Week 7 Practical Session

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Factor analysis

You might like to explore the `psych` package, which has some additional features for exploratory factor analysis.
Let's have a look at the dataset `bfi`, which are responses to a personality test. There are supposed to be five factors: Agreeableness, Conscientiousness, Extraversion, Neuroticism, and Openness. There are five items that are intended to load on each factor. More details can be found using `help('bfi')`.

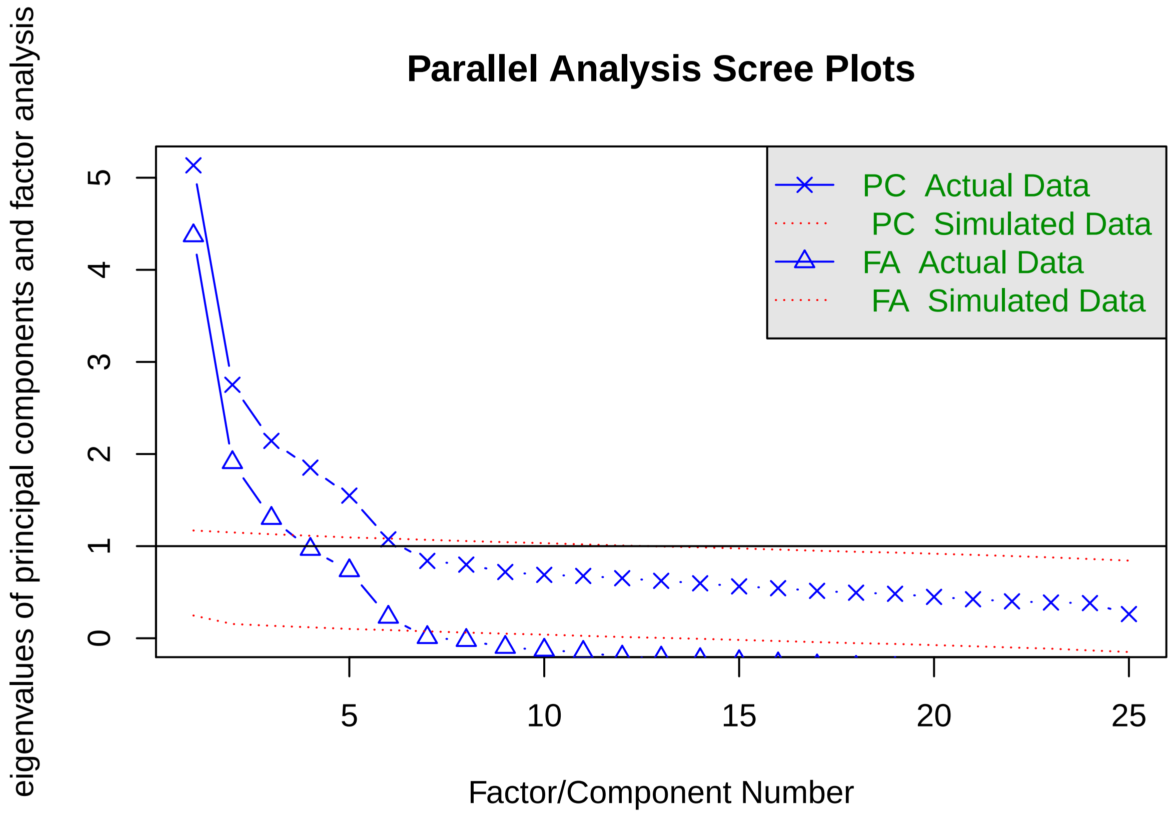
Number of factors

There is no universally agreed upon way of deciding on the number of factors. In the `psych` package there is a function `VSS` that provides some tests.

```r
library(psych)
data(bfi)
summary(bfi)
```

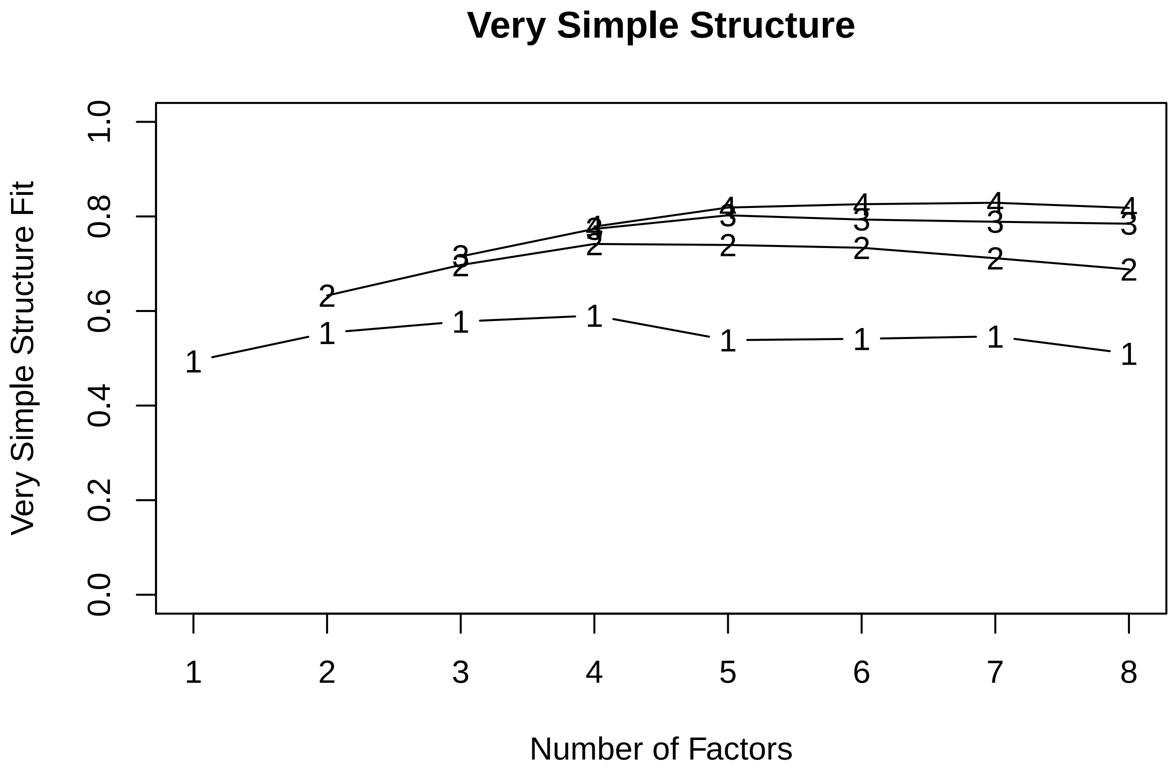
```
##        A1              A2              A3              A4     
##  Min.   :1.000   Min.   :1.000   Min.   :1.000   Min.   :1.0  
##  1st Qu.:1.000   1st Qu.:4.000   1st Qu.:4.000   1st Qu.:4.0  
##  Median :2.000   Median :5.000   Median :5.000   Median :5.0  
##  Mean   :2.413   Mean   :4.802   Mean   :4.604   Mean   :4.7  
##  3rd Qu.:3.000   3rd Qu.:6.000   3rd Qu.:6.000   3rd Qu.:6.0  
##  Max.   :6.000   Max.   :6.000   Max.   :6.000   Max.   :6.0  
##  NA's   :16      NA's   :27      NA's   :26      NA's   :19    
##        A5             C1              C2             C3       
##  Min.   :1.00   Min.   :1.000   Min.   :1.00   Min.   :1.000  
##  1st Qu.:4.00   1st Qu.:4.000   1st Qu.:4.00   1st Qu.:4.000  
##  Median :5.00   Median :5.000   Median :5.00   Median :5.000  
##  Mean   :4.56   Mean   :4.502   Mean   :4.37   Mean   :4.304  
##  3rd Qu.:5.00   3rd Qu.:5.000   3rd Qu.:5.00   3rd Qu.:5.000  
##  Max.   :6.00   Max.   :6.000   Max.   :6.00   Max.   :6.000  
##  NA's   :16     NA's   :21      NA's   :24     NA's   :20     
##        C4              C5              E1              E2       
##  Min.   :1.000   Min.   :1.000   Min.   :1.000   Min.   :1.000  
##  1st Qu.:1.000   1st Qu.:2.000   1st Qu.:2.000   1st Qu.:2.000  
##  Median :2.000   Median :3.000   Median :3.000   Median :3.000  
##  Mean   :2.553   Mean   :3.297   Mean   :2.974   Mean   :3.142  
##  3rd Qu.:4.000   3rd Qu.:5.000   3rd Qu.:4.000   3rd Qu.:4.000  
##  Max.   :6.000   Max.   :6.000   Max.   :6.000   Max.   :6.000  
##  NA's   :26      NA's   :16      NA's   :23      NA's   :16     
##        E3              E4              E5              N1       
##  Min.   :1.000   Min.   :1.000   Min.   :1.000   Min.   :1.000  
##  1st Qu.:3.000   1st Qu.:4.000   1st Qu.:4.000   1st Qu.:2.000  
##  Median :4.000   Median :5.000   Median :5.000   Median :3.000  
##  Mean   :4.001   Mean   :4.422   Mean   :4.416   Mean   :2.929  
##  3rd Qu.:5.000   3rd Qu.:6.000   3rd Qu.:5.000   3rd Qu.:4.000  
##  Max.   :6.000   Max.   :6.000   Max.   :6.000   Max.   :6.000  
##  NA's   :25      NA's   :9       NA's   :21      NA's   :22     
##        N2              N3              N4              N5      
##  Min.   :1.000   Min.   :1.000   Min.   :1.000   Min.   :1.00  
##  1st Qu.:2.000   1st Qu.:2.000   1st Qu.:2.000   1st Qu.:2.00  
##  Median :4.000   Median :3.000   Median :3.000   Median :3.00  
##  Mean   :3.508   Mean   :3.217   Mean   :3.186   Mean   :2.97  
##  3rd Qu.:5.000   3rd Qu.:4.000   3rd Qu.:4.000   3rd Qu.:4.00  
##  Max.   :6.000   Max.   :6.000   Max.   :6.000   Max.   :6.00  
##  NA's   :21      NA's   :11      NA's   :36      NA's   :29    
##        O1              O2              O3              O4      
##  Min.   :1.000   Min.   :1.000   Min.   :1.000   Min.   :1.000  
##  1st Qu.:4.000   1st Qu.:1.000   1st Qu.:4.000   1st Qu.:4.000  
##  Median :5.000   Median :2.000   Median :5.000   Median :5.000  
##  Mean   :4.816   Mean   :2.713   Mean   :4.438   Mean   :4.892  
##  3rd Qu.:6.000   3rd Qu.:4.000   3rd Qu.:5.000   3rd Qu.:6.000  
##  Max.   :6.000   Max.   :6.000   Max.   :6.000   Max.   :6.000  
##  NA's   :22                      NA's   :28      NA's   :14     
##        O5           gender        education          age       
##  Min.   :1.00   Min.   :1.000   Min.   :1.00   Min.   : 3.00  
##  1st Qu.:1.00   1st Qu.:1.000   1st Qu.:3.00   1st Qu.:20.00  
##  Median :2.00   Median :2.000   Median :3.00   Median :26.00  
##  Mean   :2.49   Mean   :1.672   Mean   :3.19   Mean   :28.78  
##  3rd Qu.:3.00   3rd Qu.:2.000   3rd Qu.:4.00   3rd Qu.:35.00  
##  Max.   :6.00   Max.   :2.000   Max.   :5.00   Max.   :86.00  
##  NA's   :20                     NA's   :223                    
```

```r
bfi.R <- cor(bfi[, 1:25], use = 'complete.obs')
p1 <- principal(bfi.R, rotate='varimax', n.obs = 2800)
fa.parallel(bfi.R, n.obs=2800, fm='ml')
```



```
## Parallel analysis suggests that the number of factors =  6  and the number of components =  5
```

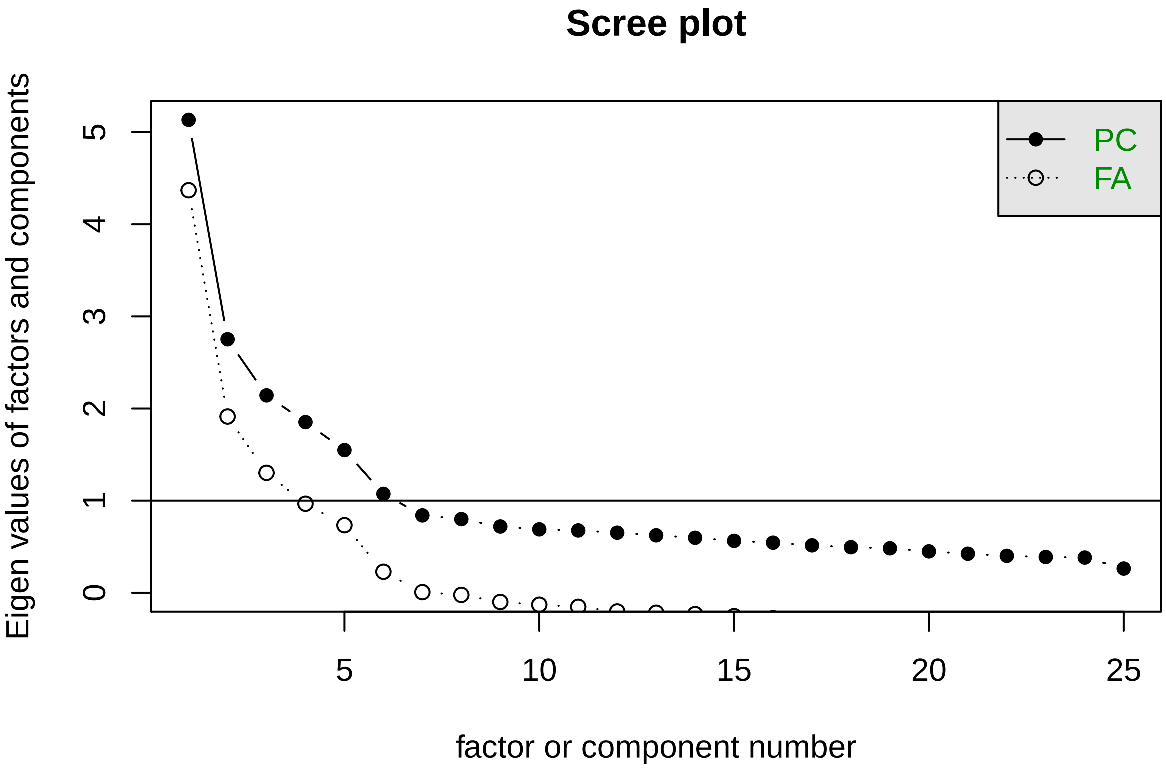
```
v1 <- VSS(bfi.R, 8, fm='ml', n.obs = 2800)
```



```
v1
```

```
##
## Very Simple Structure
## Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
##     n.obs = n.obs, plot = plot, title = title, use = use, cor = cor)
## VSS complexity 1 achieves a maximimum of 0.59  with  4  factors
## VSS complexity 2 achieves a maximimum of 0.74  with  4  factors
##
## The Velicer MAP achieves a minimum of 0.01  with  5  factors
## BIC achieves a minimum of  -511.93  with  8  factors
## Sample Size adjusted BIC achieves a minimum of  -105.23  with  8  factors
##
## Statistics by number of factors
##   vss1 vss2   map dof chisq     prob sqresid  fit  RMSEA   BIC SABIC
## 1 0.49 0.00 0.025 275 12221  0.0e+00    26.2 0.49 0.0155 10038 10912
## 2 0.55 0.63 0.019 251  7570  0.0e+00    19.0 0.63 0.0104  5578  6375
## 3 0.58 0.70 0.018 228  5163  0.0e+00    14.7 0.72 0.0077  3354  4078
## 4 0.59 0.74 0.016 206  3421  0.0e+00    11.5 0.78 0.0056  1786  2441
## 5 0.54 0.74 0.015 185  1715 1.0e-245     9.2 0.82 0.0030   246   834
## 6 0.54 0.73 0.016 165  1031 2.7e-125     8.2 0.84 0.0019  -278   246
## 7 0.55 0.71 0.019 146   712  2.2e-75     7.7 0.85 0.0014  -447    17
## 8 0.51 0.69 0.022 128   504  4.6e-46     7.3 0.86 0.0010  -512  -105
##   complex eChisq  SRMR eCRMS  eBIC
## 1     1.0  24608 0.121 0.126 22425
## 2     1.2  13022 0.088 0.096 11030
## 3     1.3   7482 0.067 0.077  5673
## 4      1.4    3787 0.047 0.057   2152
## 5      1.6    1376 0.029 0.036    -93
## 6      1.7     660 0.020 0.027   -650
## 7      1.8     441 0.016 0.023   -718
## 8      1.9     285 0.013 0.020   -731
```

```
scree(bfi.R)
```



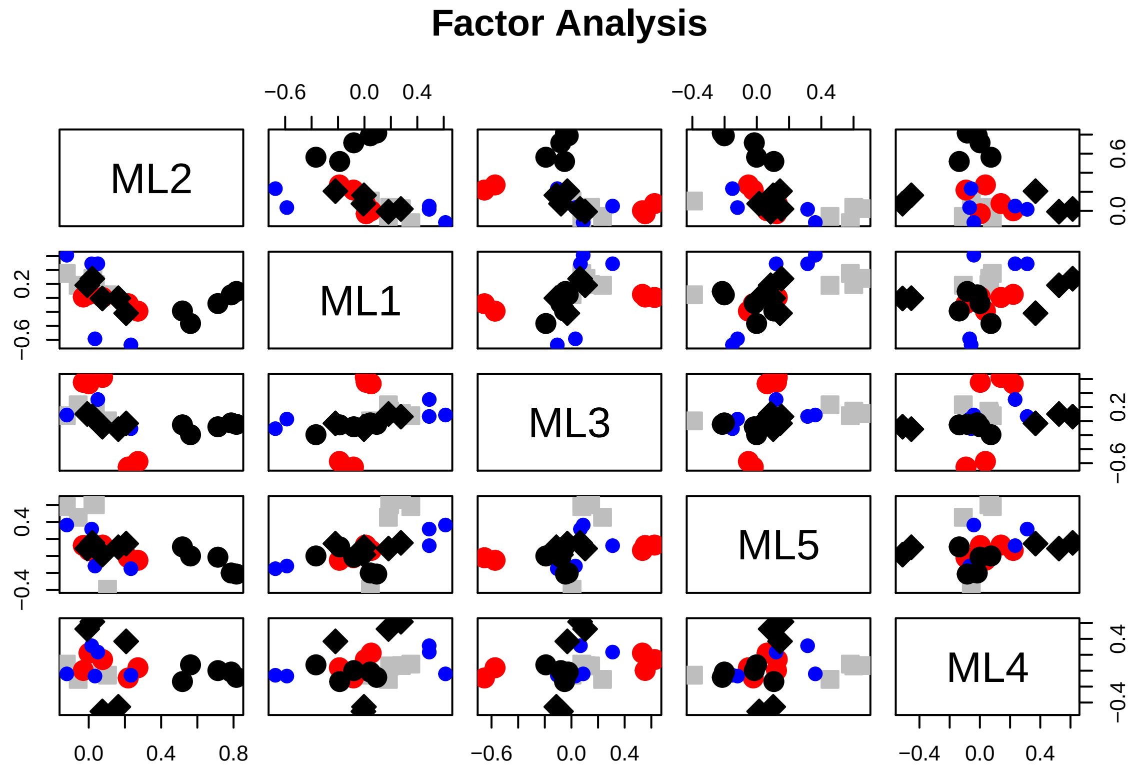
It's not very clear whether you need four or five factors. Five is the number there are supposed to be (but there are lots of reasons why this might not work in practice). Let's compare the five and four factor solutions.

Five factors

```
ml5.out <- fa(bfi.R, nfactors = 5, rotate = "varimax", fm='ml', n.obs = 2800)
ml5.out$loadings
```

```
##
## Loadings:
##    ML2    ML1    ML3    ML5    ML4
## A1  0.104               -0.393
## A2         0.191  0.144  0.601
## A3         0.280  0.110  0.662
## A4         0.181  0.234  0.454 -0.109
## A5 -0.124  0.351         0.580
## C1                0.533         0.221
## C2                0.624  0.127  0.140
## C3                 0.554  0.122
## C4   0.218        -0.653
## C5   0.272 -0.190 -0.573
## E1         -0.587        -0.120
## E2   0.233 -0.674 -0.106 -0.151
## E3          0.490         0.315  0.313
## E4  -0.121  0.613         0.363
## E5          0.491  0.310  0.120  0.234
## N1   0.816               -0.214
## N2   0.787               -0.202
## N3   0.714
## N4   0.562 -0.367 -0.192
## N5   0.518 -0.187         0.106 -0.137
## O1          0.182  0.103         0.524
## O2   0.163        -0.113  0.102 -0.454
## O3          0.276         0.153  0.614
## O4   0.207 -0.220         0.144  0.368
## O5                              -0.512
##
##                   ML2   ML1   ML3   ML5   ML4
## SS loadings    2.687 2.320 2.034 1.978 1.557
## Proportion Var 0.107 0.093 0.081 0.079 0.062
## Cumulative Var 0.107 0.200 0.282 0.361 0.423
```

```r
plot(ml5.out, cut=0.3, cex = 2)
```



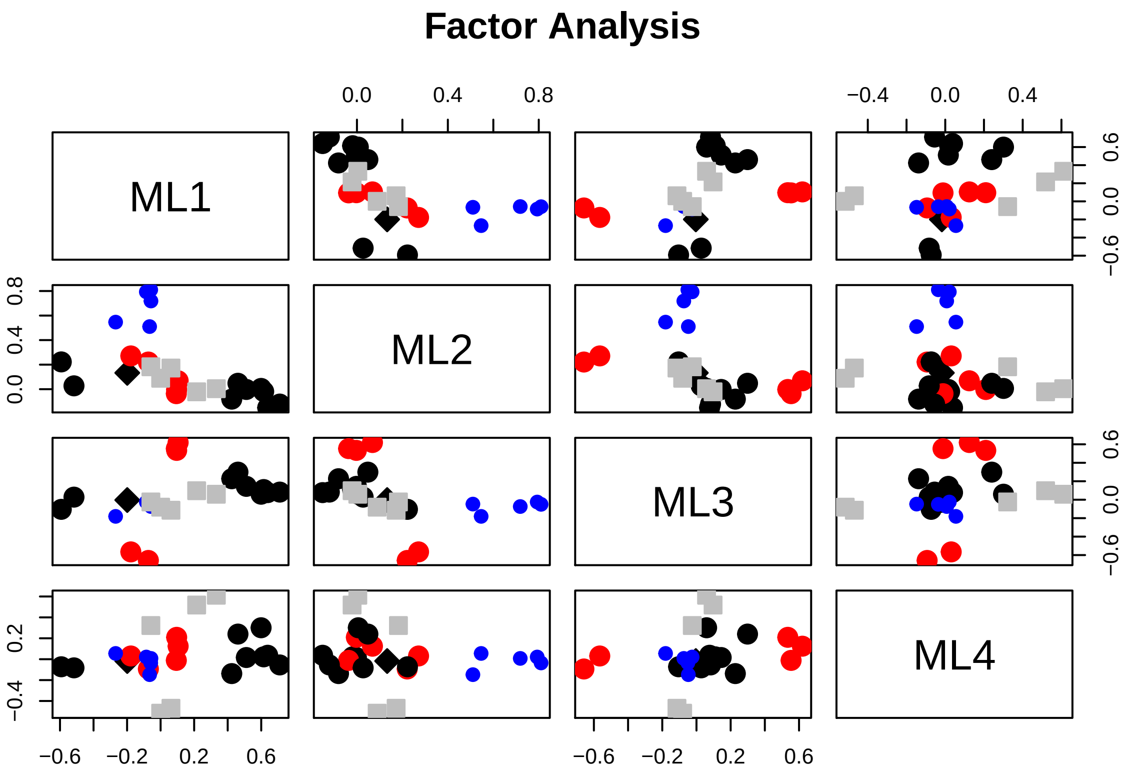
Looking at the loadings, you can see that the Openness item seems the least clear.

Four factors

```
ml4.out <- fa(bfi.R, nfactors = 4, rotate = "varimax", fm='ml', n.obs = 2800)
ml4.out$loadings
```

```
##
## Loadings:
##    ML1    ML2    ML3    ML4
## A1 -0.199  0.133
## A2  0.511         0.145
## A3  0.615         0.110
## A4  0.424         0.227 -0.138
## A5  0.639 -0.151
## C1                0.534  0.209
## C2  0.104         0.620  0.124
## C3                0.554
## C4         0.221 -0.658
## C5 -0.178  0.271 -0.566
## E1 -0.517
## E2 -0.592  0.222 -0.104
## E3  0.599                0.301
## E4  0.711 -0.121
## E5  0.461         0.299  0.240
## N1         0.810
## N2         0.793
## N3         0.719
## N4 -0.268  0.547 -0.181
## N5         0.510        -0.148
## O1  0.215                0.518
## O2         0.172 -0.114 -0.470
## O3  0.332                0.612
## O4         0.183         0.322
## O5                      -0.517
##
##                  ML1   ML2   ML3   ML4
## SS loadings    3.281 2.675 2.008 1.515
## Proportion Var 0.131 0.107 0.080 0.061
## Cumulative Var 0.131 0.238 0.319 0.379
```

```
plot(ml4.out, cut=0.3, cex = 2)
```



This solution fails to distinguish between Agreeableness and Extraversion, so the five factor solution is probably to be preferred.

We can look to see if there are differences based on gender, education and age.

```
bfi.scores <- factor.scores(bfi[1:25], ml5.out)
head(bfi.scores$scores)
```

```
##              ML2        ML1         ML3        ML5        ML4
## 61617 -0.4354757 0.36611194 -1.33325905 -0.8899694 -1.8207464
## 61618  0.0633679 0.56445587 -0.70376255 -0.1175545 -0.1533549
## 61620  0.6395802 0.48282487  0.05552719 -0.9701237  0.2594642
## 61621 -0.1553552 0.07928884 -1.18765462  0.2455958 -1.0712305
## 61622 -0.4324621 0.54197994 -0.04594893 -0.8651299 -0.7552646
## 61623  0.3331240 1.22617711  1.52918815 -0.2171455  0.3692746
```

```
for (i in 1:5){
  print(t.test(bfi.scores$scores[, i], bfi$gender))
  print(summary(lm(bfi.scores$scores[, i] ~ age + education, data = bfi)))
}
```

```
##
##  Welch Two Sample t-test
##
## data:  bfi.scores$scores[, i] and bfi$gender
## t = -74.835, df = 3341.7, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
```

```
##  -1.707024 -1.619860
## sample estimates:
##   mean of x   mean of y
## 0.008343793 1.671785714
##
##
## Call:
## lm(formula = bfi.scores$scores[, i] ~ age + education, data = bfi)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.00410 -0.78312 -0.08146  0.71940  2.79000
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.269872   0.078364   3.444 0.000584 ***
## age         -0.008189   0.002048  -3.999 6.56e-05 ***
## education   -0.011131   0.019643  -0.567 0.570996
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9998 on 2233 degrees of freedom
##   (564 observations deleted due to missingness)
## Multiple R-squared:  0.00826,    Adjusted R-squared:  0.007371
## F-statistic: 9.299 on 2 and 2233 DF,  p-value: 9.513e-05
##
##
##  Welch Two Sample t-test
##
## data:  bfi.scores$scores[, i] and bfi$gender
## t = -76.001, df = 3344.3, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.730773 -1.643718
## sample estimates:
##   mean of x   mean of y
## -0.01545947  1.67178571
##
##
## Call:
## lm(formula = bfi.scores$scores[, i] ~ age + education, data = bfi)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.6323 -0.6373  0.0664  0.7433  2.9443
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.028181   0.078375   0.360   0.7192
## age          0.002475   0.002048   1.209   0.2269
## education   -0.034711   0.019645  -1.767   0.0774 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.9999 on 2233 degrees of freedom
##   (564 observations deleted due to missingness)
## Multiple R-squared:  0.001677,   Adjusted R-squared:  0.0007831
## F-statistic: 1.876 on 2 and 2233 DF,  p-value: 0.1535
##
##
##  Welch Two Sample t-test
##
## data:  bfi.scores$scores[, i] and bfi$gender
## t = -74.8, df = 3333.5, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.713080 -1.625566
## sample estimates:
##   mean of x   mean of y
## 0.002462332 1.671785714
##
##
## Call:
## lm(formula = bfi.scores$scores[, i] ~ age + education, data = bfi)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.5472 -0.6555  0.0785  0.7557  2.3806
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.159147   0.078184  -2.036   0.0419 *
## age          0.007611   0.002043   3.725   0.0002 ***
## education   -0.008840   0.019598  -0.451   0.6520
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9975 on 2233 degrees of freedom
##   (564 observations deleted due to missingness)
## Multiple R-squared:  0.006284,   Adjusted R-squared:  0.005394
## F-statistic:  7.06 on 2 and 2233 DF,  p-value: 0.0008777
##
##
##  Welch Two Sample t-test
##
## data:  bfi.scores$scores[, i] and bfi$gender
## t = -74.86, df = 3325.2, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.721508 -1.633633
## sample estimates:
##    mean of x    mean of y
## -0.005784746  1.671785714
##
##
## Call:
## lm(formula = bfi.scores$scores[, i] ~ age + education, data = bfi)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.4183 -0.5714  0.1173  0.6996  2.5763
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.194969   0.077594  -2.513 0.012051 *
## age          0.006796   0.002028   3.352 0.000816 ***
## education    0.008162   0.019450   0.420 0.674778
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.99 on 2233 degrees of freedom
##   (564 observations deleted due to missingness)
## Multiple R-squared:  0.005749,   Adjusted R-squared:  0.004859
## F-statistic: 6.456 on 2 and 2233 DF,  p-value: 0.0016
##
##
##  Welch Two Sample t-test
##
## data:  bfi.scores$scores[, i] and bfi$gender
## t = -74.68, df = 3350.6, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -1.696244 -1.609455
## sample estimates:
##  mean of x  mean of y
## 0.01893589 1.67178571
##
##
## Call:
## lm(formula = bfi.scores$scores[, i] ~ age + education, data = bfi)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.5010 -0.6592  0.0374  0.7285  2.5794
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.357328   0.077861  -4.589 4.69e-06 ***
## age          0.001496   0.002034   0.735    0.462
## education    0.108372   0.019517   5.553 3.15e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9934 on 2233 degrees of freedom
##   (564 observations deleted due to missingness)
## Multiple R-squared:  0.0157, Adjusted R-squared:  0.01482
## F-statistic: 17.81 on 2 and 2233 DF,  p-value: 2.121e-08
```

There are significant gender differences in personality on all the dimensions. Some of them vary with age and education. It's plausible that education could be associated with personality (though you'd expect the causal direction to be the other way around), but not age!

Homework

1. Use the dataset FactorAnalysis.csv (which can be read using `read.csv()` or `read_csv`).
2. The dataset has 300 rows and 6 variables consisting of university students' ratings of their liking of six subjects on a five point scale from 1 = Strongly Dislike to 5 = Strongly Like. The six subjects are:

a. BIO (biology)
b. GEO (geology)
c. CHEM (chemistry)
d. ALG (algebra)
e. CALC (calculus)
f. STAT (statistics)

3. Conduct an exploratory factor analysis. How many factors are appropriate? How would you interpret these factors?