Designing an MPC Controller for a Simplified Rocket Landing Problem

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—MPC, Rocket Landing, Model Predictive Control, Control Theory, Control Engineering**

I. Introduction

The objective of the assignment is to implement a model-predictive control (MPC) scheme to land a simplified model of a rocket. The problem is fundamentally one of 'controllability', ie $x(0) = x_s \to x(t_f) = 0$. The assignment gives an abstracted model, we start by rewriting the model as follows:

$$\begin{bmatrix} r_x(k+1) \\ r_y(k+1) \\ r_z(k+1) \\ v_x(k+1) \\ v_y(k+1) \\ v_z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_s & 0 & 0 \\ 0 & 1 & 0 & 0 & T_s & 0 \\ 0 & 0 & 1 & 0 & 0 & T_s \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x(k) \\ r_y(k) \\ r_z(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \end{bmatrix} + \begin{bmatrix} \frac{T_s^2}{2m} & 0 & 0 \\ 0 & \frac{T_s^2}{2m} & 0 \\ 0 & 0 & \frac{T_s^2}{2m} \\ T_s & 0 & 0 \\ 0 & T_s & 0 \\ 0 & 0 & T_s \end{bmatrix} \begin{bmatrix} f_x(k) + w_x \\ f_y(k) + w_y \\ f_z(k) - mg \end{bmatrix} \quad (1)$$

Although we assume the system is reachable, it is important to check for controllability. Through MATLAB: `Rank = rank(ctrb(A, B))` we find the system is controllable, and we can proceed with designing a controller.

II. Design

A. Controller Design

Three separate controllers are considered in this report. The first controller is an unconstrained, two stage MPC. The second controller builds on the first, and introduces both state and input constraints. The third controller further builds on the first two, introducing disturbance rejection. We start by looking at the unconstrained controller.

1) Unconstrained: The Q and R matrices are chosen as follows:

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}; R = I \cdot 0.1 \text{ for } I \in \mathbb{R}^{3\times 3}$$

The Q matrix was chosen as to prioritose the vertical position and velocity. This represents the high level of importance given to landing at the correct altitude. Similarly, the prioritisation of velocity represents the importance of landing a velocity that prevents a hard landing. The lateral positions, ie r_x and r_y, are given a lower priority than the vertical position, but a higher priority than the lateral velocity, v_x and v_y. The reasoning behind this is that it is more important that the landing location is correct over the lateral velocities throughout the entire simulation.
The R matrix was chosen as to penalise the control inputs, f_x, f_y and f_z, equally. The small value given to R, 0.1, represents the low cost associated with expending fuel to control the rocket, and encourages the rocket to use fuel to exercise control in all three directions equally. Essentially, control is cheap. The horizon length N was chosen to be 5 as it provided a spectral radius of 0.5975, which is well within the stability region of the system.
The poles of the closed loop system were placed through `K_2 = -place(A, B, [0.01 0.01 0 0.01 0 0])`. The most important poles, ie the poles representing r_z, v_z were placed at zero. The limitations of the `place()` function meant that not every pole could be set to zero, hence the other poles were set to 0.01. The weighting matrix P was calculated using `P = dlyap((A+B*K_2), Q+K_2'*R*K_2)`. The prediction matrices F and G were generated using the `predict_mats()` function, and the corresponding cost matrices H, L and M were generated using the `cost_mats()` function. We assume at this point that $Q \succeq 0$ and $R \succ 0$ which implies $\vec{u}_N^*(k) = -H^{-1} \cdot Lx(k)$. As such, the unique and optimal solution to the optimal control problem is obtained and is through quadratic programming at

each time step `Uopt = quadprog(H, L*x)`

2) Constrained: The constrained controller builds on the unconstrained controller by introducing both state and input constraints. The input constraints are chosen as follows: The linear inequality constraints are based on the brief, and are chosen as follows:

$$P_x = \begin{bmatrix} I_x \\ -I_x \end{bmatrix} P_x = \begin{bmatrix} I_u \\ -I_u \end{bmatrix} \text{ for } I_x \in \mathbb{R}^{6\times6} \text{ and } I_u \in \mathbb{R}^{3\times3}$$

$$\mathrm{q}_x = \begin{bmatrix} \frac{r_z}{\tan\phi} + 50 \\ \frac{r_z}{\tan\phi} + 50 \\ 500 \\ 20 \\ 20 \\ 15 \\ \frac{r_z}{\tan\phi} + 50 \\ \frac{r_z}{\tan\phi} + 50 \\ 0 \\ 20 \\ 20 \\ 15 \end{bmatrix} q_u = \begin{bmatrix} f_z \cdot \tan\theta \\ f_z \cdot \tan\theta \\ 12 \\ f_z \cdot \tan\theta \\ f_z \cdot \tan\theta \\ 0 \end{bmatrix}$$

The r_x r_y constraints were given in the brief as $|r_x|, |r_y| \le \frac{r_z}{\tan\phi}$ where ϕ represents the glide slope angle, and is chosen as a constant $\phi = 30° = 0.52$ rad. The input constraints given in the brief as $|f_x|, |f_y| \le f_z \tan\theta$ where θ is represents the maximum allowable angle of the rocket engines, and is chosen as $\theta = 10° = 0.17$ rad.

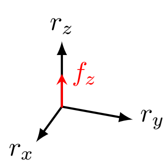

Fig. 1: f_z on the z axis.

Figure **??** provides a visualisation of how f_z acts on the point mass. Overall, this permits the implementation of Algorithm 6.1, pg. 102 from [**?**], since all the relevant optimal control and constraint matrices have been defined. The constraint matrices are obtained in code through `constraint_mats()`. Disturbances are included in the simulation despite no disturbance rejection implementation, mainly to be able to contrast the results and demonstrate the implementation of disturbance rejection.

3) Disturbance Rejection: Rank, reachability and observability tests were performed on B, E and the two pairs $(A, B); (C, A)$ and it was found that disturbance rejection was possible. The disturbance rejection controller builds on both the unconstrained and constrained controllers. No reference tracking is included in the following controller. Only wind disturbance is considered, and is modelled as a sinusoid as follows:

$$\vec{w}(k) = \begin{bmatrix} \sin\frac{50}{k} \\ \cos\frac{50}{k} \\ 0 \end{bmatrix}$$

The use of a sinusoid disturbance is intended to represent the changing windspeeds as the rocket descends through the atmosphere. The sinusoid functions were chosen to be out of phase as to yield visibly different results.
In terms of controller design, equation **??** is rewritten as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ew(k) \\ y(k) = Cx(k) + Fd(k) \end{cases}$$

where $E = B$ and $F = 0$ since we ignore output disturbance, hence:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + w(k)) \\ y(k) = Cx(k) \end{cases}$$

It is now possible to formulate the T matrix to find $x_{ss}; u_{ss}$.

$$T = \begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix}$$

In this case, we have $C \in \mathbb{R}^{6\times6} \wedge B \in \mathbb{R}^{6\times3} \therefore p < m \iff 3 < 6$. Similarly, we find that T is full row rank, confriming that for any pair (r, d) $\exists$ a pair (x_{ss}, u_{ss}). As such, $x_{ss}; u_{ss}$ are given by:

$$\begin{bmatrix} x_{ss}(k) \\ u_{ss}(k) \end{bmatrix} = T^{-1} \begin{bmatrix} B \cdot \vec{w}(k) \\ 0 \end{bmatrix}$$

From this result, the deviation variables become $z := x - x_{ss}; v := u - x_{ss}$. As a result, $v^*(k|k)$ is calculated through `quadprog(H, L*z, qc+Sc*z)` at each iteration.

B. Experiment Setup

The mass of the rocket is simplified and assumed to be constant for all experiments, and is chosen as $m = 1$ kg. The starting parameters for each experiment are as follows:

$$\vec{x}(0) = \begin{bmatrix} 600 \\ 600 \\ 500 \\ 5 \\ 5 \\ -15 \end{bmatrix} \wedge \vec{u}(0) = \vec{0}$$

These starting conditions are the limits of what the assignment permits, i.e a maximum starting altitide of 500m, and a maximum lateral distance of 600m.

III. Results

A. Unconstrained

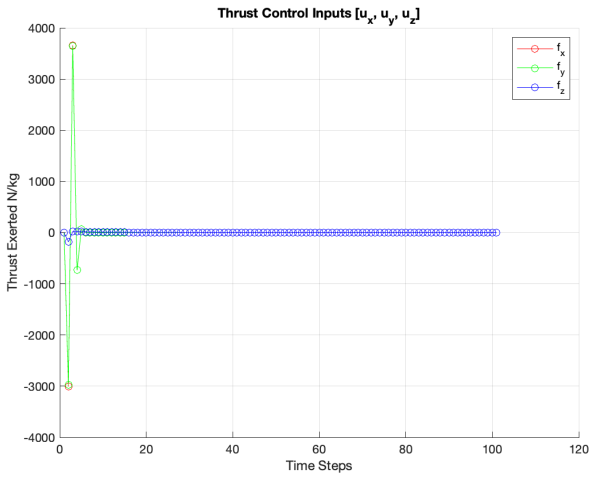

Fig. 2: Unconstrained Thrust Control Input

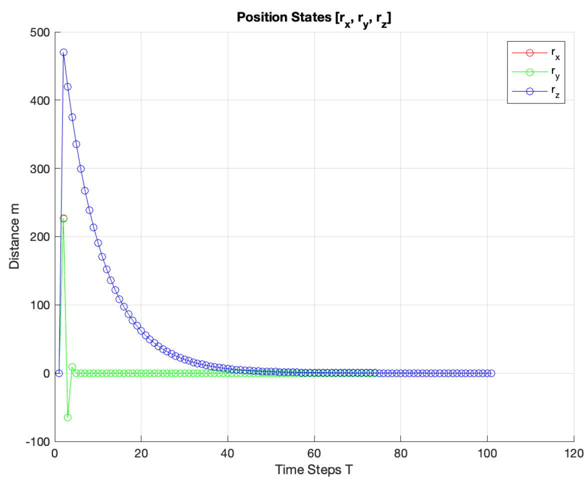

Fig. 3: Unconstrained Position State Plot

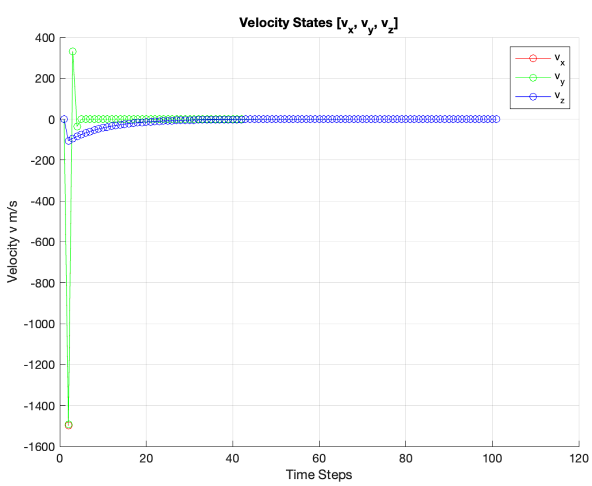

Fig. 4: Unconstrained Velocity State Plot

B. Constrained

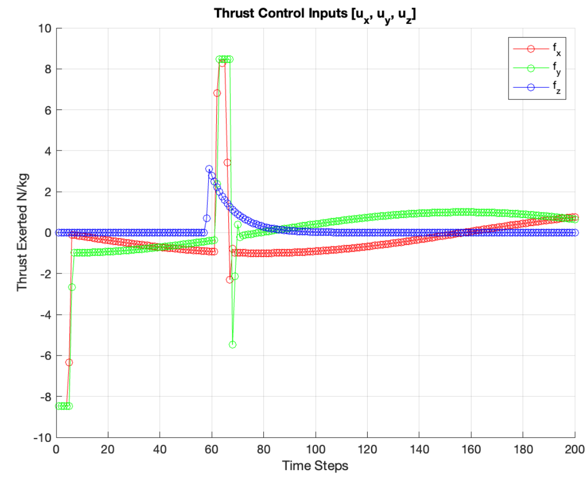

Fig. 5: Constrained Thrust Control Input

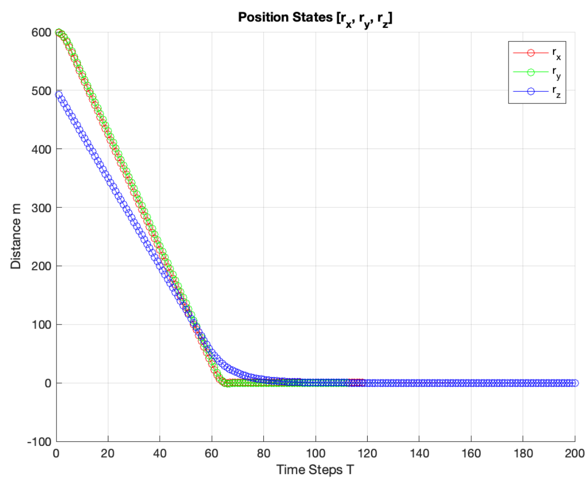

Fig. 6: Constrained Position State Plot

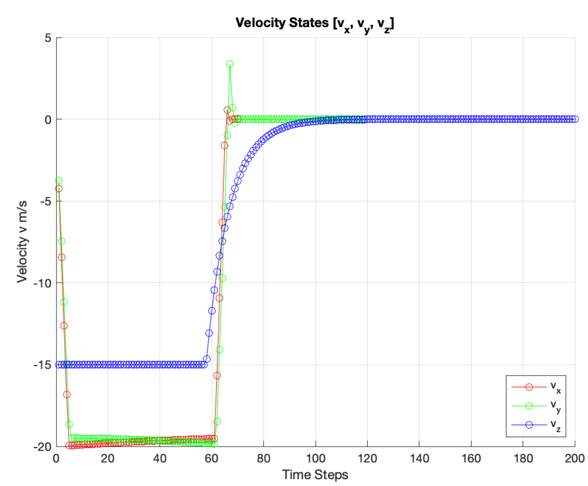

Fig. 7: Constrained Velocity State Plot

C. Disturbance Rejection

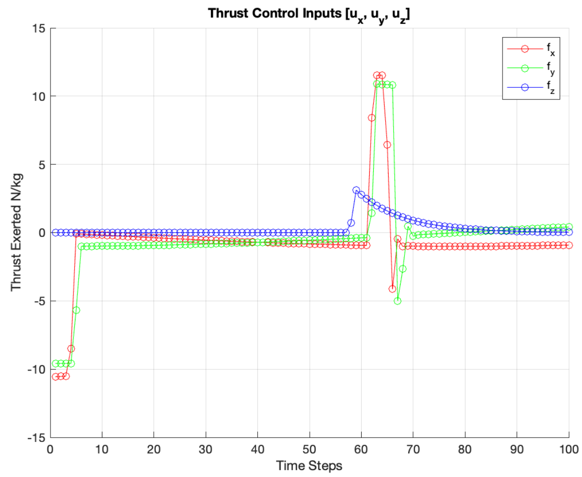

Fig. 8: Disturbance Rejection Thrust Control Input

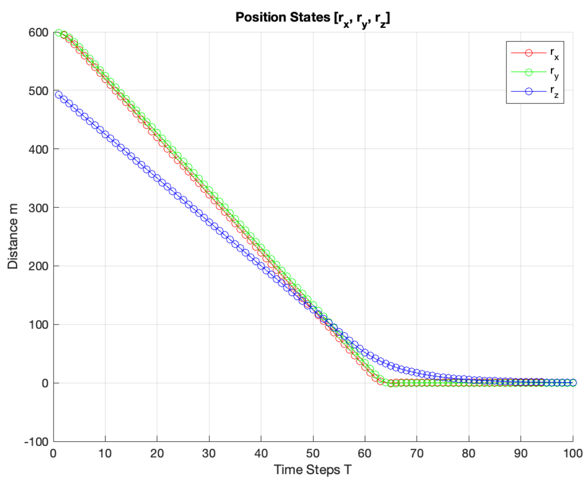

Fig. 9: Disturbance Rejection Position State Plot

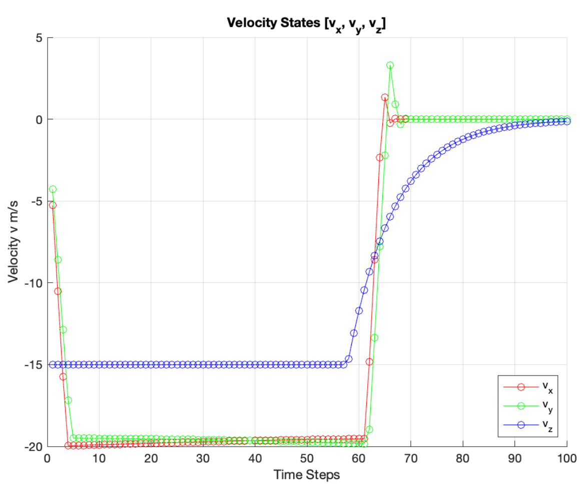

Fig. 10: Disturbance Rejection Velocity State Plot

IV. Analysis & Discussion

Due to the limitations to the number of pages, the analysis will mainly discuss the effectiveness of the controllers using constraints and disturbance rejection.

V. Conclusion

VI. Ease of Use

A. Maintaining the Integrity of the Specifications

The IEEEtran class file is used to format your paper and style the text. All margins, column widths, line spaces, and text fonts are prescribed; please do not alter them. You may note peculiarities. For example, the head margin measures proportionately more than is customary. This measurement and others are deliberate, using specifications that anticipate your paper as one part of the entire proceedings, and not as an independent document. Please do not revise any of the current designations.

VII. Prepare Your Paper Before Styling

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections **??**–**??** below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: "Wb/m^2" or "webers per square meter", not "webers/m^2". Spell out units when they appear in text: ". . . a few henries", not ". . . a few H".
- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Equations

Number equations consecutively. To make your [?] equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{2}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(**??**)", not "Eq. (**??**)" or "equation (**??**)", except at the beginning of a sentence: "Equation (**??**) is . . ."

D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

BibTeX does not work by magic. It doesn't get the bibliographic data from thin air but from .bib files. If you use BibTeX to produce a bibliography you must send the .bib files.

LaTeX can't read your mind. If you assign the same label to a subsubsection and a table, you might find that Table I has been cross referenced as Table IV-B3.

LaTeX does not have precognitive abilities. If you put a `\label` command before the command that updates the counter it's supposed to be using, the label will pick up the last counter to be cross referenced instead. In particular, a `\label` command should not go before the caption of a figure or a table.

Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won't be any anyway) and it might stop a wanted equation number in the surrounding equation.

E. Some Common Mistakes

- The word "data" is plural, not singular.
- The subscript for the permeability of vacuum μ_0, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an "inset", not an "insert". The word alternatively is preferred to the word "alternately" (unless you really mean something that alternates).

- Do not use the word "essentially" to mean "approximately" or "effectively".
- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones "affect" and "effect", "complement" and "compliment", "discreet" and "discrete", "principal" and "principle".
- Do not confuse "imply" and "infer".
- The prefix "non" is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

An excellent style manual for science writers is [**?**].

F. Authors and Affiliations

The class file is designed for, but not limited to, six authors. A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

G. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

H. Figures and Tables

a) Positioning Figures and Tables: Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. **??**", even at the beginning of a sentence.

TABLE I: Table Type Styles

Table Head	**Table Column Head**		
	Table column subhead	***Subhead***	***Subhead***
copy	More table copy[a]		

[a]Sample of a Table footnote.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization $\{A[m(1)]\}$", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

ACKNOWLEDGMENT

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks ...". Instead, try "R. B. G. thanks...". Put sponsor acknowledgments in the unnumbered footnote on the first page.

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