MAT120: Integral Calculus and Differential Equations
BRAC University

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Section - 07

Set-G

1. Evaluate the following definite integrals

a) $$\int_{1}^{2} \frac{1}{\sqrt{x}\sqrt{4-x}}\, dx$$

b) $$\int_{\frac{\pi}{2}}^{\pi} 6sinx(cosx+1)^5 dx$$

(please go to the next page)

Solution of 1(a)

$$= \int_1^2 \frac{1}{\sqrt{x(4-x)}}\, dx$$

$$= \int_1^2 \frac{1}{\sqrt{\left(4x - x^2\right)}}\, dx$$

$$= \int_1^2 \frac{1}{\sqrt{-\left(-4x + x^2 + 4\right) + 4}}\, dx$$

$$= \int_1^2 \frac{1}{\sqrt{-(x-2)^2 + (2)^2}}\, dx$$

we know that,

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$$

So,

$$\left[\sin^{-1} \frac{x-2}{2}\right]_1^2$$

$$= \left(\sin^{-1}\left(\frac{2-2}{2}\right)\right) - \left(\sin^{-1}\left(-\frac{1-2}{2}\right)\right) \qquad \left[\textit{putting the value}\right]$$

$$= \left(\sin^{-1}(0)\right) - \left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$$

$$= \left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$$

$$= \frac{\pi}{6} \qquad (\textit{Answer})$$

Solution of 1(b)

$$\int_{\frac{\pi}{2}}^{\pi} 6\sin x(\cos x + 1)\ dx$$

assuming that,

$$\cos x + 1 = u$$

$$\Rightarrow -\sin x dx = du$$

by putting the value in main function

$$\int_{1}^{0} 6u^5(-du) \qquad \textbf{[as, du = - sindx]}$$

$$= (1)^6 - (0)^6$$

$$= 1 \qquad \textbf{(answer)}$$

2. Solve for

$$\int [\ln(x)]^3 \, dx$$

$$= (\ln x)^3 x - \int 3(\ln(x))^2 dx$$

$$= x(\ln(x))^3 - 3\left(x(\ln(x))^2 - 2\int (\ln(x)) \, dx\right)$$

$$= x(\ln(x))^3 - 3x(\ln(x))^2 + 6\left(x(\ln(x)) - \int \ln(x) \, dx\right)$$

$$= x(\ln(x))^3 - 3x(\ln(x))^2 + 6x(\ln(x)) - 6x + c$$

$$= x\left(\ln(x)^3 - 3(\ln(x))^2 + 6(\ln(x)) - 6\right) + c$$

(Answer)

3. ***Evaluate*** $\int_0^3 f(x) \, dx$ ***if***

f(x) = { $x^2; x < 2$

$x - 2; x \geq 2$

Solution:

$$= \int_0^2 x^2 dx + \int_2^3 (x - 2) \, dx$$

$$= \left[\frac{x^{2+1}}{2+1}\right]_0^2 + \left[\frac{x^2}{2}\right]_2^3 - 2[x]_2^3$$

$$= \left[\frac{x^3}{3}\right]_0^2 + \left[\frac{x^2}{2}\right]_2^3 - 2(3 - 2)$$

$$= \frac{2^3}{3} + \left(\frac{3^2}{2} - \frac{2^2}{2}\right) - 2$$

$$= \frac{8}{3} + \frac{5}{2} - 2$$

$$= \frac{19}{6} \qquad (\textit{answer})$$

4. State whether the following integral converges or diverges. Why or why not? You may prove your statement via calculation.

$$\int_3^4 \frac{1}{(x-3)^2}dx$$

Solution:

$$\int_3^4 \frac{1}{(x-3)^2}\,dx$$

$$= \lim_{t \to 3^+} \int_t^4 \frac{1}{(x-3)^2}\,dx$$

$$= \lim_{t \to 3^+} \left[-\frac{1}{x-3} \right]_t^4$$

$$= \lim_{t \to 3^+} \left[-\frac{1}{4-3} + \frac{1}{t-3} \right]$$

$$= \lim_{t \to 3^+} \left[-1 + \frac{1}{t-3} \right]$$

$$= -1 + \frac{1}{3-3}$$

$$= -1 + \infty$$

$$= \infty$$

As the limit exists within the interval, the integral diverges.

5. Evaluate the following definite integrals:

$$(a)\int_0^{\frac{\pi}{4}} 4sinxcosxdx$$

$$(b)\int_1^2 \frac{1}{\sqrt{y}\sqrt{4-y}}dy$$

$$(c)\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} sin\theta\sqrt{1-4cos^2\theta}\ d\theta$$

Solution of 5(a):

assume that,

$$u = \cos x$$

$$du = -\sin(x)\,dx$$

when,

$$x = 0; \qquad x = \frac{\pi}{4}$$

$$u = \cos(0) = 1 \qquad \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\int_0^{\frac{\pi}{2}} 4\sin xcox\ dx$$

$$= -4\int_1^{\frac{1}{\sqrt{2}}} u\ dx$$

$$= \left[2u^2\right]^1_{\frac{1}{\sqrt{2}}}$$

$$= 2 \times 1^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 \qquad \textbf{\textit{(Answer)}}$$

Solution of 5(b):

$$\int_1^2 \frac{1}{\sqrt{y}\sqrt{4-y}}dy$$
$$= [\frac{x}{\sqrt{y}\sqrt{4-y}}]_1^2 \quad [according\ to\ fundamental\ theorem]$$
$$= \frac{2}{\sqrt{y}\sqrt{4-y}} - \frac{1}{\sqrt{y}\sqrt{4-y}}$$
$$= \frac{1}{\sqrt{y}\sqrt{4-y}} \qquad [Answer]$$

Solution of 5(c):

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\theta \sqrt{1-4\cos^2(\theta)}\ d\theta$$

$$= \left[x\sin(\theta)\sqrt{1-4\cos^2\theta}\right]_{\frac{\pi}{2}}^{\frac{\pi}{2}} \qquad [applying\ fundamental\ theorem\ of\ Calculus]$$

$$= \frac{1}{2}\pi \sin(\theta)\sqrt{1-4\cos^2\theta} - \frac{1}{3}\pi \sin(\theta)\sqrt{1-4\cos^2(\theta)}$$

$$= \frac{1}{6}\pi\ \sin(\theta)\sqrt{1-4\cos^2\theta} \qquad \textbf{\textit{(Answer)}}$$