BRAC University

Complex Variables & Laplace Transformations

MAT215

Assignment-01

Author:
Syed Zuhair Hossain
St.ID-19101573
Section-10

SET-A

October 28, 2020

01 **Use the help of the polar representations of complex numbers to express $(1+i)^3$ in the form a + bi, where a and b are real.**

$$\begin{aligned} Let, z &= 1+i \\ \therefore r &= \sqrt{1^2+1^2} \\ &= \sqrt{2} \\ \theta &= \frac{\pi}{4} \end{aligned}$$

According to De Moivr's theorem,

$$\begin{aligned} \therefore z &= rxos\theta + irsin\theta \\ &= \sqrt{2}cos\frac{\pi}{4} + i\left(\sqrt{2}sin\frac{\pi}{4}\right) \\ \therefore (1+i)^3 &= \left[\sqrt{2}(cos(\frac{\pi}{4}) + i\cdot\sqrt{2}\left(sin\frac{\pi}{4}\right) z\right]^3 \\ &= (\sqrt{2})^3\left(cos\frac{\pi}{4} + isin\frac{\pi}{4}\right)^3 \\ &= 2\sqrt{2}\cdot(e^{i\cdot\frac{\pi}{4}})^3 \qquad [\therefore cos\theta + sin\theta = e^{i\theta}] \\ &= 2\sqrt{2}\cdot e^{i\frac{3\pi}{4}} \\ &= 2\sqrt{2}\left(cos\frac{3\pi}{4} + isin\frac{3\pi}{4}\right) \\ &= 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \\ &= -\frac{2\sqrt{2}}{\sqrt{2}} + i\cdot\frac{2\sqrt{2}}{\sqrt{2}} \\ &= -2+2i \end{aligned}$$

$[Answer]$

02 **Express $e^{2+i\pi 2}$ in the a + bi form.**

$$
\begin{aligned}
&\textit{Given,}\\
e^{2+i\pi 2} &= e^2 \cdot e^{i(\pi 2)}
\end{aligned}
$$

$$
\begin{aligned}
&\textit{According to De Moivre's theorem,}\\
z^n &= r^n e^{in\theta}\\
&\textit{By applying this formula here -}\\
z &= e^2 e^{i(2\pi)}\\
\therefore r &= e^2\\
\theta &= 2\pi\\
&\textit{we know,}\\
X &= r cos\theta\\
Y &= r sin\theta\\
\\
z &= x + iy\\
&= r cos\theta + i r sin\theta\\
z &= x + iy\\
&= r cos\theta + i r sin\theta\\
&= e^2 cos(2\pi) + i \cdot e^2 \cdot sin(2\pi)\\
&= 7.389 + i \cdot 0\\
&= 7.389
\end{aligned}
$$

$[Answer]$

03 **Express** $\frac{1}{1+i} + \frac{2}{3+2i}$ **in the a + bi**

From the equations we get,

$$\begin{aligned}
\frac{1}{1+i} &= \frac{1}{1+i} \times \frac{1-i}{1-i} \\
&= \frac{1-i}{1-i^2} \\
&= \frac{1-i}{1-(-1)} \qquad [\therefore i^2 = -1] \\
&= \frac{1-i}{2}
\end{aligned}$$

`Again,`

$$\begin{aligned}
\frac{2}{3+2i} &= \frac{2}{3+2i} \times \frac{3-2i}{3-2i} \\
&= \frac{6-4i}{3^2-(2i)^2} \\
&= \frac{6-4i}{9-(4i)^2} \\
&= \frac{6-4i}{9+4} \qquad [\therefore i^2 = -1] \\
&= \frac{6-4i}{13}
\end{aligned}$$

$$\begin{aligned}
\therefore \frac{1}{1+i} + \frac{2}{3+2i} &= \frac{1-i}{2} + \frac{6-4i}{13} \\
&= \frac{1}{2} - \frac{i}{2} + \frac{6}{13} - \frac{4i}{13} \\
&= \frac{25}{26} - \frac{21i}{26}
\end{aligned}$$

$[Answer]$

04 **Find the distinct roots of z for the following equation, $\mathbf{z}^4 = 3i$**

$$
\begin{aligned}
z^4 &= 3i \\
\Rightarrow z^4 &= 0 + 3i \\
\Rightarrow r^4 e^{i4\theta} &= r_0 e^{i\theta_0} \\
\therefore r_0 &= r^4 \\
\Rightarrow (r_0)^{\frac{1}{4}} &= r \\
\Rightarrow r &= \left(\sqrt{(3i^2)^2}^{\frac{1}{4}}\right) \\
&= 3^{\frac{1}{4}}
\end{aligned}
$$

$$
\begin{aligned}
\theta_0 &= 4\theta \\
\Rightarrow 4\theta &= \theta_0 + 2\pi k \\
&= \frac{\pi}{2\pi k} \\
\therefore \theta &= \frac{\pi}{8} + \frac{\pi k}{2}
\end{aligned}
$$

`1st root,`

$$
\begin{aligned}
z_1 &= (3)^{\frac{1}{4}} \cdot \left[cos(\frac{\pi}{8} + \frac{\pi}{2} \times 0) + isinsin(\frac{\pi}{8} + \frac{\pi}{2} \times 0)\right] \\
&= 1.216 + 0.504i
\end{aligned}
$$

`2nd root,`

$$
\begin{aligned}
z_2 &= (3)^{\frac{1}{4}} \left[cos(\frac{\pi}{8} + \frac{\pi}{2} \times (-1)) + isin(\frac{\pi}{8} + \frac{\pi}{2} \times 1)\right] \\
&= -0.504 + 1.216i
\end{aligned}
$$

3rd root,

$$
\begin{aligned}
z_3 &= (3)^{\frac{1}{4}} \left[cos(\frac{\pi}{8} + \frac{\pi}{2} \times (-1)) + isin(\frac{\pi}{8} + \frac{\pi}{2} \times (-1))\right] \\
&= 0.504 - 1.216i
\end{aligned}
$$

4th root,

$$z_4 = (3)^{\frac{1}{4}} \left[cos(\frac{\pi}{8} + \frac{\pi}{2} \times 2) + isin(\frac{\pi}{8} + \frac{\pi}{2} \times 2)\right]$$
$$= -1.216 - 0.504i$$

$$[Answer]$$

05 `Prove by taking z = a + bi, z -` $\overline{z} = 2iImz.$

`Given that,`

$$z = a + bi$$
$$\therefore \overline{z} = a - bi$$

$$\therefore L.H.S. = z - \overline{z}$$
$$= a + bi - a + bi$$
$$= 2ib$$

$$R.H.S. = 2iIm(z)$$
$$= 2iIm(a + bi)$$
$$= 2ib$$

$$\therefore L.H.S. = R.H.S.$$

$$[Answer]$$