MAT120: Integral Calculus and Differential Equations
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You all have learnt the concept of finding arc lengths of curves that are bounded over some interval. The formula for finding the aforementioned arc length of a curve, are as follows

$$Arc\ Length = \int_a^b \sqrt{1 + .[f'(x)]^2}\ dx$$

w here $f'(x)$ denotes the first derivative of f(x).
Given the function:

$$f(x) = 9x^{\frac{3}{2}} :$$

Find out what the arc length is for the function bounded by the interval [0,1] .

Solution

$$\begin{aligned} Given, f(x) &= 9x^{\frac{3}{2}} \\ f'(x) &= 27\frac{\sqrt{x}}{2} \\ &= \frac{27}{2}\sqrt{x} \end{aligned}$$

We know that,

$$\texttt{Arc Length} = \int_a^b \sqrt{1 + .[f'(x)]^2}\ \ dx$$

$$= \int_0^1 \sqrt{1 + \left(\frac{27}{2}\sqrt{x}\right)^2}\ dx$$

$$= \int_0^1 \sqrt{1 + \frac{729}{4}x}\ dx$$

$$
\begin{aligned}
&let, \\
&\quad y = \frac{729x}{4} + 1 \\
&\frac{dy}{dx} = \frac{729}{4} \Rightarrow dx = \frac{4dy}{729} \\
&\therefore dy = \frac{729}{4} dx
\end{aligned}
$$

lower limit,

$$x \to 0; y \to 1$$

upper limit,

$$y \to 0; y \to \frac{733}{4}$$

$$
\begin{aligned}
&\therefore \int_0^1 \sqrt{1 + \frac{729}{4} x}\, dx \\
&= \int_1^{\frac{733}{4}} \sqrt{y}\, \frac{4}{729}\, dy \\
&= \frac{4}{729} \int_1^{\frac{733}{4}} \sqrt{y} dy \\
&= \frac{4}{729} \left[\frac{2y^{\frac{3}{2}}}{3} \right]_1^{\frac{733}{4}} \\
&= \left[\frac{8y^{\frac{3}{2}}}{2187} \right]_1^{\frac{733}{4}} \\
&= \frac{8(\frac{733}{4})^{\frac{3}{2}}}{2187} - \frac{8(1)^{\frac{3}{2}}}{2187} \\
&= \frac{733\sqrt{733} - 8}{2187} \\
&= 9.70517613 \\
&\approx 9.705
\end{aligned}
$$

So, 9.705 is the arc length of $f(x) = 9x^{\frac{3}{2}}$ for the function bounded by the interval [0,1]