#### HUMBOLDT-UNIVERSITÄT ZU BERLIN



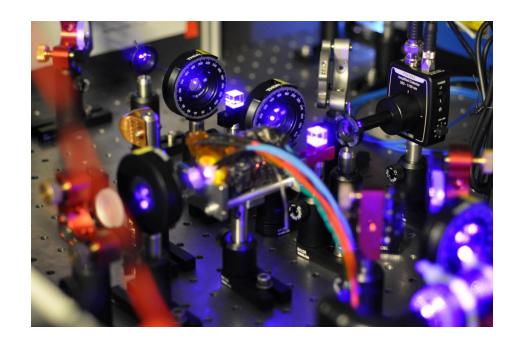
QTutorium

# Thermal attenuation noise of optical power from a 420 nm light source in hot 85,87Rb vapor

#### **Student**

Julien Kluge

564513



### The Idea

### **Experiment**

Dopplerfreie Sättigungsspektroskopie in heißen Rubidium Dampf



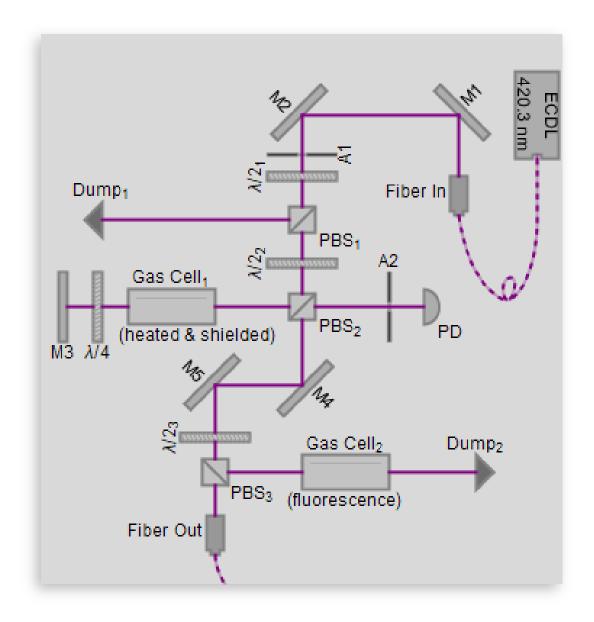


Direct optical spectroscopy of the  $6\mathrm{P}$  manifold in Rubidium

Master thesis

for obtaining the degree Master of Science (M.Sc.)

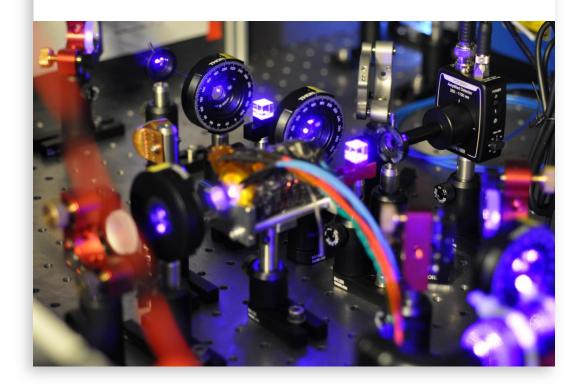
Humboldt-Universität zu Berlin Faculty of Mathematics and Natural Sciences Department of Physics AG Optical Metrology

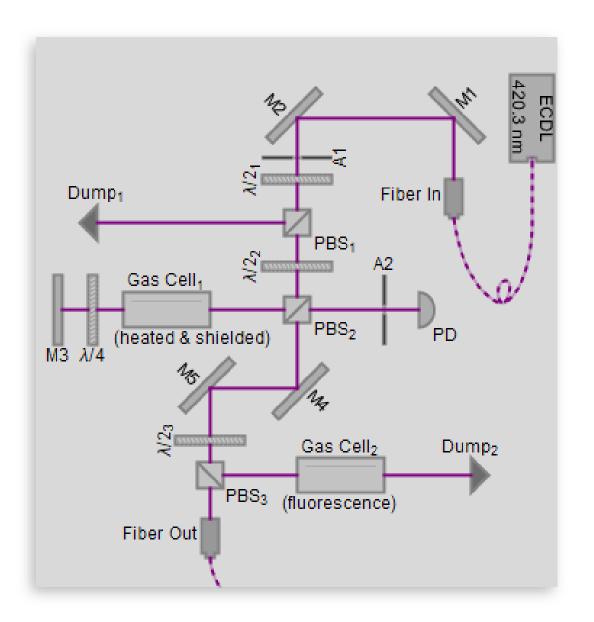


### The Idea

### **Experiment**

Dopplerfreie Sättigungsspektroskopie in heißen Rubidium Dampf

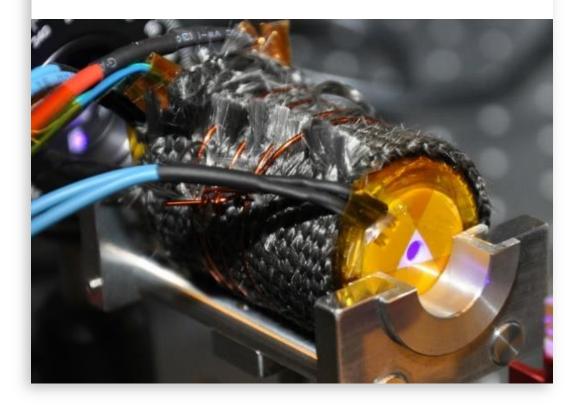


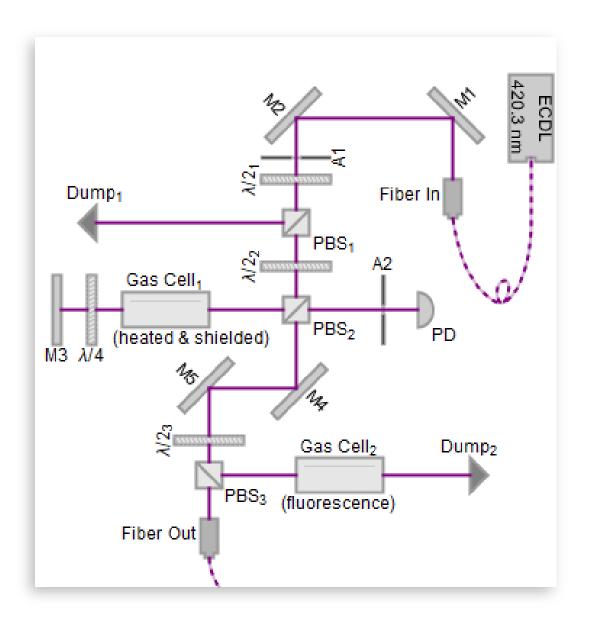


### The Idea

### **Experiment**

Dopplerfreie Sättigungsspektroskopie in heißen Rubidium Dampf





# Proposed evaluation

#### **Box Müller Transformation**

Transformation von uniform verteilten Zahlen zu Gauss-Verteilung

$$Z_1 = \sqrt{-2 \log U_1} \cos(2\pi U_2)$$

$$Z_2 = \sqrt{-2 \log U_1} \sin(2\pi U_2)$$

Scott, D. W. Box Muller transformation. *Wiley Interdisciplinary Reviews: Computational Statistics* **3**, 177 179. issn: 1939-0068 (2011).

#### **Inversion**

$$U_1 = \exp\left(\frac{-[Z_1^2 + Z_2^2]}{2}\right)$$

$$U_2 = \frac{1}{2\pi} \arccos \left( -\frac{Z_1}{\sqrt{Z_1^2 + Z_2^2}} \right)$$

# Proposed evaluation

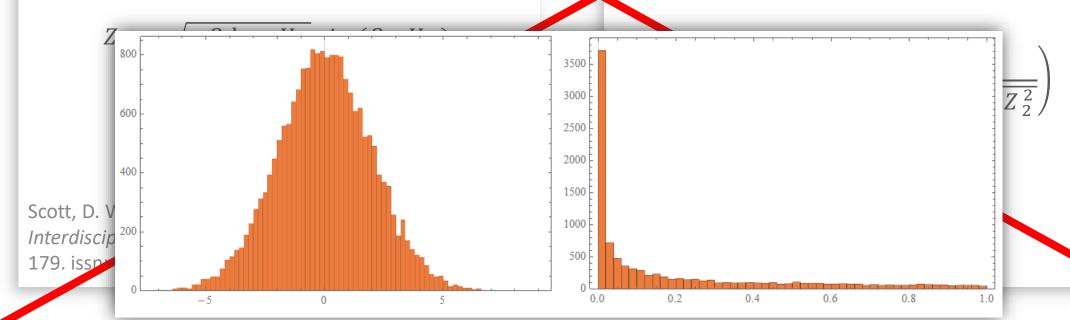
#### **Box Müller Transformation**

Transformation von uniform verteilten Zahlen zu Gauss-Verteilung

$$Z_1 = \sqrt{-2 \log U_1} \cos(2\pi U_2)$$

### **Inversion**

$$L_1 = \exp\left(\frac{-[Z_1^2 + Z_2^2]}{2}\right)$$



### New evaluation

# **Cumulative distribution function**

math.stackexchange to the rescue

Let  $X \sim \mathcal{N}(\mu, \sigma^2)$  have a normal distribution with mean  $\mu = 0$  and variance  $\sigma^2 = 0.2$ , which cumulative distribution function (CDF) is denoted by  $\Phi_X$ . The variable  $Y = 6\Phi_X(X) - 3$  has a uniform distribution over [-3, 3]. In facts,

$$egin{aligned} \mathbb{P}(Y \leq t) &= \mathbb{P}\left(\Phi_X(X) \leq rac{t+3}{6}
ight) \ &= \mathbb{P}\left(X \leq \Phi_X^{-1}\left(rac{t+3}{6}
ight)
ight) \ &= \Phi_X\left(\Phi_X^{-1}\left(rac{t+3}{6}
ight)
ight) \ &= rac{t+3}{6} \ , \end{aligned}$$

if 
$$-3 \le t \le 3$$
,  $\mathbb{P}(Y \le t) = 0$  if  $t \le -3$ , and  $\mathbb{P}(Y \le t) = 1$  if  $t \ge 3$ .

share cite edit flag

edited Jul 4 '17 at 12:41

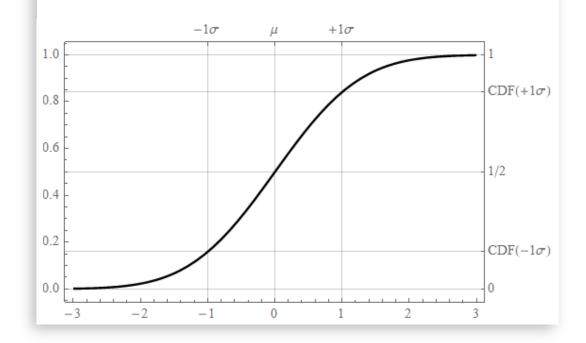
answered Jul 2 '17 at 15:59



https://math.stackexchange.com/a/2344086/373704

#### <u>CDF</u>

$$U_1(\mu, \sigma) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x - \mu}{\sqrt{2} \sigma} \right) \right]$$



### New evaluation

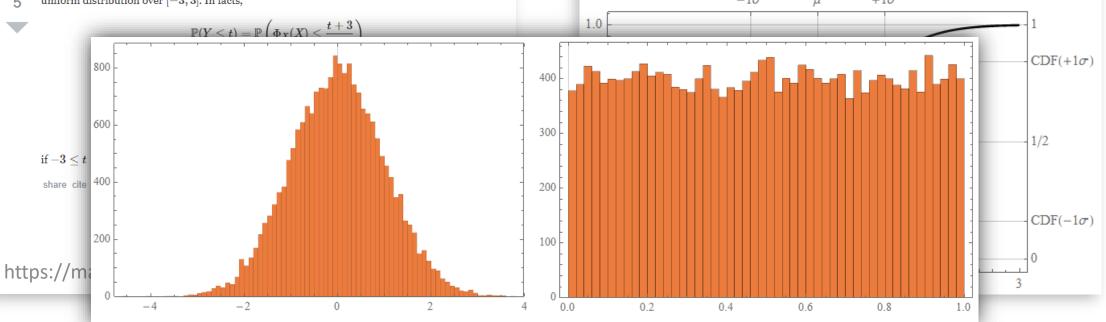
# **Cumulative distribution function**

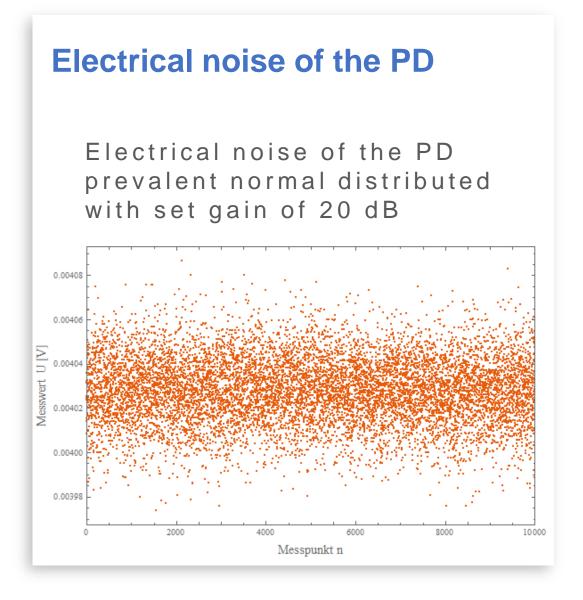
math.stackexchange to the rescue

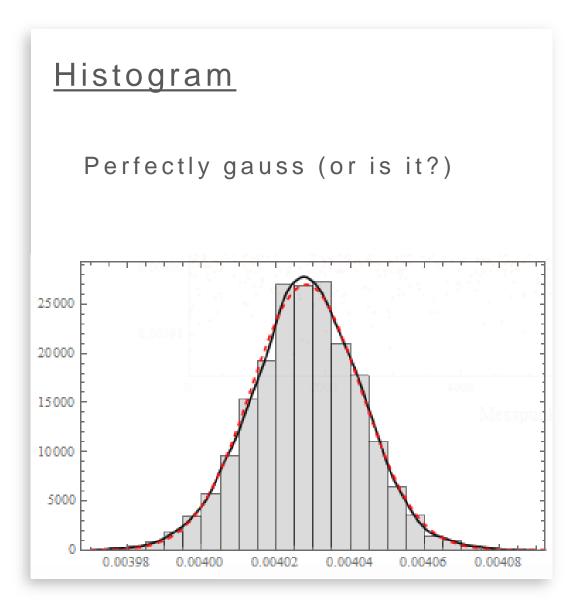
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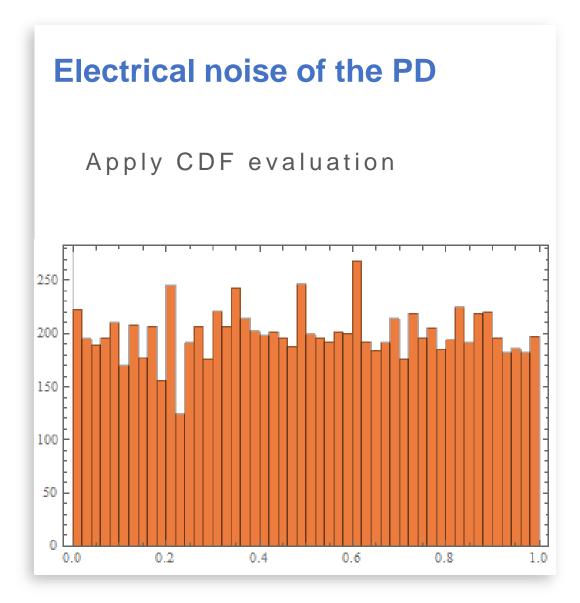
<u>CDF</u>

$$U_1(\mu, \sigma) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x - \mu}{\sqrt{2} \sigma} \right) \right]$$



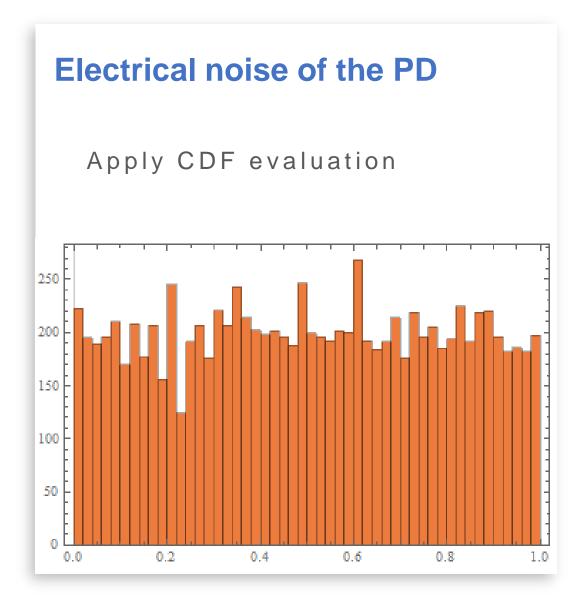






#### Result

Looks good doesn't it?



#### Result

NIST test suite with binarize (uniform \* 10 6):

Number	Name	P-Value	Time
1	MonobitFrequencyTest	3.97792×10 <sup>-111</sup>	0.03125
2	BlockFrequencyTest	2.10146 × 10 <sup>-11</sup>	0.03125
3	RunsTest	0.000148928	0.046875
4	LongestRunsOnes10000	0.000289803	0.0625
5	BinaryMatrixRankTest	0.347651	0.0625
6	SpectralTest	0.947477	0.15625
7	NonOverlappingTemplateMatching	5.52455 × 10 <sup>-42</sup>	0.015625
8	OverlapingTemplateMatching	0.000159769	0.015625
9	MaurersUniversalStatisticTest	0.0459829	0.359375
10	LinearComplexityTest	0.994662	2.75
11	SerialTest	$\{1.6818084733 \times 10^{-91482}, 2.4336268069 \times 10^{-37729}\}$	1.84375
12	ApproximateEntropyTest	1.899254225243×10 <sup>-5289</sup>	0.984375
13	CumulativeSumsTest	0.	0.015625
14	RandomExcursionsTest	{ {-4., 0.378324}, {-3., 0.784066}, {-2., 0.665949},	0.15625
		$\{-1., 0.584179\}$ , $\{1., 0.22782\}$ , $\{2., 0.189628\}$ , $\{3., 0.63857\}$ , $\{4., 0.576152\}$	
15	RandomExcursionsVariantTest	{ {-9., 0.431923}, {-8., 0.402784}, {-7., 0.368803}, {-6., 0.35212}, {-5., 0.328443}, {-4., 0.321461},	0.03125
		$\{-3., 0.300623\}, \{-2., 0.24681\}, \{-1., 0.164915\}, \{1., 0.757621\}, \{2., 0.24681\}, \{3., 0.214193\},$	
		{4., 0.243443}, {5., 0.303626}, {6., 0.35212}, {7., 0.392041}, {8., 0.425556}, {9., 0.500545}}	
16	CumulativeSumsTestReverse	0.	0.015625
17	LempelZivCompressionTest	$2.226089178210959 \times 10^{-9}$	1.

#### Wtf...

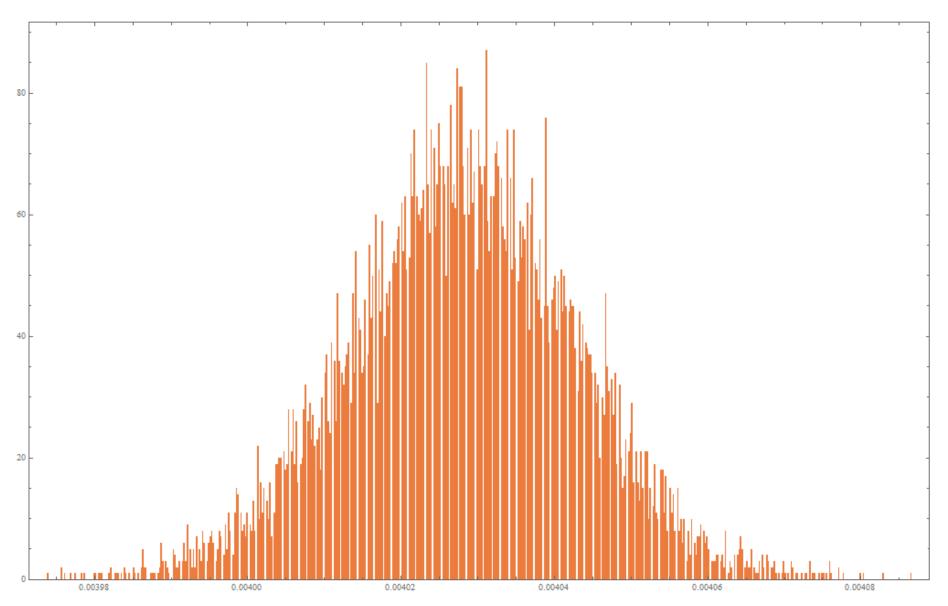
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		$\{4., 0.243443\}, \{5., 0.303626\}, \{6., 0.35212\}, \{7., 0.392041\}, \{8., 0.425556\}, \{9., 0.500545\}\}$	
16	CumulativeSumsTestReverse	0.	0.015625
17	LempelZivCompressionTest	$2.226089178210959 \times 10^{-9} \frac{221056}{10}$	1.

Again with binarize(uniform)

Better, but far less data

Number	Name	P-Value	Time
1	MonobitFrequencyTest	0.841481	0.
2	BlockFrequencyTest	0.104708	0.015625
3	RunsTest	0.936555	0.
4	LongestRunsOnes10000	0.653236	0.
5	BinaryMatrixRankTest	0.587007	0.
6	SpectralTest	0.270812	0.015625
7	NonOverlappingTemplateMatching	0.732903	0.
8	OverlapingTemplateMatching	0.106393	0.
9	MaurersUniversalStatisticTest	0.916699	0.
10	LinearComplexityTest	0.469323	0.171875
11	SerialTest	{0.619757, 0.775994}	0.171875
12	ApproximateEntropyTest	0.107842	0.046875
13	CumulativeSumsTest	0.242276	0.015625
14	RandomExcursionsTest	{ {-4., 0.31674}, {-3., 0.479924}, {-2., 0.795072},	0.015625
		$\{-1., 0.394781\}, \{1., 0.17429\}, \{2., 0.20817\}, \{3., 0.396093\}, \{4., 0.462675\}\}$	
15	RandomExcursionsVariantTest	$\{\{-9., 0.509517\}, \{-8., 0.404943\}, \{-7., 0.361695\}, \{-6., 0.340356\}, \{-5., 0.332164\}, \{-4., 0.363681\}, \{-6., 0.340356\}, \{-5., 0.332164\}, \{-4., 0.363681\}, \{-6., 0.340356\}, \{-6$	0.
		$\{-3., 0.61067\}, \{-2., 0.826581\}, \{-1., 0.254945\}, \{1., 0.0268567\}, \{2., 0.0259202\}, \{3., 0.0619348\},$	
		$\{4., 0.315382\}, \{5., 0.527089\}, \{6., 0.423185\}, \{7., 0.361695\}, \{8., 0.273909\}, \{9., 0.304075\}\}$	
16	CumulativeSumsTestReverse	0.35393	0.015625
17	LempelZivCompressionTest	$1.331464696271033 \times 10^{-14425262}$	0.046875

# Electrical noise fault



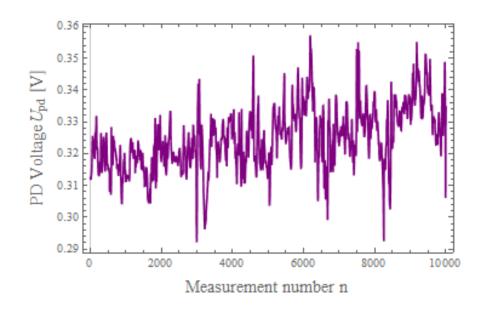
ADC discretization

387 different values where measured

equivalent: 300 nV/sample

#### **PD Power measured**

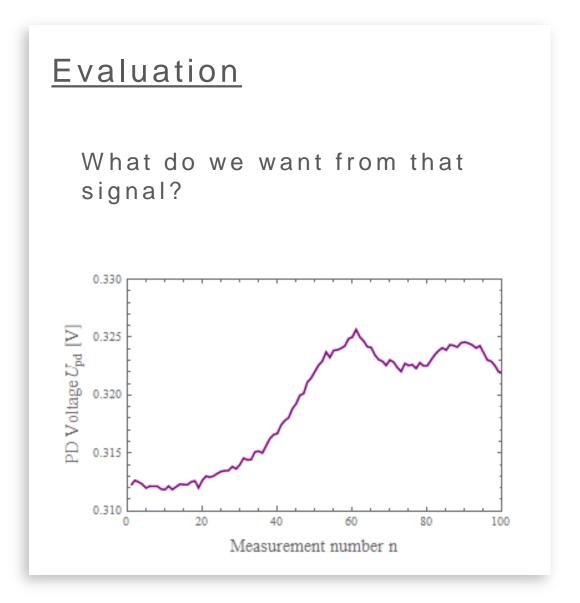
Laser power (non-resonant) measured by PD with 5% of intensity and gas cell at 75°C

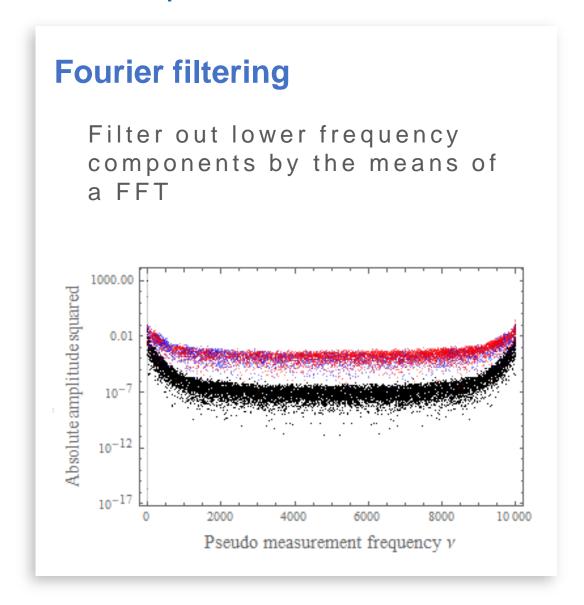


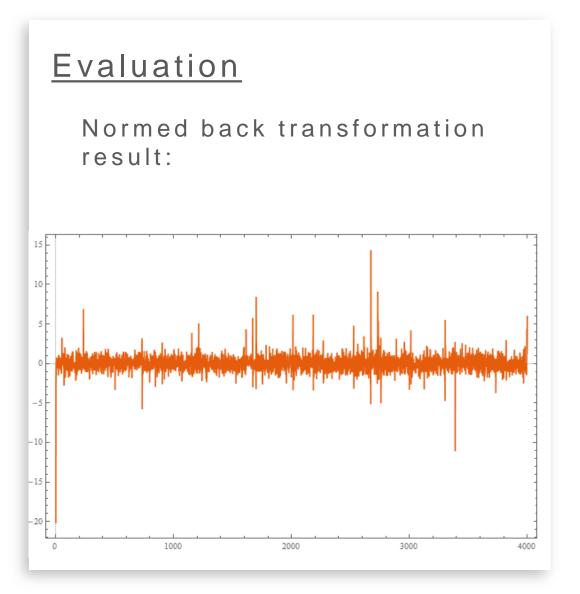
#### **Evaluation**

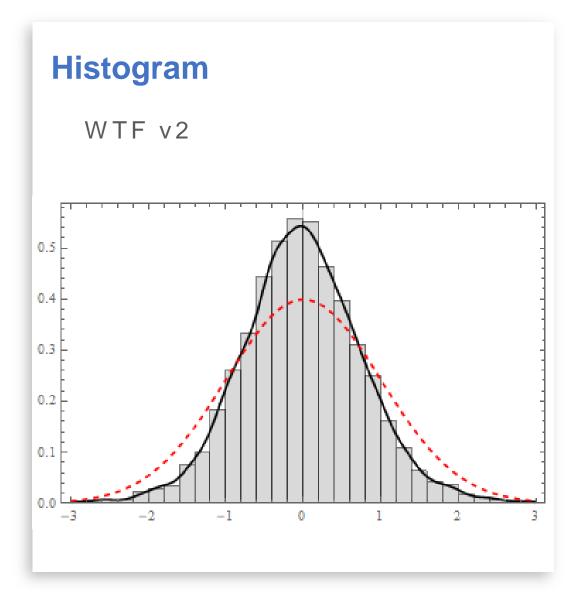
What do we want from that signal?

### PD Power measured Laser power (non-resonant) measured by PD with 5% of intensity and gas cell at 75°C 0.35 PD Voltage Upd [V] 0.30 2000 4000 6000 8000 10000 Measurement number n

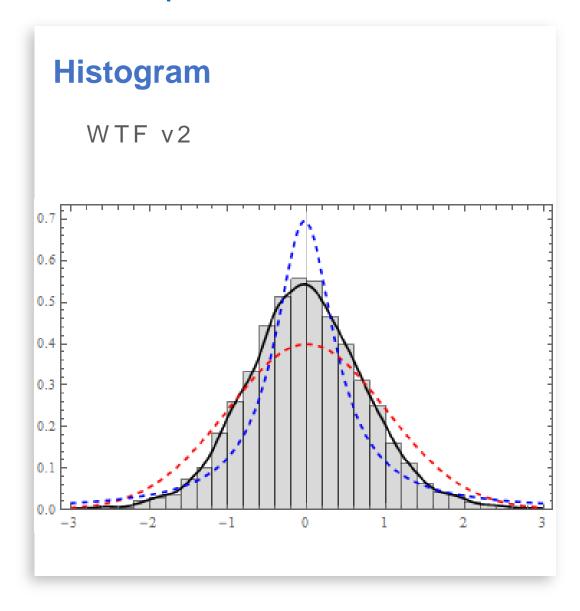




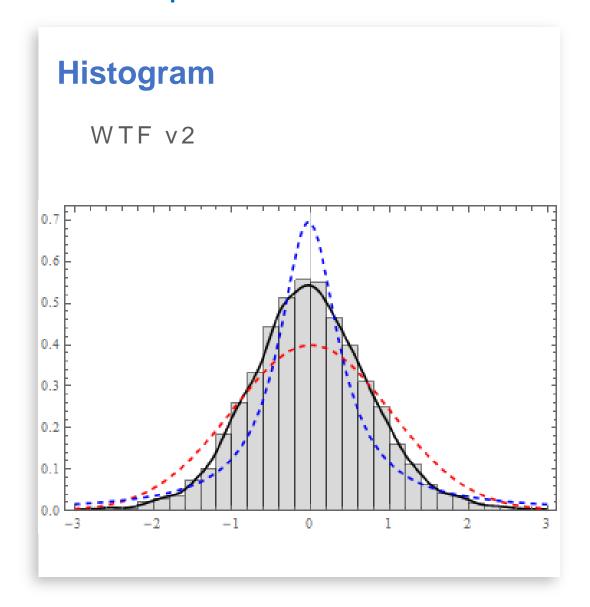


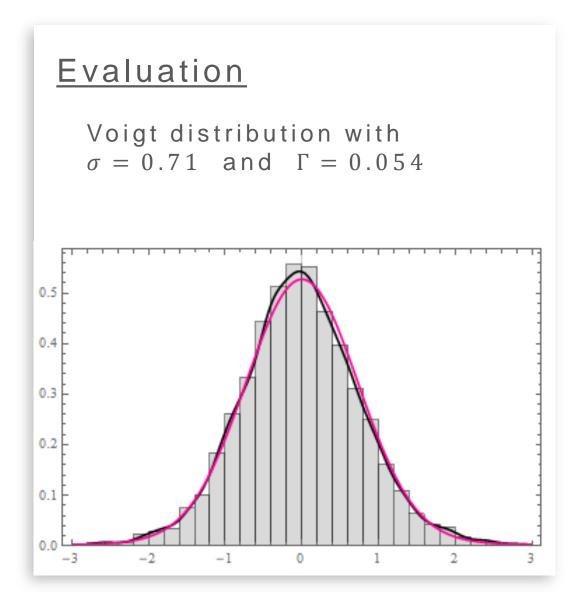


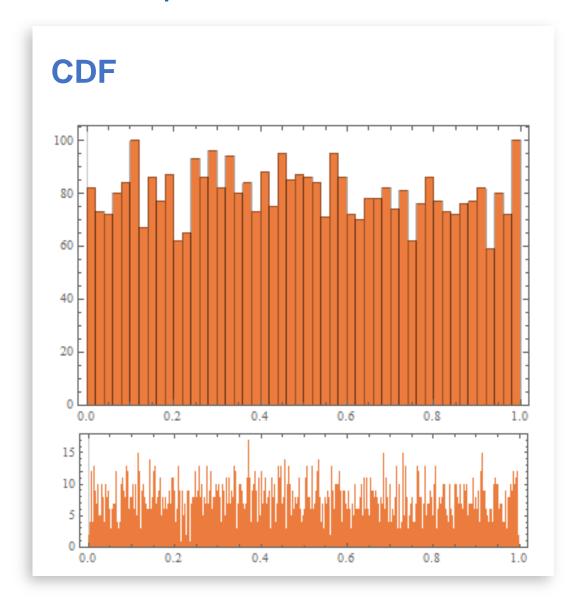
Evaluation



**Evaluation** 









Number	Name	P-Value	Time
1	MonobitFrequencyTest	$8.52249 \times 10^{-29}$	0.015625
2	BlockFrequencyTest	0.346865	0.03125
3	RunsTest	0.302571	0.046875
4	LongestRunsOnes10000	0.0596088	0.046875
5	BinaryMatrixRankTest	0.601812	0.046875
6	SpectralTest	0.804172	0.109375
7	NonOverlappingTemplateMatching	0.0795404	0.015625
8	OverlapingTemplateMatching	0.0133263	0.
9	MaurersUniversalStatisticTest	0.955476	0.3125
10	LinearComplexityTest	0.419316	2.67188
11	SerialTest	{0.245029, 0.0560255}	1.89063
12	ApproximateEntropyTest	0.0316065	0.890625
13	CumulativeSumsTest	$-2.65683 \times 10^{-245}$	0.03125
14	RandomExcursionsTest	{ {-4., 0.143588}, {-3., 0.769867}, {-2., 0.250748},	0.09375
		$\{-1., 0.369626\}$ , $\{1., 0.545633\}$ , $\{2., 0.426357\}$ , $\{3., 0.220249\}$ , $\{4., 0.382739\}$	
15	RandomExcursionsVariantTest	{{-9., 0.454735}, {-8., 0.769371}, {-7., 0.718894}, {-6., 0.27111}, {-5., 0.15187}, {-4., 0.177367},	0.03125
		$\{-3., 0.118813\}, \{-2., 0.241706\}, \{-1., 0.935354\}, \{1., 0.33039\}, \{2., 0.281447\}, \{3., 0.179554\},$	
		$\{4., 0.326581\}, \{5., 0.892466\}, \{6., 0.281903\}, \{7., 0.149939\}, \{8., 0.160569\}, \{9., 0.18099\}\}$	
16	CumulativeSumsTestReverse	$-1.53393 \times 10^{-253}$	0.03125
17	LempelZivCompressionTest	$2.231650903975532 \times 10^{-9852947}$	0.796875

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#### Results

For every measurement:
Monobit test & CumSum test
fails

Always compressible

Correlation of successive values suspected

Temperature not dependent

Worse quality than pi or e

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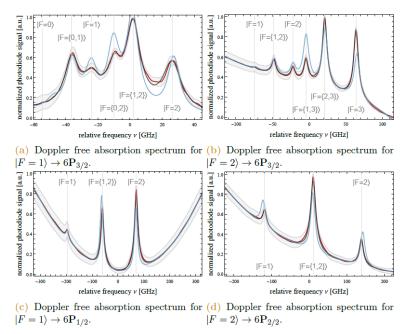
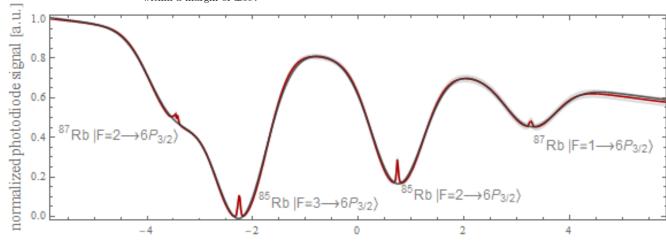


Figure 12: Doppler free absorption spectroscopy signals (red) of  $^{87}$ Rb with Lorentz fits (gray) and theoretical spectrum (blue). The error interval (light gray) is given within a margin of  $\pm 5\sigma$ .



relative frequency v [GHz]

# Vielen Dank für die Aufmerksamkeit