Assignment # 1 Solutions

. Given: $f(x) = e^x$ and data sets $(0,1)$, $(0.6, 1.8221)$, $(1.2, 3.3201)$ and $(1.8, 6.0496)$.

1 For $x \in [-0.5, 1.5]$, first 3 data sets/points can be chosen to interpolate the given function. ✓

2 Here: $x_0 = 0$, $x_1 = 0.6$ and $x_2 = 1.2$

$f(x_0) = 1$, $f(x_1) = 1.8221$, $f(x_2) = 3.3201$

$$\therefore l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0.6)(x-1.2)}{(-0.6)(-1.2)} = \frac{25}{18}(x^2 - 1.8x + 0.72)$$

$$= \frac{25}{18}x^2 - \frac{5}{2}x + 1 \quad ✓$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{x(x-1.2)}{(0.6)(0.6-1.2)} = -\frac{25}{9}x^2 + \frac{10}{3}x \quad ✓$$

and
$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{x(x-0.6)}{(1.2)(0.6)} = \frac{25}{18}x^2 - \frac{5}{6}x. \quad ✓$$

3
$$P_2(x) = l_0(x)f(x_0) + l_1(x)f(x_1) + l_2(x)f(x_2)$$

$$= \left(\frac{25}{18}x^2 - \frac{5}{2}x + 1\right)1 + \left(-\frac{25}{9}x^2 + \frac{10}{3}x\right)(1.8221) + \left(\frac{25}{18}x^2 - \frac{5}{6}x\right)(3.3201)$$

$$= \left(\frac{25}{18} - \frac{25}{9}\times 1.8221 + \frac{25}{18}\times 3.3201\right)x^2$$

$$+ \left(-\frac{5}{2} + \frac{10}{3}\times 1.8221 - \frac{5}{6}\times 3.3201\right)x + 1$$

$$\Rightarrow \boxed{P_2(x) = 0.9386x^2 + 0.8069x + 1}$$

4
$f(x_0) - P_2(x_0) = e^0 - (0.9386\times 0 + 0.8069\times 0 + 1) = 0$. ✓ checked

$f(x_1) - P_2(x_1) = e^{0.6} - (0.9386\times .6^2 + 0.8069\times 0.6 + 1) \simeq 0.00008 \simeq 0$ ✓ checked

$f(x_2) - P_2(x_2) = e^{1.2} - (0.9386\times 1.2^2 + 0.8069\times 1.2 + 1) \approx 0.00025 \approx 0$ ✓ checked

5
$|f(0.75) - P_2(0.75)| = |e^{0.75} - (0.9386\times 0.75^2 + 0.8069\times 0.75 + 1)| \approx 0.01614$. ✓