CS 325 - More Practice Problems for the Final

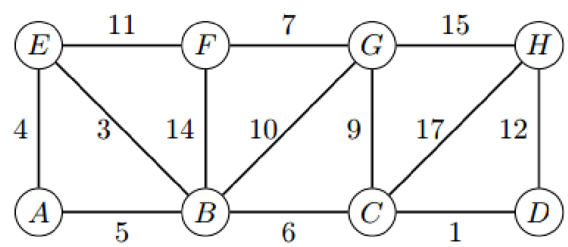
1. Graph-Coloring. Mapmakers try to use as few colors as possible when coloring countries on a map, as long as no two countries that share a border have the same color. We can model this problem with an undirected graph G = (V,E) in which each vertex represents a country and vertices whose respective countries share a border are adjacent. A k-coloring is a function c: V -> {1, 2, … , k} such that $c(u) \neq c(v)$ for every edge $(u,v) \in E$. In other words the number 1, 2, .., k represent the k colors and adjacent vertices must have different colors. The graph-coloring problem is to determine the minimum number of colors needed to color a given graph.

(a) Give an efficient algorithm to determine a 2-coloring of a graph, if one exists.

(b) State the graph-coloring problem as a decision problem K-COLOR.

(c) Use the fact that 3-COLOR is NP-complete to show that 4-COLOR is NP-complete

2. Consider the weighted graph below:



(a) Demonstrate Prim's algorithm starting from vertex A.

(b) Demonstrate Dijkstra's algorithm on the graph, using vertex A as the source.

3. Suppose you have four production plants for making cars. Each works a little differently in terms of labor needed, materials, and energy used per car:

Location	labor hrs	materials	energy
plant1	2	3	15
plant2	3	4	10
plant3	4	5	9
plant4	5	6	7

Suppose we need to produce at least 400 cars at plant 3 according to a labor agreement. We have 3300 hours of labor and 4000 units of material available. We are allowed to use at most 12000 units of energy, and we want to maximize the number of cars produced. Formulate this as a linear programming problem: (1) what are the variables, (2) what is the objective in terms of these variables, and (3) what are the constraints.