

Designing an MPC Controller for a Simplified Rocket Landing Problem

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Abstract—This document is a model and instructions for L^AT_EX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.

Index Terms—MPC, Rocket Landing, Model Predictive Control, Control Theory, Control Engineering

I. INTRODUCTION

The objective of the assignment is to implement a model-predictive control (MPC) scheme to land a simplified model of a rocket. The problem is fundamentally one of 'controllability', ie $x(0) = x_s \rightarrow x(t_f) = 0$. The assignment gives an abstracted model, we start by rewriting the model as follows:

$$(1) \begin{bmatrix} r_x(k+1) \\ r_y(k+1) \\ r_z(k+1) \\ v_x(k+1) \\ v_y(k+1) \\ v_z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x(k) \\ r_y(k) \\ r_z(k) \\ v_x(k) \\ v_y(k) \\ v_z(k) \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2m} & 0 & 0 \\ 0 & \frac{T^2}{2m} & 0 \\ 0 & 0 & \frac{T^2}{2m} \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} f_x(k) + w_x \\ f_y(k) + w_y \\ f_z(k) - mg \end{bmatrix}$$

Although we assume the system is reachable, it is important to check for controllability. Through MATLAB: `Rank = rank(ctrb(A, B))` we find the system is controllable, and we can proceed with designing a controller.

II. DESIGN

A. Controller Design

Three separate controllers are considered in this report. The first controller is an unconstrained, two stage MPC. The second controller builds on the first, and introduces both state and input constraints. The third controller further builds on

the first two, introducing disturbance rejection. We start by looking at the unconstrained controller.

1) *Unconstrained*: The Q and R matrices are chosen as follows:

$$Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}; R = I \cdot 0.1 \text{ for } I \in \mathbb{R}^{3 \times 3}$$

The Q matrix was chosen as to prioritise the vertical position and velocity. This represents the high level of importance given to landing at the correct altitude. Similarly, the prioritisation of velocity represents the importance of landing a velocity that prevents a hard landing. The lateral positions, ie r_x and r_y , are given a lower priority than the vertical position, but a higher priority than the lateral velocity, v_x and v_y . The reasoning behind this is that it is more important that the landing location is correct over the lateral velocities throughout the entire simulation.

The R matrix was chosen as to penalise the control inputs, f_x , f_y and f_z , equally. The small value given to R , 0.1, represents the low cost associated with expending fuel to control the rocket, and encourages the rocket to use fuel to exercise control in all three directions equally. Essentially, control is cheap. The horizon length N was chosen to be 5 as it provided a spectral radius of 0.5975, which is well within the stability region of the system.

2) *Constrained*: The constrained controller builds on the unconstrained controller by introducing both state and input constraints. The input constraints are chosen as follows:

$$u_{max} \begin{bmatrix} f_z \cdot \tan \theta \\ f_z \cdot \tan \theta \\ 12 \end{bmatrix}; u_{min} = \begin{bmatrix} -f_z \cdot \tan \theta \\ -f_z \cdot \tan \theta \\ 0 \end{bmatrix}$$

the input constraints given in the brief as $|f_x|, |f_y| \leq f_z \tan \theta$. The angle θ is represents the maximum allowable angle of the rocket engines, and is chosen as $\theta = 10^\circ = 0.17$ rad.

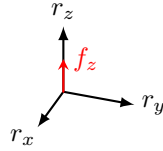


Fig. 1. f_z on the z axis.

Figure ?? provides a visualisation of how f_z acts on the point mass.

The state constraints are chosen as follows:

$$x_{max} = \begin{bmatrix} \frac{r_z}{\tan \phi} + 50 \\ \frac{r_z}{\tan \phi} + 50 \\ 500 \\ 20 \\ 20 \\ 15 \end{bmatrix}; x_{min} = \begin{bmatrix} -\frac{r_z}{\tan \phi} - 50 \\ -\frac{r_z}{\tan \phi} - 50 \\ 0 \\ -20 \\ -20 \\ -15 \end{bmatrix}$$

The r_x r_y constraints were given in the brief as $|r_x|, |r_y| \leq \frac{r_z}{\tan \phi}$. The angle ϕ represents the glide slope angle, and is chosen as a constant $\phi = 30^\circ = 0.52$ rad.

3) *Disturbance Rejection:* The disturbance rejection controller builds on both the unconstrained and constrained controllers. Only wind disturbance is considered, and is modelled as a sinusoid as follows:

$$\vec{w}(k) = \begin{bmatrix} \sin \frac{50}{k} \\ \cos \frac{50}{k} \\ 0 \end{bmatrix}$$

In terms of controller design, we start by rewriting equation ?? as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ew(k) \\ y(k) = Cx(k) + Fd(k) \end{cases}$$

where $E = B$ and $F = 0$ since we ignore output disturbance, hence:

$$\begin{cases} x(k+1) = Ax(k) + B(u(k) + w(k)) \\ y(k) = Cx(k) \end{cases}$$

It is now possible to formulate the T matrix to find $x_{ss}; u_{ss}$.

$$T = \begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix}$$

In this case, we have $C \in \mathbb{R}^{6 \times 6} \wedge B \in \mathbb{R}^{6 \times 3} \therefore p < m \iff 3 < 6$. As such, we expect only some as such, $x_{ss}; u_{ss}$ are given by:

$$\begin{bmatrix} x_{ss}(k) \\ u_{ss}(k) \end{bmatrix} = \begin{bmatrix} T^{-1}B \cdot \vec{w}(k) \\ 0 \end{bmatrix}$$

B. Experiment Setup

First, it is important to state simplifications assumed. Let the starting parameters be as follows:

$$\vec{x}(0) = \begin{bmatrix} 600 \\ 600 \\ 500 \\ 5 \\ 5 \\ -15 \end{bmatrix} \wedge \vec{u}(0) = \vec{0}$$

These starting conditions are the limits of what the assignment permits, i.e a maximum starting altitude of 500m, and a maximum lateral distance of 600m.

III. RESULTS

IV. ANALYSIS & DISCUSSION

V. CONCLUSION

VI. EASE OF USE

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ACKNOWLEDGMENT

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