CS 325 HW 6

You may solve Problems 1 to 3 using your choice of software, state which software package/language(s) you used and provide the code or spreadsheet.

***<u>Note</u>**: There is no submission to TEACH this week.*

1. <u>Shortest Paths using LP</u>: **(7 points)**

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

  max dt
  subject to
  ds = 0
  dv – du ≤ w(u,v) for all (u,v)∈E

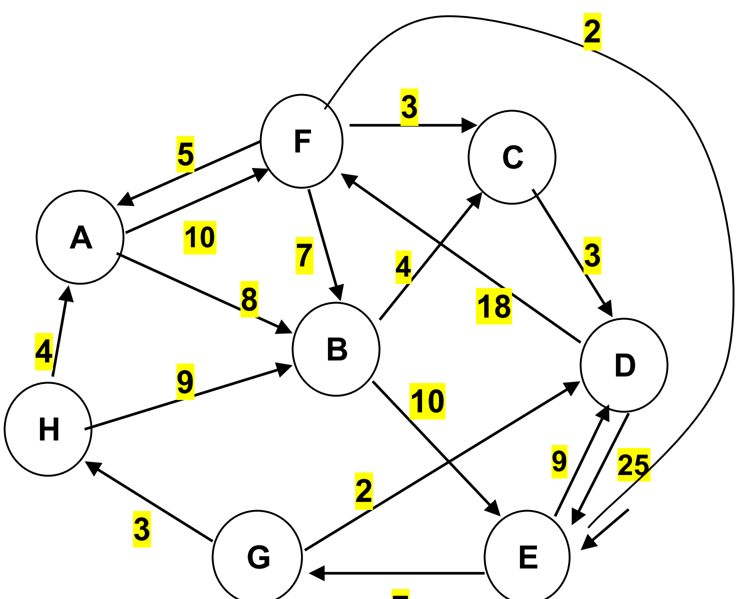
- We can compute the single-source by changing the objective function to

  $$\max \sum_{v\in V} dv$$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

a) Find the distance of the shortest path from A to G in the graph below.

b) Find the distances of the shortest paths from A to all other vertices.



SOLUTION 1a:

LP Optimum found at Step 5
Objective Function Value: 19.00000
Number of iterations: 5
Shortest path is 19 A->F(10)_>E(12)->G(19)

SOLUTION 1b:

Objective Function Value: 98.00000
Number of Iterations: 14

da → da = 0
da → db = 8
da → dc = 12
da → dd = 15
da → de = 12
da → df = 10
da → dg = 19
da → dh = 22

CS 325 HW 6

2. Product Mix: **(7 points)**
Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	Cost per yard	Yards available per month
Silk	$20	1,000
Polyester	$6	2,050
Cotton	$12	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	$6.75	$3.50	$4.31	$4.81
Monthly Minimum units	6,000	10,000	14,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

SOLUTION:
Formulate the problem as a linear program with an objective function and all constraints.

Max 3.50s + 2.27p + 2.66b + 3.04c

ST
0.125s <= 1000 : silk
0.08p + 0.05b + 0.03c <= 2050 : poly
0.05b + 0.07c <= 1250 :cotton
S >= 6000 ; S <= 7000
P >= 10,000 ; p <= 14,000
B >= 14,000; b <= 16000
C >= 6000; c <= 8500

Ideal production for each type of tie is:
Silk: 7,000
Poly: 13,928
Blend 1: 14,001
Blend 2: 7,856
Total profit: $117,241.50

CS 325 HW 6

3. Making Change **(6 points)**

Given coins of denominations (value) $1 = v_1 < v_2 < \ldots < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that v_i's and A are integers. Since $v_1 = 1$ there will always be a solution. Solve the coin change using integer programming. For each of the following denomination sets and amounts, formulate the problem as an integer program with an objective function and constraints. Determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

```
<untitled>
min V1+V2+V3+V4
ST
     1V1+5V2+10V3+25V4 = 202
     V1>=0
     V2>=0
     V3>=0
     V4>=0
END
GIN V1
GIN V2
GIN V3
GIN V4
```

The minimum number of coins is 10.

2 of 1 and 8 of 25 coins are used.

```
LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      10.00000

  VARIABLE        VALUE          REDUCED COST
        V1         2.000000          1.000000
        V2         0.000000          1.000000
        V3         0.000000          1.000000
        V4         8.000000          1.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.000000
        3)         2.000000          0.000000
        4)         0.000000          0.000000
        5)         0.000000          0.000000
        6)         8.000000          0.000000

NO. ITERATIONS=      32
BRANCHES=     6 DETERM.=  1.000E    0
```

a) V = [1, 5, 10, 25] and A = 202.
   Minimum number of coins is 10

b) V = [1, 3, 7, 12, 27] and A = 293
   Minimum number of coins is 14

4. Consider the following linear program.

a) Write the following linear program in slack form. **(4 points)**

b) Please state what are the basic and non-basic variables in your slack form. **(1 points)**

Maximize $2x_1 - 6x_3$

Subject to

$x_1 + x_2 - x_3 \le 14$

$6x_1 - x_2 \ge 8$

$-x_1 + 2x_2 + 2x_3 \ge 0$

$x_1 \ge 0$

$x_2 \ge 0$

$x_3 \ge 0$

Solution of part a. First multiply the second and the third constrains by -1 to get to the standard form. Thus, we have the following:

Maximize $2x_1 - 6x_3$

Subject to

$x_1 + x_2 - x_3 <= 14$

$-6x_1 + x_2 <= -8$

$-x_1 - 2x_2 - 2x_3 <= 0$

$x_1, x_2, x_3 >= 0$

Then, we introduce three slack variables x_4, x_5, x_6, where $x_4, x_5, x_6 >= 0$. Thus, we have

maximize $2x_1 - 6x_3$

Subject to

$x_4 = -14 + x_1 + x_2 - x_3$

$x_5 = 8 - 6x_1 + x_2$

$x_6 = x_1 - 2x_2 - 2x_3$

$x_1, x_2, x_3, x_4, x_5, x_6 >= 0$

Solution of part b. The basic variables are x_4, x_5, x_6 and the non-basic variables include x_1, x_2, x_3.