Solutions

Q#1: Consider the function $f(x) = 5x^2 + 3x - 2$. This function is replaced by a interpolating function $p_4(x)$ for the interval $[-10, 10]$. Which of the following is the best estimate of the interpolation error?

- ◯ Error will be minimum at the middle of the interval.
- ◯ Error will diverse at the end point.
- ◯ 0
- ◯ It cannot be determined. Need to know the exact nodes.

Ans: 0

Q#2: If the function $\tan(x)$ is interpolated by a quadratic polynomial using the nodes $(-\pi/4, 0, \pi/4)$, the polynomial coefficient a_2 will be

- ◯ a positive number
- ◯ a negative number
- ◯ 0
- ◯ a non-real number

Answer 0

Q#3: Which of the following avoids the occurance of Runge phenomenon?

- ◯ Increase the number of nodes.
- ◯ More nodes at the ends of the interval.
- ◯ More nodes at the middle of the interval.
- ◯ None of the above

Ans More nodes at the ends of interval

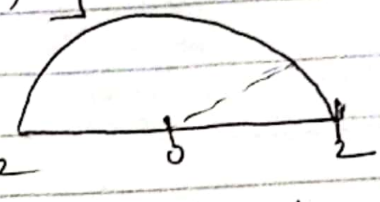
In the lecture note ans also during the video lecture, we have discussed the Chebyshev nodes and used it to interpolate the Runge function. Based on the notes and lecture, answer the following questions:

1. [2 marks] If the Runge function $f(x) = \frac{1}{1+25x^2}$ is considered on the interval $[-2, 2]$, then how do we define the Chebyshev points?
2. [2 points] Write down the Chebyshev points for the interval $[-2, 2]$ for $n = 7$.
3. [2 points] Compute the Chebyshev nodes for $n = 7$.
4. [1 point] Write down the expression for the Lagrange basis $L_7(x)$ using the Chebyshev nodes. You do not have to simplify the expression.
5. Submit your solution to the above questions according to the instructions given in this link.

* PDF Submission

$[-2,2]$

$$f(x) = \frac{1}{1+25x^2}$$



$|x| = 2\cos(\theta_j)$

$\theta_j = \frac{(2j+1)\pi}{2(n+1)}$

- The function provided is a runge function. Hence, rather than taking equally spaced nodes, we will be considering chebyshev nodes → equal theta intervals. This ensures ~~nodes~~ more nodes at the ends of the given interval. Thus error at ends will be lower.

$\boxed{2}$

$$\theta_j = \frac{(2j+1)\pi}{2(n+1)} \qquad j = 0,1,2,3,4,5,6,7$$

$n=7$

$$\theta_0 = \frac{(2(0)+1)\pi}{2(7+1)} = \frac{1}{16}\pi$$

$$\theta_1 = \frac{3}{16}\pi \qquad \theta_7 = \frac{15}{16}\pi$$

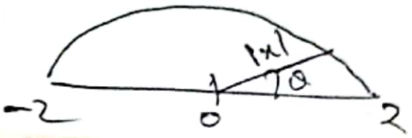
$$\theta_2 = \frac{5}{16}\pi$$

$$\theta_3 = \frac{7}{16}\pi$$

$$\theta_4 = \frac{9}{16}\pi$$

$$\theta_5 = \frac{11}{16}\pi$$

$$\theta_6 = \frac{13}{16}\pi$$



$|x| = 2\cos\theta_j$

$\boxed{3}$ $x = 2\cos(\theta_j) \Rightarrow$

$x_0 = 2\cos\left(\frac{1}{16}\pi\right) = 1.9616$

$x_1 = 1.6629$

$x_2 = 1.1111$

$x_3 = 0.3902$

$x_4 = -0.3902$

$x_5 = -1.1111$

$x_6 = -1.6629$

$x_7 = -1.9616$

$\boxed{4}$ $L_7(x) = \frac{(x-x_0)}{(x_7-x_0)} \frac{(x-x_1)}{(x_7-x_1)} \frac{(x-x_2)}{(x_7-x_2)} \frac{(x-x_3)}{(x_7-x_3)} \frac{(x-x_4)}{(x_7-x_4)} \frac{(x-x_5)}{(x_7-x_5)} \frac{(x-x_6)}{(x_7-x_6)}$

$= \left(\frac{x-1.9616}{-1.9616-1.9616}\right)\left(\frac{x-1.6629}{-1.9616-1.6629}\right)\left(\frac{x-1.1111}{-1.9616-1.1111}\right)\left(\frac{x-0.3902}{-1.9616-0.3902}\right)\left(\frac{x+0.3902}{-1.9616+0.3902}\right)$

$\left(\frac{x+1.1111}{-1.9616+1.1111}\right)\left(\frac{x+1.6629}{-1.9616+1.6629}\right)$

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