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Lab 1. Introduction to modelling

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Outline

1. Theory part
2. Example

Theory part

Objective

Familiarize yourself with the Simulink software and basic techniques for modeling linear electrical circuits.

Goal

To know how to build and simulate electrical circuits in Simulink by 3 ways:

- topological diagram (Simscape circuit);
- input – state – output form (state-space model);
- input-output form (transfer function).

Theory part

A mathematical model of a linear electric circuit as a linear stationary system can be represented in the form of a scalar differential equation of the nth order (input-output model) or in the form of a system of n differential equations of the 1st order (input-state-output model).

The input-output model has the follows form:

$$y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 \dot{y} + a_0 y = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \ldots + b_1 \dot{u} + b_0 u$$

where y is the output variable, u is the input signal, n is the order of the system, m is the order of the derivative of the output variable, which explicitly depends on u ($m \le n$), a_j, b_j are constant coefficients.

Theory part

Provided that *m* ≤ *n*, the input-state-output model can be represented as

$$\begin{cases} \dot{x}_1 = \alpha_{11}x_1 + \alpha_{12}x_2 + \ldots + \alpha_{1n}x_n + \beta_1 u, \\ \dot{x}_2 = \alpha_{21}x_1 + \alpha_{22}x_2 + \ldots + \alpha_{2n}x_n + \beta_2 u, \\ \ldots \\ \dot{x}_n = \alpha_{n1}x_1 + \alpha_{n2}x_2 + \ldots + \alpha_{nn}x_n + \beta_n u, \\ y = c_1x_1 + c_2x_2 + \ldots + c_nx_n, \end{cases}$$

where x_j are the coordinates of the state vector, α_{ij} and β_j are constant coefficients. The system of differential equations can be also represented in a compact vector-matrix form.

Theory part

Electrical system dynamic elements and equations

Phase coordinates

- Current, I, [A];
- Voltage, U, [V].

Elements with linear dynamic

- Capacity, C, [F];
- Resistance, R, [Ohm];
- Inductance, L, [H].

Theory part

Component equations

Capacity

$$U_C = \frac{\int I_C dt}{C}$$

or

$$\frac{dU_C}{dt} = \frac{I_C}{C}$$

Theory part

Component equations

<u>**Resistance**</u>

$$U_R = R \cdot I_R$$

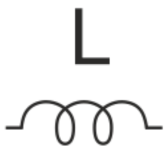
or

$$I_R = \frac{U_R}{R}$$

Theory part

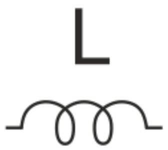
Component equations

Inductance



$$U_L = L\frac{dI_L}{dt}$$

or



$$I_L = \frac{\int U_L dt}{L}$$

Theory part

Topological equations

1st Kirchhoff`s law

$$\sum_k I_k = 0$$

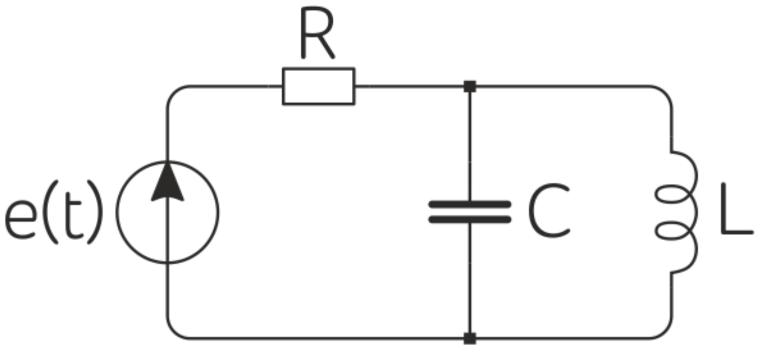
2nd Kirchhoff`s law

$$\sum_i U_i = 0$$

Initial data

Initial data

R = 240; L = 24 mH; C = 12 μF



Source voltage waveform:

1. $e(t) = E_m = 50$
2. $e(t) = E_m \sin(50 \cdot 2 \cdot \pi \cdot t)$

Models input-output:

$$W_1(s) = \frac{u_C(s)}{e(s)}; \quad W_2(s) = \frac{i_L(s)}{e(s)}$$

State vector and initial conditions:

$$x = \begin{bmatrix} i_L & u_C \end{bmatrix}^T; \quad x = \begin{bmatrix} 0.5 & 10 \end{bmatrix}^T$$

Task

1. In accordance with the task option (Table 1), build a simulation circuit of a linear electrical circuit using the elements of the Simscape library Electrical (Simscape / Foundation Library / Electrical).
2. Write down all component equations for this circuit.
3. Write down all topological equations for this circuit.
4. Get the state-space model of the electric circuit with the given coordinates of the state vector (Table 2).
5. Carry out Simulink simulation of the circuit and the state-space model under the input signals indicated in Table 1 and zero initial conditions. The model must be compiled using integration, summation and amplification blocks.

Task

6. Obtain "input-output" model for the given characteristics of the electrical circuit in the form of transfer functions.

7. Carry out Simulink simulation of the circuit and the resulting transfer functions under the input signals specified in Table 1 (source voltage waveform) and zero initial conditions. The duration of the observation interval is chosen independently.

8. Carry out the simulation of the circuit and the state-space model with zero input signals and non-zero initial conditions specified in Table 2.

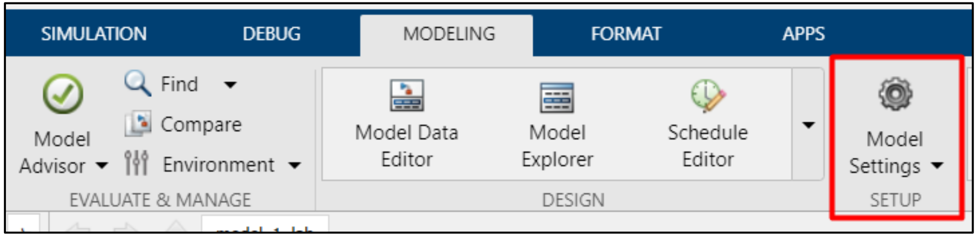
Example

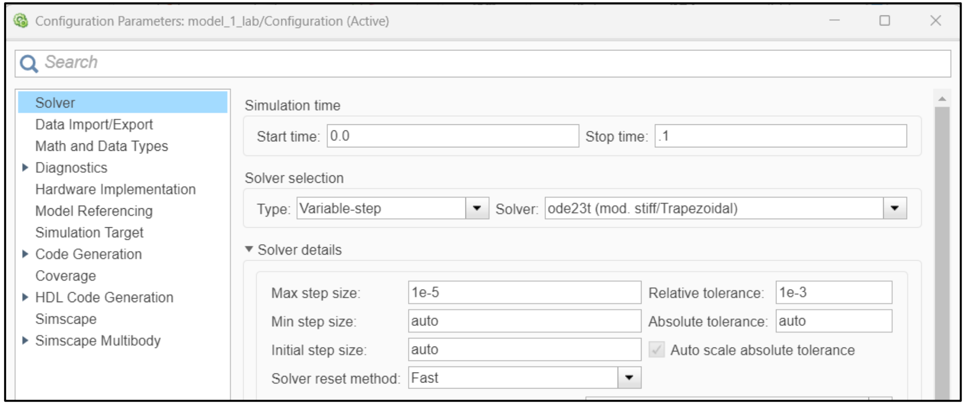
Create the script with initial data for your variant:

```
R = 240;  % Resistence
L = 24e-3; % Inducrance
C = 12e-6; % Capacitance
Em = 50; % Magnitude of source voltage
w = 50 * 2 * pi; % Frequency
I_L_0 = 0.5; % Initial condition of the current throw the inductance
U_C_0 = 10; % Initial condition of the voltage drop on the capacitance
```

Example

Open new model, create new Simulink model, tune the solver:

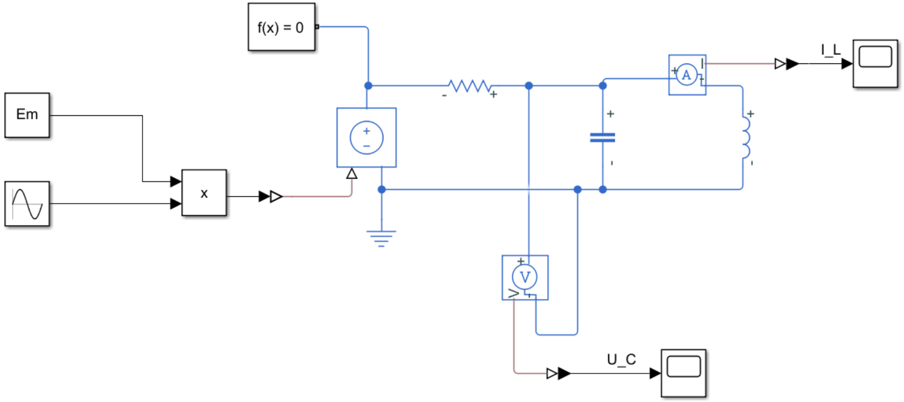




Example

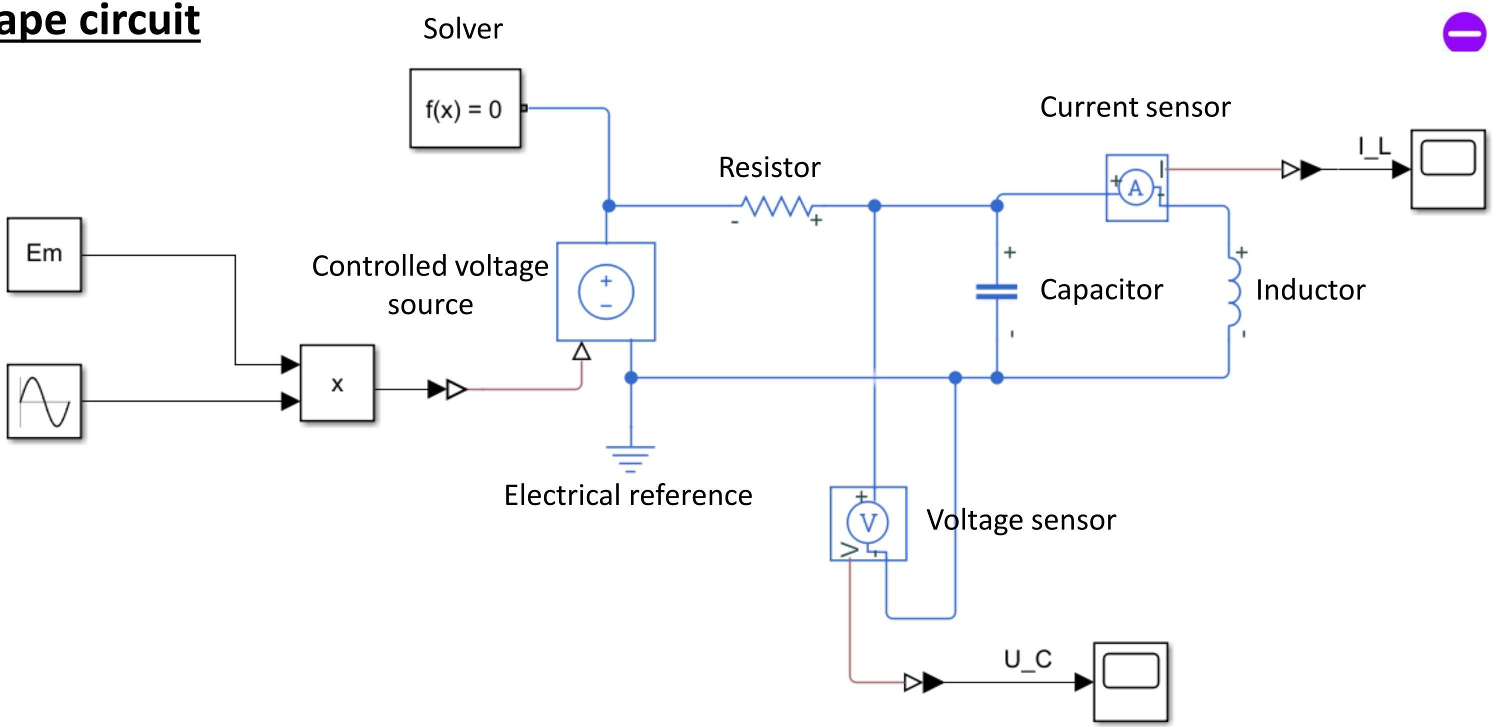
Simscape circuit

- Simscape
  - Foundation Library
    - Electrical
      - Electrical Elements
      - Electrical Sensors
      - Electrical Sources



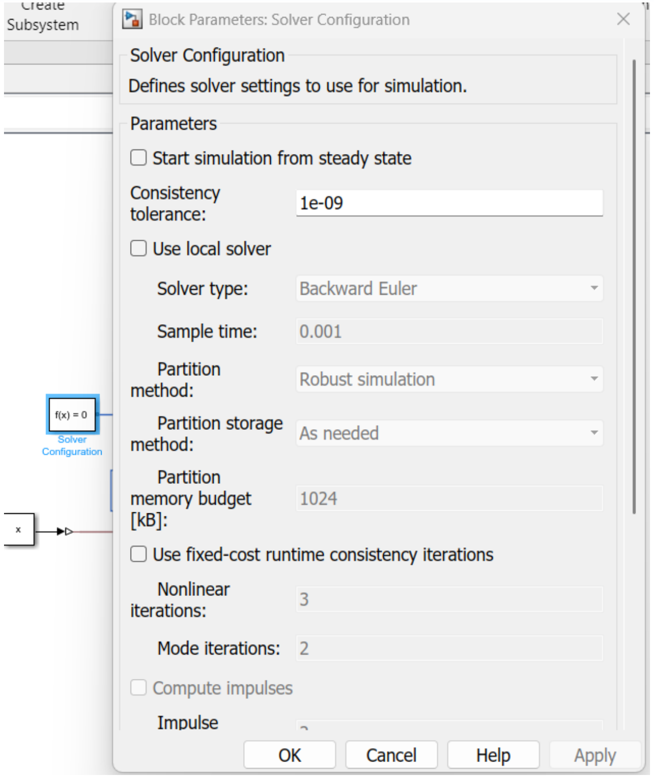
Example

Simscape circuit



Example

Simscape solver:



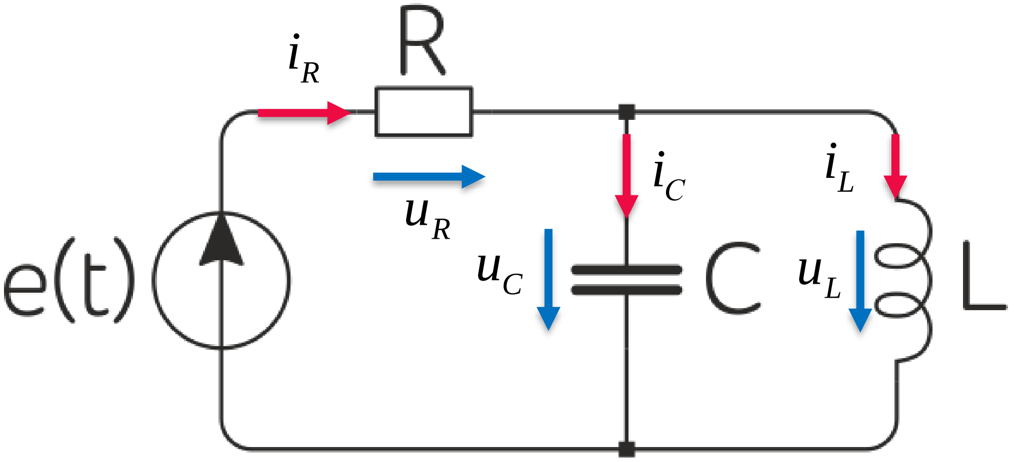
Example

Component equations

$$i_c = C\frac{du_c}{dt}$$

$$i_R = \frac{u_R}{R}$$

$$i_L = \frac{\int u_L dt}{L} = \frac{\int u_C dt}{L}$$



Example

Topological equations

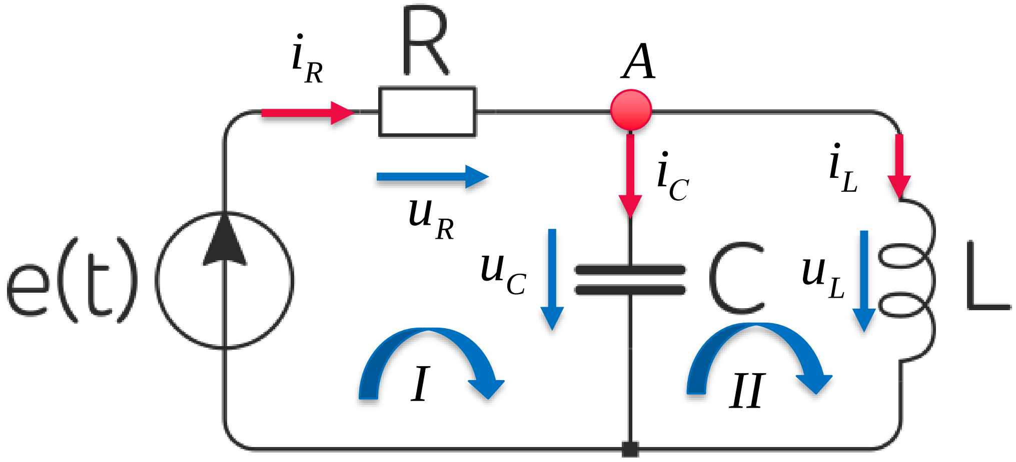
Kirchhoff's second law:

$$I: \quad u_C(t) + u_R(t) = e(t)$$

$$II: \quad u_C(t) - u_L(t) = 0$$

Kirchhoff's first law:

$$A: \quad i_R - i_L - i_C = 0$$



Example

State space form

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

State vector:

$$\mathbf{x} = \begin{bmatrix} i_L & u_C \end{bmatrix}^T; \quad \mathbf{x}_0 = \begin{bmatrix} 0.5 & 10 \end{bmatrix}^T$$

Control signal:

$$\mathbf{u} = e(t)$$

$$\boxed{\begin{cases} \dfrac{di_L}{dt} = a_{11} i_L + a_{12} u_C + b_{11} e \\ \dfrac{du_C}{dt} = a_{12} i_L + a_{22} u_C + b_{21} e \end{cases}}$$

Example

State space form

From topological equations:

$$u_C(t) - u_L(t) = 0$$

$$u_C(t) = u_L(t)$$

From component equations:

$$i_L = L^{-1}\int u_L dt$$

$$i_L = L^{-1}\int u_C dt$$

$$\frac{di_L}{dt} = \frac{u_C}{L}$$

$$a_{11} = 0;$$

$$a_{12} = \frac{1}{L};$$

$$b_{11} = 0;$$

Example

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<u>**State space form**</u>

From topological equations:

$$u_C(t)+u_R(t)=e(t)$$

$$u_R=e(t)-u_C(t)$$

$$Ri_r=e(t)-u_C(t)$$

$$R\left(i_C+i_L\right)=e(t)-u_C(t)$$

From component equations:

$$i_c=C\frac{du_c}{dt}$$

$$R\left(C\frac{du_C}{dt}+i_L\right)=e-u_C$$

$$RC\frac{du_C}{dt}=e-u_C-Ri_L$$

$$\frac{du_C}{dt}=-\frac{i_L}{C}-\frac{u_C}{RC}+\frac{e}{RC}$$

$$a_{21}=-\frac{1}{C};\;\; a_{22}=-\frac{1}{RC};\;\; b_{21}=\frac{1}{RC};$$

Example

Input-output form

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{u_C}{L} \\ \dfrac{du_C}{dt} = -\dfrac{i_L}{C} - \dfrac{u_C}{RC} + \dfrac{e}{RC} \end{cases}$$

Differential equations in time domain

$$\Rightarrow \qquad \frac{d}{dt} = s$$

$$\begin{cases} si_L = \dfrac{u_C}{L} \\ su_C = -\dfrac{i_L}{C} - \dfrac{u_C}{RC} + \dfrac{e}{RC} \end{cases}$$

Algebraic equations in Laplace domain

Example

Input-output form

$$i_L = \frac{u_C}{Ls}$$

$$su_C(s) = \frac{\left(-\frac{u_C}{R} - \frac{u_C}{Ls} + \frac{e(s)}{R}\right)}{C}$$

$\Rightarrow$

$$\left(s + \frac{1}{CR} + \frac{1}{CLs}\right)u_C(s) = \frac{1}{CR}e(s)$$

$$u_C(s) = \frac{1}{CR\left(s + \frac{1}{CR} + \frac{1}{CLs}\right)}e(s)$$

$$u_C(s) = \frac{s}{CR\left(s^2 + \frac{s}{CR} + \frac{1}{CL}\right)}e(s)$$

Example

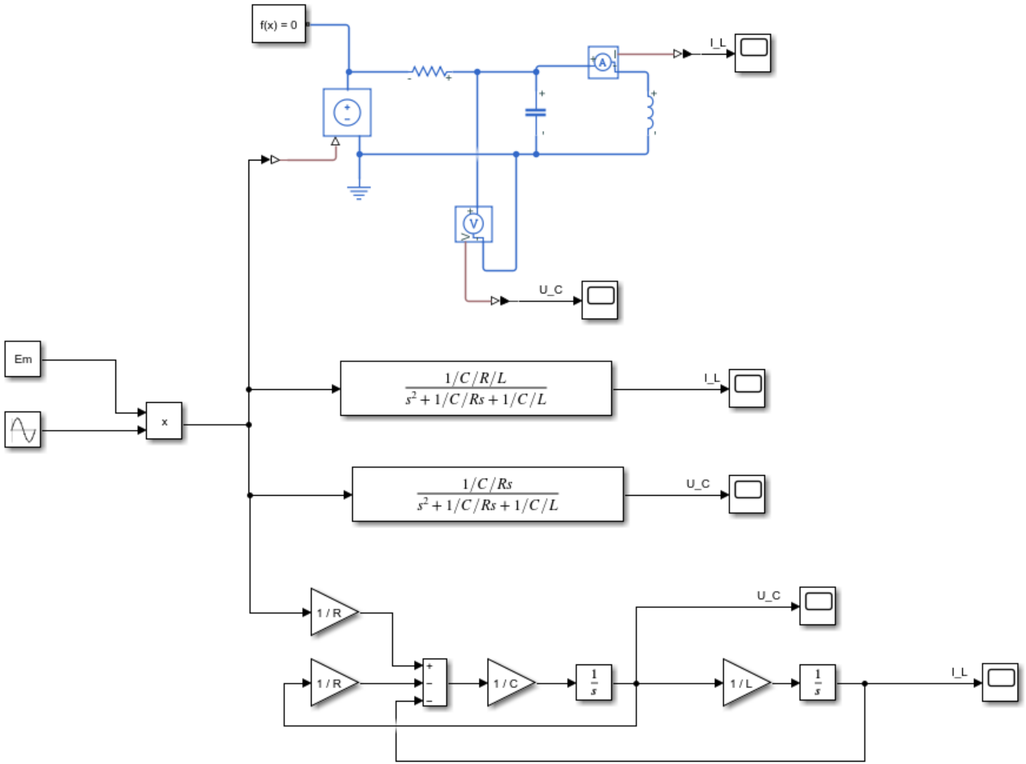
Input-output form

$$u_C(s) = \frac{s}{CR\left(s^2 + \frac{s}{CR} + \frac{1}{CL}\right)} e(s)$$

$$i_L(s) = \frac{u_C}{Ls} = \frac{1}{CRL\left(s^2 + \frac{s}{CR} + \frac{1}{CL}\right)} e(s)$$

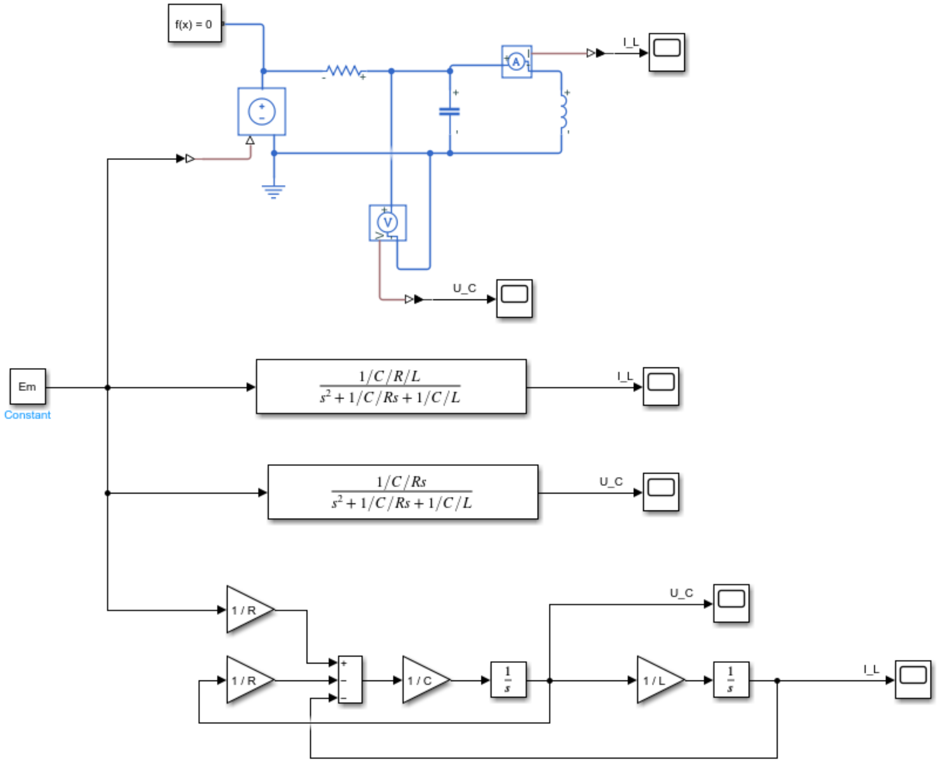
Example

<u>**Modelling with sinusoidal input voltage**</u>



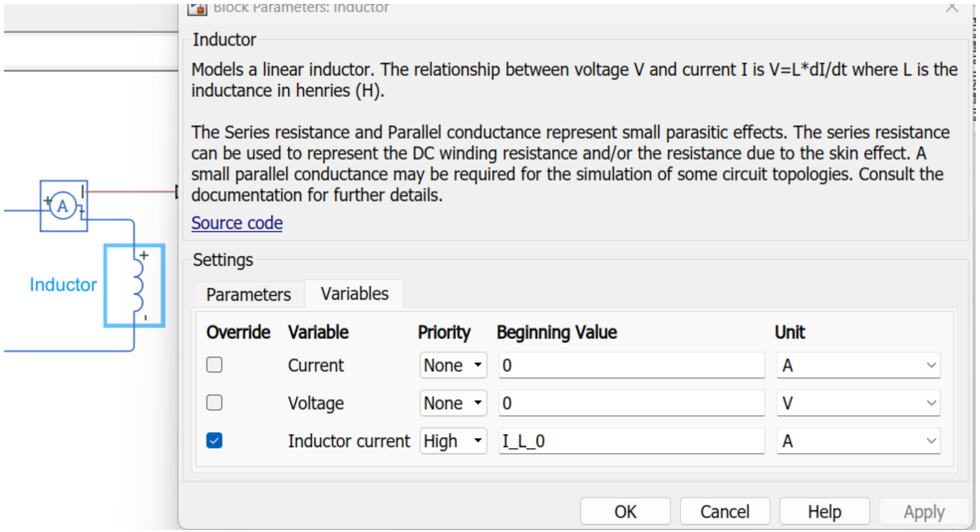
Example

Modelling with constant voltage



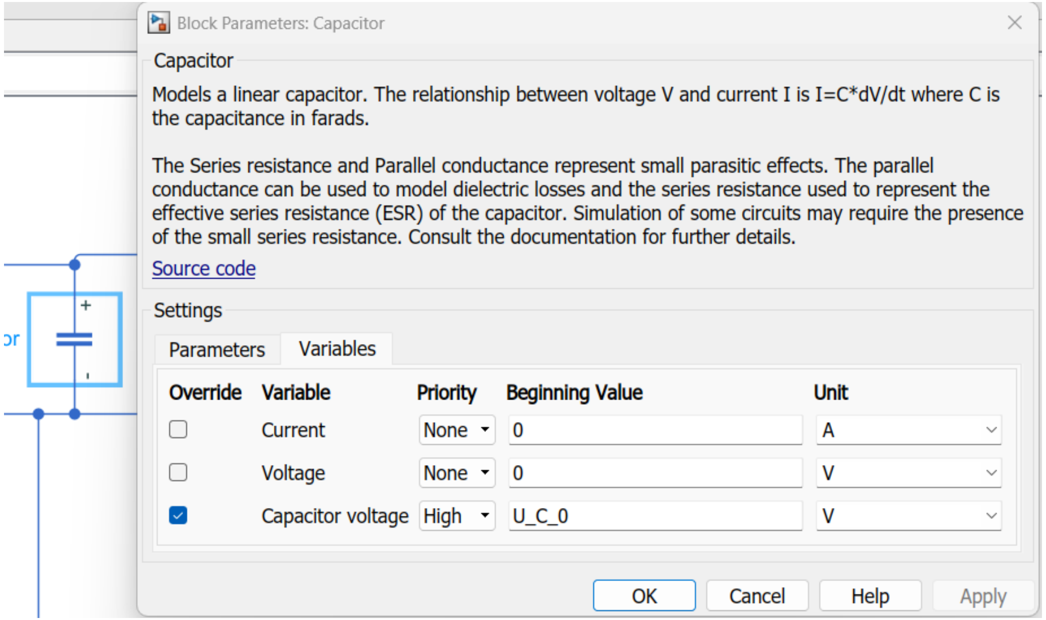
Example

Initial conditions



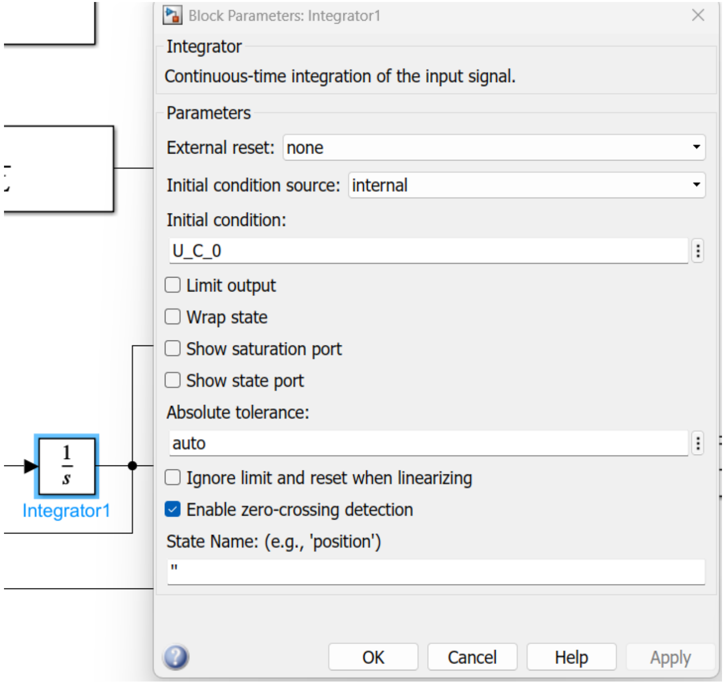
Example

Initial conditions



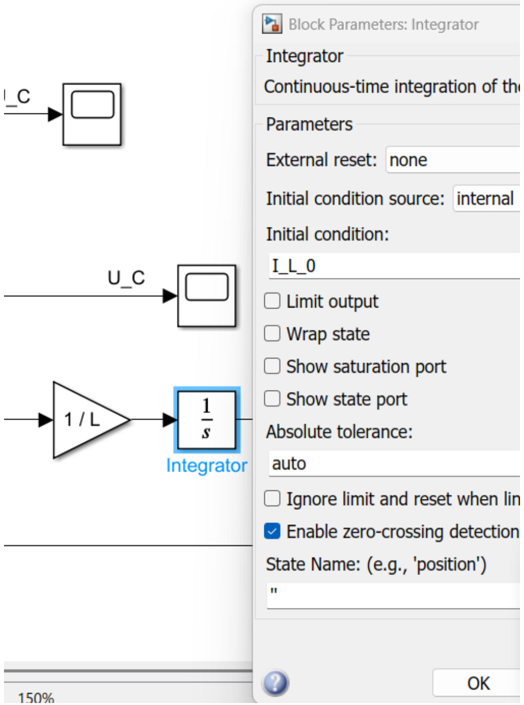
Example

Initial conditions



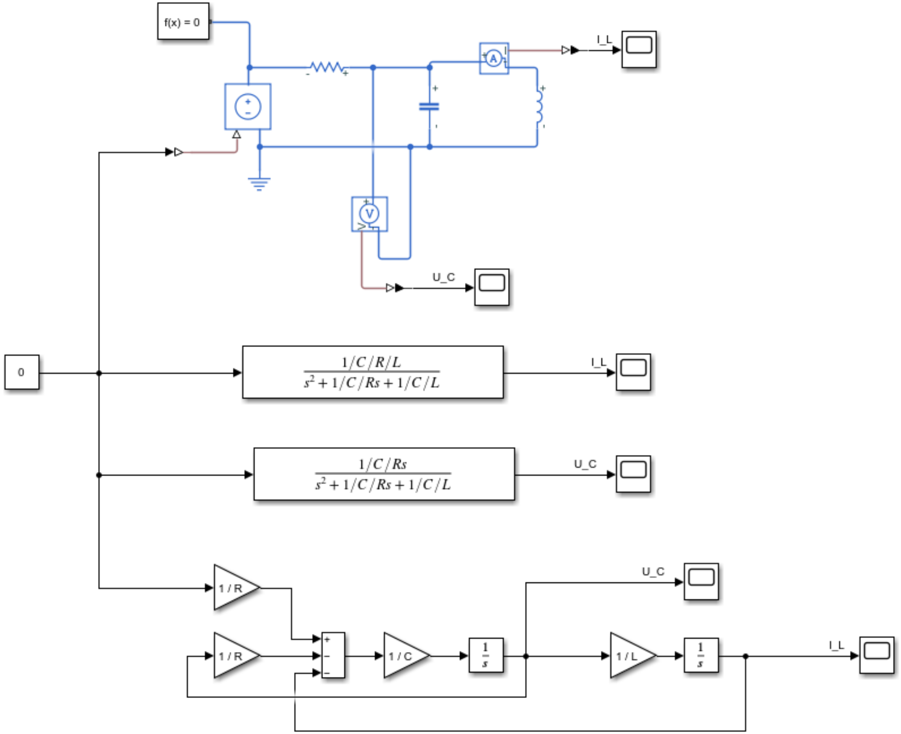
Example

Initial conditions



Example

<u>Modelling with zero voltage and nonzero initial conditions</u>



Report content

1. Equivalent circuit and simulation circuit of a linear electrical circuit.
2. Description of the procedure for obtaining models "input-output" (point 6 of the lab work task).
3. Simulation results (point 6 of the lab work task). Compare the graphs of transients of the simulation circuit and the "input-output" models.
4. Description of the procedure for obtaining the state-space model (point 4 of the lab work task).
5. Simulation results (points 7 and 8 of the lab work task). Compare the graphs of transients of the simulation circuit and the state-space models.
6. Conclusions.

THANK YOU
FOR YOUR TIME!

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