Lab 5. Simulation electrical systems dynamic

Name: Zhu Chenhao

ITMO ID: 375462

Specialization: Automation

Objective

Familiarize yourself with the methods for determining the parameters of the model by frequency characteristics using the example of electrical circuits.

Initial data

An electric filter is given, the circuit of which is shown in Figure 1. The input signal of the filter is EMF *E*, and the output signal is voltage U_R. The frequency response is also given in the form of a data set for the options (Figure 2a) and the response to the step signal of EMF *E* (Figure 2b) to verify the results of identifying the parameters of the electric filter model.

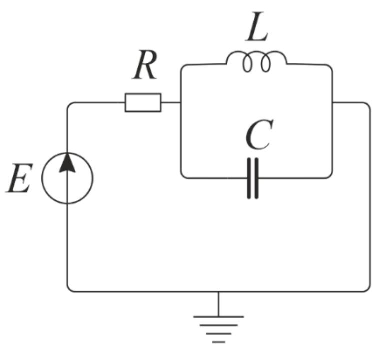
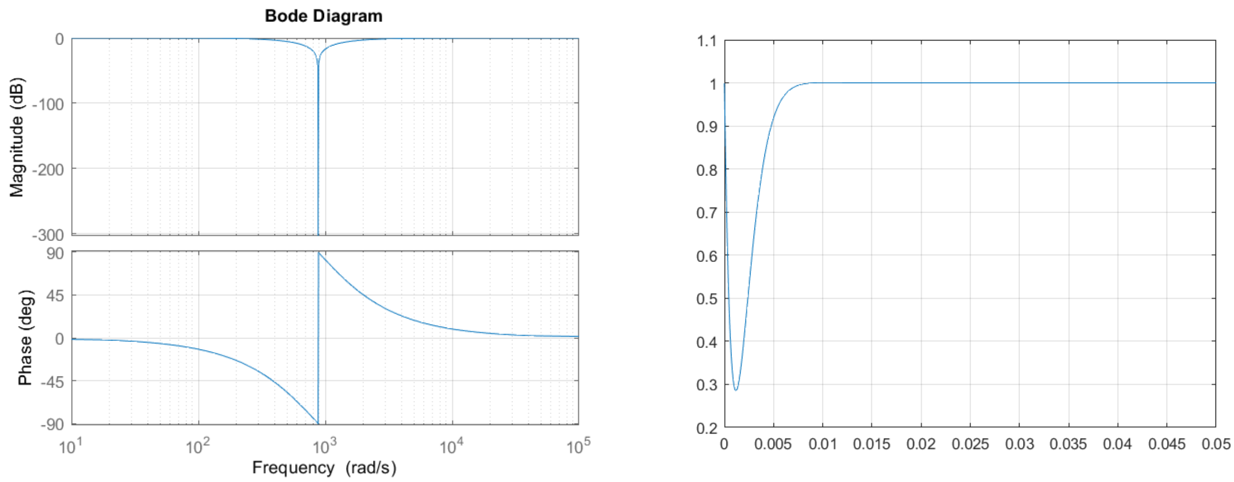


Figure 1. Electric filter equivalent circuit.



a) b)

Figure 2. Frequency response of voltage U_R (a) and response to the step signal of EMF *E* (b).

1. Build a simulation circuit.

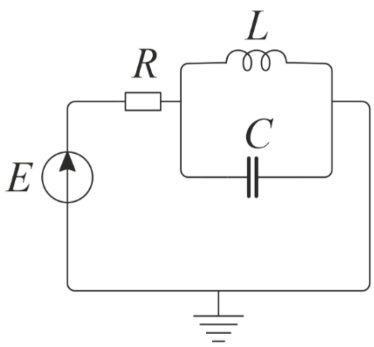


Figure 1. Equivalent circuit.

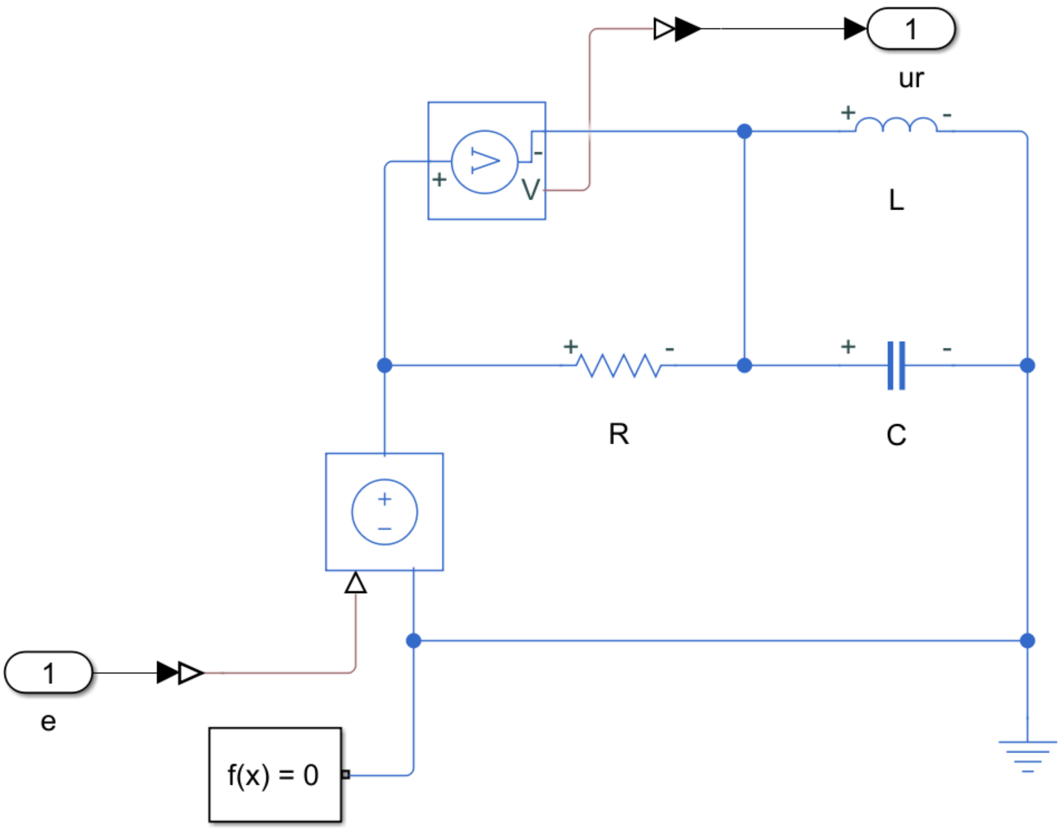


Figure 2. Simulation circuit.

2. Component equations.

$$u_R = R \cdot i_R$$
$$u_L = L \frac{di_L}{dt}$$
$$u_c = \frac{1}{C} \int i_c dt$$

3. Topological equations.

$$KVL:$$
$$u_R + u_c = e$$
$$-u_c + u_L = 0$$
$$KCL:$$
$$i_R - i_L - i_c = 0$$

4. State-space model.

$$\begin{cases} u_R = R i_R \quad \checkmark \\ u_L = L\dfrac{di_L}{dt} \quad \checkmark \\ u_c = C^{-1}\int i_c dt \quad \checkmark \end{cases} \qquad x = \begin{bmatrix} i_L \\ u_c \end{bmatrix}, \quad y = u_R$$

$$\begin{cases} u_R + u_c = e \quad \checkmark \\ -u_c + u_L = 0 \quad \checkmark \\ i_R - i_L - i_c = 0 \quad \checkmark \end{cases} \Rightarrow \begin{cases} i_c = C\dfrac{du_c}{dt} \quad \checkmark \\ i_R = i_L + i_c = i_L + C\dfrac{du_c}{dt} = \\ u_L = u_c \quad \checkmark \\ u_R = e - u_c \quad \checkmark \end{cases}$$

C

u_L

$-\frac{1}{C} i_L -$

$$\Rightarrow e - u_c = R \cdot i_L + RC\frac{du_c}{dt} \Rightarrow \begin{cases} \dfrac{du_c}{dt} = -\dfrac{1}{C} i_L - \dfrac{1}{RC} u_c + \dfrac{e}{RC} \\ \dfrac{di_L}{dt} = \dfrac{1}{L} u_c \end{cases}$$

$$u_R = -u_c + e$$

5. Transfer function.

$$\begin{cases} sU_c = -\frac{1}{C}I_L - \frac{1}{RC}U_c + \frac{E}{RC} \\ sI_L = \frac{1}{L}U_c \\ U_R = -U_c + E \end{cases} \Rightarrow \begin{cases} (s+\frac{1}{RC})U_c = -\frac{1}{C}I_L + \frac{E}{RC} \\ I_L = \frac{1}{sL}U_c \end{cases}$$

$$\Rightarrow (s+\frac{1}{RC}+\frac{1}{sCL})U_c = \frac{E}{RC}$$

$$\Rightarrow U_c = \frac{s\cdot E}{RC\cdot s^2+s+\frac{R}{L}} \Rightarrow W(s) = \frac{U_R(s)}{E(s)} = \frac{RC\cdot s^2+R/L}{RC\cdot s^2+s+R/L} \equiv \frac{T^2s^2+1}{T^2s^2+2T\xi s+1}$$

6. Calculation of R, L and C using frequency response.

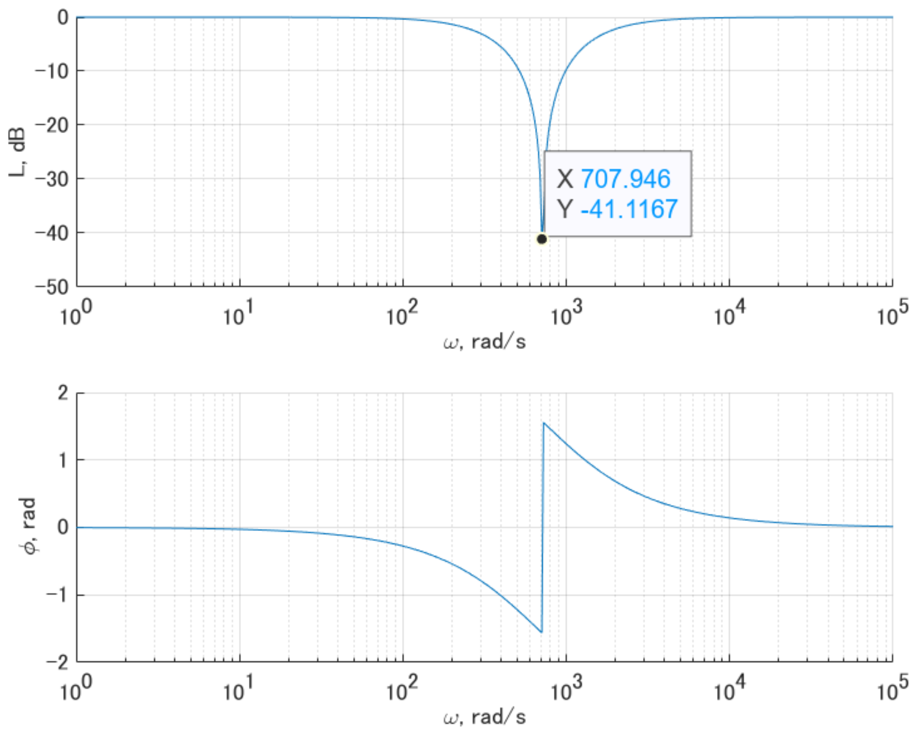


Figure 3. Frequency response.

$$\Rightarrow \begin{cases} T = \sqrt{LC} \\ 2T\xi = \frac{L}{R} \\ \xi = \frac{1}{2R}\sqrt{\frac{L}{C}} \end{cases}$$

$$H_1 = -41.12\ dB$$

$$K_1 = 10^{\frac{H_1}{20}} =$$

$$\xi = \sqrt{\frac{1}{K_1^2} - 1} \cdot \frac{1 - T^2 \omega_r^2}{2\omega_r T}$$

$$\omega_r = 707.95$$

$$T = \frac{1}{\omega_r} = 0.0014 \Rightarrow \begin{cases} R = \sqrt{\frac{L}{C}} \cdot \frac{1}{2\xi} \\ L = 2\xi R \cdot T \\ C = \frac{T}{2\xi R} \end{cases}$$

$$\xi = 1.0188$$

$$R = 0.6933 \quad \text{Ohm}$$

$$L = 0.0020 \quad H$$

$$C = 0.0010 \quad F$$

7. Comparing of transients

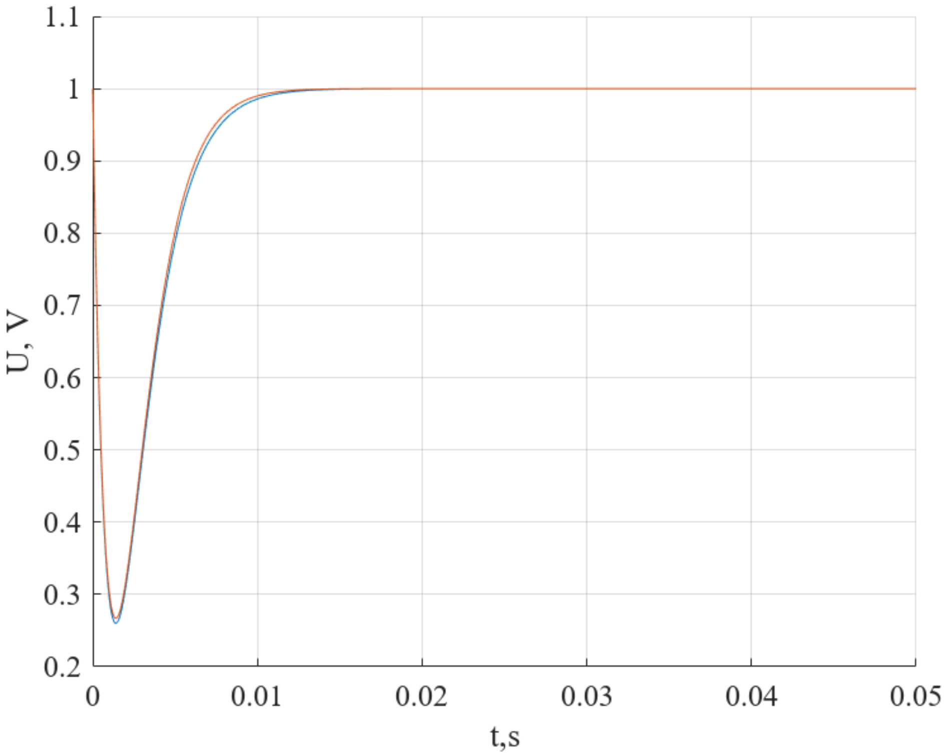


Figure 4. Simulation results of the state space model and the given transient response.

RMS value of error: **0.0040**.

8. Comparing of frequency responses.

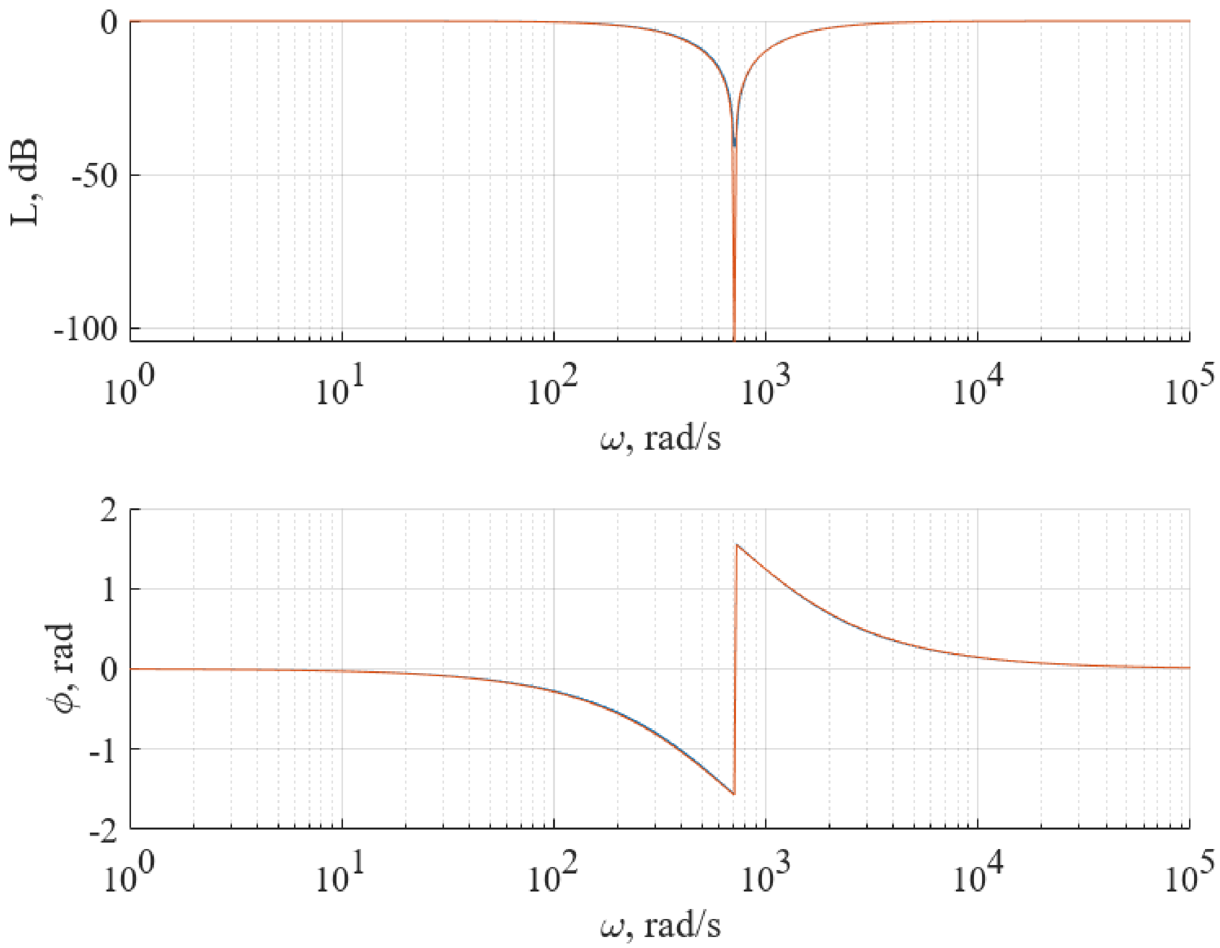


Figure 5. Given frequency response and frequency response received experimentally.