

# Actuators

## Modelling of mechanics of the actuators

LAB1

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HDU-ITMO Joint Institute

2024

# LAB #1 Modelling of mechanics of the actuators

Task to be completed

## LAB#1 Modelling of mechanics of the actuators

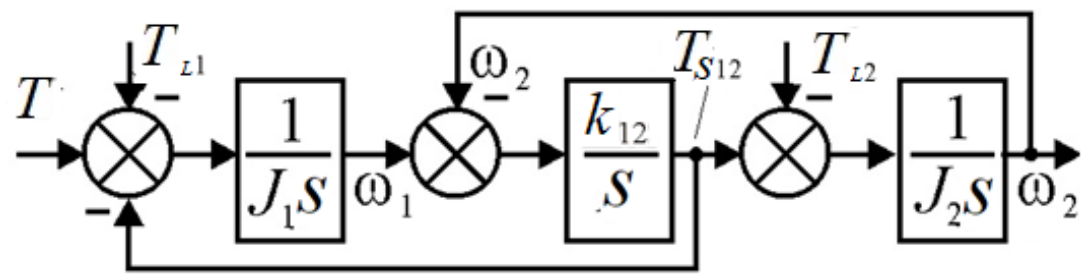
- ✓ LAB#1 is aimed at checking the theoretical data and relationships presented in theory materials (Lectures 1, Practice 1 of course “Actuators”)
- ✓ LAB#1 is performed in MATLAB / Simulink
- ✓ LAB#1 consists two parts:

**Part1 . Mathematical modelling of two-mass mechanism**

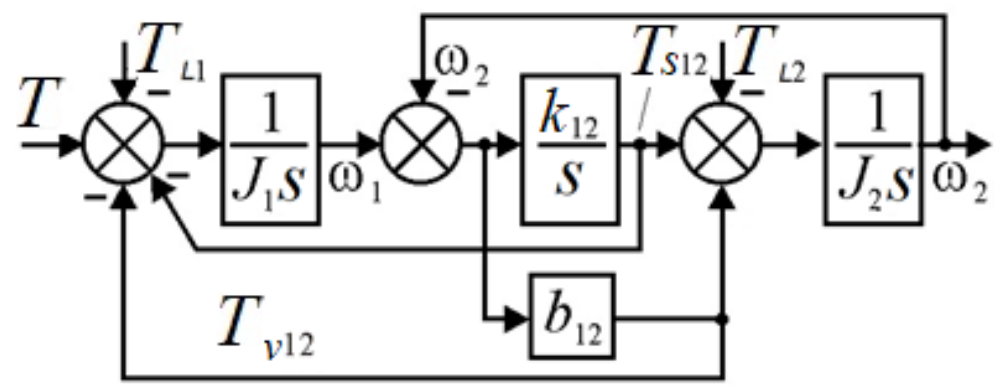
**Part2. Mathematical modelling of DC-motor with two-mass mechanism**

# Schemes under consideration

## Part 1. Mathematical modelling of two-mass mechanism

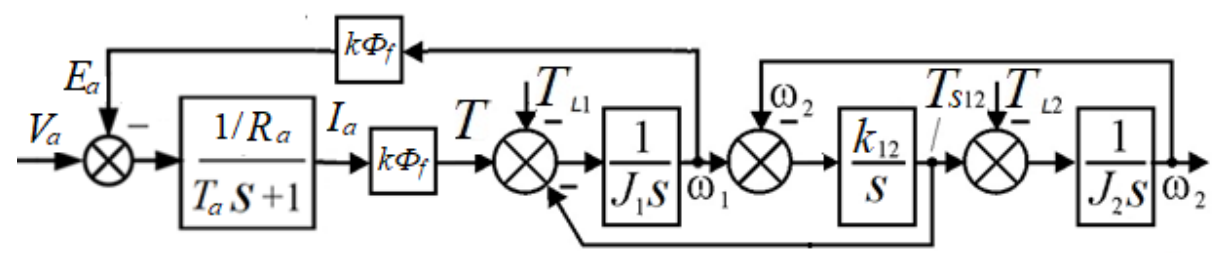


a)



b)

## Part 2. Mathematical modelling of DC-motor with two-mass mechanism (not necessary – this is additional option)



c)

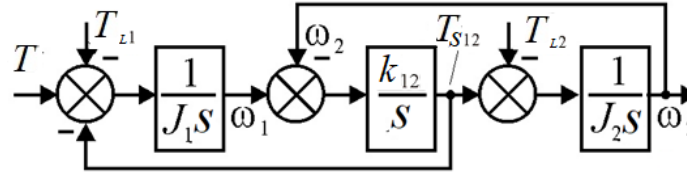
Table 1 –The data for the LAB

Two-mass mechanism					DC-motor				
N	J1 kgm2	J2 kgm2	k12 Nm/ rad	Δφ rad	Va V	Ra Ohm	Ta s	kΦf	Trated
One option for all	0.183	0.055	3800	0,3 9	400	21,45	0,004	12	47,7

# Tasks for part 1 (modelling of two-mass mechanism )

## Part 1. Mathematical modelling of two-mass mechanism

Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions).

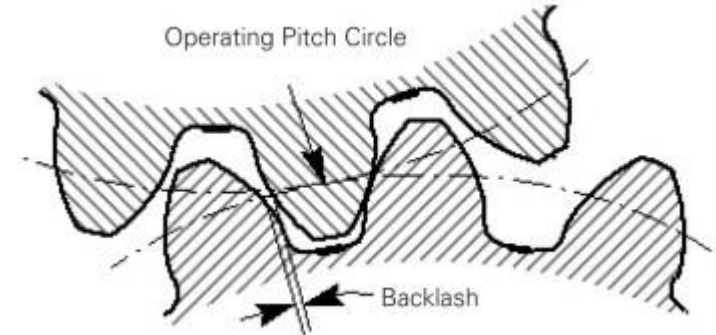


1. Show on plot transient response of  $\omega_1(t)$ ,  $\omega_2(t)$ ,  $T_{s12}(t)$  by the step reference signal  $T$  with value  $0.1T_{rated}$  (at  $T_{L1}=0$ ,  $T_{L2}=0$ ). Please put  $\omega_1(t)$ ,  $\omega_2(t)$  on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration
2. Plot the Bode diagram of the two-mass mechanism and determine resonance frequency.
3. Compare calculated parameters of transient and parameters got by simulation
4. Change the parameters of the two-mass bodies' system: mass inertia ratio ((3 values  $J_2$  to get  $\gamma$  from 1 to 2) , link stiffnesses coeff (3 values:  $k_{12min}$ ,  $k_{12medium}$ ,  $k_{12max}$ ). Draw a conclusion about the comparison of the processes

# Tasks for part 1 (modelling of two-mass mechanism )

## Part 1. Mathematical modelling of two-mass mechanism

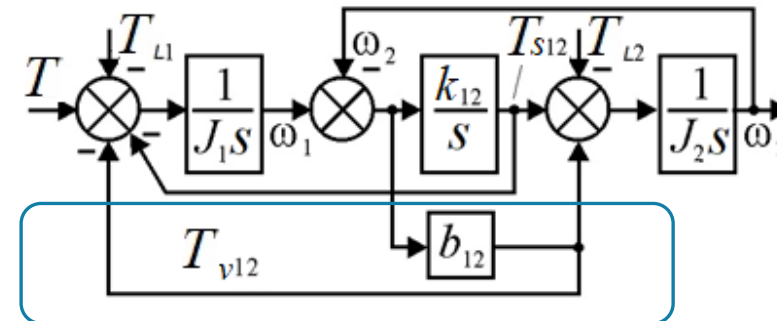
### Task 1.2. Add backlash in a model of the two-mass mechanism



1. Show on plot transient response of  $\omega_1(t)$ ,  $\omega_2(t)$ ,  $T_{s12}(t)$  by the step reference signal  $T$  with value  $0.1T_{rated}$  (at  $T_{L1}=0$ ,  $T_{L2}=0$ ). Please put  $\omega_1(t)$ ,  $\omega_2(t)$  on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration but with deadzones
2. Compare  $T_{s12}(t)$  in mechanism without and with backlash in gearbox
3. Draw conclusions

### Task 1.3. Add torque of viscous friction in a model of the two-mass mechanism.

1. The viscous damping coefficient  $b$  should be chosen considering that the oscillation damp in 5 periods.
2. Get results. Draw conclusions



# LAB #1 Modelling of mechanics of the actuators

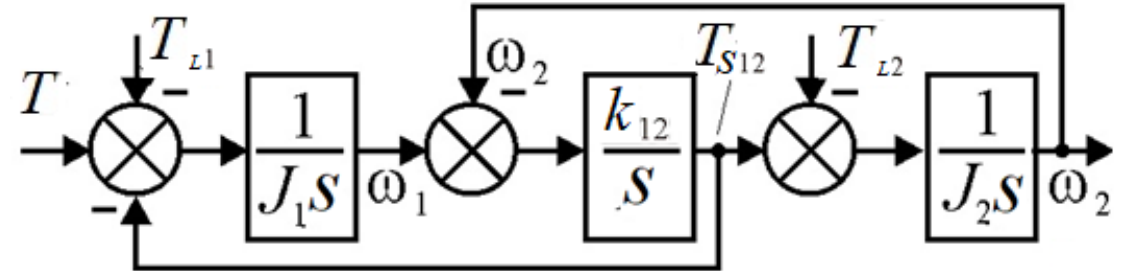
## Execution steps

# Tasks for part 1 (modelling of two-mass mechanism )

## Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

Mathematic model of two-mass mechanism

$$\begin{cases} \text{XXX} \\ \text{XXX} \\ \text{XXX} \end{cases} \quad \boxed{\text{Please fill in by yourself}} \quad (1)$$



Scheme of the system (Fig.)

*Please, design it by yourself*

Figure : Math model of the two-mass mechanism in Simulink



# Tasks for part 1 (modelling of two-mass mechanism )

## Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

1. Show transient response of  $\omega_1(t)$ ,  $\omega_2(t)$ ,  $T_{s12}(t)$  by the step reference signal  $T$  with value  $0.1T_{rated}$  ( $T_{L1}=0$ ,  $T_{L2}=0$ ).

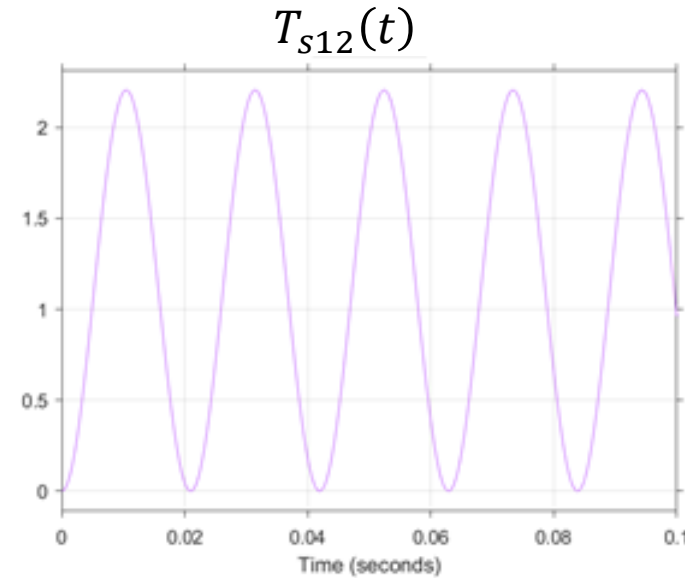
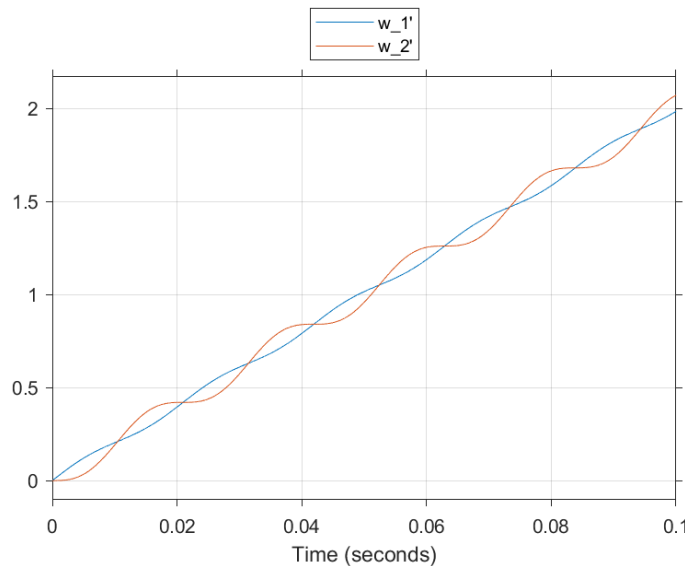


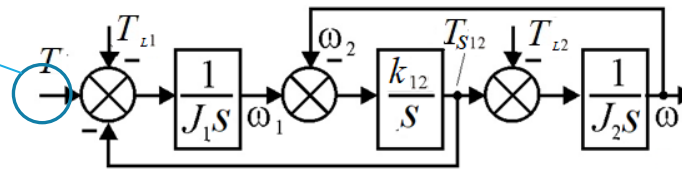
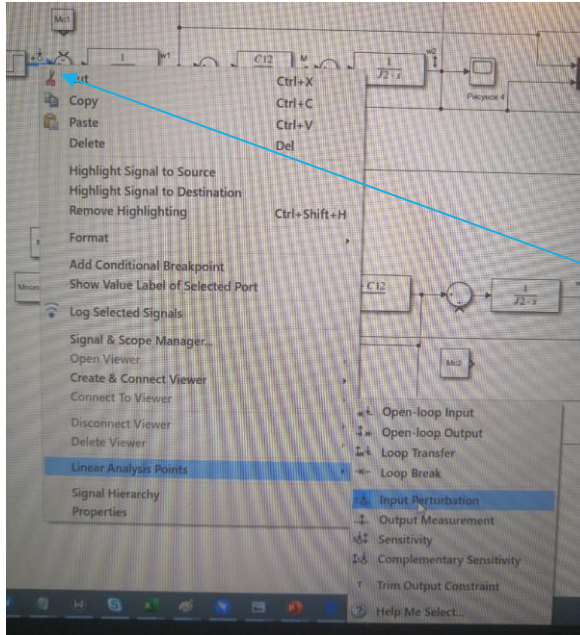
Figure : transient response of  $\omega_1(t)$ ,  $\omega_2(t)$  (a),  $T_{s12}(t)$  (b)

*Please show  $\omega_1(t)$ ,  $\omega_2(t)$  on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration*

# Tasks for part 1 (modelling of two-mass mechanism )

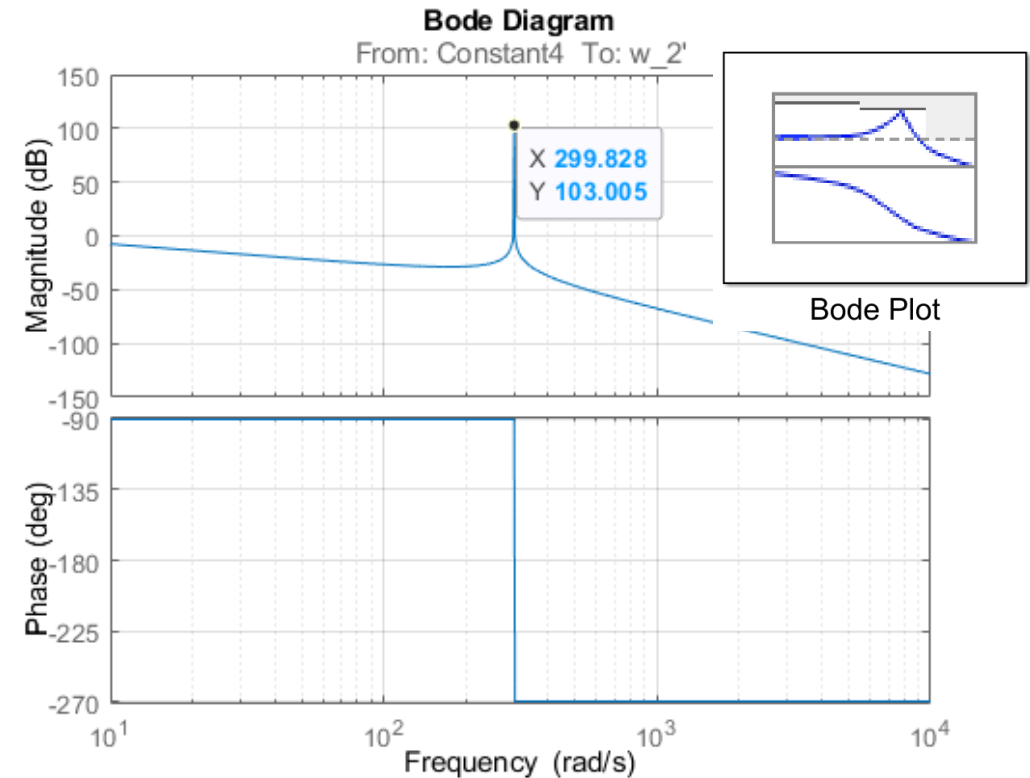
## Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

### 2. Plot the Bode diagram of the two-mass mechanism



In Simulink Library Browser:

Simulink Control Design/Linear Analysis Plots/Bode Plot.



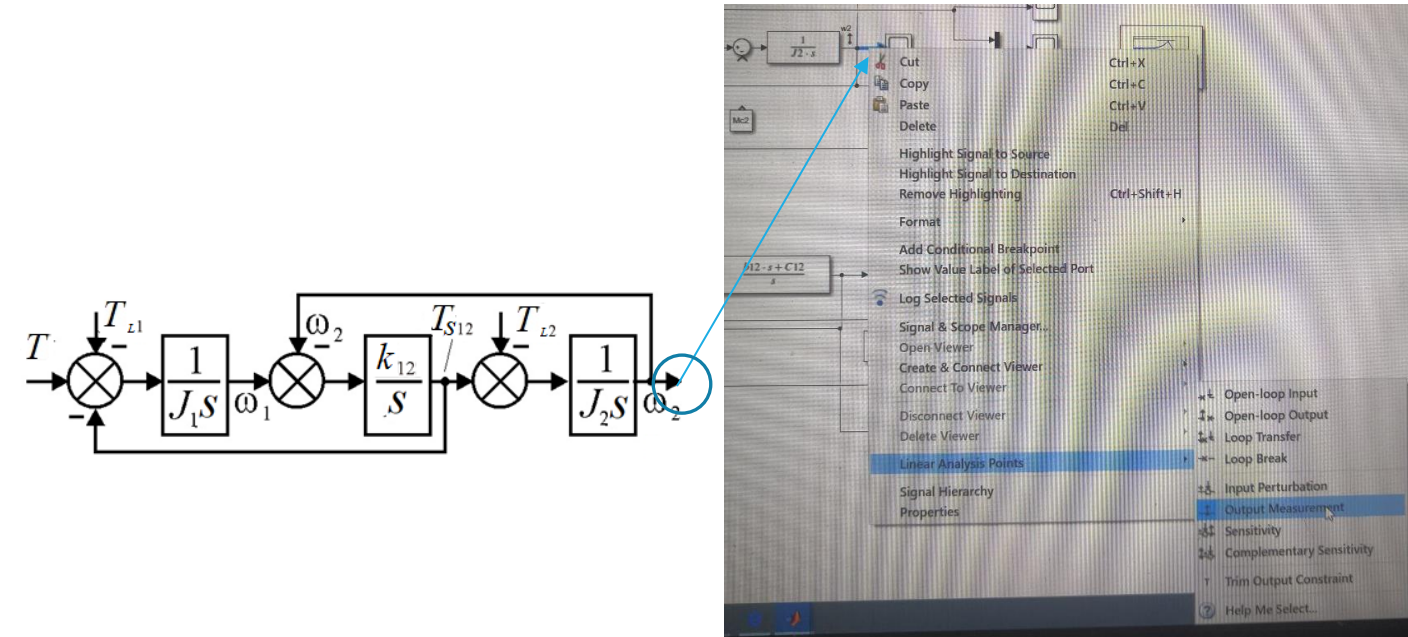
Point – Right mouse  
button → Linear  
Analysis Points → Input  
Perturbation

# Tasks for part 1 (modelling of two-mass mechanism )

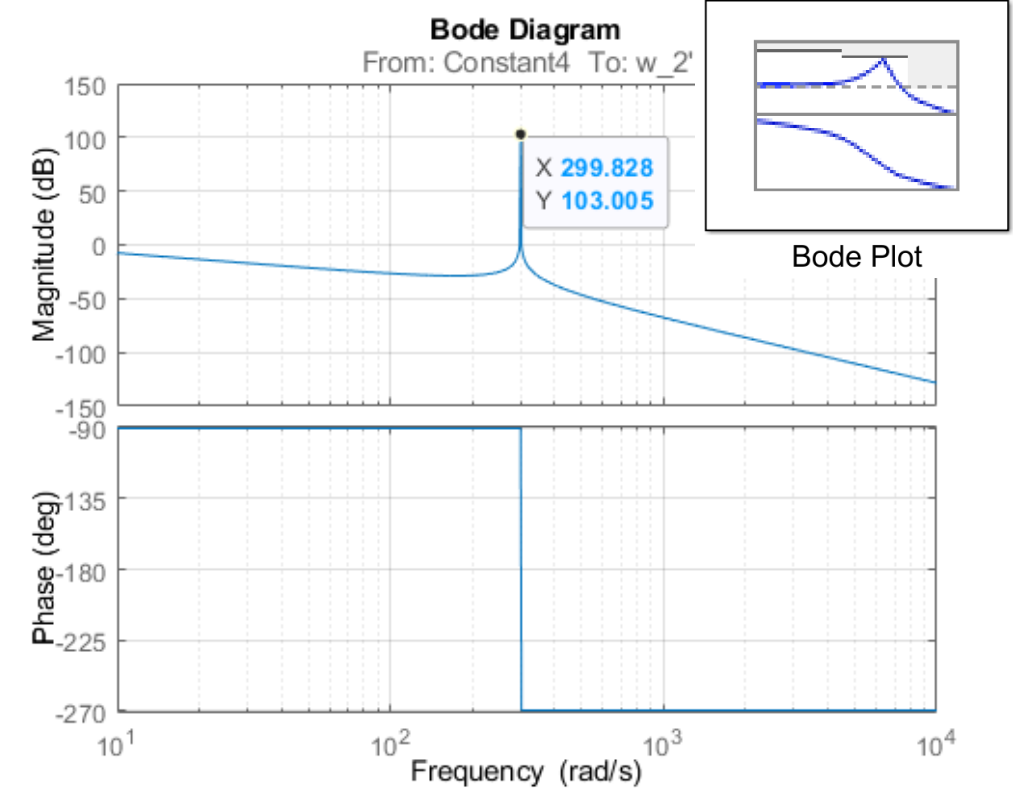
## Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

### 2. Plot the Bode diagram of the two-mass mechanism

Simulink Control Design/Linear Analysis Plots/Bode Plot.



Point – Right mouse button →  
Linear Analysis Points → Output  
Measurement

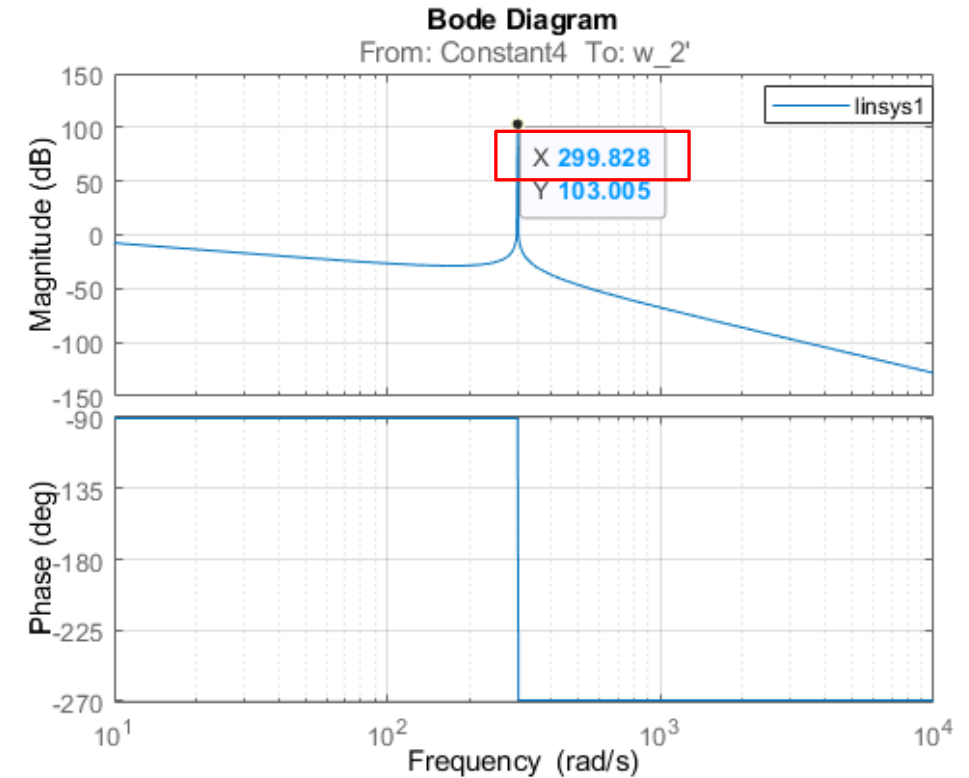


# Tasks for part 1 (modelling of two-mass mechanism )

## Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

- The resonance frequency:

$$\omega_{R1} = \sqrt{k_{12} \frac{J_1 + J_2}{J_1 J_2}} = 299,75 c^{-1}$$



# Tasks for part 1 (modelling of two-mass mechanism )

## Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)

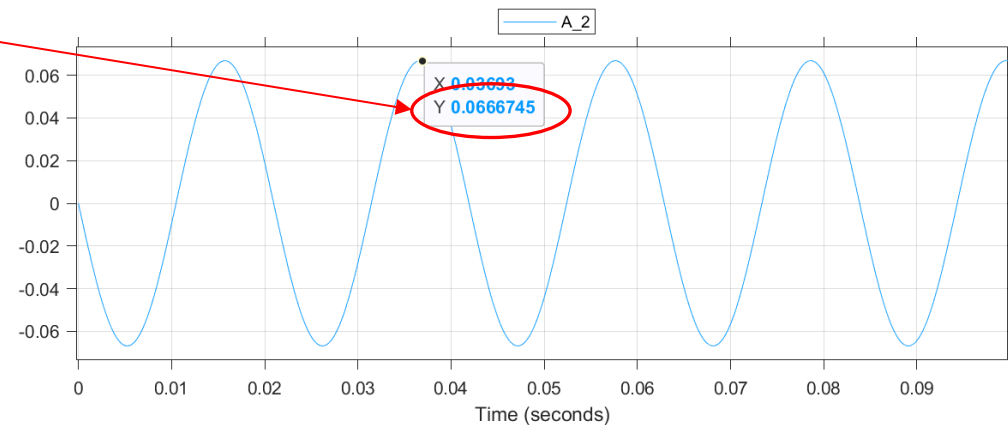
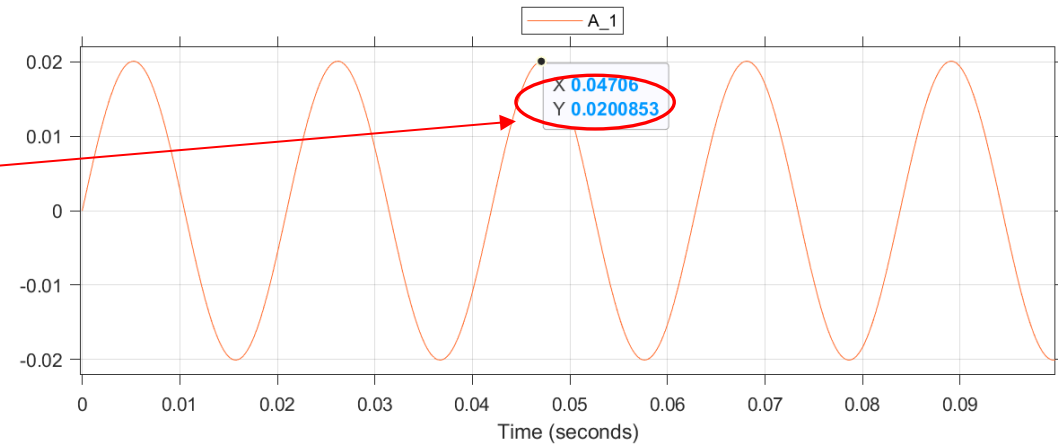
- The magnitudes of bodies fluctuation:

$$A_1 = \frac{J_2 \varepsilon_{av}}{J_1 \omega_{R1}} = 0,0201 \text{ rad/s}$$

$$A_2 = \frac{\varepsilon_{av}}{\omega_{R1}} = 0,0669 \text{ rad/s}$$

- Exclude** the average angular acceleration:

$$\varepsilon_{av} = \frac{T}{J_1 + J_2} = \frac{0.1 \times T_{\text{rated}}}{J_1 + J_2} = 20 \text{ rad/s}^2$$



# Tasks for part 1 (modelling of two-mass mechanism )

## **Task 1.1. Design a model of the two-mass mechanism without any disturbances (load torques, frictions)**

1. Changing the parameters of the two-mass motor-mechanism system

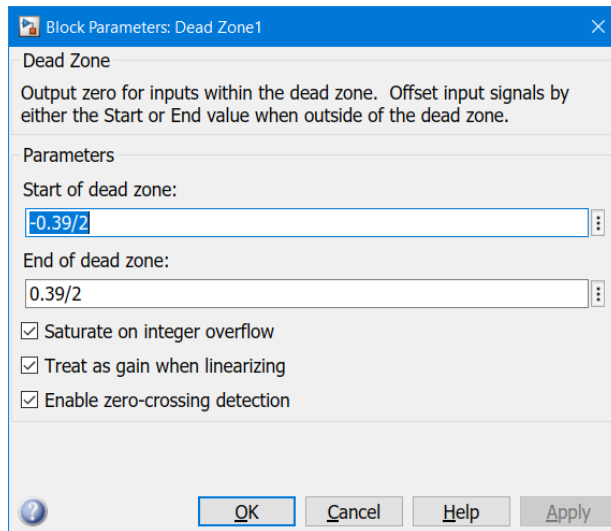
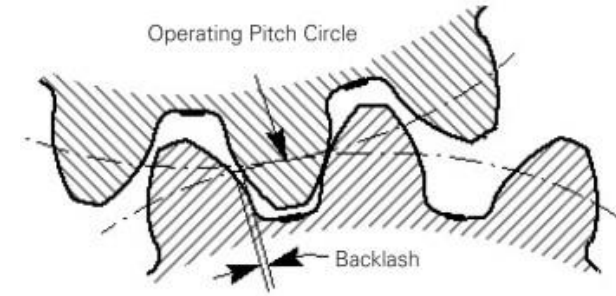
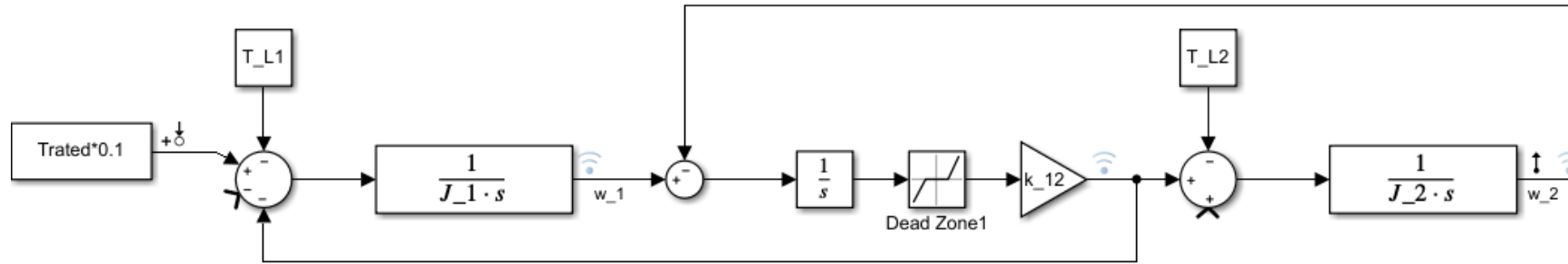
a) Show transient process in a two-mass load at various values of mass inertia ratio  $\gamma = \frac{J_1 + J_2}{J_1}$  (3 values  $J_2$  to get  $\gamma$  from 1 to 2)

b) Show transient process in a two-mass load at various values of link stiffnesses (3 values:  $k_{12\min}$ ,  $k_{12\text{medium}}$ ,  $k_{12\max}$ )

Draw a conclusion about the comparison of the processes in one-mass and two-mass systems.

# Tasks for part 1 (modelling of two-mass mechanism )

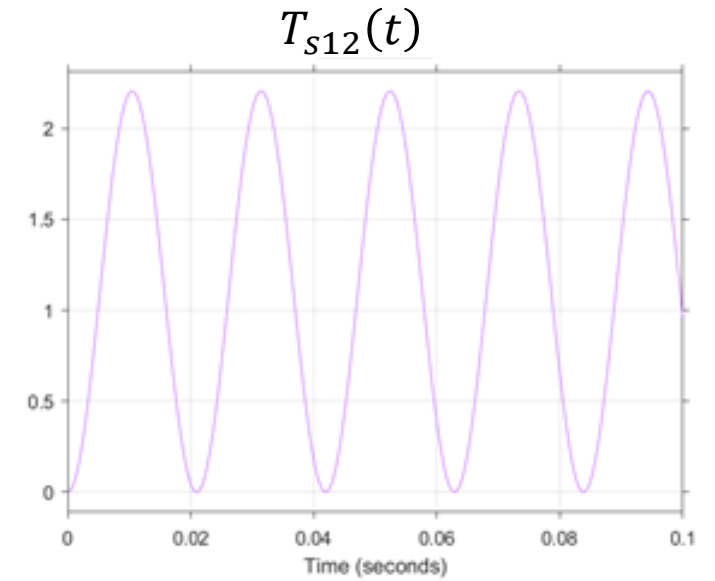
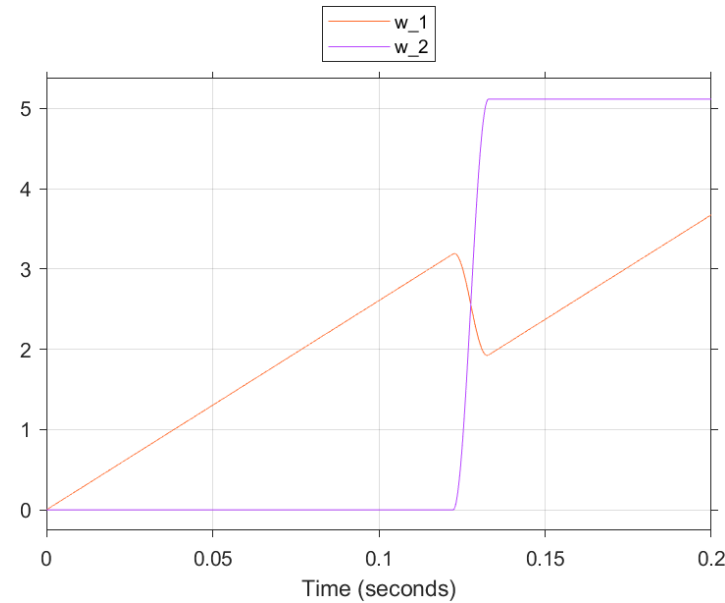
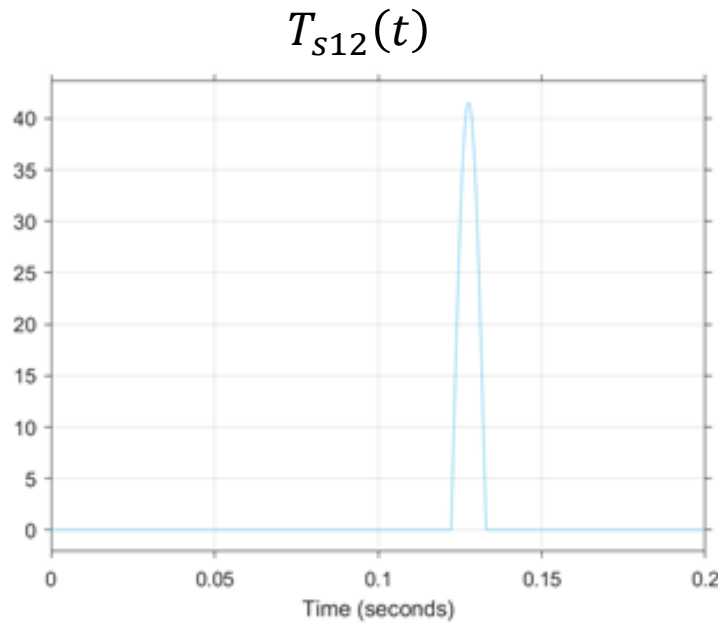
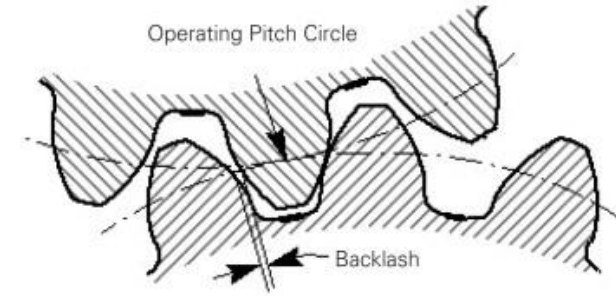
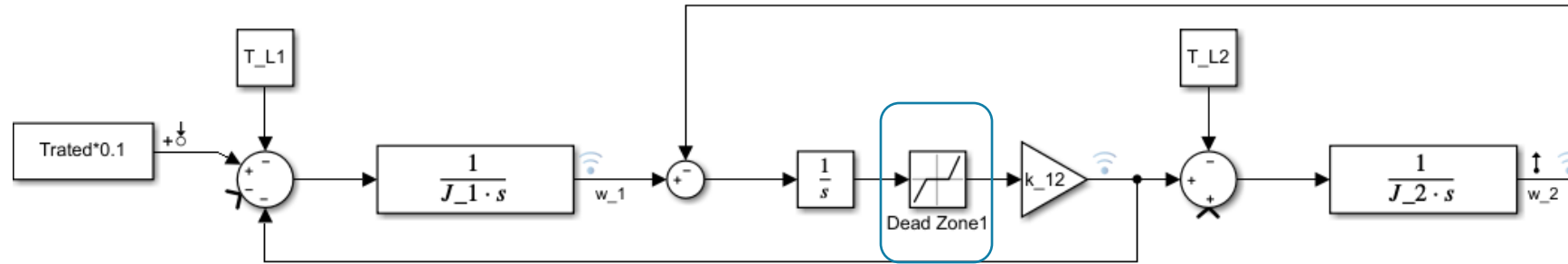
## Task 1.2. Add backlash in a model of the two-mass mechanism



$$\begin{cases} T - T_{L1} - T_{s12} = J_1 s w_1 \\ T_{s12} - T_{L2} = J_2 s w_2 \\ T_{s12} = k_{12}(\varphi_1 - \varphi_2 \pm \Delta \varphi / 2), |\varphi_1 - \varphi_2| > \Delta \varphi / 2 \\ T_{s12} = 0, |\varphi_1 - \varphi_2| \leq \Delta \varphi / 2 \end{cases}$$

# Tasks for part 1 (modelling of two-mass mechanism )

## Task 1.2. Add backlash in a model of the two-mass mechanism



$T_{s12}(t)$  in the mechanism  
without backlash



# Tasks for part 1 (modelling of two-mass mechanism )

## Task 1.3 Design a model of the two-mass mechanism with viscous frictions

$$b_{12} = \frac{2a_v J_1 J_2}{J_1 + J_2} = \frac{6\omega_{R1} J_1 J_2}{10\pi(J_1 + J_2)} = 2,4211$$

$$\text{where } a_v \approx \frac{3\lambda_v \cdot \omega_{R1}}{2\pi} = \frac{3\omega_{R1}}{10\pi}$$

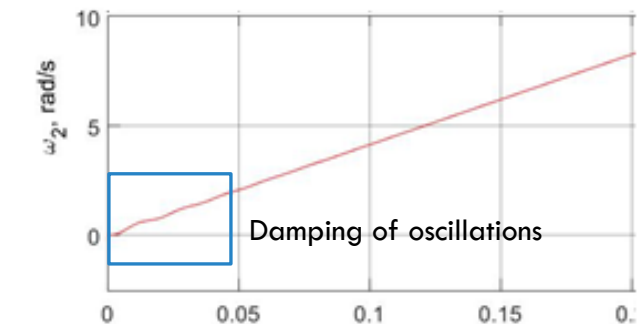
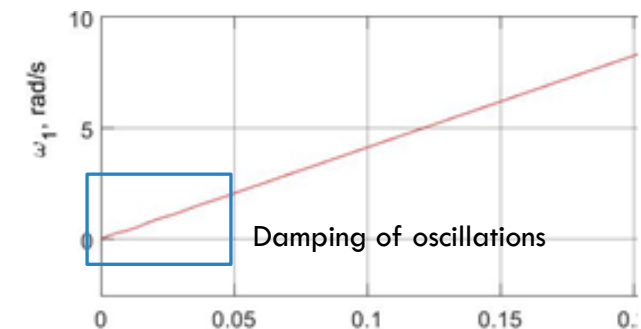
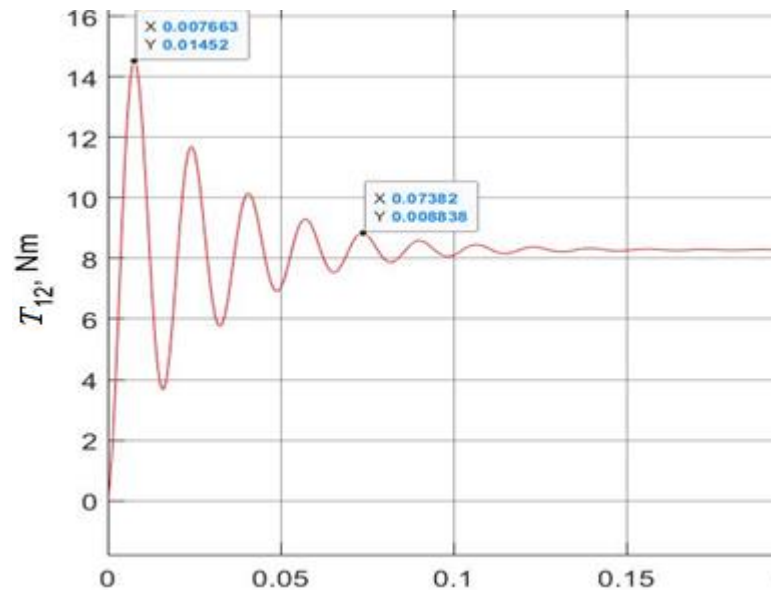
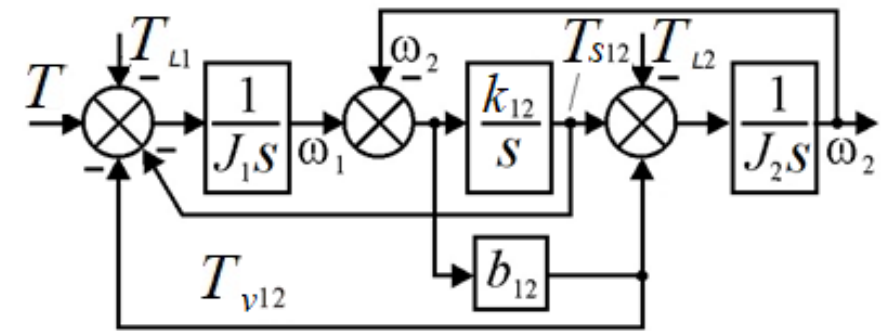
$\lambda_v$  - logarithmic decrement

$a_v$  - attenuation coefficient

$$\lambda_v = a_v T = \frac{T}{\tau} = \frac{1}{n}$$

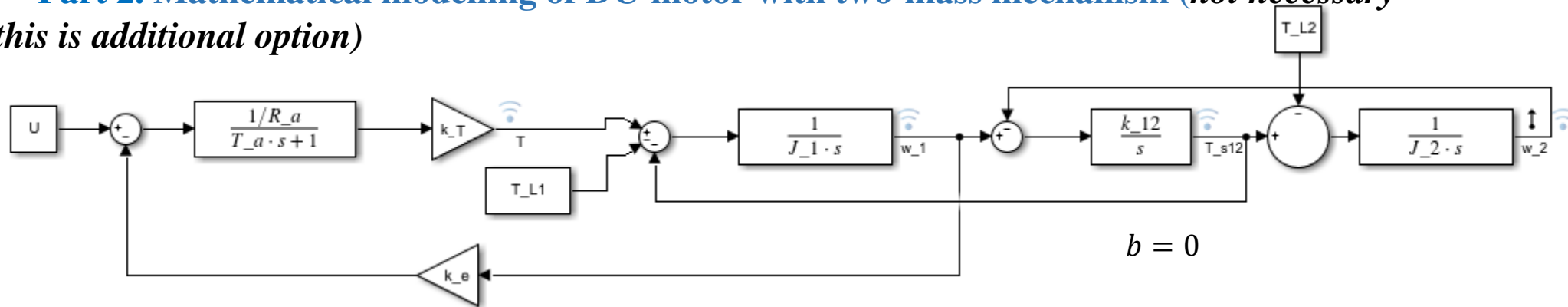
$n$  - number of harmonic oscillations during relaxation  $\tau$  (the amplitude decreases  $e$  times)

$$A_0 e^{-a_v t} \rightarrow t_{res} = 3 \frac{1}{a_v} = nT = 5T \text{ time response}$$



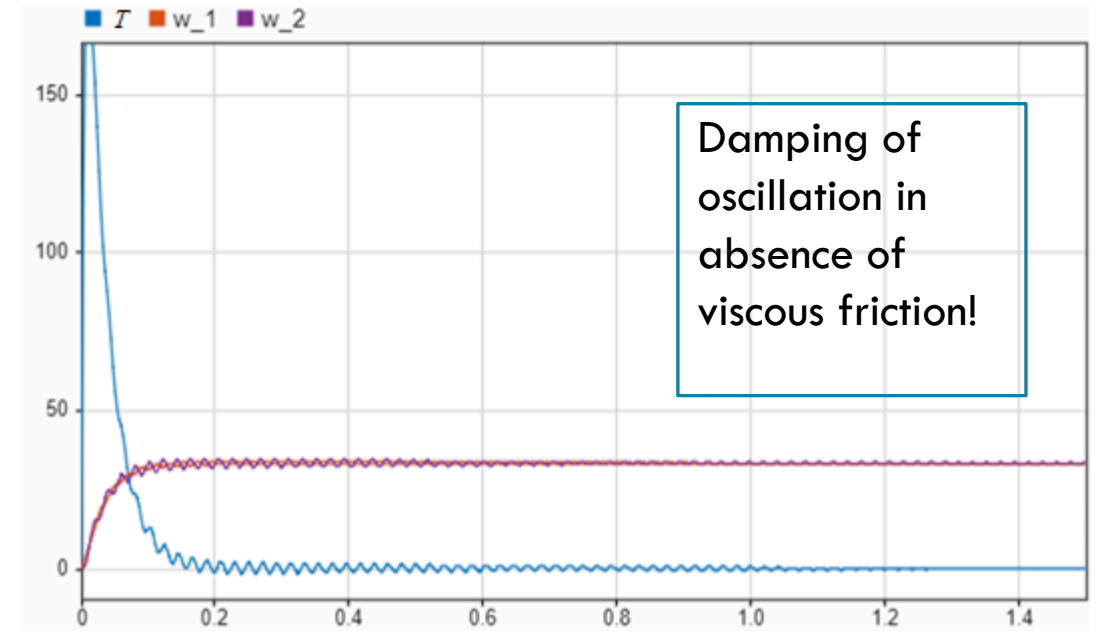
# Tasks for part 2 (modelling of motor with two-mass load) - *not necessary*

## Part 2. Mathematical modelling of DC-motor with two-mass mechanism (*not necessary – this is additional option*)



### Task 2.1 Modelling of the DC-motor with two-mass mechanism.

1. Design a model of the DC-motor with two-body mechanism.
2. Show plots  $T(t)$ ,  $T_{s12}(t)$ ,  $\omega_1(t)$ ,  $\omega_2(t)$



## Tasks for part 2 (modelling of motor with two-mass load) - *not necessary*

### Task 2.2 Modelling of the DC-motor with two-body mechanism with another parameters

1. Changing the parameters of the two-mass motor-mechanism system

a) Show transient process of start mode of a motor with a linear torque-speed curve with a two-mass load at various values

of mass inertia ratio  $\gamma = \frac{J_1 + J_2}{J_1}$  (3 values  $J_2$  to get  $\gamma$  from 1 to 2)

b) Show transient process of start mode of a motor with a linear torque-speed curve with a two-mass load at various values of link stiffnesses (3 values:  $k_{12\min}$ ,  $k_{12\text{medium}}$ ,  $k_{12\max}$ )

Draw a conclusion about the comparison of the processes in one-mass and two-mass systems.

**Thank you for your attention**