Actuators

Report for

Lab#1 Modelling of mechanics of the actuators

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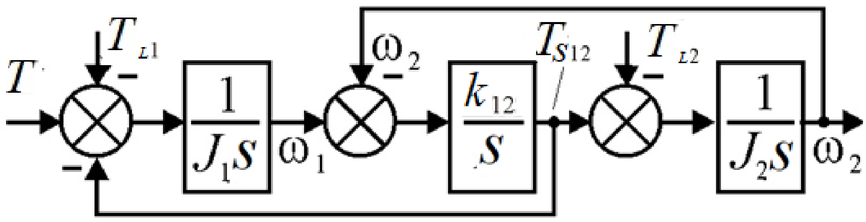
- ✓ LAB#1 is aimed at checking the theoretical data and relationships presented in theory materials (Lectures#2 of course “Actuators”) when considering dynamic processes in the mechanical part of an electric drive (*as well as in a motor, as well as in a two-mass motor-engine system – additional option*)
- ✓ LAB#1 is performed in MATLAB / Simulink
- ✓ LAB#1 consists two parts:

Table 1 –The data for the LAB#1

Var. No	V_a	R_a	T_a	$k\Phi f$	T_{rated}	J_1	J_2	k_{12}	Δφ
	V	*Ohm*	*s*			kgm^2	kgm^2	Nm/rad	rad
	Input armature voltage	armature resistance	Electromagnetic time constant armature	EMF/torque constant of DC-motor	Rated value of DC-motor torque	Moment of inertia of the 1st mass	Moment of inertia of the 2nd mass	Stiffness coeff.	Value of blacklash
Example	400	21.45	3	15	100	1.72	0.7	7846	0.39

Part 1. Mathematical modelling of two-mass mechanism

Task 1.1. Research processes in a model of the two-mass mechanism without any disturbances (load torques, friction torques) (*you have done it with A.Mamatov in “ElMechSystDynamic”*)



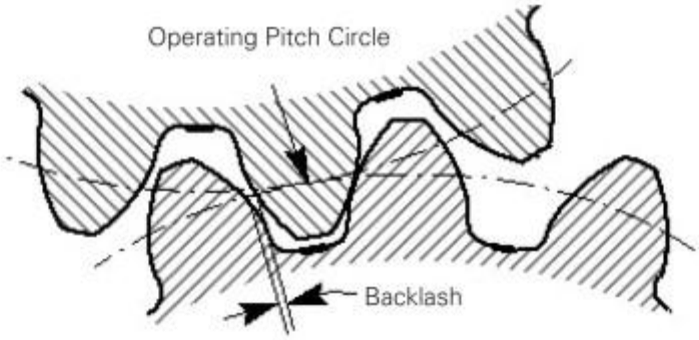
1. Design a model of the two-mass mechanism.

2. Show on plot transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal *T* with value $0.1T_{rated}$ (at T_{L1}=0, T_{L2}=0). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration. Display $T_{s12}(t)$ on the plot.

3. Display the Bode diagram of the two-mass mechanism and determine resonance frequency.

4. Compare calculated parameters of transient and parameters got by simulation

Task 1.2. Research the effect of backlash in a model of the two-mass mechanism



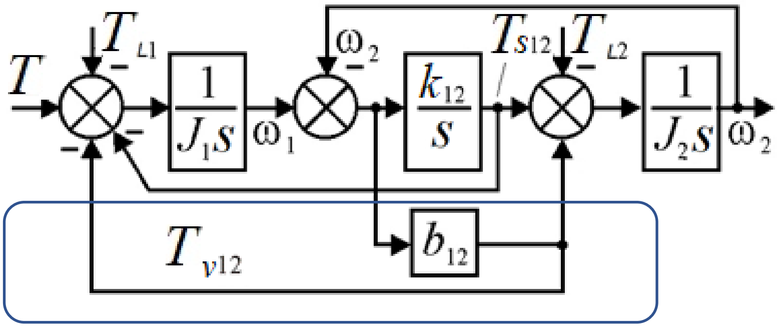
1. Add backlash in a model of the two-mass mechanism

2. Show on plot transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value $0.1T_{rated}$ (at $T_{L1}=0$, $T_{L2}=0$). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration but with deadzones

2. Compare $T_{s12}(t)$ in mechanism without and with backlash in gearbox

3. Draw conclusions

Task 1.3. Research the effect of viscous friction torque in a model of the two-mass mechanism



1. Add torque of viscous friction in a model of the two-mass mechanism
2. The viscous damping coefficient b should be chosen
considering that the oscillation damp in 5 periods.
3. Get results. Draw conclusions

Part2. Mathematical modelling of DC-motor with two-mass mechanism (*not necessary – this is additional option)*

Task 2.1 Modelling of the DC-motor with two-mass mechanism
1. Design a model of the DC-motor with two-mass mechanism.
2. Show plots $T(t)$, $T_{s12}(t)$, $\omega_1(t)$, $\omega_2(t)$

Part 1. Mathematical modelling of two-mass mechanism

Task 1.1. Research processes in a model of the two-mass mechanism without any disturbances (load torques, frictions)

Mathematic model of two-mass mechanism without any disturbances (load torques, frictions) (1)

$$\left.\begin{aligned} T - k_{12}(\omega_1 - \omega_2)/s &= J_1 s\omega_1; \\ k_{12}(\omega_1 - \omega_2)/s &= J_2 s\omega_2. \end{aligned}\right\} \qquad (1)$$

Scheme of the system (Fig.1)

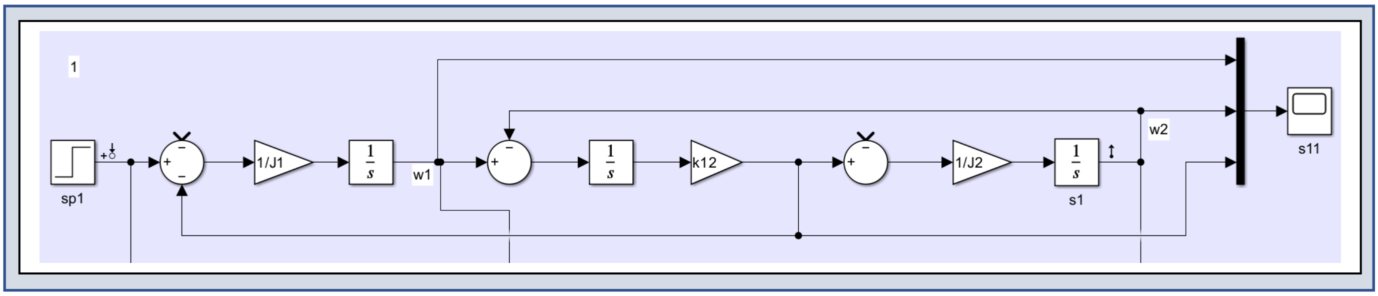


Figure 1 : Math model of the two-mass mechanism in Simulink

Let's consider the effect of elasticity. Consider the reaction to a torque step $T(t)$ from 0 to $0.1T_{rated}$ in a two-mass system at zero loads T_{L1}=0, T_{L2}=0 and zero initial conditions.

Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration.

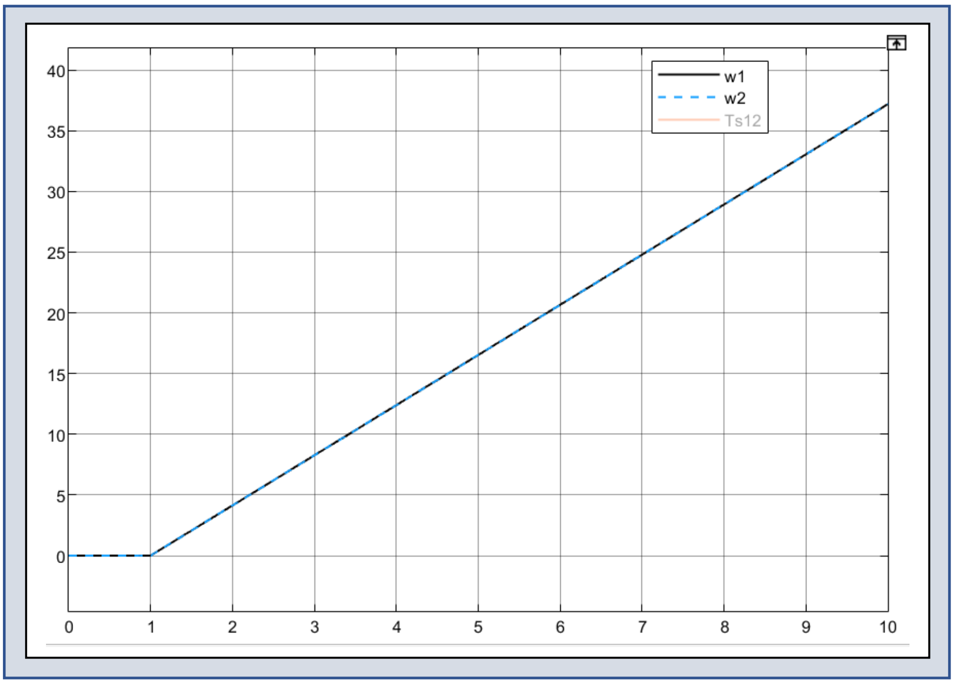


Figure 2: The plot of angular speed of the 1st and 2nd body versus time

Write expression for $\omega_1(t), \omega_2(t)$

$$\omega_1(t) = \varepsilon_{av}t + \frac{\varepsilon_{av}}{\omega_{R1}}(\gamma - 1)\, sin\, \omega_{R1}\, t$$

$$\omega_2(t) = \varepsilon_{av}t - \frac{\varepsilon_{av}}{\omega_{R1}} sin\, \omega_{R1}\, t$$

Design the scheme for the checking amplitude values of harmonic component (to exclude the average angular acceleration from expressions $\omega_1(t), \omega_2(t)$ *above)*

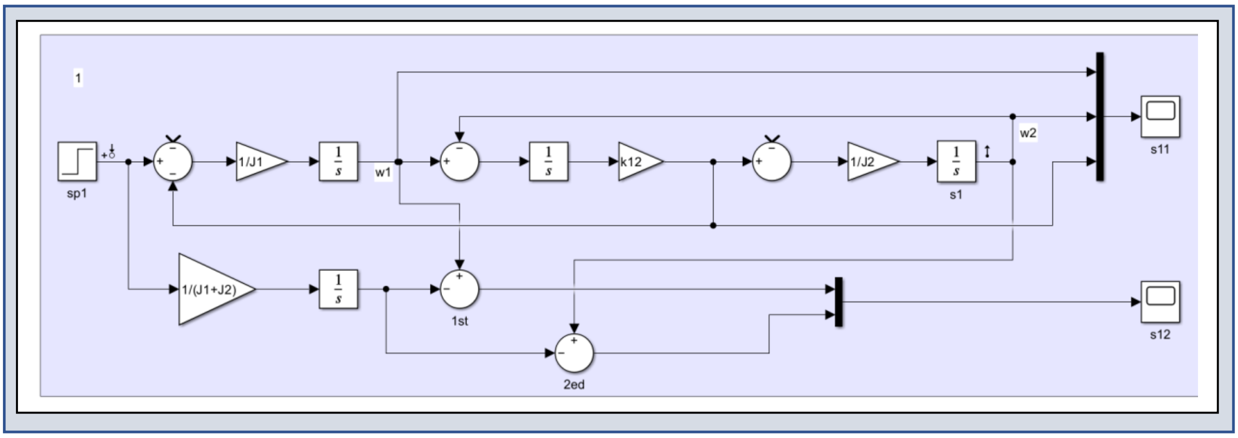


Figure : The math model for the checking amplitude values of harmonic component

The obtaining results of the simulation are presented in Fig. below

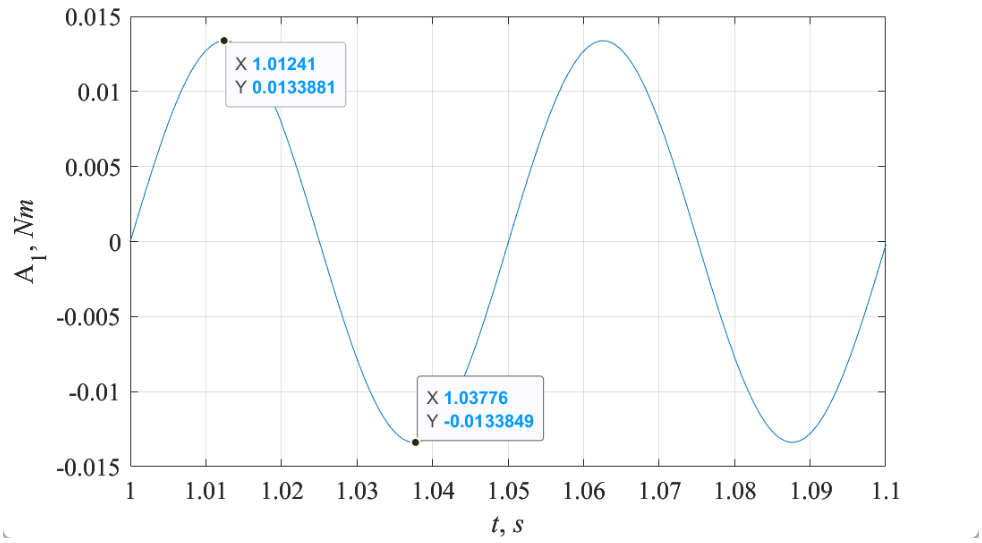


Figure: Obtaining of amplitude value of the 1st body oscillation

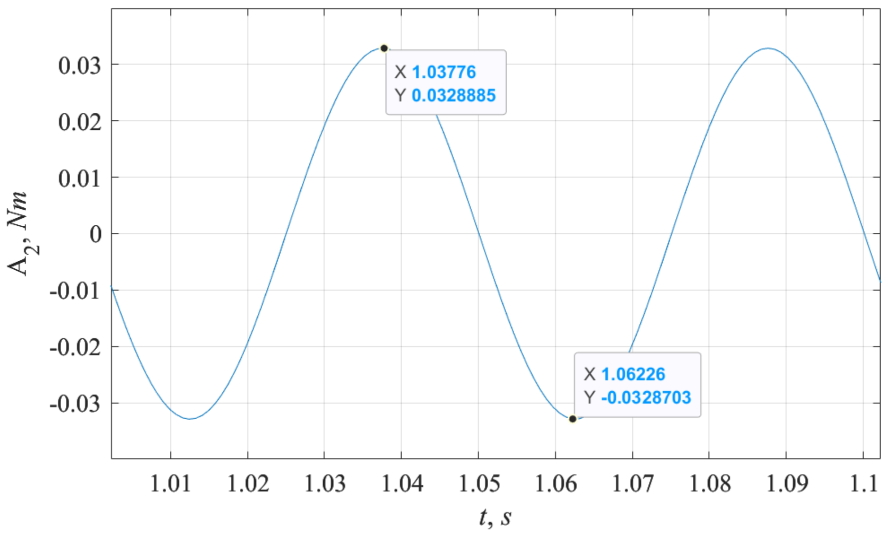


Figure: Obtaining of amplitude value of the 2nd body oscillation

Measure the magnitudes of bodies oscillations and compare with calculated parameters.

- The average angular acceleration:

$$\varepsilon_{\text{av}} = \frac{T}{J_1 + J_2} = \frac{0.1 \times T_{\text{rated}}}{J_1 + J_2} = 41.3223\ \text{rad/s}^2$$

- The magnitudes of bodies fluctuation:

$$A_1 = \frac{J_2 \varepsilon_{av}}{J_1 \omega_{R1}} = 0.0133881\ \text{rad/s}$$

$$A_2 = \frac{\varepsilon_{\text{av}}}{\omega_{R1}} = 0.0328885\ \text{rad/s}$$

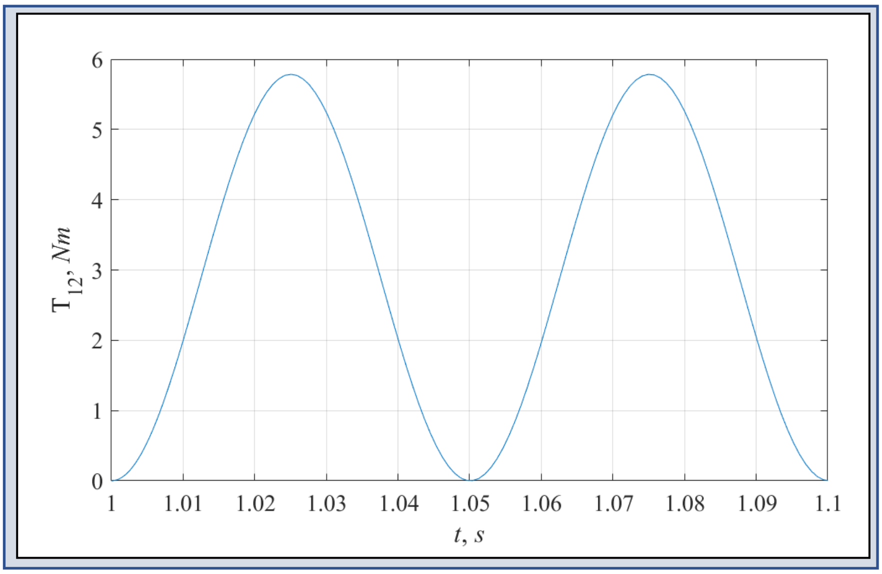


Figure: The plot of torque of elastic bonding forces between bodies versus time

Draw conclusions:

The computed amplitudes match the theoretical values, the system behaves as expected.

Bode diagram of the two-mass mechanism

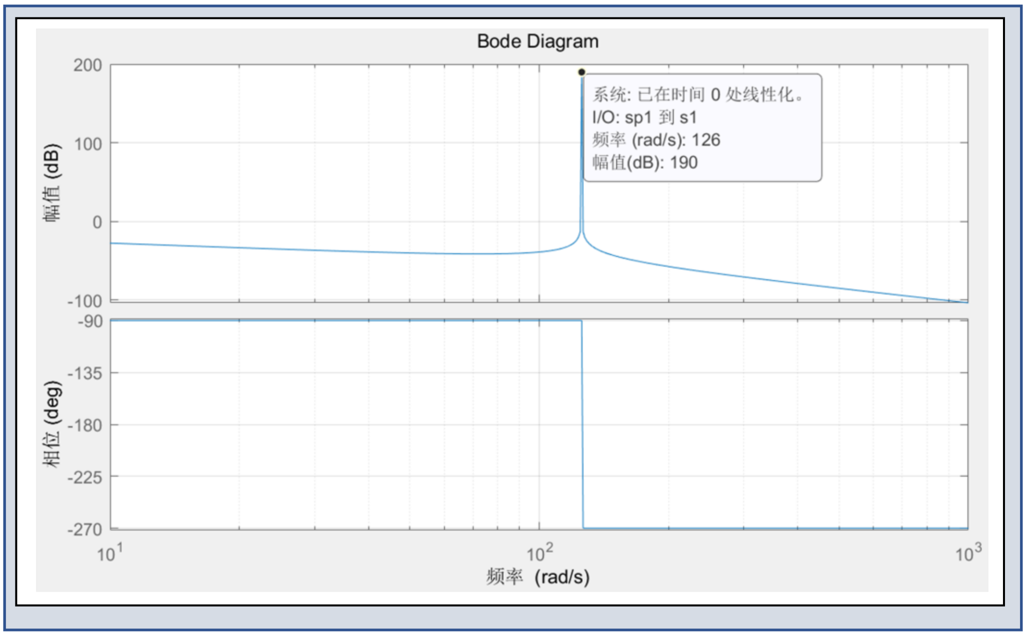


Figure 4: The Bode diagram

Show the resonance frequency on this diagram and compare with calculated parameters

*The resonance frequency:**126 rad/s** **is close to** **125.57 rad/s** **(calculation)***

Show plots $T_{s12}(t)$, $\omega_1(t)$, $\omega_2(t)$ when:

- *varying mass ratio γ (three meaning to get different transients)*
- *varying stiffness k_{12} (three meaning to get different transients)*

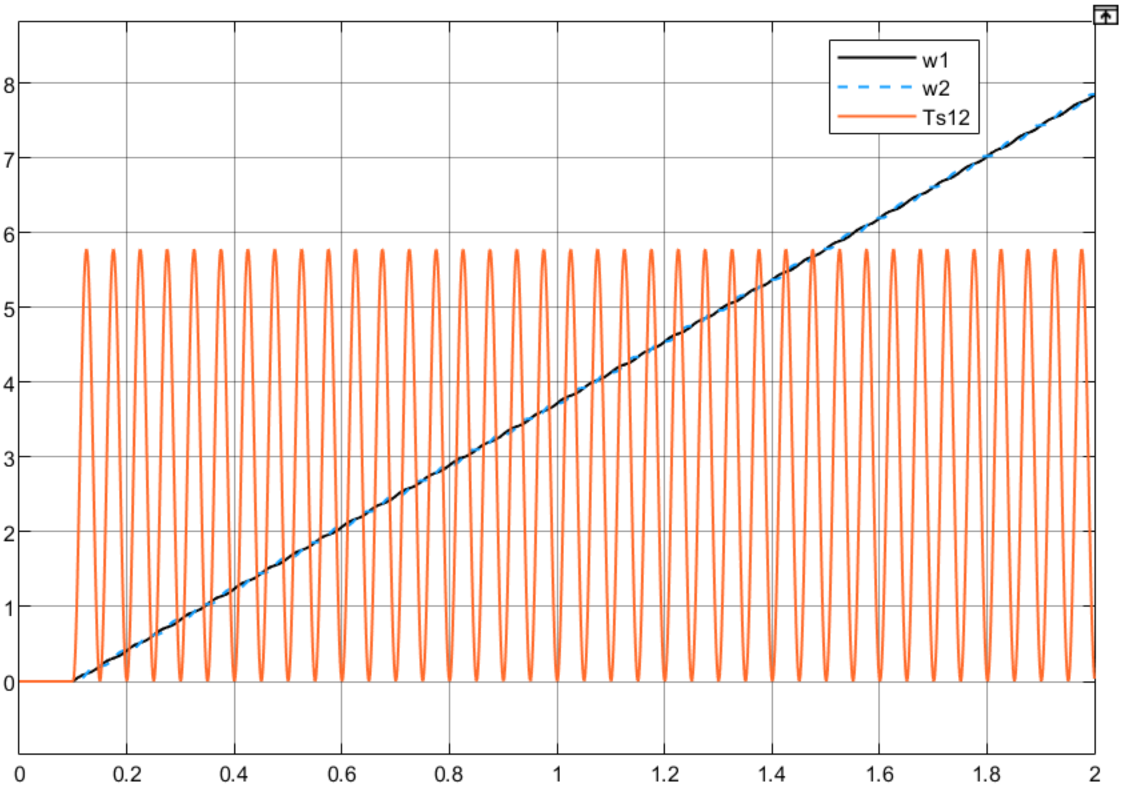


Figure: $\gamma = 1.2$, *J2 = 0.2*J1*

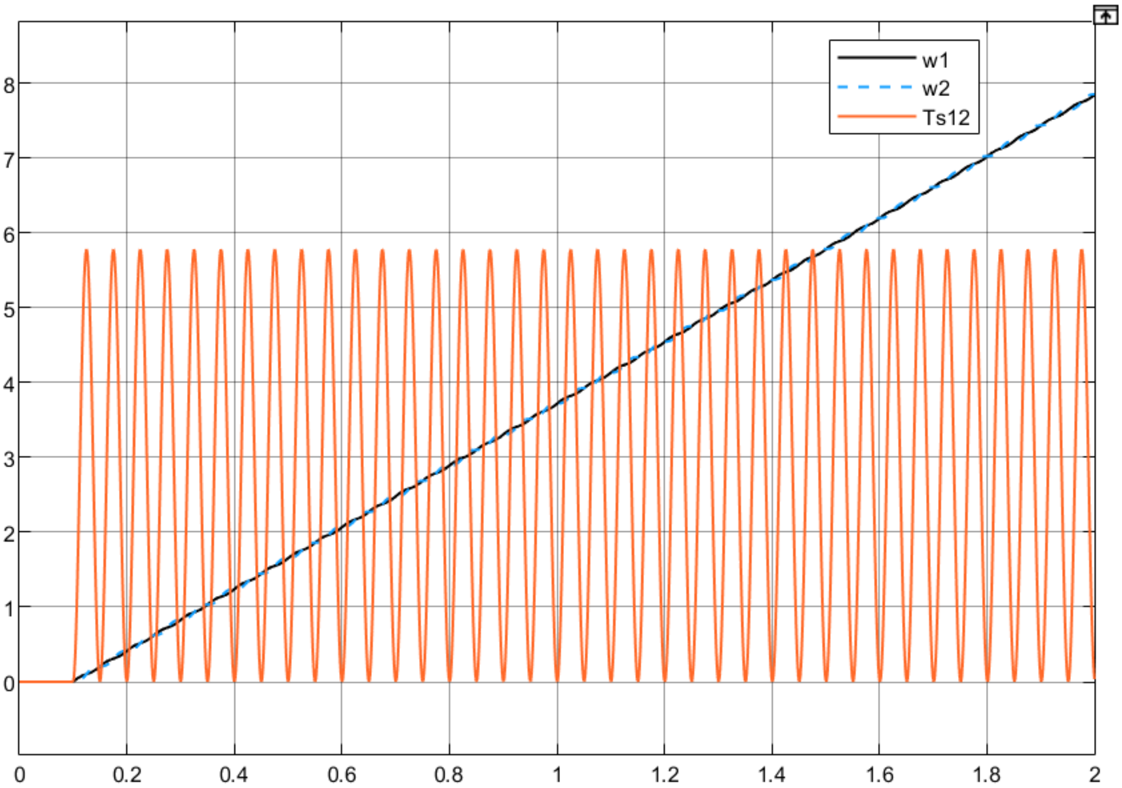


Figure: $\gamma = 1.5$, *J2 = 0.5*J1*

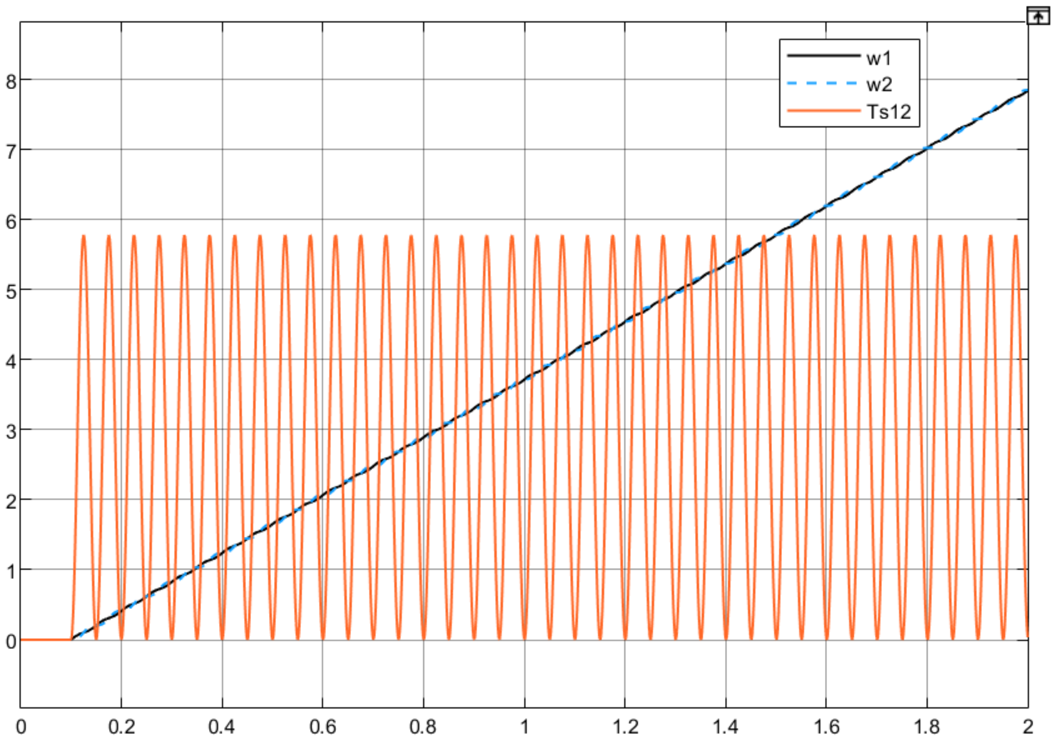


Figure: $\gamma = 1.2$, $J2 = J1$

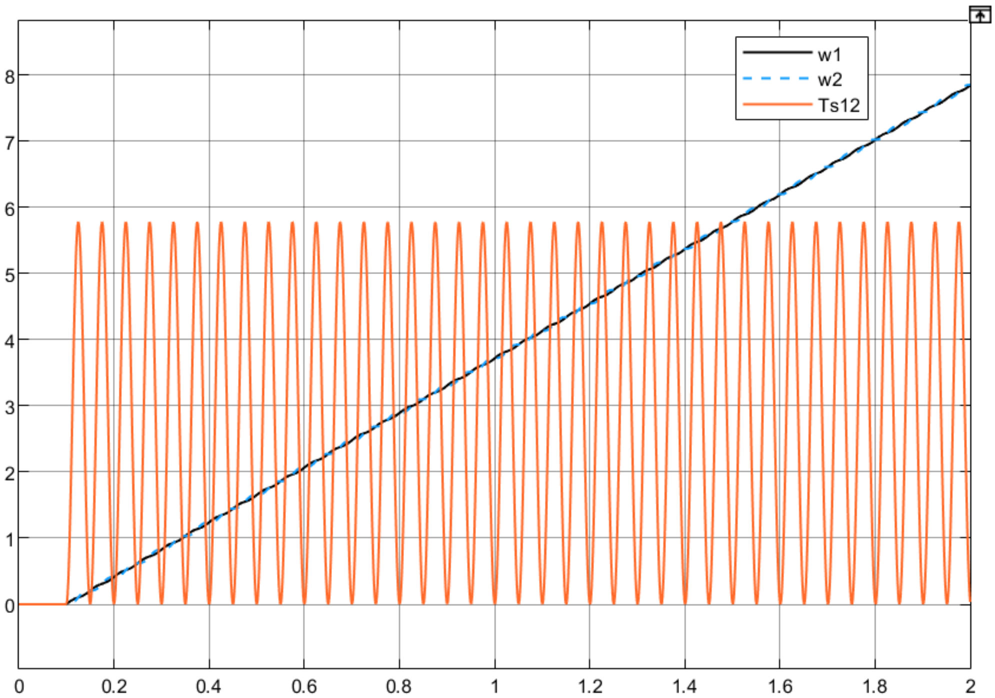


Figure: $k12 = 5000$

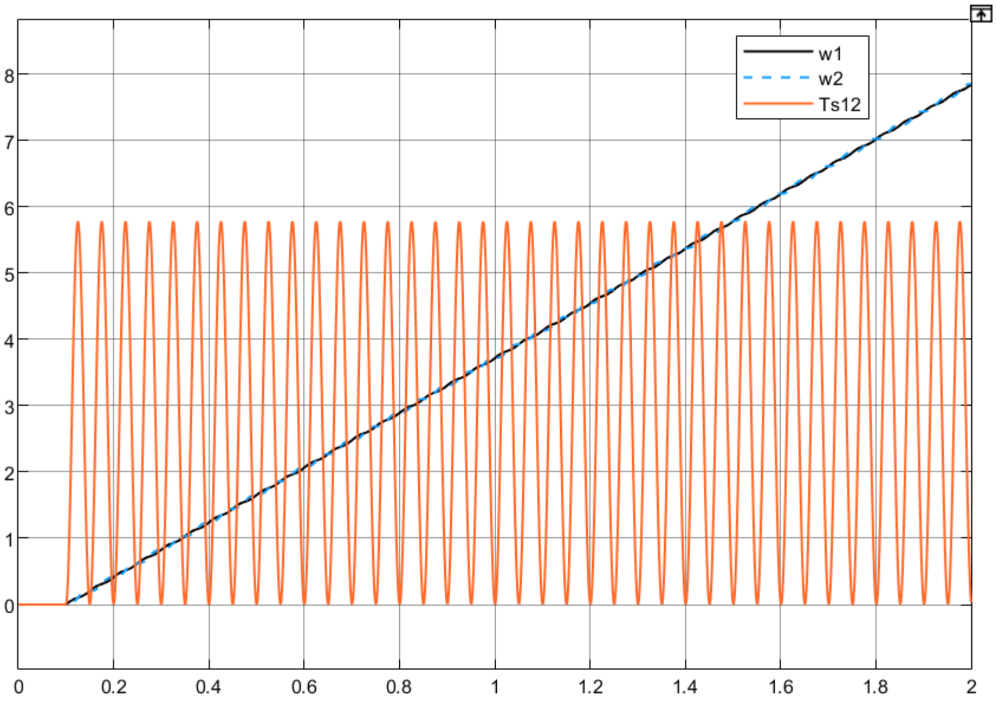


Figure: $k12 = 7500$

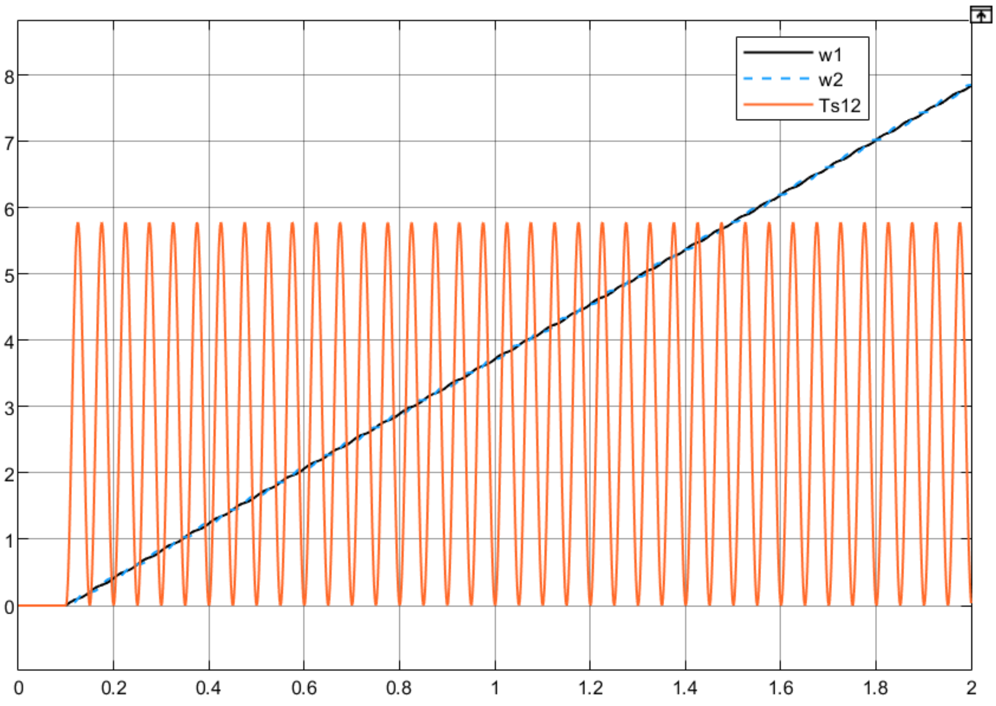
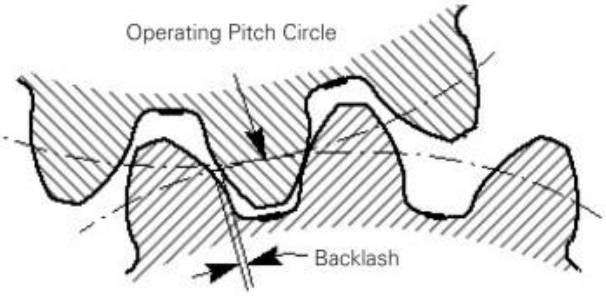


Figure: $k12 = 9000$

Draw conclusions:

Almost don't change when varying stiffness or mass ratio.

Task 1.2. Research the effect of backlash in a model of the two-mass mechanism



Mathematic model of two-mass mechanism with backlash:

$$\begin{cases} T - T_{L1} - T_{s12} = J_1\, sw_1 \\ T_{s12} - T_{L2} = J_2\, sw_2 \\ T_{s12} = k_{12}(\varphi_1 - \varphi_2 \pm \Delta\,\varphi/2), |\varphi_1 - \varphi_2| > \Delta\,\varphi/2 \\ T_{s12} = 0, |\varphi_1 - \varphi_2| \le \Delta\,\varphi/2 \end{cases}$$

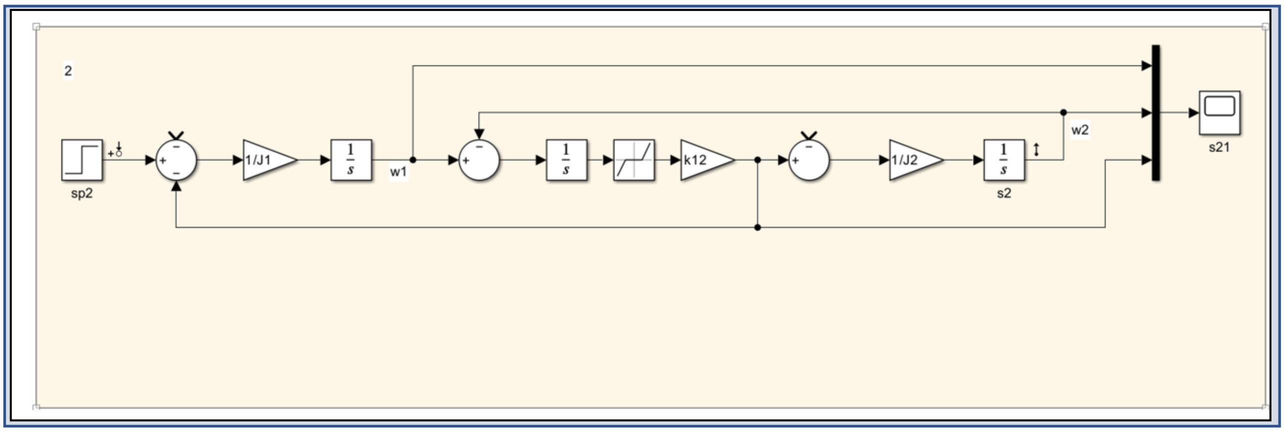


Figure: Math model of the two-mass mechanism with backlash in Simulink

Show transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value 0.1T_{rated} (at T_{L1}=0, T_{L2}=0). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses

Compare $T_{s12}(t)$ in mechanism without and with backlash in gearbox

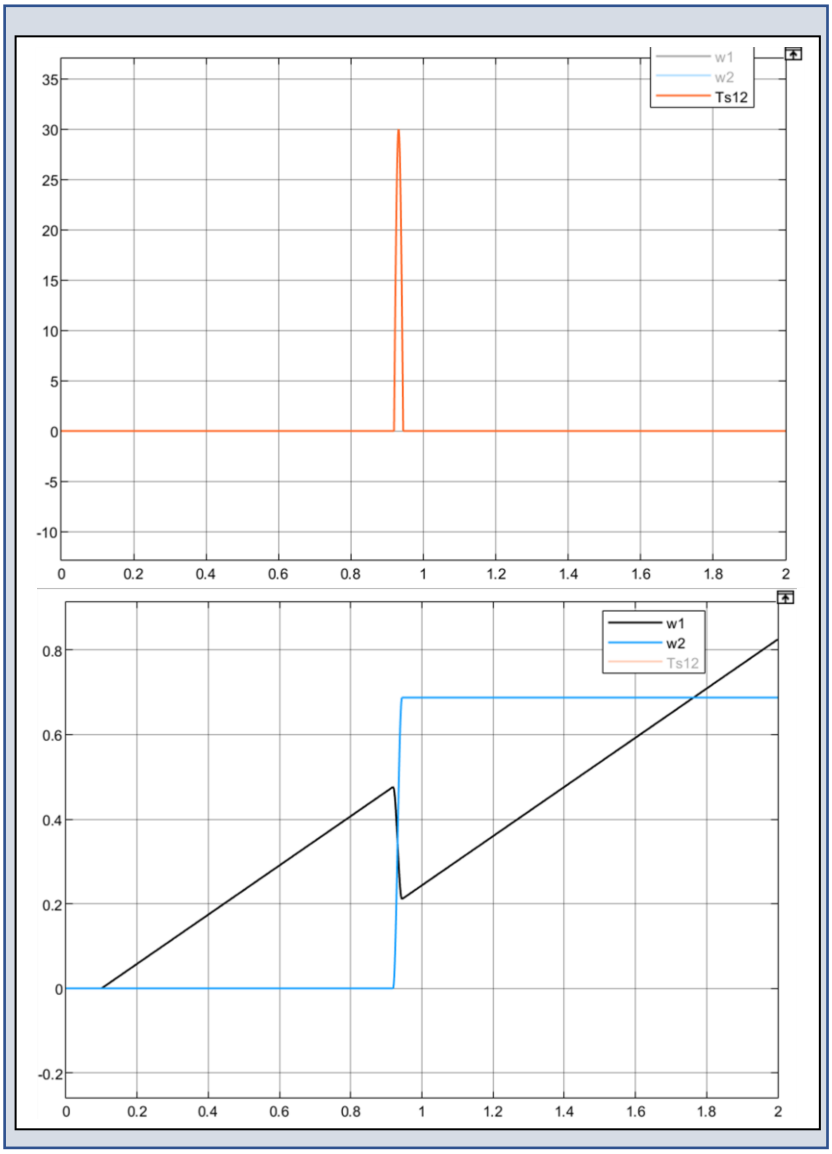


Figure: The plot of torque of elastic bonding forces between bodies versus time

Draw conclusions

Under impact, Ts12 experiences sudden changes in w1 and w2, with w2 having a significant angular acceleration and a high likelihood of damage

Task 1.3. Research the effect of viscous friction torque in a model of the two-mass mechanism

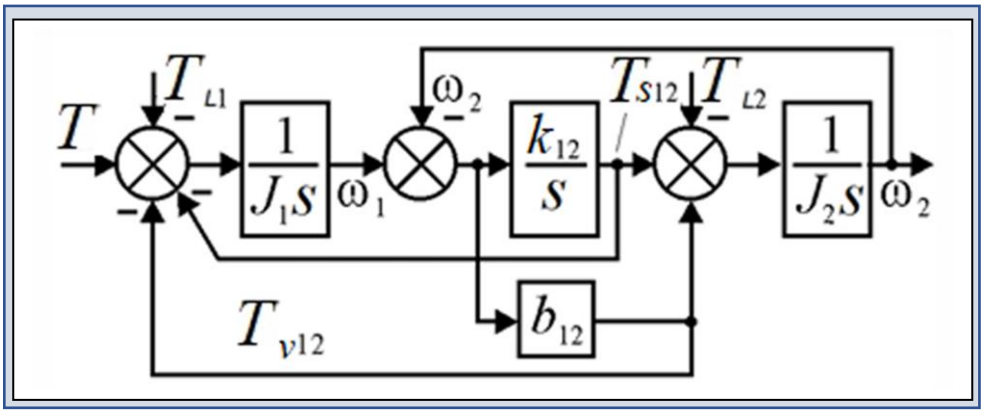


Figure: Scheme of the system with viscous friction

Mathematic model of the system with viscous friction

$$\left.\begin{aligned} &T - b_{12}(\omega_1 - \omega_2) - k_{12}(\omega_1 - \omega_2)/s - T_{L1} = J_1 s\omega_1; \\ &b_{12}(\omega_1 - \omega_2) + k_{12}(\omega_1 - \omega_2)/s - T_{L2} = J_2 s\omega_2. \end{aligned}\right\}$$

Where b_{12} is calculated as the following (The viscous damping coefficient *b* should be chosen considering that the oscillation damp in 5 periods):

$$b12 = \frac{2a_v J_1 J_2}{J_1 + J_2} = 11.9325$$

where $a_v \approx \frac{3\lambda_v \cdot \omega_{R1}}{2\pi} = \frac{3\omega_{R1}}{10\pi}$ - attenuation coefficient

$\lambda_v = a_v\, T = \frac{T}{\tau} = \frac{1}{n}$ - logarithmic decrement

n - number of harmonic oscillations during relaxation τ (the amplitude decreases *e* times)

$tres = 3\frac{1}{av} = nT$=5*T* - time response

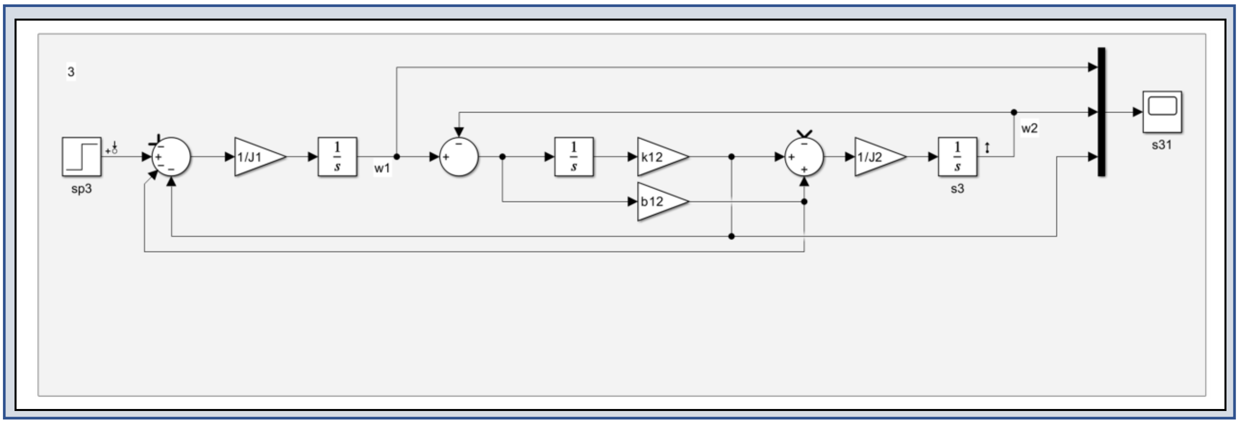


Figure: Math model of the two-mass mechanism in Simulink

Show transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value 0.1T_{rated} (at T_{L1}=0, T_{L2}=0). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot

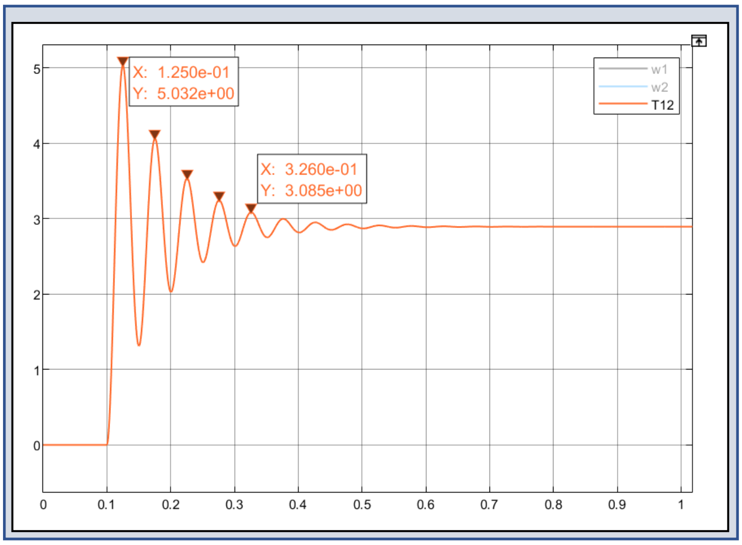


Figure: The plot of torque of elastic bonding forces between bodies versus time

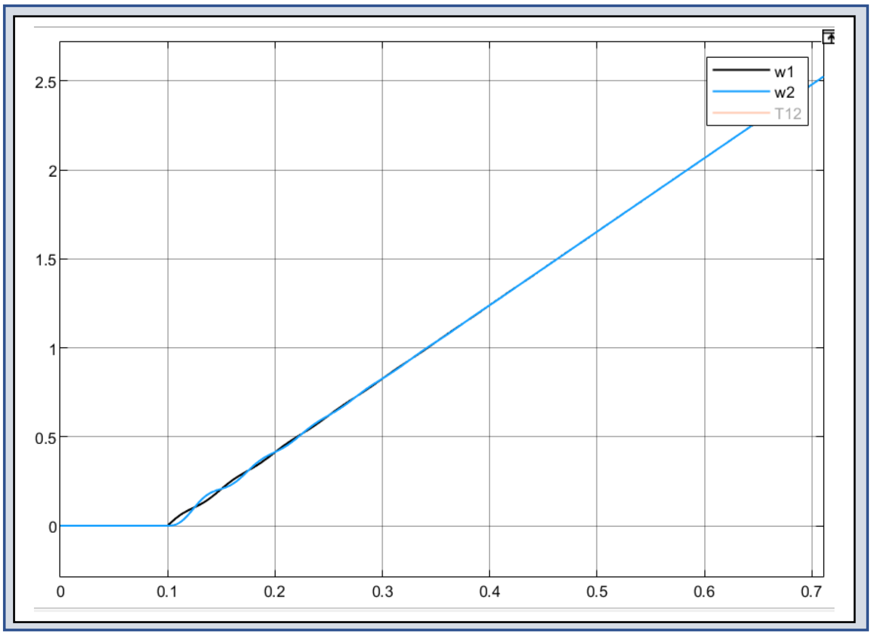


Figure: The plot of angular speed of the 1st and 2nd body versus time

Draw conclusions

There are damping of oscillations happened in w1 and w2.

Part 2. Mathematical modelling of DC-motor with two-mass mechanism (*not necessary – this is additional option)*

<u>Task 2.1 Modelling of the DC-motor with two-body mechanism.</u>

Design a model of the DC-motor with two-body mechanism.

Mathematical model of DC motor with two-mass mechanism:

$$\begin{cases} J_1\ddot{\theta}_1 = T_m - T_s - b_1\dot{\theta}_1 \\ J_2\ddot{\theta}_2 = T_s - b_2\dot{\theta}_2 \\ T_s = k(\theta_1 - \theta_2) + c(\dot{\theta}_1 - \dot{\theta}_2) \end{cases}$$

Scheme of the system is presented in Fig

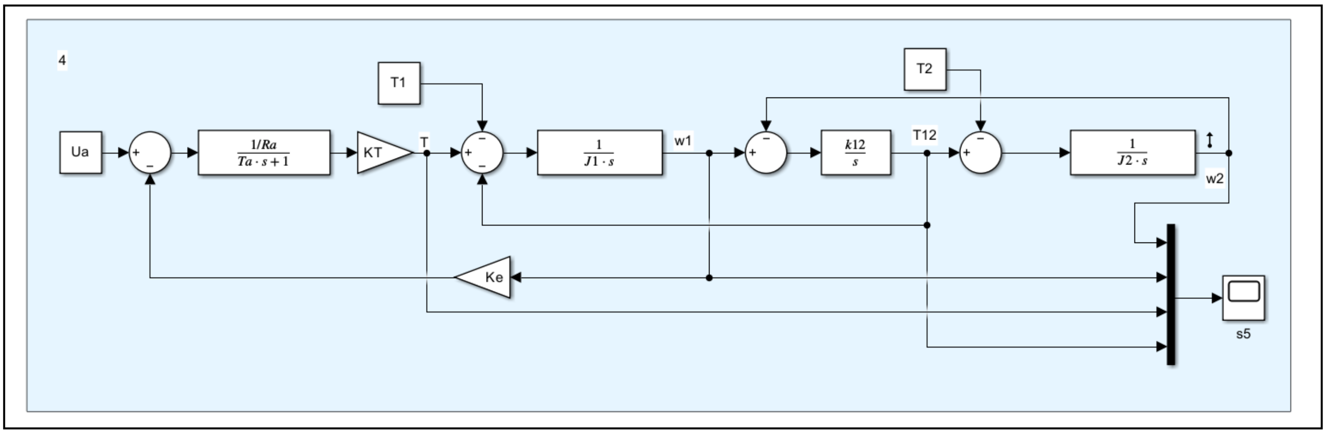


Figure: The model of DC-motor with two-mass mechanism.

$$\gamma = \frac{J_1+J_2}{J_1}$$

Show plots T(t) , $T_{s12}(t)$, $\omega_1(t)$, $\omega_2(t)$ when:

- *varying mass ratio γ (three meaning to get different transients)*
- *varying stiffness* k_{12} *(three meaning to get different transients)*
- *varying resistance R / rigidity β (three meaning to get different transients)*

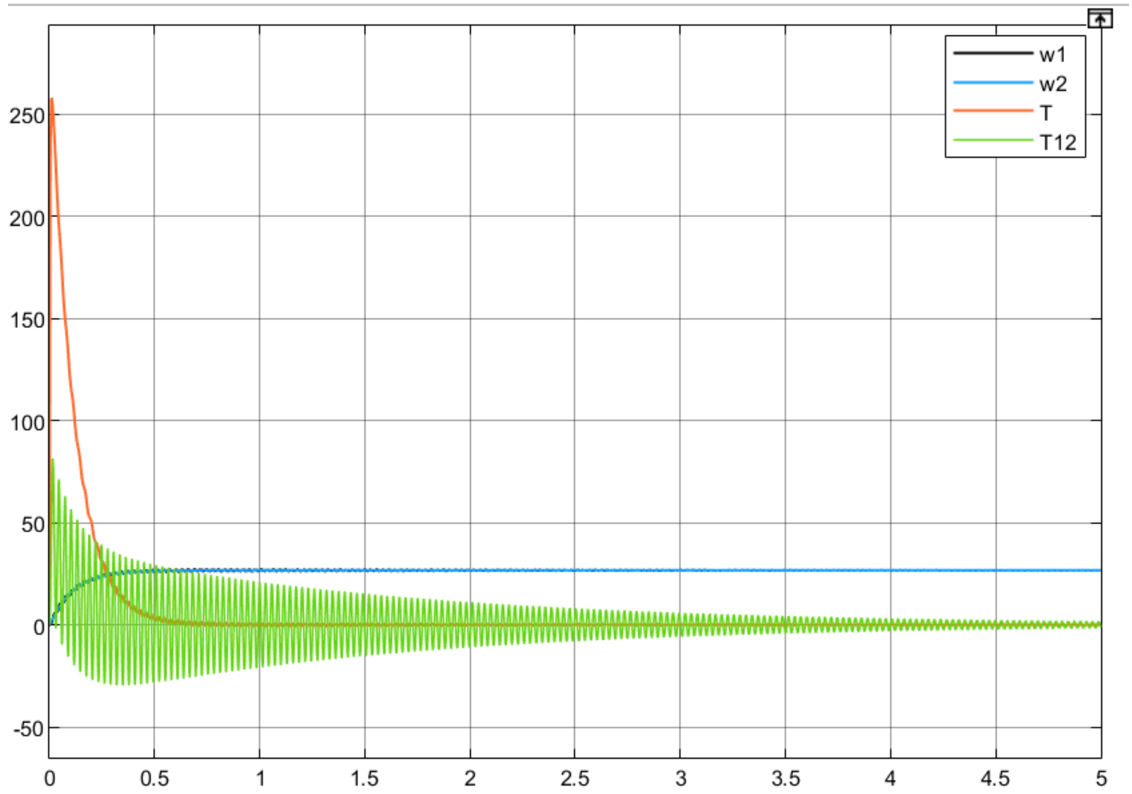


Figure: $\gamma = 1.2$

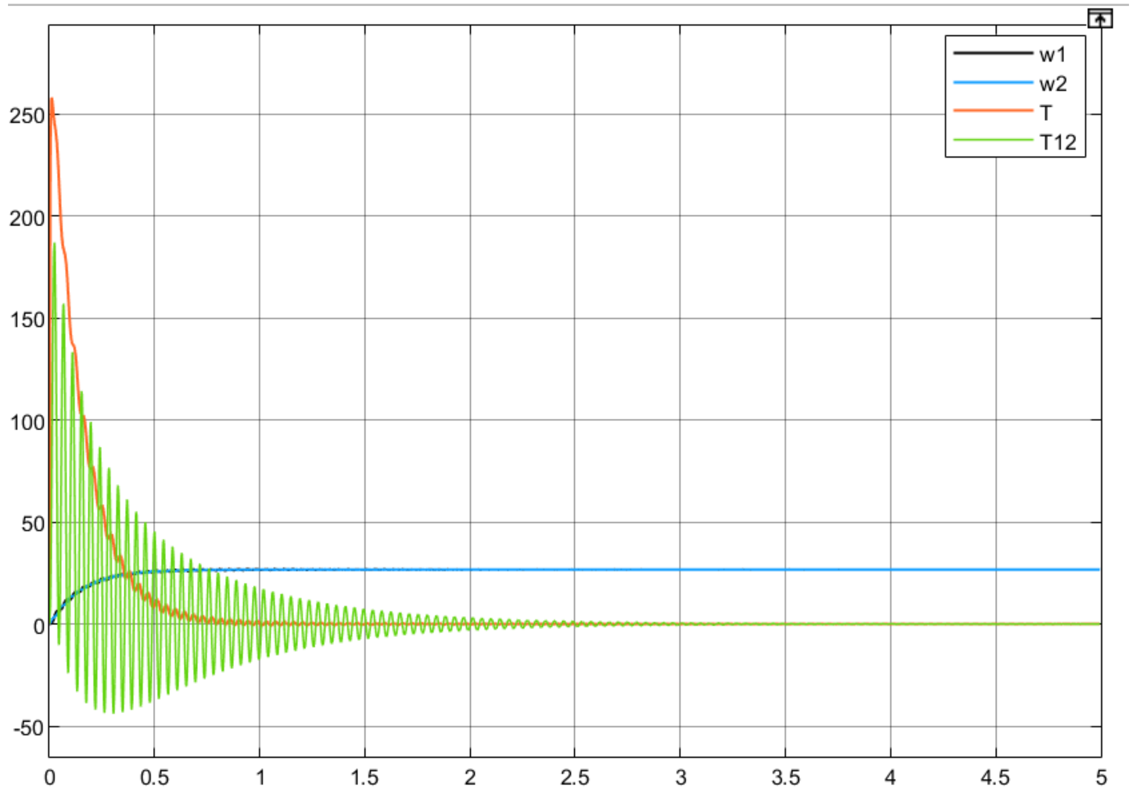


Figure: $\gamma = 1.6$

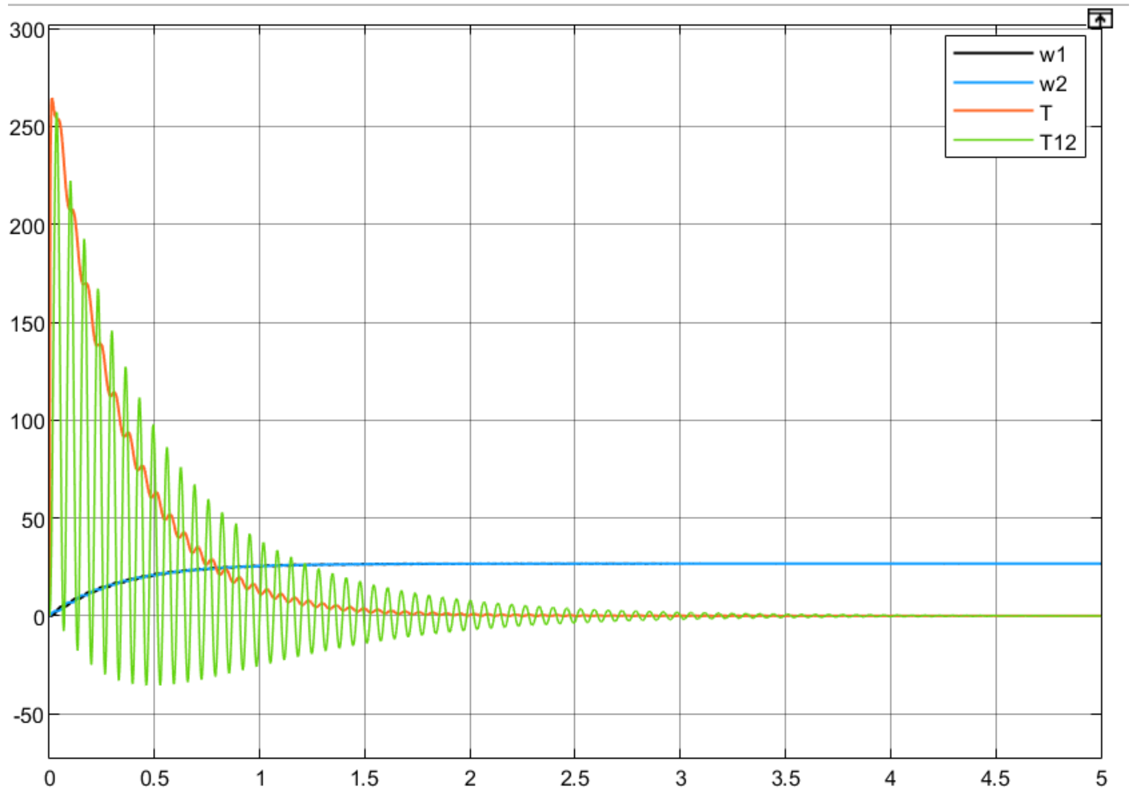


Figure: $\gamma = 2$

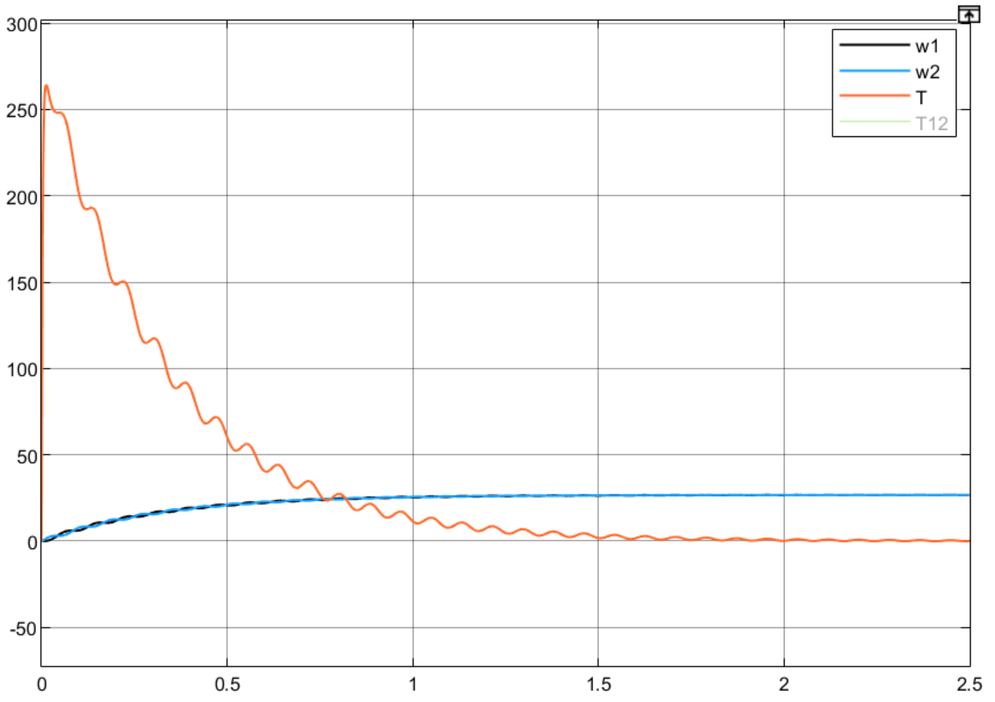


Figure: $k12 = 5000$

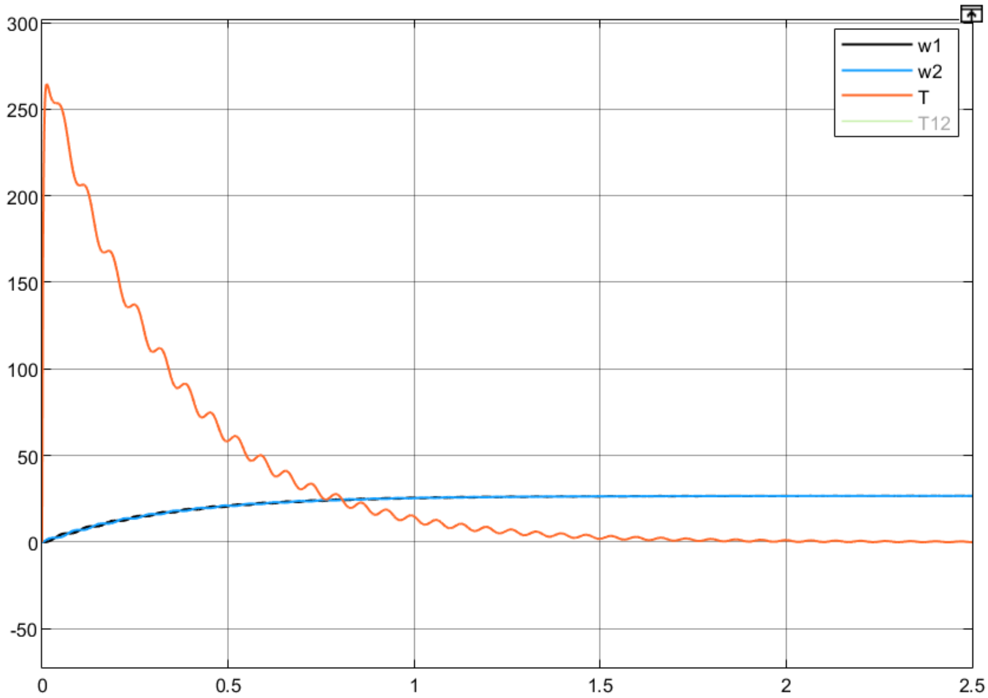


Figure: $k12 = 7500$

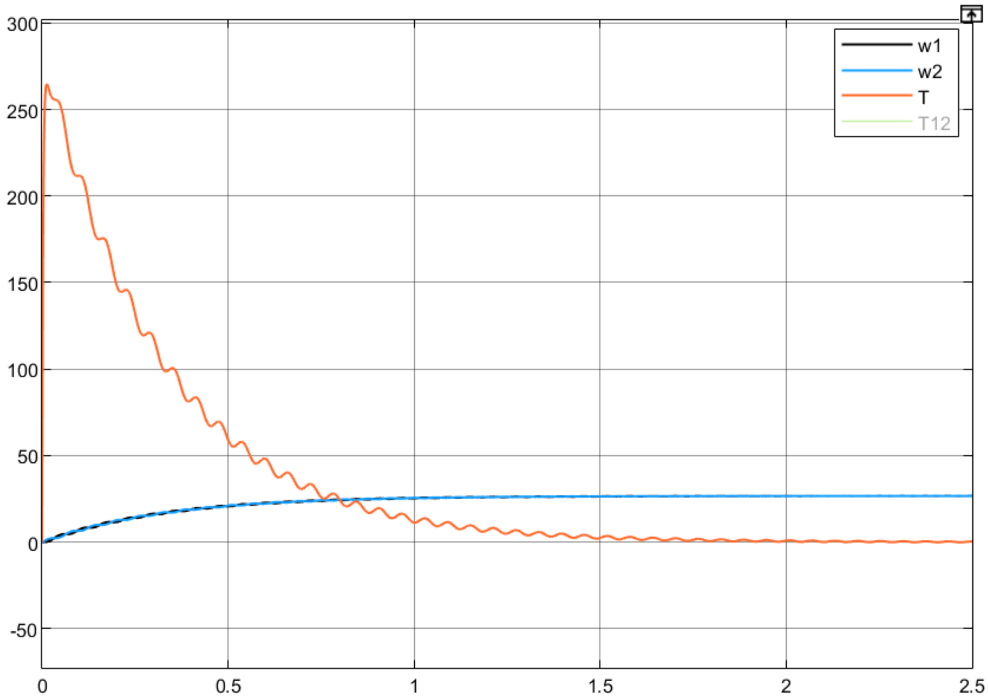


Figure: $k12 = 9000$

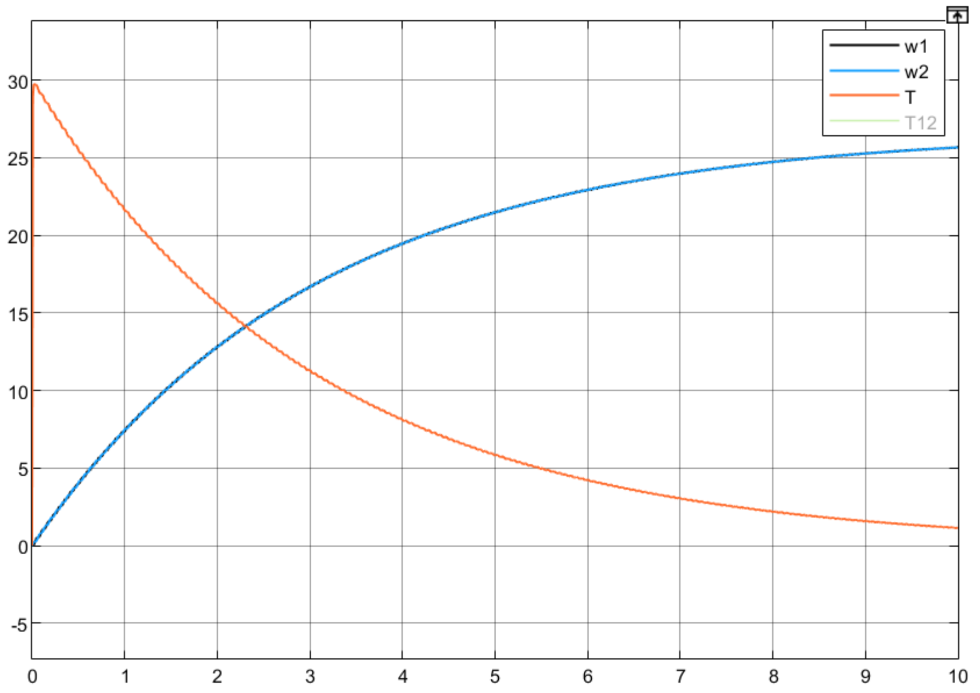


Figure: $R = 200\ Ohm$

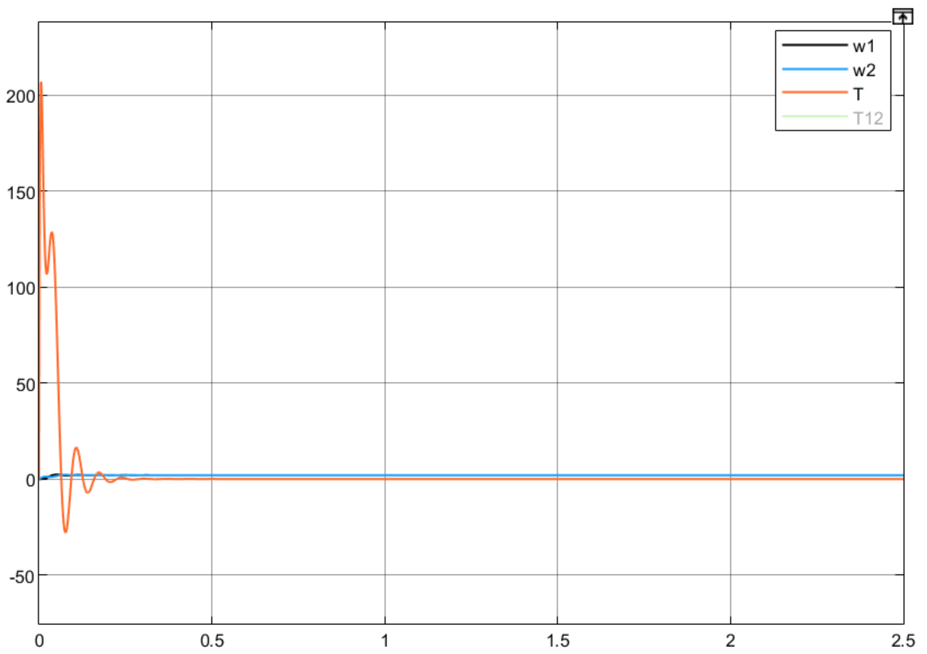


Figure: $Ke = 2$

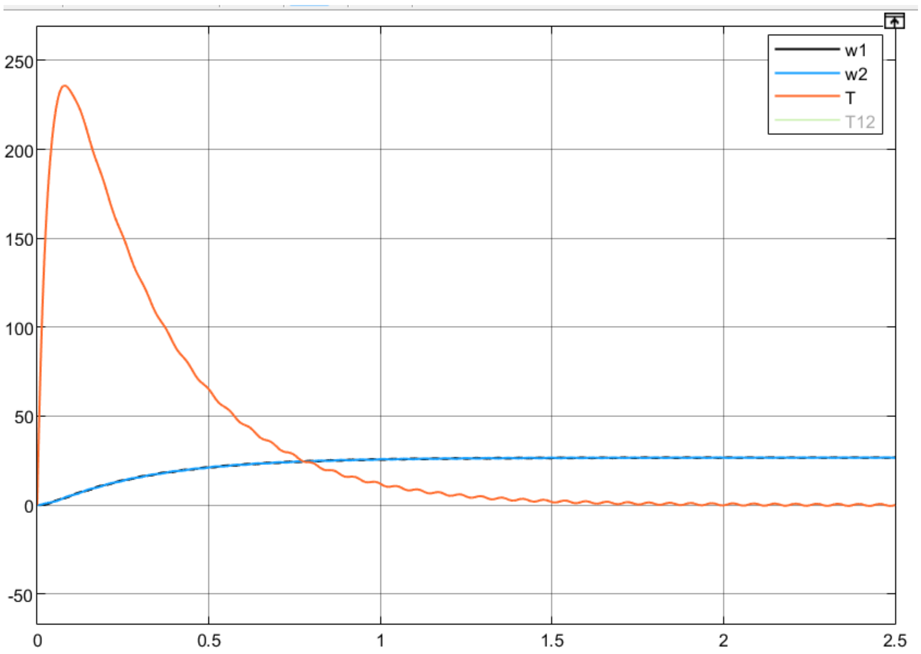


Figure: $Ta = 0.03$

Draw conclusions

When changing γ, the Shaft Torque changes significantly, and the area around the x-axis becomes similar to T, while changing k12 changes very little; increasing the resistance R will make the transition state very long, and reducing ke will cause oscillation; changing the motor torque, the system becomes stable.