Practice 5

Name: Zhu Chenhao

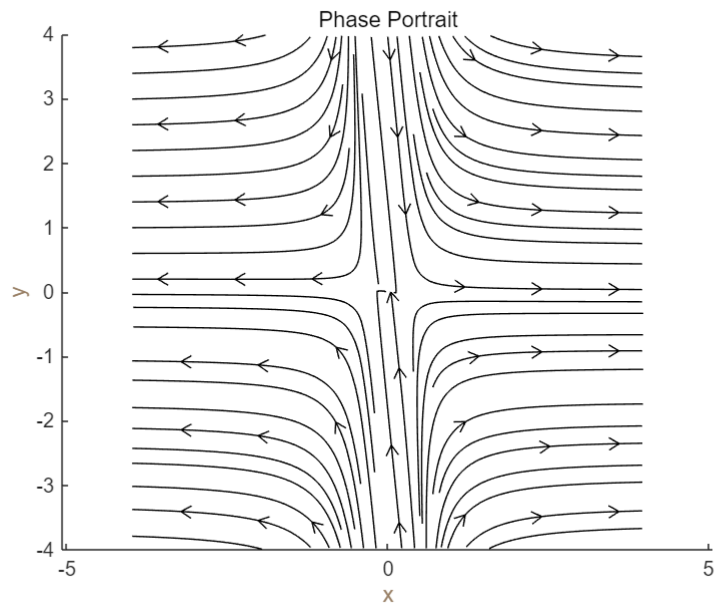
HDU ID: 22320630

ITMO ID: 375462

System	Stability/Behavior	Linearization (Jacobian at (0,0))	Eigenvalues	Equilibrium Points	Notes
$\frac{dx_1}{dt} = -x_1 + 20x_1^3 + x_2$ $\frac{dx_2}{dt} = -x_1 - 10x_2$	**Stable node**	$J = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}$	$\lambda = \frac{-11 \pm \sqrt{77}}{2}$	(0,0), $(\pm\frac{\sqrt{22}}{20}, \mp\frac{\sqrt{22}}{200})$	Nonlinear terms (x_1^3) dominate away from origin
$\frac{dx_1}{dt} = 10x_1 + x_1 * x_2$ $\frac{dx_2}{dt} = -x_2 + x_2{}^2 + x_1 * x_2 - x_1{}^3$	**Saddle point**	$J = \begin{bmatrix} 10 & 0 \\ 0 & -1 \end{bmatrix}$	$\lambda = 10, -1$	(0,0), (0,1)	Unstable due to positive eigenvalue
$\frac{dx_1}{dt} = 10x_2$ $\frac{dx_2}{dt} = -x_1 + x_2 * (1 - x_1{}^2 + x_1^4)$	**Unstable focus**	$J = \begin{bmatrix} 0 & 10 \\ -1 & 1 \end{bmatrix}$	$\lambda = \frac{1 \pm i\sqrt{39}}{2}$	(0,0)	Spirals outward
$\frac{dx_1}{dt} = 10(x_1 + x_2) * (1 - x_1{}^2 - x_2{}^2)$ $\frac{dx_2}{dt} = (x_1 + x_2) * (1 - x_1{}^2 - x_2{}^2)$	**Saddle point**	$J = \begin{bmatrix} 10 & -10 \\ 1 & 1 \end{bmatrix}$	$\lambda = 5.5 \pm \sqrt{20.25}$	(0,0), (-1,0),(1,0)	Limit cycle behavior on unit circle
$\frac{dx_1}{dt} = -x_1{}^3 + 10x_2$ $\frac{dx_2}{dt} = 10x_1 - x_2{}^3$	**Linearization fails** (nonlinear center likely)	$J = \begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$	$\lambda = \pm 10$	(0,0)	Cubic terms stabilize trajectories

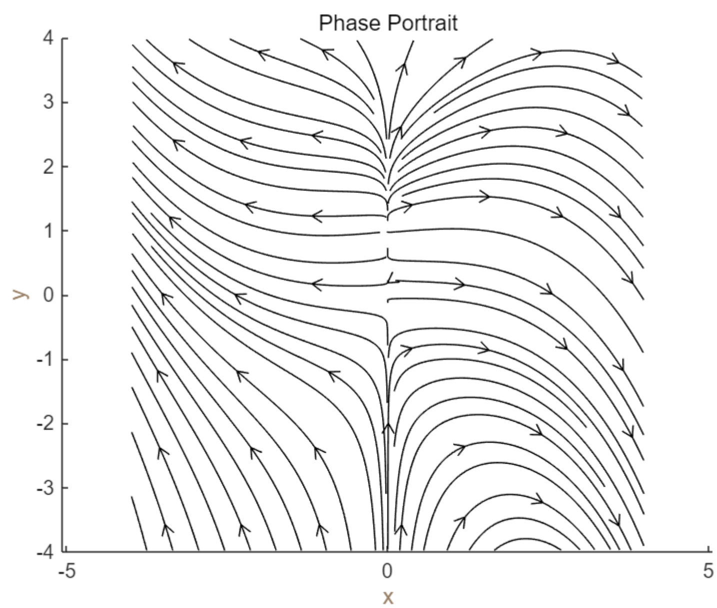
System1

Draw phase-portrait



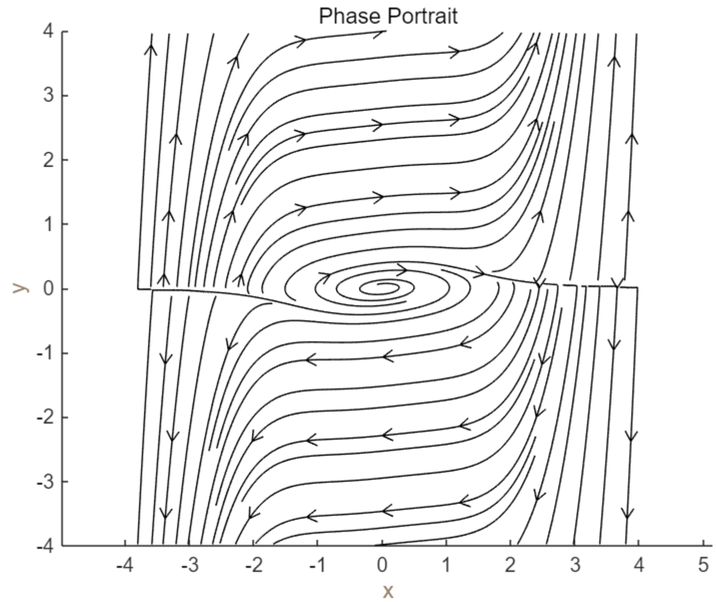
System2

Draw phase-portrait



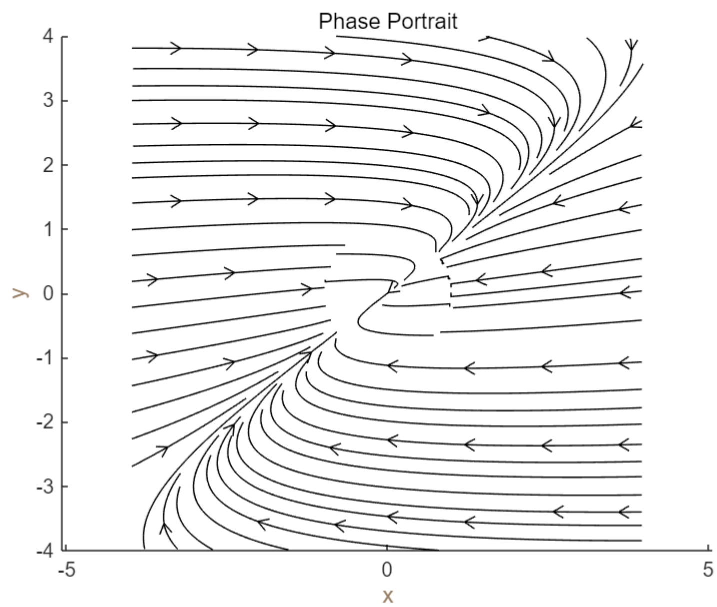
System3

Draw phase-portrait



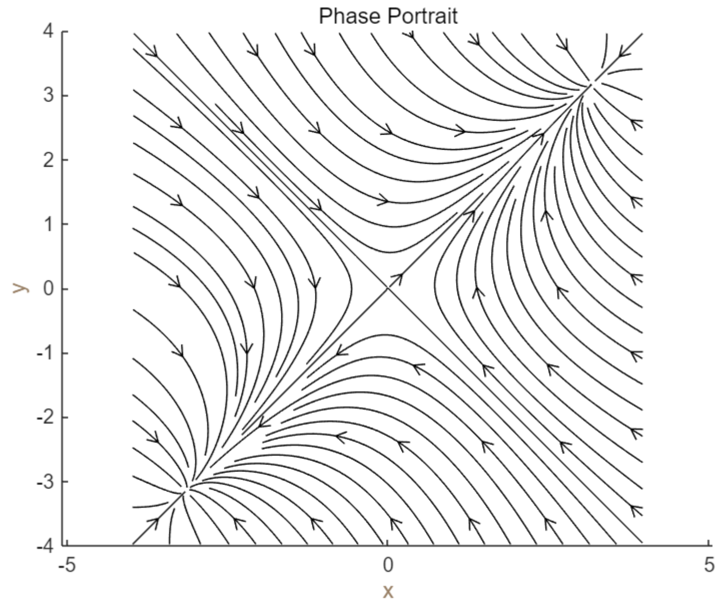
System4

Draw phase-portrait



System5

Draw phase-portrait



Conclusions

A linear system is a system whose mathematical model consists solely of linear expressions. In contrast, a nonlinear system is a system whose mathematical model includes at least one nonlinear expression.

In the analysis of nonlinear systems, the Jacobian matrix can be used to linearize the system around equilibrium points. By examining the eigenvalues and eigenvectors of the Jacobian matrix, we can determine the stability and behavior of these equilibrium points.