CS 325 Midterm Practice Problems

1. (True/False) The running time of a dynamic programming algorithm is always Θ(P) where P is the number of subproblems.

2. (True/False) If f(n) = O(g(n)) and g(n) = O(h(n)), then f(n) = O(h(n))

3. Give asymptotic bounds. Make as tight as possible.

T(n) = 2 T(n/3) + nlgn

4. What does the fact given below imply regarding big-O, big-Ω and/or big-Θ relationships between the functions.

For all n > 40, 3g(n) ≤ f(n) ≤ 5g(n)

5. Order the following functions in increasing order of asymptotic (big-O) complexity.

$$f(n) = 2^{2^{1000}}, \quad g(n) = \sum_{i=1}^{n}(i+1), \quad h(n) = 2^n, \quad p(n) = 10^{10}n, \quad q(x) = n2^{n/2}$$

6. Show log(n!) is O(nlogn)

7. What is the asymptotical running-time complexity of Find-Array-Max?

```
Function FIND-ARRAY-MAX(A, n)
if (n = 1) then
    return(A[1])
else
    return(max(A[n], FIND-ARRAY-MAX (A, n − 1) ))
end if
```

8. Mr. Smith has an algorithm which he has proved (correctly) to run in time O(2^n). He coded the algorithm correctly in C, yet he was surprised when it ran quickly on inputs of size up to a million. What are at least two plausible explanations of this behavior?

9. Given a set $\{x_1 \leq x_2 \leq \ldots \leq x_n\}$ of points on the real line, determine the smallest set of unit-length closed intervals (e.g. the interval [1.25,2.25] includes all x_i such that $\{1.25 \leq x_i \leq 2.25\}$) that contains all of the points. Give the most efficient algorithm you can to solve this problem, prove it is correct and analyze the time complexity.

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10. You are going on another long trip (this time your headlights are working). You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1, < a_2 < \ldots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance an), which is your destination.

You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the penalty for that day is $(200-x)^2$. You want to plan your trip so as to minimize the total penalty – that is the sum, over all travel days, of daily penalties. Give an efficient algorithm that determines the minimum penalty for the optimal sequence of hotels at which to stop.

11. (True / False)

a) $n^2 + n = O(n^2)$?

b) $\lg(n^2) = O(n)$?

c) A function that calls a O(n) function three times and has constant time for the rest of the algorithm. The overall asymptotic run-time of the algorithm is O(n).

12. Suppose we have an alphabet with only five letters A, B, C, D, E which occur with the following frequencies. Construct a Huffman code.

Letter	A	B	C	D	E
frequency	0.35	0.12	0.18	0.05	0.30

13. For each of the following give a tight Θ( ) bound on the number of times the $z \leftarrow z + 1$ statement is executed and justify your solution.

$j \leftarrow 0$
while $(j < n)$ **do**
  $j \leftarrow j + 2$
  $z \leftarrow z + 1$

for $k \leftarrow 0$ **to** n **do**
  for $j \leftarrow 0$ **to** k **do**
    $z \leftarrow z + 1$

$i \leftarrow n$
while $(i > 1)$ **do**
  $i \leftarrow \lfloor i/2 \rfloor$
  $z \leftarrow z + 1$

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14. You just started a consulting business where you collect a fee for completing various types of projects (the fee is different for each project). You can select in advance the projects you will work on during some finite time period. You work on only one project at a time and once you start a project it must be completed to receive your fee. There is a set of n projects $p_1, p_2, .. p_n$ each with a duration $d_1, d_2, .. d_n$ (in days) and you receive the fee $f_1, f_2, .., f_n$ (in dollars) associated with it. That is project p_i takes d_i days and you collect f_i dollars after it is completed.

Each of the n projects must be completed in the next D days or you lose its contract. Unfortunately, you do not have enough time to complete all the projects. Your goal is to select a subset S of the projects to complete that will maximize the total fees you earn in D days.

a) What type of algorithm would you use to solve this problem? Divide and Conquer, Greedy or Dynamic Programming. Why?

b) Describe the algorithm verbally. If you select a DP algorithm give the formula used to fill the table or array.

c) What is the running time of your algorithm?

15. Canoe Rental Problem**:** There are n trading posts numbered 1 to n as you travel downstream. At any trading post i you can rent a canoe to be returned at any of the downstream trading posts j, where $j \geq i$. You are given an array R[i, j] defining the costs of a canoe which is picked up at post i and dropped off at post j, for $1 \leq i \leq j \leq n$. Assume that R[i,i] = 0 and that you can't take a canoe upriver. Your problem is to determine a sequence of rentals which start at post 1 and end at post n, and that has the minimum total cost. There are really two problems: determine the cost of a cheapest sequence and determine the sequence itself.

16. Product Sum

Given a list of n integers, $v_1, \ldots, v_n$, the product-sum is the largest sum that can be formed by multiplying adjacent elements in the list. Each element can be matched with at most one of its neighbors.

For example, given the list 1, 2, 3, 1 the product sum is $8 = 1 + (2 \times 3) + 1$, and given the list 2, 2, 1, 3, 2, 1, 2, 2, 1, 2 the product sum is $19 = (2 \times 2) + 1 + (3 \times 2) + 1 + (2 \times 2) + 1 + 2$.

a) Compute the product-sum of 1, 4, 3, 2, 3, 4, 2.

b) Give the optimization formula OPT[j] for computing the product-sum of the first j elements.