Designing an MPC Controller for a Simplified Rocket Landing Problem

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—MPC, Rocket Landing, Model Predictive Control, Control Theory, Control Engineering**

I. Introduction

The objective of the assignment is to implement a model-predictive control (MPC) scheme to land a simplified model of a rocket. The problem is fundamentally one of 'controllability', ie $x(0) = x_s \rightarrow x(t_f) = 0$. The assignment gives an abstracted model, we start by rewriting the model as follows:

$$\begin{bmatrix} r_x(k+1) \\ r_y(k+1) \\ r_z(k+1) \\ v_x(k+1) \\ v_y(k+1) \\ v_z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 & T_s & 0 \\ 0 & 0 & 1 & 0 & 0 & T_s \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x(k) \\ r_y(k) \\ r_z(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \end{bmatrix} + \begin{bmatrix} \frac{T_s^2}{2m} & 0 & 0 \\ 0 & \frac{T_s^2}{2m} & 0 \\ 0 & 0 & \frac{T_s^2}{2m} \\ T_s & 0 & 0 \\ 0 & T_s & 0 \\ 0 & 0 & T_s \end{bmatrix} \begin{bmatrix} f_x(k) + w_x \\ f_y(k) + w_y \\ f_z(k) - mg \end{bmatrix} \tag{1}$$

Although we assume the system is reachable, it is important to check for controllability. Through MATLAB: `Rank = rank(ctrb(A, B))` we find the system is controllable, and we can proceed with designing a controller.

II. Design

A. Controller Design

Three separate controllers are considered in this report. The first controller is an unconstrained, two stage MPC. The second controller builds on the first, and introduces both state and input constraints. The third controller further builds on the first two, introducing disturbance rejection. We start by looking at the unconstrained controller.

1) Unconstrained: The Q and R matrices are chosen as follows:

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}; R = I \cdot 0.1 \text{ for } I \in \mathbb{R}^{3\times 3}$$

The Q matrix was chosen as to prioritose the vertical position and velocity. This represents the high level of importance given to landing at the correct altitude. Similarly, the prioritisation of velocity represents the importance of landing a velocity that prevents a hard landing. The lateral positions, ie r_x and r_y, are given a lower priority than the vertical position, but a higher priority than the lateral velocity, v_x and v_y. The reasoning behind this is that it is more important that the landing location is correct over the lateral velocities throughout the entire simulation.

The R matrix was chosen as to penalise the control inputs, f_x, f_y and f_z, equally. The small value given to R, 0.1, represents the low cost associated with expending fuel to control the rocket, and encourages the rocket to use fuel to exercise control in all three directions equally. Essentially, control is cheap. The horizon length N was chosen to be 5 as it provided a spectral radius of 0.5975, which is well within the stability region of the system.

The poles of the closed loop system were placed through `K_2 = -place(A, B, [0.01 0.01 0 0.01 0 0])`. The most important poles, ie the poles representing r_z, v_z were placed at zero. The limitations of the `place()` function meant that not every pole could be set to zero, hence the other poles were set to 0.01. The weighting matrix P was calculated using `P = dlyap((A+B*K_2), Q+K_2'*R*K_2)`. The prediction matrices F and G were generated using the `predict_mats()` function, and the corresponding cost matrices H, L and M were generated using the `cost_mats()` function. We assume at this point that $Q \succeq 0$ and $R \succ 0$ which implies $\vec{u}_N^*(k) = -H^{-1} \cdot Lx(k)$. As such, the unique and optimal solution to the optimal control problem is obtained and is through quadratic programming at each time step `Uopt = quadprog(H, L*x)`

2) Constrained: The constrained controller builds on the unconstrained controller by introducing both state and input constraints. The input constraints are chosen as follows:

$$u_{max}\begin{bmatrix} f_z \cdot \tan\theta \\ f_z \cdot \tan\theta \\ 12 \end{bmatrix}; u_{min} = \begin{bmatrix} -f_z \cdot \tan\theta \\ -f_z \cdot \tan\theta \\ 0 \end{bmatrix}$$

the input constraints given in the brief as $|f_x|, |f_y| \leq f_z \tan\theta$ The angle θ is represents the maximum allowable angle of the rocket engines, and is chosen as $\theta = 10° = 0.17$ rad.

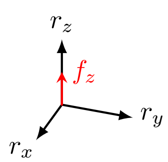

Fig. 1. f_z on the z axis.

Figure **??** provides a visualisation of how f_z acts on the point mass.

The state constraints are chosen as follows:

$$x_{max} = \begin{bmatrix} \frac{r_z}{\tan\phi} + 50 \\ \frac{r_z}{\tan\phi} + 50 \\ 500 \\ 20 \\ 20 \\ 15 \end{bmatrix}; x_{min} = \begin{bmatrix} -\frac{r_z}{\tan\phi} - 50 \\ -\frac{r_z}{\tan\phi} - 50 \\ 0 \\ -20 \\ -20 \\ -15 \end{bmatrix}$$

The r_x r_y constraints were given in the brief as $|r_x|, |r_y| \leq \frac{r_z}{\tan\phi}$. The angle ϕ represents the glide slope angle, and is chosen as a constant $\phi = 30° = 0.52$ rad.

3) Disturbance Rejection: The disturbance rejection controller builds on both the unconstrained and constrained controllers. Only wind disturbance is considered, and is modelled as a sinusoid as follows:

$$\vec{w}(k) = \begin{bmatrix} \sin\frac{50}{k} \\ \cos\frac{50}{k} \\ 0 \end{bmatrix}$$

In terms of controller design, we start by rewriting equation **??** as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ew(k) \\ y(k) = Cx(k) + Fd(k) \end{cases}$$

where $E = B$ and $F = 0$ since we ignore output disturbance, hence:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + w(k)) \\ y(k) = Cx(k) \end{cases}$$

It is now possible to formulate the T matrix to find $x_{ss}; u_{ss}$.

$$T = \begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix}$$

In this case, we have $C \in \mathbb{R}^{6\times6} \land B \in \mathbb{R}^{6\times3} \therefore p < m \iff 3 < 6$. As such, we expect only some as such, $x_{ss}; u_{ss}$ are given by:

$$\begin{bmatrix} x_{ss}(k) \\ u_{ss}(k) \end{bmatrix} = \begin{bmatrix} T^{-1}B \cdot \vec{w}(k) \\ 0 \end{bmatrix}$$

B. Experiment Setup

First, it is important to state simplifications assumed.

Let the starting parameters be as follows:

$$\vec{x}(0) = \begin{bmatrix} 600 \\ 600 \\ 500 \\ 5 \\ 5 \\ -15 \end{bmatrix} \land \vec{u}(0) = \vec{0}$$

These starting conditions are the limits of what the assignment permits, i.e a maximum starting altitide of 500m, and a maximum lateral distance of 600m.

III. Results

IV. Analysis & Discussion

V. Conclusion

VI. Ease of Use

A. Maintaining the Integrity of the Specifications

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Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
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- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Equations

Number equations consecutively. To make your [?] equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{2}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(**??**)", not "Eq. (**??**)" or "equation (**??**)", except at the beginning of a sentence: "Equation (**??**) is . . ."

D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

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- The word "data" is plural, not singular.
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- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
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- In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.
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- Do not confuse “impl
- The prefix “non” is nc word it modifies, usu
- There is no period aft “et al.”.
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Text heads organize the basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

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a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. **??**”, even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
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[a]Sample of a Table footnote.

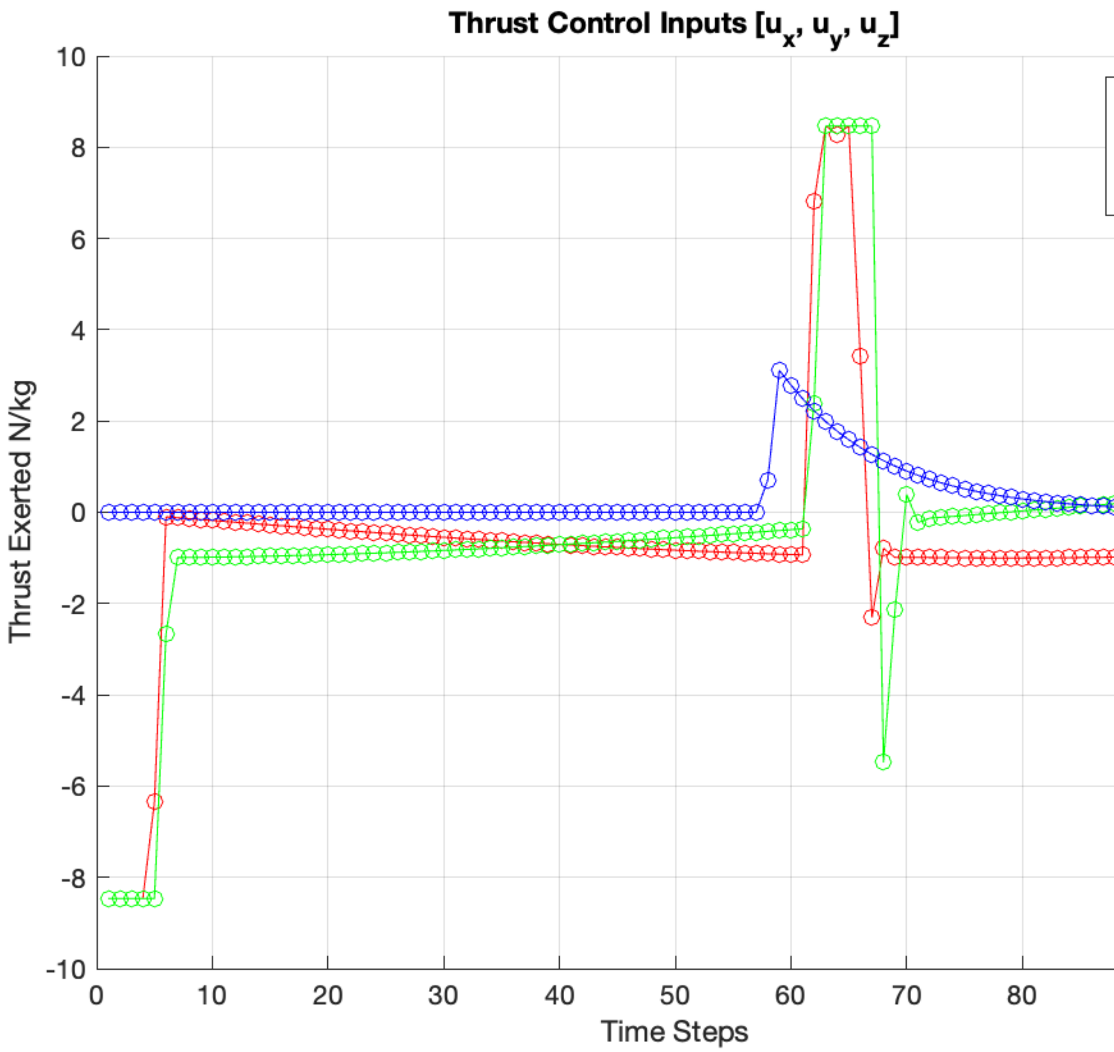

Fig. 2. Example of a figure caption.

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Acknowledgment

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks . . .”. Instead, try “R. B. G. thanks. . .”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

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