

Problem 1

verify $[w^2, T^{m0}] = 0$, where $w^2 = w_\mu w^\mu$, and $w_\mu = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} T^\nu T^{\rho\sigma}$

known:

- $[T^{\mu\nu}, T^\sigma] = \eta^{\nu\sigma} T^\mu - \eta^{\mu\sigma} T^\nu$
- $[T^{\mu\nu}, T^{\rho\sigma}] = \eta^{\sigma\nu} T^{\mu\rho} + \eta^{\rho\mu} T^{\nu\sigma} + \eta^{\mu\sigma} T^{\rho\nu} + \eta^{\nu\rho} T^{\sigma\mu}$

$$\begin{aligned}
[w^2, T^{m0}] &= w^2 T^{m0} - T^{m0} w^2 \\
&= \langle \text{part.1} \rangle T^{m0} - T^{m0} \langle \text{part.1} \rangle \\
&= \left(\frac{1}{2} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} \right) T^{m0} - T^{m0} \left(\frac{1}{2} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} \right) \\
&= \frac{1}{2} [T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma}, T^{m0}] + [T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho}, T^{m0}] \\
&= \frac{1}{2} \langle \text{part.2} \rangle + \langle \text{part.3} \rangle \\
&= \frac{1}{2} \times 0 + 0 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
&\langle \text{part.3} \rangle \\
&= T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} T^{m0} - T^{m0} T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} \\
&= \langle \text{part.3.1} \rangle - \langle \text{part.3.2} \rangle \\
&= T^\nu T^{m\sigma} T_\sigma T_\nu^0 + T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^\nu T^{0\sigma} T_\sigma T_\nu^m - T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^\nu T^{\rho m} T^0 T_{\nu\rho} - T^\nu T^{\rho 0} T^m T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho} \\
&\quad - (T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^\nu T^{m\sigma} T_\sigma T_\nu^0 + T^\nu T^{0\sigma} T_\sigma T_\nu^m - T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho}) \\
&= T^\nu T^{m\sigma} T_\sigma T_\nu^0 + T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^\nu T^{0\sigma} T_\sigma T_\nu^m - T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^\nu T^{\rho m} T^0 T_{\nu\rho} - T^\nu T^{\rho 0} T^m T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho} \\
&\quad - (T^\nu T^{m\sigma} T_\sigma T_\nu^0 + T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^\nu T^{0\sigma} T_\sigma T_\nu^m - T^m T^{\rho\sigma} T_\sigma T_\rho^0 - T^\nu T^{m\rho} T^0 T_{\nu\rho} + T^\nu T^{0\rho} T^m T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho}) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.3.1} \rangle &= T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} T^{m0} \\
&= T^\nu T^{\rho\sigma} T_\sigma [T_{\nu\rho}, T^{m0}] + T^\nu T^{\rho\sigma} T_\sigma T^{m0} T_{\nu\rho} \\
&= T^\nu T^{\rho\sigma} T_\sigma (T_\nu^0 \delta_\rho^m + T_\rho^m \delta_\nu^0 - T_\nu^m \delta_\rho^0 - T_\rho^0 \delta_\nu^m) + T^\nu T^{\rho\sigma} [T_\sigma, T^{m0}] T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho} \\
&= T^\nu T^{m\sigma} T_\sigma T_\nu^0 + T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^\nu T^{0\sigma} T_\sigma T_\nu^m \\
&\quad - T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^\nu T^{\rho\sigma} (\delta_\sigma^m T^0 - \delta_\sigma^0 T^m) T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho} \\
&= T^\nu T^{m\sigma} T_\sigma T_\nu^0 + T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^\nu T^{0\sigma} T_\sigma T_\nu^m \\
&\quad - T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^\nu T^{\rho m} T^0 T_{\nu\rho} - T^\nu T^{\rho 0} T^m T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho}
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.3.2} \rangle &= T^{m0} T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} \\
&= [T^{m0}, T^\nu] T^{\rho\sigma} T_\sigma T_{\nu\rho} + T^\nu T^{m0} T^{\rho\sigma} T_\sigma T_{\nu\rho} \\
&= (\eta^{\nu 0} T^m - \eta^{\nu m} T^0) T^{\rho\sigma} T_\sigma T_{\nu\rho} + T^\nu [T^{m0}, T^{\rho\sigma}] T_\sigma T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho} \\
&= -T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^0 T^{\rho\sigma} T_\sigma T_\rho^m \\
&\quad + T^\nu (T^{m\sigma} \eta_\rho^{0\sigma} + T^{0\rho} \eta_\sigma^{m\sigma} - T^{m\rho} \eta_\sigma^{0\sigma} - T^{0\sigma} \eta_\nu^{m\rho}) T_\sigma T_{\nu\rho} + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho} \\
&= T^0 T^{\rho\sigma} T_\sigma T_\rho^m - T^m T^{\rho\sigma} T_\sigma T_\rho^0 + T^\nu T^{m\sigma} T_\sigma T_\nu^0 \\
&\quad + T^\nu T^{0\rho} T^m T_{\nu\rho} - T^\nu T^{m\rho} T^0 T_{\nu\rho} - T^\nu T^{0\sigma} T_\sigma T_\nu^m + T^\nu T^{\rho\sigma} T^{m0} T_\sigma T_{\nu\rho}
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.2} \rangle &= T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} T^{m0} - T^{m0} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} \\
&= \langle \text{part.2.1} \rangle - \langle \text{part.2.2} \rangle \\
&= 2T^\nu T^{0\sigma} T_\nu T_\sigma^m - 2T^\nu T^{m\sigma} T_\nu T_\sigma^0 + T^m T^{\rho\sigma} T^0 T_{\rho\sigma} - T^0 T^{\rho\sigma} T^m T_{\rho\sigma} + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma} \\
&\quad - (2T^\nu T^{0\rho} T_\nu T_\rho^m - 2T^\nu T^{m\rho} T_\nu T_\rho^0 + T^m T^{\rho\sigma} T^0 T_{\rho\sigma} - T^0 T^{\rho\sigma} T^m T_{\rho\sigma} + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma}) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.2.1} \rangle &= T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} T^{m0} \\
&= T^\nu T^{\rho\sigma} T_\nu [T_{\rho\sigma}, T^{m0}] + T^\nu T^{\rho\sigma} T_\nu T^{m0} T_{\rho\sigma} \\
&= T^\nu T^{\rho\sigma} T_\nu (-\delta_\rho^m T_\sigma^0 + \delta_\rho^0 T_\sigma^m + \delta_\sigma^m T_\rho^0 - \delta_\sigma^0 T_\rho^m) + T^\nu T^{\rho\sigma} T_\nu T^{m0} T_{\rho\sigma} \\
&= -T^\nu T^{m\sigma} T_\nu T_\sigma^0 + T^\nu T^{0\sigma} T_\nu T_\sigma^m + T^\nu T^{\rho m} T_\nu T_\rho^0 - T^\nu T^{\rho 0} T_\nu T_\rho^m + T^\nu T^{\rho\sigma} T_\nu T^{m0} T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma} T_\nu T_\sigma^m - 2T^\nu T^{m\sigma} T_\nu T_\sigma^0 + T^\nu T^{\rho\sigma} T_\nu T^{m0} T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma} T_\nu T_\sigma^m - 2T^\nu T^{m\sigma} T_\nu T_\sigma^0 + T^\nu T^{\rho\sigma} [T_\nu, T^{m0}] T_{\rho\sigma} + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma} T_\nu T_\sigma^m - 2T^\nu T^{m\sigma} T_\nu T_\sigma^0 + T^\nu T^{\rho\sigma} (\delta_\nu^m T^0 - \delta_\nu^0 T^m) T_{\rho\sigma} + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma} T_\nu T_\sigma^m - 2T^\nu T^{m\sigma} T_\nu T_\sigma^0 + T^m T^{\rho\sigma} T^0 T_{\rho\sigma} - T^0 T^{\rho\sigma} T^m T_{\rho\sigma} + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma}
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.2.2} \rangle &= T^{m0} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} \\
&= [T^{m0}, T^\nu] T^{\rho\sigma} T_\nu T_{\rho\sigma} + T^\nu T^{m0} T^{\rho\sigma} T_\nu T_{\rho\sigma} \\
&= (\eta^{0\nu} T^m - \eta^{m\nu} T^0) T^{\rho\sigma} T_\nu T_{\rho\sigma} + T^\nu [T^{m0}, T^{\rho\sigma}] T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma} \\
&= T^m T^{\rho\sigma} T^0 T_{\rho\sigma} - T^0 T^{\rho\sigma} T^m T_{\rho\sigma} + T^\nu (T^{m\sigma} \eta^{0\rho} + T^{0\rho} \eta^{m\sigma} - T^{m\rho} \eta^{0\sigma} - T^{0\sigma} \eta^{m\rho}) T_\nu T_{\rho\sigma} \\
&\quad + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma} \\
&= T^m T^{\rho\sigma} T^0 T_{\rho\sigma} - T^0 T^{\rho\sigma} T^m T_{\rho\sigma} - T^\nu T^{m\sigma} T_\nu T_\sigma^\rho \\
&\quad + T^\nu T^{0\rho} T_\nu T_\rho^m - T^\nu T^{m\rho} T_\nu T_\rho^0 + T^\nu T^{0\sigma} T_\nu T_\sigma^m + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma} \\
&= T^m T^{\rho\sigma} T^0 T_{\rho\sigma} - T^0 T^{\rho\sigma} T^m T_{\rho\sigma} + 2T^\nu T^{0\rho} T_\nu T_\rho^m - 2T^\nu T^{m\rho} T_\nu T_\rho^0 + T^\nu T^{\rho\sigma} T^{m0} T^\nu T_{\rho\sigma}
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.1} \rangle &= w_\mu w^\mu \\
&= \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} T^\nu T^{\rho\sigma} \times \frac{1}{2} \epsilon^{\mu\lambda\kappa\tau} T_\lambda T_{\kappa\tau} \\
&= \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} \epsilon^{\mu\lambda\kappa\tau} T^\nu T^{\rho\sigma} T_\lambda T_{\kappa\tau} \\
&= \frac{1}{4} (\delta_\nu^\lambda \delta_\rho^\kappa \delta_\sigma^\tau + \delta_\nu^\kappa \delta_\rho^\tau \delta_\sigma^\lambda + \delta_\nu^\tau \delta_\rho^\lambda \delta_\sigma^\kappa - \delta_\nu^\lambda \delta_\rho^\tau \delta_\sigma^\kappa - \delta_\nu^\kappa \delta_\rho^\lambda \delta_\sigma^\tau - \delta_\nu^\tau \delta_\rho^\kappa \delta_\sigma^\lambda) T^\nu T^{\rho\sigma} T_\lambda T_{\kappa\tau} \\
&= \frac{1}{4} T^\nu T^{\rho\sigma} (T_\nu T_{\rho\sigma} + T_\sigma T_{\nu\rho} + T_\rho T_{\sigma\nu} - T_\nu T_{\sigma\rho} - T_\sigma T_{\rho\nu} - T_\rho T_{\nu\sigma}) \\
&= \frac{1}{4} T^\nu T^{\rho\sigma} (2T_\nu T_{\rho\sigma} + 2T_\sigma T_{\nu\rho} + 2T_\rho T_{\sigma\nu}) \\
&= \frac{1}{2} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} + \frac{1}{2} T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} + \frac{1}{2} T^\nu T^{\rho\sigma} T_\rho T_{\sigma\nu} \\
&= \frac{1}{2} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} + \frac{1}{2} (T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} + T^\nu T^{\rho\sigma} T_\rho T_{\sigma\nu}) \\
&= \frac{1}{2} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} + \frac{1}{2} (T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho} + T^\nu T^{\sigma\rho} T_\rho T_{\nu\sigma}) \\
&= \frac{1}{2} T^\nu T^{\rho\sigma} T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma} T_\sigma T_{\nu\rho}
\end{aligned}$$

problem 2

should know

$$H = \int d^3\vec{x} \left(\frac{1}{2} \pi^2 + \frac{1}{2} (\nabla\phi)^2 + \frac{1}{2} m^2 \phi^2 + V(\phi) \right)$$

$$\begin{aligned}
&\hat{\phi}(x)|\phi(x)\rangle = \phi(x)|\phi(x)\rangle \text{ and } \hat{\phi}(y)|\phi(x)\rangle = 0 \\
&\hat{\pi}(x)|\pi(x)\rangle = \pi(x)|\pi(x)\rangle \text{ and } \hat{\pi}(y)|\pi(x)\rangle = 0
\end{aligned}$$

$$\langle \phi(x)|\pi(x)\rangle = \exp\left(i \int d^3x \{ \phi(x)\pi(x) \}\right)$$

and I can't handle the $\langle \phi(x)|\pi(x)\rangle$ using creation/annihilation operator ,only learn it from some text.

Calculate:

$$\begin{aligned}
& \langle \phi(\vec{x}_f, t_f) | e^{-iH(t_f - t_i)} | \phi(\vec{x}_i, t_i) \rangle \\
&= \int \prod_{n'=1}^{N-1} d\phi_{n'} \{ \langle \phi_N(\vec{x}_N) | e^{-iH\epsilon} | \phi_{N-1}(\vec{x}_{N-1}) \rangle \dots \langle \phi_1(\vec{x}_1) | e^{-iH\epsilon} | \phi_0(\vec{x}_0) \rangle \} \\
&= \prod_{l=1}^3 \int \prod_{n'=1}^{N-1} d\phi(x_l)_{n'} \{ \langle \phi_N((x_l)_N) | e^{-iH\epsilon} | \phi_{N-1}((x_l)_{N-1}) \rangle \dots \langle \phi_1((x_l)_1) | e^{-iH\epsilon} | \phi_0((x_l)_0) \rangle \} \\
&= \prod_{l=1}^3 \int \prod_{n'=1}^{N-1} d\phi_{ln'} \{ \langle \phi_{lN} | e^{-iH\epsilon} | \phi_{lN-1} \rangle \dots \langle \phi_{ln} | e^{-iH\epsilon} | \phi_{ln-1} \rangle \dots \langle \phi_{l1} | e^{-iH\epsilon} | \phi_{l0} \rangle \} \\
&= \prod_{l=1}^3 \int \prod_{n'=1}^{N-1} d\phi_{ln'} \{ \langle \text{part.1} \rangle_N \dots \langle \text{part.1} \rangle_n \dots \langle \text{part.1} \rangle_1 \} \text{ (see below)} \\
&= \prod_{l=1}^3 \int \prod_{n'=1}^{N-1} d\phi_{ln'} \left\{ \prod_{n=1}^N \int d\pi_{ln} \left\{ \exp \left(i\epsilon \left(\frac{\pi_{ln}(\phi_{ln} - \phi_{ln-1})}{\epsilon} - \mathcal{H}_{ln} \right) \right) \right\} \right\} \\
&= \prod_{l'=1}^3 \int \prod_{n'=1}^{N-1} d\phi_{l'n'} d\pi_{l'n'} \left\{ \exp \left(i\epsilon \sum_{n=1}^N \sum_{l=1}^{3N} \left(\frac{\pi_{ln}(\phi_{ln} - \phi_{ln-1})}{\epsilon} - \mathcal{H}_{ln} \right) \right) \right\} \\
&= \int \prod_{l'=1}^{3N} \prod_{n=1}^N d\phi_{l'n'} d\pi_{l'n'} \left\{ \exp \left(i\epsilon \sum_{n=1}^N \sum_{l=1}^{3N} \left(\frac{\pi_{ln}(\phi_{ln} - \phi_{ln-1})}{\epsilon} - \mathcal{H}_{ln} \right) \right) \right\} \\
&\rightarrow \lim_{\epsilon \rightarrow 0} \int \prod_{l'=1}^{3N} \prod_{n=1}^N d\phi_{l'n'} d\pi_{l'n'} \left\{ \exp \left(i\epsilon \sum_{n=1}^N \sum_{l=1}^{3N} \left(\frac{\pi_{ln}(\phi_{ln} - \phi_{ln-1})}{\epsilon} - \mathcal{H}_{ln} \right) \right) \right\} \\
&= \int \mathcal{D}\phi \mathcal{D}\pi \left\{ \prod_{l=1}^{3N} \exp \left(i \sum_{l=1}^{3N} \int dt \{ \pi_l \partial_0 \phi - \mathcal{H}_l \} \right) \right\} \\
&\rightarrow \int \mathcal{D}\phi \mathcal{D}\pi \left\{ \exp \left(i \int d^4x \{ \pi \partial_0 \phi - \mathcal{H} \} \right) \right\} \\
&= \int \mathcal{D}\phi \mathcal{D}\pi \left\{ \exp \left(i \int d^4x \left\{ \pi \partial_0 \phi - \frac{1}{2} \pi^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 - V(\phi) \right\} \right) \right\} \\
&= \int \mathcal{D}\phi \left\{ \exp \left(i \int d^4x \left\{ -\frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 - V(\phi) \right\} \right) \right\} \int \mathcal{D}\pi \left\{ \exp \left(i \int d^4x \left\{ \pi \partial_0 \phi - \frac{1}{2} \pi^2 \right\} \right) \right\} \\
&= \langle \text{term.1} \rangle \int \mathcal{D}\pi \left\{ \exp \left(-\frac{i}{2} \int d^4x \{ (\pi - \partial_0 \phi)^2 \} - \frac{i}{2} \int d^4x \{ (\partial_0 \phi)^2 \} \right) \right\} \\
&= \int \mathcal{D}\phi \left\{ \exp \left(-\frac{i}{2} \int d^4x \{ -(\partial_0 \phi)^2 + (\nabla \phi)^2 + m^2 \phi^2 + 2V(\phi) \} \right) \right\} \int \mathcal{D}\pi \left\{ \exp \left(-\frac{i}{2} \int \pi^2 d^4x \right) \right\} \\
&= \mathcal{N} \int \mathcal{D}\phi \left\{ \exp \left(-\frac{i}{2} \int d^4x \{ \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 + 2V(\phi) \} \right) \right\} \\
&= \mathcal{N} \int \mathcal{D}\phi \left\{ \exp \left(-\frac{i}{2} \int d^4x \{ \phi(\square + m^2) \phi + 2V(\phi) \} \right) \right\} \\
&= \mathcal{N} \int \mathcal{D}\phi \{ e^{iS} \}
\end{aligned}$$

$$\begin{aligned}
\langle \text{part.1} \rangle_n &= \langle \phi_{ln} | e^{-iH\epsilon} | \phi_{ln-1} \rangle \\
&= e^{-i\epsilon \left(\frac{1}{2} (\nabla \phi_{ln})^2 + \frac{1}{2} m^2 \phi_{ln}^2 + V(\phi_{ln}) \right)} \langle \phi_{ln} | e^{-i\epsilon \int d^3x \left\{ \frac{1}{2} \pi^2 \right\}} | \phi_{ln-1} \rangle \\
&= e^{-i\epsilon \left(\frac{1}{2} (\nabla \phi_{ln})^2 + \frac{1}{2} m^2 \phi_{ln}^2 + V(\phi_{ln}) \right)} \int d\pi_{ln} \langle \phi_{ln} | \pi_{ln} \rangle \langle \pi_{ln} | e^{-i\epsilon \int d^3x \left\{ \frac{1}{2} \pi^2 \right\}} | \phi_{ln-1} \rangle \\
&= e^{-i\epsilon \left(\frac{1}{2} (\nabla \phi_{ln})^2 + \frac{1}{2} m^2 \phi_{ln}^2 + V(\phi_{ln}) \right)} \int d\pi_{ln} \left\{ e^{i\pi_{ln} \phi_{ln}} e^{-i\epsilon \frac{1}{2} \pi_{ln}^2} e^{-i\pi_{ln} \phi_{ln-1}} \right\} \\
&= \int d\pi_{ln} \left\{ e^{-i\epsilon \left(\frac{1}{2} \pi_{ln}^2 + \frac{1}{2} (\nabla \phi_{ln})^2 + \frac{1}{2} m^2 \phi_{ln}^2 + V(\phi_{ln}) \right)} e^{i\pi_{ln}(\phi_{ln} - \phi_{ln-1})} \right\} \\
&= \int d\pi_{ln} \{ \exp(i(-\mathcal{H}_{ln}\epsilon + \pi_{ln}(\phi_{ln} - \phi_{ln-1}))) \} \\
&= \int d\pi_{ln} \left\{ \exp \left(i\epsilon \left(\frac{\pi_{ln}(\phi_{ln} - \phi_{ln-1})}{\epsilon} - \mathcal{H}_{ln} \right) \right) \right\}
\end{aligned}$$