end{bmatrix} \begin{bmatrix} f_x(k) + w_x \\ f_y(k) + w_y \\ f_z(k) - mg \end{bmatrix} \quad (1)
$$

Although we assume the system is reachable, it is important to check for controllability. Through MATLAB: `Rank = rank(ctrb(A, B))` we find the system is controllable, and we can proceed with designing a controller.

II. DESIGN

A. Controller Design

Three separate controllers are considered in this report. The first controller is an unconstrained, two stage MPC. The second controller builds on the first, and introduces both state and input constraints. The third controller further builds on the first two, introducing disturbance rejection. We start by looking at the unconstrained controller.

1) Unconstrained: The Q and R matrices are chosen as follows:

$$
Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} ; R = I \cdot 0.1 \text{ for } I \in \mathbb{R}^{3 \times 3}
$$

The Q matrix was chosen as to prioritose the vertical position and velocity. This represents the high level of importance given to landing at the correct altitude. Similarly, the prioritisation of velocity represents the importance of landing a velocity that prevents a hard landing. The lateral positions, ie r_x and r_y, are given a lower priority than the vertical position, but a higher priority than the lateral velocity, v_x and v_y. The reasoning behind this is that it is more important that the landing location is correct over the lateral velocities throughout the entire simulation.

The R matrix was chosen as to penalise the control inputs, f_x, f_y and f_z, equally. The small value given to R, 0.1, represents the low cost associated with expending fuel to control the rocket, and encourages the rocket to use fuel to exercise control in all three directions equally. Essentially, control is cheap. The horizon length N was chosen to be 5 as it provided a spectral radius of 0.5975, which is well within the stability region of the system.

The poles of the closed loop system were placed through `K_2 = -place(A, B, [0.01 0.01 0 0.01 0 0])`. The most important poles, ie the poles representing r_z, v_z were placed at zero. The limitations of the `place()` function meant that not every pole could be set to zero, hence the other poles were set to 0.01. The weighting matrix P was calculated using `P = dlyap((A+B*K_2), Q+K_2'*R*K_2)`. The prediction matrices F and G were generated using the `predict_mats()` function, and the corresponding cost matrices H, L and M were generated using the `cost_mats()` function. We assume at this point that $Q \succeq 0$ and $R \succ 0$ which implies $\vec{u}_N^*(k) = -H^{-1} \cdot Lx(k)$. As such, the unique and optimal solution to the optimal control problem is obtained and is through quadratic programming at

each time step `Uopt = quadprog(H, L*x)`

2) Constrained: The constrained controller builds on the unconstrained controller by introducing both state and input constraints. The input constraints are chosen as follows: The linear inequality constraints are based on the brief, and are chosen as follows:

$$P_x = \begin{bmatrix} I_x \\ -I_x \end{bmatrix} P_x = \begin{bmatrix} I_u \\ -I_u \end{bmatrix} \text{ for } I_x \in \mathbb{R}^{6\times6} \text{ and } I_u \in \mathbb{R}^{3\times3}$$

$$\mathsf{q}_x = \begin{bmatrix} \frac{r_z}{\tan\phi} + 50 \\ \frac{r_z}{\tan\phi} + 50 \\ 500 \\ 20 \\ 20 \\ 15 \\ \frac{r_z}{\tan\phi} + 50 \\ \frac{r_z}{\tan\phi} + 50 \\ 0 \\ 20 \\ 20 \\ 15 \end{bmatrix} \mathsf{q}_u = \begin{bmatrix} f_z \cdot \tan\theta \\ f_z \cdot \tan\theta \\ 12 \\ f_z \cdot \tan\theta \\ f_z \cdot \tan\theta \\ 0 \end{bmatrix}$$

The r_x r_y constraints were given in the brief as $|r_x|, |r_y| \leq \frac{r_z}{\tan\phi}$ where ϕ represents the glide slope angle, and is chosen as a constant $\phi = 30° = 0.52$ rad. The input constraints given in the brief as $|f_x|, |f_y| \leq f_z \tan\theta$ where θ is represents the maximum allowable angle of the rocket engines, and is chosen as $\theta = 10° = 0.17$ rad.

Fig. 1. f_z on the z axis.

Figure **??** provides a visualisation of how f_z acts on the point mass. Overall, this permits the implementation of Algorithm 6.1, pg. 102 from [**?**], since all the relevant optimal control and constraint matrices have been defined. The constraint matrices are obtained in code through `[Pc, qc, Sc] = constraint_mats(F, G, Pu, qu,` `Px, qx, Px, qx);`

3) Disturbance Rejection: The disturbance rejection controller builds on both the unconstrained and constrained controllers. No reference tracking is included in the following controller. Only wind disturbance is considered, and is modelled as a sinusoid as follows:

$$\vec{w}(k) = \begin{bmatrix} \sin\frac{50}{k} \\ \cos\frac{50}{k} \\ 0 \end{bmatrix}$$

The use of a sinusoid disturbance is intended to represent the changing windspeeds as the rocket descends. The sinusoid

functions were chosen to be out of phase as to yield visibly different results.

In terms of controller design, equation **??** is rewritten as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ew(k) \\ y(k) = Cx(k) + Fd(k) \end{cases}$$

where $E = B$ and $F = 0$ since we ignore output disturbance, hence:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + w(k)) \\ y(k) = Cx(k) \end{cases}$$

It is now possible to formulate the T matrix to find $x_{ss}; u_{ss}$.

$$T = \begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix}$$

In this case, we have $C \in \mathbb{R}^{6\times6} \wedge B \in \mathbb{R}^{6\times3} \therefore p < m \iff 3 < 6$. As such, we expect only some as such, $x_{ss}; u_{ss}$ are given by:

$$\begin{bmatrix} x_{ss}(k) \\ u_{ss}(k) \end{bmatrix} = \begin{bmatrix} T^{-1}B \cdot \vec{w}(k) \\ 0 \end{bmatrix}$$

B. Experiment Setup

First, it is important to state simplifications assumed. Let the starting parameters be as follows:

$$\vec{x}(0) = \begin{bmatrix} 600 \\ 600 \\ 500 \\ 5 \\ 5 \\ -15 \end{bmatrix} \wedge \vec{u}(0) = \vec{0}$$

These starting conditions are the limits of what the assignment permits, i.e a maximum starting altitide of 500m, and a maximum lateral distance of 600m.

III. RESULTS
IV. ANALYSIS & DISCUSSION
V. CONCLUSION

VI. Ease of Use

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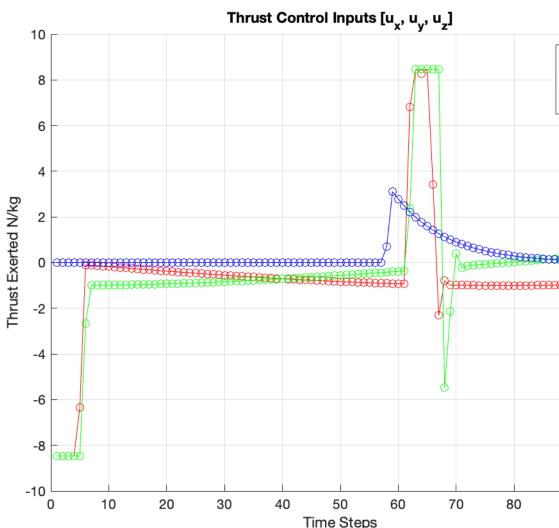| Table Head	Table Column Head		
	Table column subhead	*Subhead*	*Subhead*
copy	More table copy[a]		

[a]Sample of a Table footnote.

Fig. 2. Example of a figure caption.

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ACKNOWLEDGMENT

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