# Designing an MPC Controller for a Simplified Rocket Landing Problem

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—MPC, Rocket Landing, Model Predictive Control, Control Theory, Control Engineering**

## I. Introduction

The objective of the assignment is to implement a model-predictive control (MPC) scheme to land a simplified model of a rocket. The problem is fundamentally one of 'controllability', ie $x(0) = x_s \rightarrow x(t_f) = 0$. The assignment gives an abstracted model, we start by rewriting the model as follows:

$$\begin{bmatrix} r_x(k+1) \\ r_y(k+1) \\ r_z(k+1) \\ v_x(k+1) \\ v_y(k+1) \\ v_z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x(k) \\ r_y(k) \\ r_z(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2m} & 0 & 0 \\ 0 & \frac{T^2}{2m} & 0 \\ 0 & 0 & \frac{T^2}{2m} \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} f_x(k) + w_x \\ f_y(k) + w_y \\ f_z(k) - mg \end{bmatrix} \tag{1}$$

Although we assume the system is reachable, it is important to check for controllability. Through MATLAB: `Rank = rank(ctrb(A, B))` we find the system is controllable, and we can proceed with designing a controller.

## II. Design

### A. Controller Design

Three separate controllers are considered in this report. The first controller is an unconstrained, two stage MPC. The second controller builds on the first, and introduces both state and input constraints. The third controller further builds on the first two, introducing disturbance rejection. We start by looking at the unconstrained controller.

*1) Unconstrained:* The $Q$ and $R$ matrices are chosen as follows:

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}; R = I \cdot 0.1 \text{ for } I \in \mathbb{R}^{3\times3}$$

The $Q$ matrix was chosen as to prioritose the vertical position and velocity. This represents the high level of importance given to landing at the correct altitude. Similarly, the prioritisation of velocity represents the importance of landing a velocity that prevents a hard landing. The lateral positions, ie $r_x$ and $r_y$, are given a lower priority than the vertical position, but a higher priority than the lateral velocity, $v_x$ and $v_y$. The reasoning behind this is that it is more important that the landing location is correct over the lateral velocities throughout the entire simulation.
The $R$ matrix was chosen as to penalise the control inputs, $f_x$, $f_y$ and $f_z$, equally. The small value given to $R$, 0.1, represents the low cost associated with expending fuel to control the rocket, and encourages the rocket to use fuel to exercise control in all three directions equally. Essentially, control is cheap. The horizon length $N$ was chosen to be 5 as it provided a spectral radius of 0.5975, which is well within the stability region of the system.

*2) Constrained:* The constrained controller builds on the unconstrained controller by introducing both state and input constraints. The input constraints are chosen as follows:

$$u_{max} \begin{bmatrix} f_z \cdot \tan\theta \\ f_z \cdot \tan\theta \\ 0 \end{bmatrix}; u_{min} = \begin{bmatrix} -f_z \cdot \tan\theta \\ -f_z \cdot \tan\theta \\ -12 \end{bmatrix}$$

the input constraints given in the brief as $|f_x|, |f_y| \leq f_z \tan\theta$ The angle $\theta$ is represents the maximum allowable angle of the rocket engines, and is chosen as $\theta = 10° = 0.17$ rad.

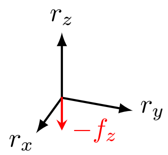


Fig. 1. $f_z$ on the $z$ axis.

Figure **??** shows the justification for setting the $f_z$ constraint in the negative $z$ direction. Since the rocket must exert thrust to decelerate, the $f_z$ input must be negative.

The state constraints are chosen as follows:

$$x_{max} = \begin{bmatrix} \frac{r_z}{\tan\phi} + 50 \\ \frac{r_z}{\tan\phi} + 50 \\ 500 \\ 20 \\ 20 \\ 15 \end{bmatrix} ; x_{min} = \begin{bmatrix} -\frac{r_z}{\tan\phi} - 50 \\ -\frac{r_z}{\tan\phi} - 50 \\ 0 \\ -20 \\ -20 \\ -15 \end{bmatrix}$$

The $r_x$ $r_y$ constraints were given in the brief as $|r_x|, |r_y| \leq \frac{r_z}{\tan\phi}$. The angle $\phi$ represents the glide slope angle, and is chosen as a constant $\phi = 30° = 0.52$ rad.

*3) Disturbance Rejection:* The disturbance rejection controller builds on both the unconstrained and constrained controllers. Only wind disturbance is considered, and is modelled as a sinusoid as follows:

$$\vec{w}(k) = \begin{bmatrix} \sin\frac{50}{k} \\ \cos\frac{50}{k} \\ 0 \end{bmatrix}$$

In terms of controller design, we start by rewriting equation **??** as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ew(k) \\ y(k) = Cx(k) + Fd(k) \end{cases}$$

where $E = B$ and $F = 0$ since we ignore output disturbance, hence:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + w(k)) \\ y(k) = Cx(k) \end{cases}$$

It is now possible to formulate the $T$ matrix to find $x_{ss}; u_{ss}$. Since no reference tracking is used, $C = 0$ obtaining $T$ as follows:

$$T = \begin{bmatrix} I - A & -B \end{bmatrix}$$

In this case, we have $pm$ since we are not performing reference tracking.

as such, $x_{ss}; u_{ss}$ are given by:

$$\begin{bmatrix} x_{ss}(k) \\ u_{ss}(k) \end{bmatrix} = T^{-1}B \cdot \vec{w}(k)$$

## B. Experiment Setup

First, it is important to state simplifications assumed.

Let the starting parameters be as follows:

$$\vec{x}(0) = \begin{bmatrix} 600 \\ 600 \\ 500 \\ 5 \\ 5 \\ -15 \end{bmatrix} \wedge \vec{u}(0) = \vec{0}$$

These starting conditions are the limits of what the assignment permits, i.e a maximum starting altitide of 500m, and a maximum lateral distance of 600m.

# III. Results

# IV. Analysis & Discussion

# V. Conclusion

## VI. Ease of Use

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Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

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Number equations consecutively. To make your [**?**] equations more compact, you may use the solidus ( / ), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{2}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(**??**)", not "Eq. (**??**)" or "equation (**??**)", except at the beginning of a sentence: "Equation (**??**) is . . ."

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*a) Positioning Figures and Tables:* Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. **??**”, even at the beginning of a sentence.

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| Table Head | Table Column Head | | |
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[a]Sample of a Table footnote.

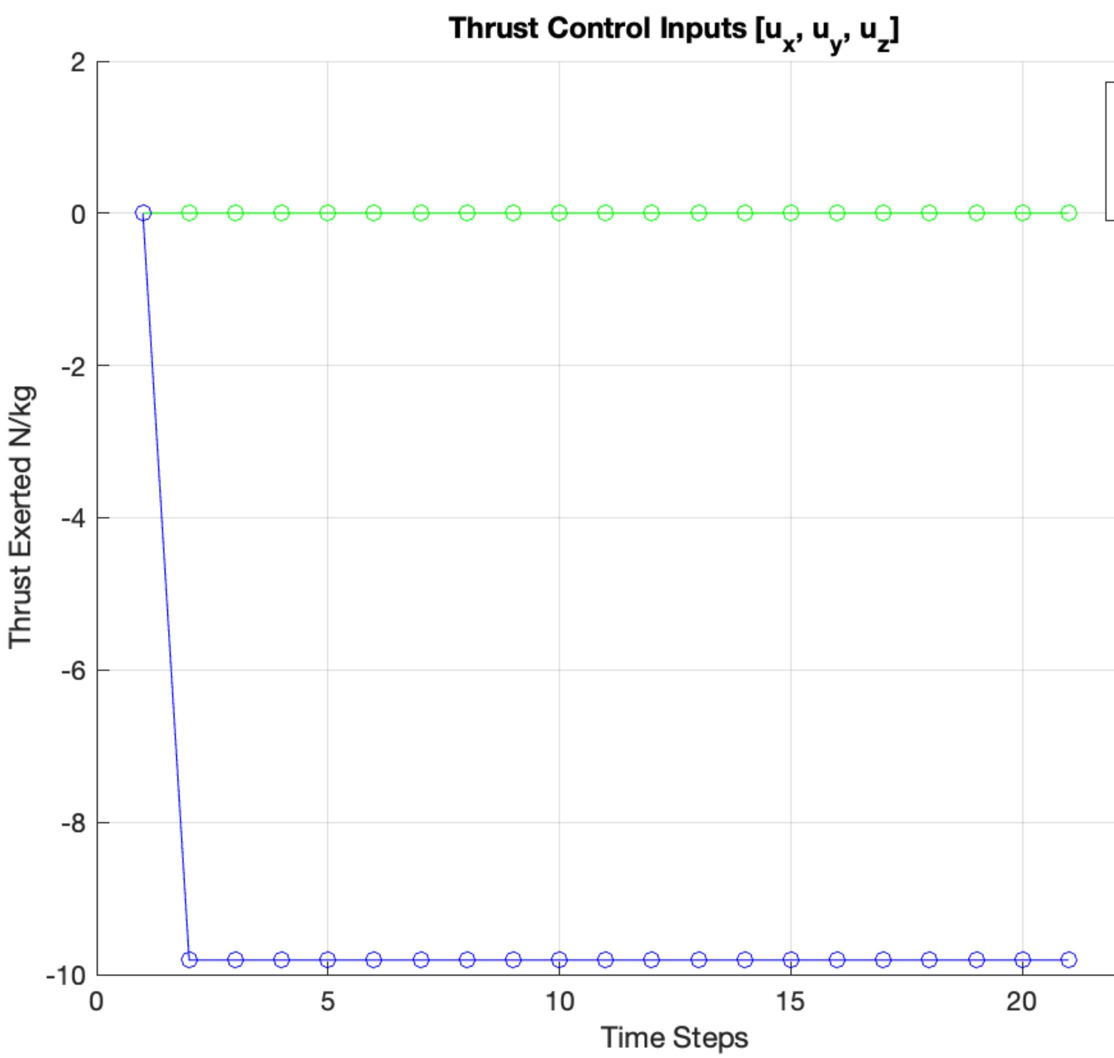


Fig. 2. Example of a figure caption.

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## Acknowledgment

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