BRAC UNIVERSITY

CSE330: NUMERICAL METHODS

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Theory Sec-02

ASSIGNMENT-02

1) Given function,

$$f(x) = \sin x$$

Given nodes, $[0, \pi/2, \pi]$

$x_0 = 0$; $f(x_0) = \sin x_0 = 0$

$x_1 = \pi/2$; $f(x_1) = \sin x_1 = 1$

$x_2 = \pi$; $f(x_2) = \sin x_2 = 0$

So, the points are - $(0, 0)$, $(\pi/2, 1)$, $(\pi, 0)$

$x_0 = 0; f(x_0) = 0;$

$f[x_0, x_1] = \dfrac{1-0}{\pi/2 - 0} = 0.6366$

$x_1 = \pi/2 ; f(x_1) = 1;$

$f[x_1, x_2] = \dfrac{0-1}{\pi - \pi/2} = -0.6366$

$x_2 = \pi; f(x_2) = 0;$

$f[x_0, x_1, x_2] = \dfrac{-0.6366 - .6366}{\pi - 0}$

$= -0.4052$

$P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0\, x_1, x_2](x - x_0)(x - x_1)$

~~$= 0 + 0.6366 \cdot (x - x$~~

$= 0 + 0.6366 \cdot (x - 0) + \cancel{(-0.6366)}(-0.4052)(x - 0)(x - \pi/2)$

... (i)

here,

$a_0 = 0$

$a_1 = 0.6366$

$a_2 = -0.4052$

2)

From (1) we get,

eq. → (i)

$$P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$$= 0 + 0.6366 \cdot x - 0.4052\, x\left(x - \frac{\pi}{2}\right)$$

$$= 0.6366\, x - 0.4052\, x\left(x - \frac{\pi}{2}\right)$$

[Ans]

3)

~~For~~ We have to add a new node $\left(\frac{3\pi}{2}\right)$ with the above nodes

from (1) we get,

$x_0 = 0;\ f(x_0) = 0;$

$f[x_0, x_1] = 0{\cdot}6366$

$f[x_0, x_1, x_2] = 0{\cdot}4052$

$x_1 = \pi/2;\ f(x_1) = 1;$

$f[x_1, x_2] = -0{\cdot}6366$

$x_2 = \pi;\ f(x_2) = 0;$

$f[x_1, x_2\ x_3] = 0$

$f[x_2, x_3] = -0{\cdot}6366$

$x_3 = \frac{3\pi}{2};\ f(x_3) = -1;$

$$\frac{-{\cdot}4052}{3\pi/2 - 0} = -0{\cdot}086$$

$$\therefore P_3(x) = f[x_0] + f[x_0\ x_1](x - x_0) + f[x_0\ x_1\ x_2](x - x_0)(x - x_1) + f[x_0\ x_1\ x_2\ x_3](x - x_0)(x - x_1)(x - x_2)$$

$$= 0{\cdot}6366x - 0{\cdot}4052x\left(x - \frac{\pi}{2}\right) - \cancel{0{\cdot}0806}\ {\cdot}086x\left(x - \frac{\pi}{2}\right)(x - \pi)$$

~~Q~~ 4)

As we know, the interpolation error term for above polinomial is -

$$\frac{f^{n+1}(z)}{(n+1)!} \times (x-x_0)\,\cancel{(x}\ \cdots$$

for. $n = 3;$

$$\text{error term} = \frac{f^4(z)}{4!} \times \left(x - \frac{\pi}{2}\right)(x-x_1)(x-x_2)(x-x_3)$$

$$= \frac{\sin(z)}{24}\ \underbrace{x\left(x-\frac{\pi}{2}\right)(x-\pi)\left(x-\frac{3\pi}{2}\right)}_{\omega(x)}$$

~~$\omega'(x) =$~~

As we know,

max limit of $\sin(z) = \sin(1)$

$$\therefore \omega(x) = x\left(x-\frac{\pi}{2}\right)(x-\pi)\left(x-\frac{3\pi}{2}\right)$$

$$= x^4 - 3\pi x^3 + \frac{11x^2}{4}\pi^2 - \cancel{\frac{3\pi^3 x}{4}}\ \frac{3\pi^3}{4}x$$

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$$\omega'(x) = 4x^3 - 9\pi x^2 + \frac{11\pi^2}{2}x - \frac{3\pi^3}{4}$$

$x_1 = 0.599$; $\omega(0.599) = -6.088$

$x_2 = 2.35$; $\omega(2.35) = 3.423$

as $2.35 > 1$; $\omega(1) = -4.53$

$x_3 = 4.11 > -1$; $\omega(-1) = 60.821$

So, the maximum error is $= \dfrac{\sin(1)}{24} \times 60.82$

$= 2.13242$

(Ans)