BRAC UNIVERSITY

CSE330: NUMERICAL METHODS

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Theory Sec-02

ASSIGNMENT-01

A function $f(x)$ passes through the points $(0,1)$ $(0.6, 1.8221)$, $(1.2, 3.201)$ & $(1.8, 6.0496)$

1] if the function $f(x)$ is interpolated by a polynomial for $x \in [-0.5, 1.5]$ which points can be chosen as nodes?

we can take the points $(0,1)$, $(0.6, 1.8221)$, ~~+5~~ $(1.2, 3.201)$

$\therefore x_0 = 0$; $x_1 = 0.6$; $x_2 = 1.2$

2) Compute Lagrange Basis for the interpolating function $P_2(x)$

from (1) we get,

$x_0 = 0$

$x_1 = 0.6$; $x_2 = 1.2$

So, the lagrange basis for function, $P_2(x) \rightarrow$

$$\therefore L_0(x) = \frac{x - x_1}{x_0 - x_1} \times \frac{x - x_2}{x_0 - x_2}$$

$$= \frac{x - 0.6}{0 - 0.6} \times \frac{x - 1.2}{0 - 1.2} = \frac{(x-6)(x-1.2)}{0.72}$$

Again,

$$L_1(x) = \frac{x - x_0}{x_1 - x_0} \times \frac{x - x_2}{x_1 - x_2}$$

$$= \frac{x - 0}{0.6 - 0} \times \frac{x - 1.2}{0.6 - 1.2}$$

$$= \frac{x(x - 1.2)}{-.36}$$

$$L_2(x) = \frac{x - x_0}{x_2 - x_0} \times \frac{x - x_1}{x_2 - x_1} = \frac{x - 0}{1.2 - 0} \times \frac{x - 0.6}{1.2 - 0.6}$$

$$= \frac{x(x - 0.6)}{0.72}$$

3)

Write down the algebric expression of of $p_2(x)$, and simply the expression to write the result in natural basis.

$$p_2(x) = L_0(x) f(x_0) + L_1(x) f(x_1) + L_2 f(x_2)$$

$$L_0(x) = \frac{(x-0.6)(x-1.2)}{0.72}$$

$$L_1(x) = \frac{-x(x-1.2)}{0.36}$$

$$L_2(x) = \frac{x(x-0.6)}{0.72}$$

$f(x_0) = 1$

$f(x_1) = 1.8221$

$f(x_2) = 3.3201$

$$\therefore p_2(x) = \frac{(x-0.6)(x-1.2)}{0.72} \times 1 + \frac{-x(x-1.2)}{0.36} \times 1.8221 + \frac{x(x-0.6)}{0.272} \times 3.3201$$

$$= \frac{x^2 - 1.8x + 0.72}{0.72} + \frac{-x^2 + 1.2x}{0.36\ 0.19757} + \frac{x^2 - 0.6x}{0.21686}$$

$$= x^2\left(\frac{1}{0.72} - \frac{1}{0.19757} + \frac{1}{.21686}\right) + x\left(\frac{1.2}{.19757} - \frac{1.8}{0.72} - \frac{0.6}{0.21686}\right) + 1$$

$\therefore P_2(x) = 0.93866x^2 + 0.807x + 1$ ✓

4) verify at the nodal points $|f(x) - P_2(x)| = 0$

from (1) & (2) we got the nodal points are

$x_0 = 0$; $x_1 = 0.6$; $x_2 = 1.2$

$f(x_0) = 1$ $f(x_1) = 1.8221$ $f(x_2) = 3.3201$

from (3) we got,

$P_2(x) = 0.93866x^2 + 0.807x + 1$

$\Rightarrow x_0 = 0$

$\therefore P_2(x_0) = 0.93866x_0^2 + 0.807x_0 + 1$

$= 0 + 0 + 1 = 1$

$\therefore$ ~~f ll x~~ $|f(x) - P_2(x)| = 0$

for,

$\therefore f(x_0) - P_2(x_0) = 1 - 1 = 0$

$\therefore$ L.H.S. = R.H.S.

Again,

$x_1 = 0.6$;

$P_2(x_1) = 0.93866(0.6)^2 + 0.807(0.6) + 1$

$= 1.8221$

$\therefore f(x_1) - P_2(x_1) = 1.8221 - 1.8221 = 0$

$x_2 = 1.2$;

$P_2(x_2) = 0.93866(1.2)^2 + .807 \times 1.2 + 1$

$= $ ~~3.30~~ $3.3200704 = 3.3201$

$\therefore f(x_2) - P_2(x_2) = 3.3201 - 3.3201$

$= 0$

$\therefore |f(x) - P_2(x)| = |f(x_0) - P_2(x_0)| = |f(x_1) - P_2(x_1)| =$

$|f(x_2) - P_2(x_2)| = 0$

[Verified]

5]

Compute the error

$$|f(0.75) - P_2(0.75)| =$$

$$\therefore f(x) = e^x \quad ; f(0.75) = e^{0.75}$$

$$= 2.117$$

$$P_2(x) = 0.93866\,x^2 + 0.807x + 1$$

$$P_2(0.75) = 0.93866(0.75)^2 + 0.807(0.75) + 1$$

$$= 2.13324625$$

$$\therefore \text{error} = |f(0.75) - P_2(0.75)|$$

$$= |2.117 - 2.13324625|$$

$$= |-0.016246233 38|$$

$$\simeq 0.016$$

(Ans)