1. Consider the following three definitions in a connected undirected graph $G = (V, E)$:

   - **Minimum Vertex Cover** (MVC) is a subset V' of V with minimum size such that every edge (u, v) in E has at least one end in V'.
   - **Maximum Clique** (MaxCliq) is a complete subgraph of G that has a maximum size.
   - **Maximum Independent Set** (MIS) is a subset V' of V with maximum size such that for all u and v in V', edge (u, v) is not in E, called the *independence* property.

   - Explain how these problems relate with each other.

   - Write the decision problem of each one of the problems.

2. Prove that MIS is NP-complete.

3. Prove the NP-completeness of MaxCliq by a reduction from MVC.

4. The SAT problem is NP-complete, in general. A 2-SAT formula is a conjunction of a set of disjunctions (i.e., clauses) such that each clause has at most two literals. The decision problem of 2-SAT asks whether or not a given 2-SAT formula is satisfiable.

   - Prove that the 2-SAT problem is in P.

   - Now, consider the following decision problem:

     MAX-2-SAT: Given a 2-SAT formula with n propositional variables and m clauses, does there exist a value assignment that satisfies *at least* k clauses, where $k \leq m$?

     Prove that MAX-2-SAT is NP-complete.

5. Consider the k-cluster problem defined as follows:

   - **Input**: A set of points $P = \{p_1, \cdots, p_n\}$ in the Cartesian space (where $n > 1$) and a positive integer $k \leq n$.
   - **Output**: A partition of the points into k clusters $C_1, \cdots, C_k$ such that the diameter of the clusters is **minimized**. The *diameter* of the clusters is defined as follows:

$\Delta = \text{Max}_j\ (\text{Max}_{x,y \in C_j}\ d(x,y))$ where $d(x,y)$ is the Euclidean distance of x and y.

The k-cluster is known to be NP-hard. An approximation algorithm for solving this problem is as follows. Let $d[x]$ denote the distance of any point $x \in P$ to its closest center.

Step 1: For any point $x \in P$, set $d[x]$ to infinity. Also, let S be the set of the k center. Initially, $S = \emptyset$.

Step 2: For $i := 1$ to k do
- Let $x \in P$ be the point with maximum distance;
  i.e., $d[x]$ is max amongst all n points.
- Add x to S.
- For $i := 1$ to n do
  Set $d[p_i]$ to the minimum of $d[p_i]$ and distance(x, p_i).
- Set the diameter of the cluster, denoted Δ, to the max
  of all $d[x]$, for any $x \in P$.

Step 3: return S and Δ.

Prove that the above algorithm gives a 2-approximation solution; i.e., the resulting diameter is at most twice as the optimal diameter.