Assignment1-1 Segway

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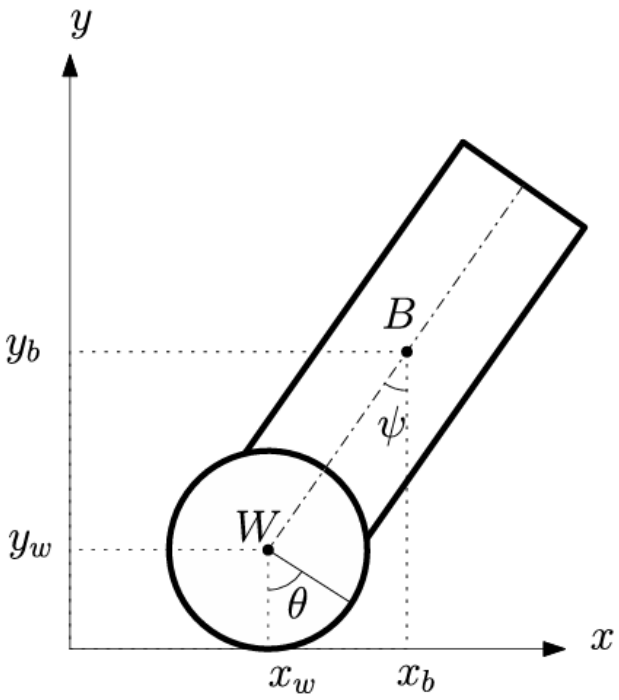


Figure 1: Kinematic description of the segway robot

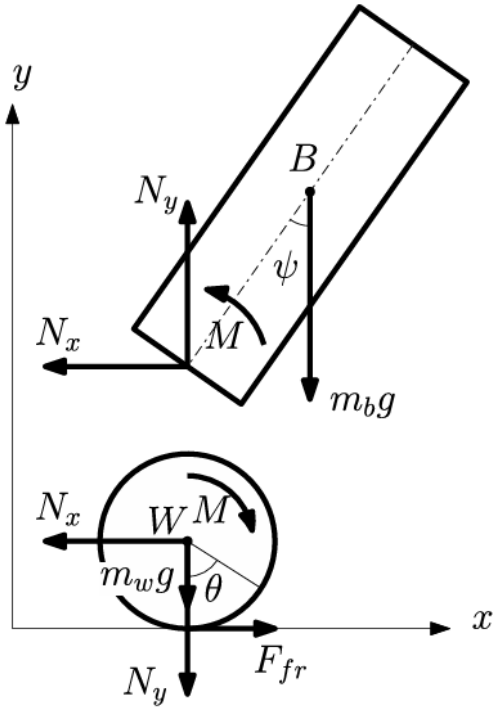


Figure 2: Forces on the segway robot

Calculations:

$$x_w = r\theta \qquad y_w = r \qquad x_b = r\theta + l\sin\psi \qquad y_b = r + l\cos\psi$$
$$\dot{x}_w = r\dot{\theta} \qquad \dot{y}_w = 0 \qquad \dot{x}_b = r\dot{\theta} + l\dot{\psi}\cos\psi \qquad \dot{y}_b = -l\dot{\psi}\sin\psi$$
$$\ddot{x}_w = r\ddot{\theta} \qquad \ddot{y}_w = 0 \qquad \ddot{x}_b = r\ddot{\theta} + l\ddot{\psi}\cos\psi - l\dot{\psi}^2\sin\psi \qquad \ddot{y}_b = -l\ddot{\psi}\sin\psi - l\dot{\psi}^2\cos\psi$$

(1.1) (1.2) (1.3) (1.4)

$$m_b\ddot{x}_b = N_x$$
$$m_b\ddot{y}_b = N_y - m_b g$$
$$J_b\ddot{\psi} = -N_x l\cos\psi + N_y l\sin\psi - 2M$$

$$2m_w\ddot{x}_w = -N_x + F_{fr}$$
$$2J_w\ddot{\theta} = 2M - F_{fr} r$$

$$N_x = m_b\ddot{x}_b$$
$$N_y = m_b\ddot{y}_b + m_b g$$
$$F_{fr} = 2m_w\ddot{x}_w + m_b\ddot{x}_b$$

$$\begin{bmatrix} (m_b r^2 + 2m_w r^2 + 2J_w) & m_b rl\cos\psi \\ m_b rl\cos\psi & (m_b l^2 + J_b) \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} -m_b rl\dot{\psi}^2\sin\psi \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -m_b gl\sin\psi \end{bmatrix} = \begin{bmatrix} 2M \\ 0 \end{bmatrix}$$

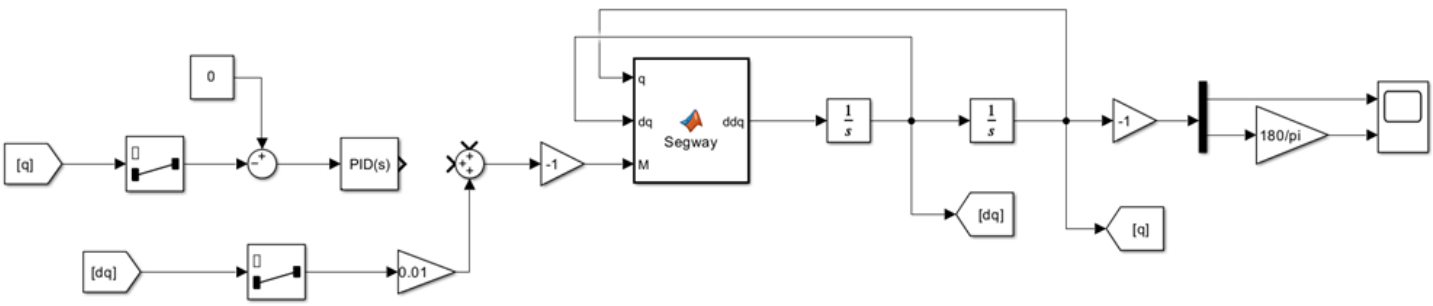
$$D(q) = \begin{bmatrix} (m_b r^2 + 2m_w r^2 + 2J_w) & m_b rl\cos\psi \\ m_b rl\cos\psi & (m_b l^2 + J_b) \end{bmatrix}, \quad C(\dot{q}, q) = \begin{bmatrix} -m_b rl\dot{\psi}^2\sin\psi \\ 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 0 \\ -m_b gl\sin\psi \end{bmatrix}, \quad F = \begin{bmatrix} 2M \\ 0 \end{bmatrix}$$

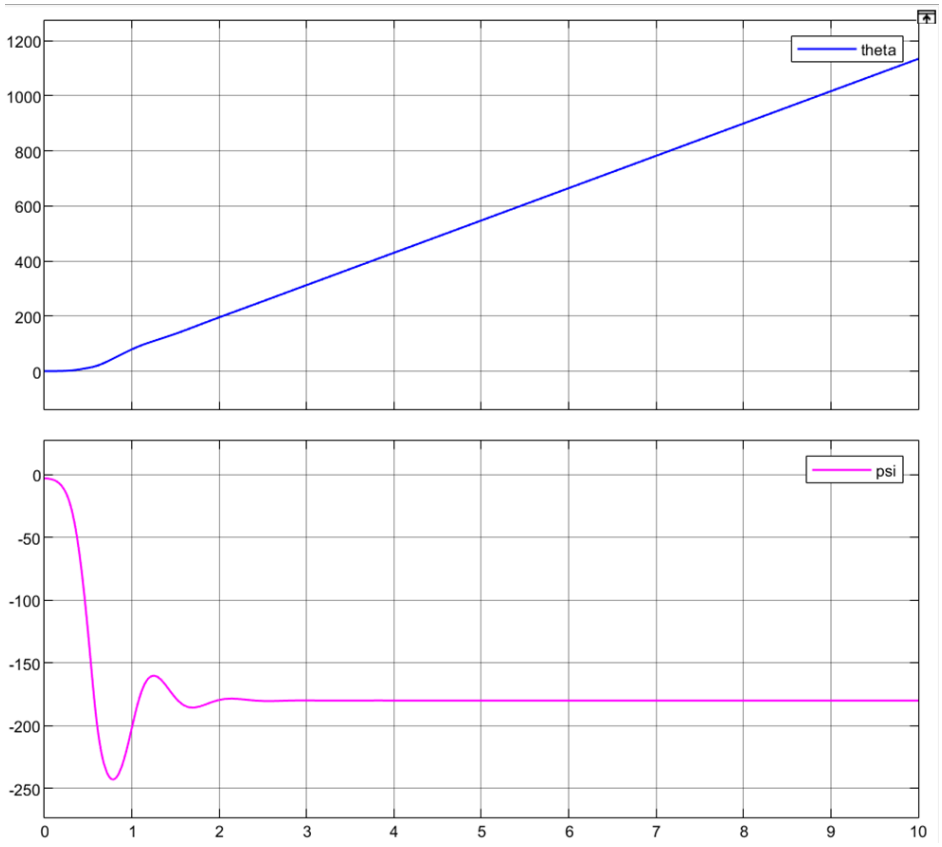
$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} m_b r^2 + 2m_w r^2 + 2J_w & m_b rl\cos\psi \\ m_b rl\cos\psi & m_b l^2 + J_b \end{bmatrix}^{-1}\left(\begin{bmatrix} 2M \\ 0 \end{bmatrix} - \begin{bmatrix} -m_b rl\dot{\psi}^2\sin\psi \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -m_b gl\sin\psi \end{bmatrix}\right)$$

5. Construct the model in Matlab Simulink with initial condition $\theta = 0, \psi = 0.05, \dot{\theta} = 0, \dot{\psi} = 0$. Use $r = 0.025$ and $l = 0.12$, $m_w = 0.01$, $m_b = 0.5$, $J_w = 10^{-4}$, $J_b = 0.0072$, $g = 9.81$. To simulate the system please use next equation:

$$\ddot{q} = D^{-1}(F - C - G) \tag{5}$$



6. Apply $M = -0.01 \cdot \dot{\theta}$ and show the graph of $\psi(t)$ (it should converge to $-\pi$). You may need to simulate about 1000 seconds.



7. Add the PID controller in format

$$M = \mathrm{PID}(0-\psi) + \mathrm{PID}\left(0-\dot{\theta}\right) \tag{6}$$

and find the coefficients such that $e = -\psi$ converges to zero.