Problem 1

verify $[w^2, T^{m0}] = 0$,where $w^2 = w_\mu w^\mu$, and $w_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}T^\nu T^{\rho\sigma}$

known:

- $[T^{\mu\nu}, T^\sigma] = \eta^{\nu\sigma}T^\mu - \eta^{\mu\sigma}T^\nu$
- $[T^{\mu\nu}, T^{\sigma\rho}] = \eta^{\sigma\nu}T^{\mu\rho} + \eta^{\rho\mu}T^{\nu\sigma} + \eta^{\mu\sigma}T^{\rho\nu} + \eta^{\nu\rho}T^{\sigma\mu}$

$$
\begin{aligned}
[w^2, T^{m0}] &= w^2T^{m0} - T^{m0}w^2 \\
&= \langle\text{part.1}\rangle T^{m0} - T^{m0}\langle\text{part.1}\rangle \\
&= \left(\frac{1}{2}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho}\right)T^{m0} - T^{m0}\left(\frac{1}{2}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho}\right) \\
&= \frac{1}{2}[T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma}, T^{m0}] + [T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho}, T^{m0}] \\
&= \frac{1}{2}\langle\text{part.2}\rangle + \langle\text{part.3}\rangle \\
&= \frac{1}{2}\times 0 + 0 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.3}\rangle &= T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho}T^{m0} - T^{m0}T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho} \\
&= \langle\text{part.3.1}\rangle - \langle\text{part.3.2}\rangle \\
&= T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^\nu T^{\rho m}T^0T_{\nu\rho} - T^\nu T^{\rho 0}T^mT_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho} \\
&\quad -(T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^\nu T^{0\rho}T^mT_{\nu\rho} - T^\nu T^{m\rho}T^0T_{\nu\rho} - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho}) \\
&= T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^\nu T^{\rho m}T^0T_{\nu\rho} - T^\nu T^{\rho 0}T^mT_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho} \\
&\quad -(T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 - T^\nu T^{m\rho}T^0T_{\nu\rho} + T^\nu T^{0\rho}T^mT_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho}) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.3.1}\rangle &= T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho}T^{m0} \\
&= T^\nu T^{\rho\sigma}T_\sigma[T_{\nu\rho}, T^{m0}] + T^\nu T^{\rho\sigma}T_\sigma T^{m0}T_{\nu\rho} \\
&= T^\nu T^{\rho\sigma}T_\sigma(T_\nu{}^0\delta^m_\rho + T_\rho{}^m\delta^0_\nu - T_\nu{}^m\delta^0_\rho - T_\rho{}^0\delta^m_\nu) + T^\nu T^{\rho\sigma}[T_\sigma, T^{m0}]T_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho} \\
&= T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^\nu T^{\rho\sigma}(\delta^m_\sigma T^0 - \delta^0_\sigma T^m)T_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho} \\
&= T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^\nu T^{\rho m}T^0T_{\nu\rho} - T^\nu T^{\rho 0}T^mT_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho}
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.3.2}\rangle &= T^{m0}T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho} \\
&= [T^{m0}, T^\nu]T^{\rho\sigma}T_\sigma T_{\nu\rho} + T^\nu T^{m0}T^{\rho\sigma}T_\sigma T_{\nu\rho} \\
&= (\eta^{\nu 0}T^m - \eta^{\nu m}T^0)T^{\rho\sigma}T_\sigma T_{\nu\rho} + T^\nu[T^{m0}, T^{\rho\sigma}]T_\sigma T_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho} \\
&= -T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^0T^{\rho\sigma}T_\sigma T_\rho{}^m + T^\nu(T^{m\sigma}\eta^{0\rho} + T^{0\rho}\eta^{m\sigma} - T^{m\rho}\eta^{0\sigma} - T^{0\sigma}\eta^{m\rho})T_\sigma T_{\nu\rho} + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho} \\
&= T^0T^{\rho\sigma}T_\sigma T_\rho{}^m - T^mT^{\rho\sigma}T_\sigma T_\rho{}^0 + T^\nu T^{m\sigma}T_\sigma T_\nu{}^0 + T^\nu T^{0\rho}T^mT_{\nu\rho} - T^\nu T^{m\rho}T^0T_{\nu\rho} - T^\nu T^{0\sigma}T_\sigma T_\nu{}^m + T^\nu T^{\rho\sigma}T^{m0}T_\sigma T_{\nu\rho}
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.2}\rangle &= T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma}T^{m0} - T^{m0}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} \\
&= \langle\text{part.2.1}\rangle - \langle\text{part.2.2}\rangle \\
&= 2T^\nu T^{0\sigma}T_\nu T_\sigma{}^m - 2T^\nu T^{m\sigma}T_\nu T_\sigma{}^0 + T^mT^{\rho\sigma}T^0T_{\rho\sigma} - T^0T^{\rho\sigma}T^mT_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma} \\
&\quad -(2T^\nu T^{0\rho}T_\nu T_\rho{}^m - 2T^\nu T^{m\rho}T_\nu T_\rho{}^0 + T^mT^{\rho\sigma}T^0T_{\rho\sigma} - T^0T^{\rho\sigma}T^mT_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma}) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.2.1}\rangle &= T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma}T^{m0} \\
&= T^\nu T^{\rho\sigma}T_\nu[T_{\rho\sigma}, T^{m0}] + T^\nu T^{\rho\sigma}T_\nu T^{m0}T_{\rho\sigma} \\
&= T^\nu T^{\rho\sigma}T_\nu(-\delta^m_\rho T_\sigma{}^0 + \delta^0_\rho T_\sigma{}^m + \delta^m_\sigma T_\rho{}^0 - \delta^0_\sigma T_\rho{}^m) + T^\nu T^{\rho\sigma}T_\nu T^{m0}T_{\rho\sigma} \\
&= -T^\nu T^{m\sigma}T_\nu T_\sigma{}^0 + T^\nu T^{0\sigma}T_\nu T_\sigma{}^m + T^\nu T^{\rho m}T_\nu T_\rho{}^0 - T^\nu T^{\rho 0}T_\nu T_\rho{}^m + T^\nu T^{\rho\sigma}T_\nu T^{m0}T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma}T_\nu T_\sigma{}^m - 2T^\nu T^{m\sigma}T_\nu T_\sigma{}^0 + T^\nu T^{\rho\sigma}T_\nu T^{m0}T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma}T_\nu T_\sigma{}^m - 2T^\nu T^{m\sigma}T_\nu T_\sigma{}^0 + T^\nu T^{\rho\sigma}[T_\nu, T^{m0}]T_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma}T_\nu T_\sigma{}^m - 2T^\nu T^{m\sigma}T_\nu T_\sigma{}^0 + T^\nu T^{\rho\sigma}(\delta^m_\nu T^0 - \delta^0_\nu T^m)T_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma} \\
&= 2T^\nu T^{0\sigma}T_\nu T_\sigma{}^m - 2T^\nu T^{m\sigma}T_\nu T_\sigma{}^0 + T^mT^{\rho\sigma}T^0T_{\rho\sigma} - T^0T^{\rho\sigma}T^mT_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma}
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.2.2}\rangle &= T^{m0}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} \\
&= [T^{m0}, T^\nu]T^{\rho\sigma}T_\nu T_{\rho\sigma} + T^\nu T^{m0}T^{\rho\sigma}T_\nu T_{\rho\sigma} \\
&= (\eta^{0\nu}T^m - \eta^{m\nu}T^0)T^{\rho\sigma}T_\nu T_{\rho\sigma} + T^\nu[T^{m0}, T^{\rho\sigma}]T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma} \\
&= T^mT^{\rho\sigma}T^0T_{\rho\sigma} - T^0T^{\rho\sigma}T^mT_{\rho\sigma} + T^\nu(T^{m\sigma}\eta^{0\rho} + T^{0\rho}\eta^{m\sigma} - T^{m\rho}\eta^{0\sigma} - T^{0\sigma}\eta^{m\rho})T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma} \\
&= T^mT^{\rho\sigma}T^0T_{\rho\sigma} - T^0T^{\rho\sigma}T^mT_{\rho\sigma} - T^\nu T^{m\sigma}T_\nu T_\sigma{}^\rho + T^\nu T^{0\rho}T_\nu T_\rho{}^m - T^\nu T^{m\rho}T_\nu T_\rho{}^0 + T^\nu T^{0\sigma}T_\nu T_\sigma{}^m + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma} \\
&= T^mT^{\rho\sigma}T^0T_{\rho\sigma} - T^0T^{\rho\sigma}T^mT_{\rho\sigma} + 2T^\nu T^{0\rho}T_\nu T_\rho{}^m - 2T^\nu T^{m\rho}T_\nu T_\rho{}^0 + T^\nu T^{\rho\sigma}T^{m0}T^\nu T_{\rho\sigma}
\end{aligned}
$$

$$
\begin{aligned}
\langle\text{part.1}\rangle &= w_\mu w^\mu \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}T^\nu T^{\rho\sigma} \times \frac{1}{2}\epsilon^{\mu\lambda\kappa\tau}T_\lambda T_{\kappa\tau} \\
&= \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\epsilon^{\mu\lambda\kappa\tau}T^\nu T^{\rho\sigma}T_\lambda T_{\kappa\tau} \\
&= \frac{1}{4}(\delta^\lambda_\nu\delta^\kappa_\rho\delta^\tau_\sigma + \delta^\kappa_\nu\delta^\tau_\rho\delta^\lambda_\sigma + \delta^\tau_\nu\delta^\lambda_\rho\delta^\kappa_\sigma - \delta^\lambda_\nu\delta^\tau_\rho\delta^\kappa_\sigma - \delta^\tau_\nu\delta^\kappa_\rho\delta^\lambda_\sigma - \delta^\kappa_\nu\delta^\lambda_\rho\delta^\tau_\sigma)T^\nu T^{\rho\sigma}T_\lambda T_{\kappa\tau} \\
&= \frac{1}{4}T^\nu T^{\rho\sigma}\big(T_\nu T_{\rho\sigma} + T_\sigma T_{\nu\rho} + T_\rho T_{\sigma\nu} - T_\nu T_{\sigma\rho} - T_\sigma T_{\rho\nu} - T_\rho T_{\nu\sigma}\big) \\
&= \frac{1}{4}T^\nu T^{\rho\sigma}\big(2T_\nu T_{\rho\sigma} + 2T_\sigma T_{\nu\rho} + 2T_\rho T_{\sigma\nu}\big) \\
&= \frac{1}{2}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} + \frac{1}{2}T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho} + \frac{1}{2}T^\nu T^{\rho\sigma}T_\rho T_{\sigma\nu} \\
&= \frac{1}{2}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} + \frac{1}{2}(T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho} + T^\nu T^{\rho\sigma}T_\rho T_{\sigma\nu}) \\
&= \frac{1}{2}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} + \frac{1}{2}(T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho} + T^\nu T^{\sigma\rho}T_\rho T_{\nu\sigma}) \\
&= \frac{1}{2}T^\nu T^{\rho\sigma}T_\nu T_{\rho\sigma} + T^\nu T^{\rho\sigma}T_\sigma T_{\nu\rho}
\end{aligned}
$$

problem 2

should know

$$H \;=\; \int \mathrm{d}^3\vec{x}\left(\tfrac{1}{2}\pi^2+\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{2}m^2\phi^2+V(\phi)\right)$$

- $\hat{\phi}(x)|\phi(x)\rangle=\phi(x)|\phi(x)\rangle$ and $\hat{\phi}(y)|\phi(x)\rangle=0$
  $\hat{\pi}(x)|\pi(x)\rangle=\pi(x)|\pi(x)\rangle$ and $\hat{\pi}(y)|\pi(x)\rangle=0$

- $\langle\phi(x)|\pi(x)\rangle \;=\; \exp\left(i\int \mathrm{d}^3x\{\phi(x)\pi(x)\}\right)$

and I can't handle the $\langle\phi(x)|\pi(x)\rangle$ using creation/annihilation operator ,only learn it from some text.

Calculate:

as in QM $\langle\vec{x}\,|\vec{x}\,'\rangle=\langle(x_1,x_2,x_3)|(x_1',x_2',x_3')\rangle=\delta(\vec{x}-\vec{x}\,')=\delta(x_1-x_1')\delta(x_2-x_2')\delta(x_3-x_3')=\langle x_1|x_1'\rangle\langle x_2|x_2'\rangle\langle x_3|x_3'\rangle$

$$\begin{aligned}
\langle\phi(\vec{x}_f,t_f)|e^{-iH(t_f-t_i)}|\phi(\vec{x}_i,t_i)\rangle \;&=\; \prod_{l=1}^{3}\langle\phi((x_l)_f,t_f)|e^{-iH(t_f-t_i)}|\phi((x_l)_i,t_i)\rangle\\
&=\; \prod_{l_f,l_i=1}^{3N}\langle\phi_{l_f}(t_f)|e^{-iH(t_f-t_i)}|\phi_{l_i}(t_i)\rangle\\
&=\; \prod_{l_*=1}^{3N}\int\prod_{n'=1}^{N-1}\mathrm{d}\phi_{l_i n'}\{\langle\phi_{l_f N}|e^{-iH\epsilon}|\phi_{lN-1}\rangle\ldots\langle\phi_{ln}|e^{-iH\epsilon}|\phi_{ln-1}\rangle\ldots\langle\phi_{l1}|e^{-iH\epsilon}|\phi_{l_i 0}\rangle\}\\
&=\; \prod_{l_*=1}^{3N}\int\prod_{n'=1}^{N-1}\mathrm{d}\phi_{ln'}\{\langle\text{part.1}\rangle_N\ldots\langle\text{part.1}\rangle_n\ldots\langle\text{part.1}\rangle_1\}(\text{see below})\\
&=\; \prod_{l=1}^{3N}\int\prod_{n'=1}^{N-1}\mathrm{d}\phi_{ln'}\left\{\prod_{n=1}^{N}\int\mathrm{d}\pi_{ln}\left\{\exp\left(i\epsilon\left(\frac{\pi_{ln}(\phi_{ln}-\phi_{ln-1})}{\epsilon}-\mathcal{H}_{ln}\right)\right)\right\}\right\}\\
&=\; \prod_{l'=1}^{3N}\int\prod_{n'=1}^{N-1}\mathrm{d}\phi_{l'n'}\mathrm{d}\pi_{l'n'}\left\{\exp\left(i\epsilon\sum_{n=1}^{N}\sum_{l=1}^{3N}\left(\frac{\pi_{ln}(\phi_{ln}-\phi_{ln-1})}{\epsilon}-\mathcal{H}_{ln}\right)\right)\right\}\\
&=\; \int\prod_{l'=1}^{3N}\prod_{n=1}^{N-1}\mathrm{d}\phi_{l'n'}\mathrm{d}\pi_{l'n'}\left\{\exp\left(i\epsilon\sum_{n=1}^{N}\sum_{l=1}^{3N}\left(\frac{\pi_{ln}(\phi_{ln}-\phi_{ln-1})}{\epsilon}-\mathcal{H}_{ln}\right)\right)\right\}\\
&\to\; \lim_{\epsilon\to0}\int\prod_{l'=1}^{3N}\prod_{n'=1}^{N-1}\mathrm{d}\phi_{l'n'}\mathrm{d}\pi_{l'n'}\left\{\exp\left(i\epsilon\sum_{n=1}^{N}\sum_{l=1}^{3N}\left(\frac{\pi_{ln}(\phi_{ln}-\phi_{ln-1})}{\epsilon}-\mathcal{H}_{ln}\right)\right)\right\}\\
&=\; \int\mathcal{D}\phi\mathcal{D}\pi\left\{\prod_{l=1}^{3N}\exp\left(i\sum_{l=1}^{3N}\int\mathrm{d}t\{\pi_l\partial_0\phi-\mathcal{H}_l\}\right)\right\}\\
&\to\; \int\mathcal{D}\phi\mathcal{D}\pi\left\{\exp\left(i\int\mathrm{d}^4x\{\pi\partial_0\phi-\mathcal{H}\}\right)\right\}\\
&=\; \int\mathcal{D}\phi\mathcal{D}\pi\left\{\exp\left(i\int\mathrm{d}^4x\left\{\pi\partial_0\phi-\frac{1}{2}\pi^2-\frac{1}{2}(\nabla\phi)^2-\frac{1}{2}m^2\phi^2-V(\phi)\right\}\right)\right\}\\
&=\; \int\mathcal{D}\phi\left\{\exp\left(i\int\mathrm{d}^4x\left\{-\frac{1}{2}(\nabla\phi)^2-\frac{1}{2}m^2\phi^2-V(\phi)\right\}\right)\right\}\int\mathcal{D}\pi\left\{\exp\left(i\int\mathrm{d}^4x\left\{\pi\partial_0\phi-\frac{1}{2}\pi^2\right\}\right)\right\}\\
&=\; \langle\text{term.1}\rangle\int\mathcal{D}\pi\left\{\exp\left(-\frac{i}{2}\int\mathrm{d}^4x\{(\pi-\partial_0\phi)^2\}-\frac{i}{2}\int\mathrm{d}^4x\{(\partial_0\phi)^2\}\right)\right\}\\
&=\; \int\mathcal{D}\phi\left\{\exp\left(-\frac{i}{2}\int\mathrm{d}^4x\{-(\partial_0\phi)^2+(\nabla\phi)^2+m^2\phi^2+2V(\phi)\}\right)\right\}\int\mathcal{D}\pi\left\{\exp\left(-\frac{i}{2}\int\pi^2\mathrm{d}^4x\right)\right\}\\
&=\; \mathcal{N}\int\mathcal{D}\phi\left\{\exp\left(-\frac{i}{2}\int\mathrm{d}^4x\{\partial_\mu\phi\partial^\mu\phi+m^2\phi^2+2V(\phi)\}\right)\right\}\\
&=\; \mathcal{N}\int\mathcal{D}\phi\left\{\exp\left(-\frac{i}{2}\int\mathrm{d}^4x\{\phi(\Box+m^2)\phi+2V(\phi)\}\right)\right\}\\
&=\; \mathcal{N}\int\mathcal{D}\phi\{e^{iS}\}
\end{aligned}$$

$$\begin{aligned}
\langle\text{part.1}\rangle_n \;&=\; \langle\phi_{ln}|e^{-iH\epsilon}|\phi_{ln-1}\rangle\\
&=\; e^{-i\epsilon\left(\frac{1}{2}(\nabla\phi_{ln})^2+\frac{1}{2}m^2\phi_{ln}^2+V(\phi_{ln})\right)}\langle\phi_{ln}|e^{-i\epsilon\int\mathrm{d}^3x\left\{\frac{1}{2}\pi^2\right\}}|\phi_{ln-1}\rangle\\
&=\; e^{-i\epsilon\left(\frac{1}{2}(\nabla\phi_{ln})^2+\frac{1}{2}m^2\phi_{ln}^2+V(\phi_{ln})\right)}\int\mathrm{d}\pi_{ln}\langle\phi_{ln}|\pi_{ln}\rangle\langle\pi_{ln}|e^{-i\epsilon\int\mathrm{d}^3x\left\{\frac{1}{2}\pi^2\right\}}|\phi_{ln-1}\rangle\\
&=\; e^{-i\epsilon\left(\frac{1}{2}(\nabla\phi_{ln})^2+\frac{1}{2}m^2\phi_{ln}^2+V(\phi_{ln})\right)}\int\mathrm{d}\pi_{ln}\left\{e^{i\pi_{ln}\phi_{ln}}e^{-i\epsilon\frac{1}{2}\pi_{ln}^2}e^{-i\pi_{ln}\phi_{ln-1}}\right\}\\
&=\; \int\mathrm{d}\pi_{ln}\left\{e^{-i\epsilon\left(\frac{1}{2}\pi_{ln}^2+\frac{1}{2}(\nabla\phi_{ln})^2+\frac{1}{2}m^2\phi_{ln}^2+V(\phi_{ln})\right)}e^{i\pi_{ln}(\phi_{ln}-\phi_{ln-1})}\right\}\\
&=\; \int\mathrm{d}\pi_{ln}\{\exp\left(i(-\mathcal{H}_{ln}\epsilon+\pi_{ln}(\phi_{ln}-\phi_{ln-1}))\right)\}\\
&=\; \int\mathrm{d}\pi_{ln}\left\{\exp\left(i\epsilon\left(\frac{\pi_{ln}(\phi_{ln}-\phi_{ln-1})}{\epsilon}-\mathcal{H}_{ln}\right)\right)\right\}
\end{aligned}$$