Collaboration is encouraged; however, students must submit their own work.

Mathematics Department Kwantlen Polytechnic University

MATH 4280 | Assignment #1 | **Due on January 22**
Graph Theory & Applications | Spring 2026

1. Model a scenario in your life using a graph G with at least 10 vertices and try to answer some meaningful questions about your graph. Explicitly indicate what the vertices and edges are.

   Watch the video to get started with making a graph and studying it in SageMath. Then based on the nature of information you are modeling, you can answer these kinds of questions about the graph G:

   - Is my graph connected? $G.is\_connected()$
   - Size of the largest group of vertices that are mutually adjacent[1]? $G.clique\_number()$

     Present such cliques. $G.cliques\_maximum()$

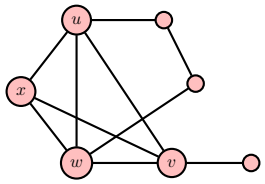

Figure 1: Vertices u, v, w and x form a clique because each pair of these vertices is adjacent

   - How many vertices to remove in order to get a disconnected graph? $G.vertex\_connectivity()$
   - More... you can ask meaningful questions about your graph and then look up the code for getting that info.

> - To start a blank notebook on CoCalc, follow the steps:
>
> New $\longrightarrow$ Jupyter notebook $\longrightarrow$ Suggested kernel SageMath
>
> - You can enter the info about your graph as in the GT_A1_St.ipynb file, or store your graph info in a csv file (see my sample in the .zip file), include the file in the same folder as your Notebook and use the sample code in the .zip file to import your graph data.

[1] Such a group is called a clique and the largest is maximum clique; they could show, e.g., largest groups of people who are mutually friends in a network.

2. (a) Draw graphs on 3, 4, and 5 vertices so that the degrees of vertices are as diverse as possible.

(a') Draw a graph on 8 vertices so that the degrees of vertices are as diverse as possible.

(a") Were you able to find a graph in which all the vertex degrees are distinct? Does such a graph exist?

(b) Generalize your observation above for graphs of any order n and formulate it as a proposition. Prove your proposition.

3. (a) Consider a graph of order $n = 3$. How many edges (minimum) must be there to be sure the graph is connected.

(a') Now let $n = 4$. How many edges (minimum) must be there to be sure the graph is connected.

(a") Repeat this for $n = 5$ and $n = 6$.

(b) Generalize your observation above for graphs of any order n and formulate it as a proposition. Prove your proposition.

4. (a) Draw a 2–regular graph and call it G. Then draw a 3–regular graph that has G as a subgraph and call it H.

(a') Now draw a 4–regular graph that has H as a subgraph.

(b) Let G be an r–regular graph. Present a process that builds an $(r+1)$–regular graph that has G as a subgraph.

5. (a) Draw an arbitrary graph on eight vertices. Find $\delta(G), \Delta(G)$ and $\frac{2m}{n}$ and write them in order of non-decreasing.

(a') Repeat (a) for another arbitrary graph on eight vertices.

(a") Are the orders of elements in (a) and (a') the same? Try to explain why.

(b) Show that in a graph G, $\delta(G) \leq \frac{2\,m}{n} \leq \Delta(G)$.

6. We can build graphs in however way we wish. One of them is to assume vertices represent sets and then define when two vertices are adjacent. For example, we can take two vertices to be adjacent if their corresponding edges share no elements; see Figure 2 for a example illustration.

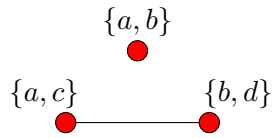

Figure 2: Vertices are sets; two vertices are adjacent if their corresponding sets do not intersect

(a) Let G be a graph in which V is all the 2–subsets of the set $\{a, b, c, d, e\}$ and two vertices are adjacent if their corresponding subsets are disjoint (their intersection is empty). Draw G and find the followings.

(i) Order of G.

(ii) Degree of each vertex.

(iii) Size of G.

(b) Now consider the graph G in which V is all the k–subsets of the set $\{1, 2, 3, \ldots, \ell\}$, for $k \leq \ell$, and two vertices are adjacent if their corresponding subsets are disjoint. Find order of G, degree of each vertex and size of G.