CS 325 - Homework 7

1. *(7 pts)* Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following can we infer? Explain

   a. If Y is NP-complete then so is X.

   b. If X is NP-complete then so is Y.

   c. If Y is NP-complete and X is in NP then X is NP-complete.

   d. If X is NP-complete and Y is in NP then Y is NP-complete.

   e. X and Y can't both be NP-complete.

   f. If X is in P, then Y is in P.

   g. If Y is in P, then X is in P.

2. *(4 pts)* Consider the problem COMPOSITE: given an integer *y*, does *y* have any factors other than one and itself? For this exercise, you may assume that COMPOSITE is in NP, and you will be comparing it to the well-known NP-complete problem SUBSET-SUM: given a set S of *n* integers and an integer target *t*, is there a subset of *S* whose sum is exactly *t*? Clearly explain whether or not each of the following statements follows from that fact that COMPOSITE is in NP and SUBSET-SUM is NP-complete:

   a. SUBSET-SUM ≤p COMPOSITE.

   b. If there is an $O(n^3)$ algorithm for SUBSET-SUM, then there is a polynomial time algorithm for COMPOSITE.

   c. If there is a polynomial algorithm for COMPOSITE, then P = NP.

   d. If P ≠ NP, then ***no*** problem in NP can be solved in polynomial time.

3. *(3 pts)* Two well-known NP-complete problems are 3-SAT and TSP, the traveling salesman problem. The 2-SAT problem is a SAT variant in which each clause contains at most two literals. 2-SAT is known to have a polynomial-time algorithm. Is each of the following statements true or false? Justify your answer.

   a. 3-SAT ≤p TSP.

   b. If P ≠ NP, then 3-SAT ≤p 2-SAT.

   c. If P ≠ NP, then no NP-complete problem can be solved in polynomial time.

4. *(6 pts)* A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Show that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G} is NP-complete. You may use the fact that HAM-CYCLE is NP-complete

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5. *(5 pts)* LONG-PATH is the problem of, given (G, u, v, k) where G is a graph, u and v vertices and k an integer, determining if there is a simple path in G from u to v of length at least k. Prove that LONGPATH is NP-complete.