Linear Systems- Lab 2

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Case 4. Mass-Spring-Damper system with free base

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Damper and spring are rigidly fixed with a zero block at the top in position $x_0 = 0$ and with beam in position $x = x_1(0)$. Control signal $u = 0$, autonomous system. A gravitational force F_g acts on the beam, at the same time, the spring and the damper exert their influence.

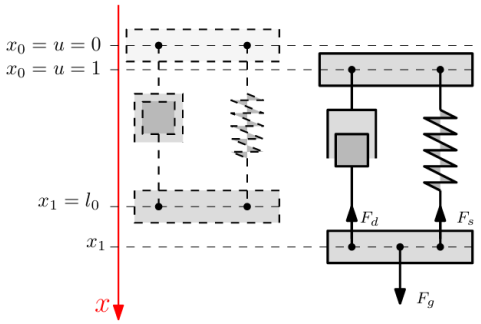

Figure 1: System scheme

Similar system like the first lab:

- Block 0 position: $x_0 = u$
- Gravity force: $F_g = m_1 \cdot g$
- Damping force: $F_d = K_d \cdot (\dot{x}_1 - \dot{x}_0)$

but the spring force is more complex:

- $F_s = K_{s1} \cdot (1 - K_{s2}^2 \cdot x_1^2) \cdot x_1$, where added a new coefficient K_{s2}.

$$mg - kd\dot{x}_1 - k_{s1}(1 - k_{s2}^2 x_1^2)x_1 = m_1\ddot{x}_1$$

$$\begin{cases} \frac{dz_1}{dt} = z_2 \\ \frac{dz_2}{dt} = \frac{k_{s1}}{m_1}(k_{s2}^2 z_1^2 - 1)z_1 - \frac{kd}{m_1}z_2 + g \quad (u=0) \end{cases}$$

$z_e = \begin{bmatrix} a \\ 0 \end{bmatrix}$ $(\dot{z}_2 = 0,\ \dot{z}_1 = 0)$ which a satisfy: $g - \frac{k_{s1}}{m_1}(1 - k_{s2}^2 a^2)a = 0$

one numerical solutions: $a = -0.0697$, $z_e = \begin{bmatrix} -0.0697 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 1 \\ 474 & -1.7 \end{bmatrix}$, $\det(\lambda I - A) = \lambda^2 + 1.7\lambda - 474$

$\Rightarrow \lambda_1 \approx 20$, $\lambda_2 \approx -22$.

Since $\lambda_1 > 0$ the sys is unstable.

```
Connection established


Equilibrium point:

  z1 = -0.0697

  z2 = 0.0000

Jacobian matrix:

         0    1.0000
  474.5342   -1.7063

System is stable: 0

ans = 3×1 complex
  -0.0697 + 0.0000i
   0.0348 + 0.0537i
   0.0348 - 0.0537i
```

警告: 在 t=1.842386e-01 处失败。在时间 t 处，步长必须降至所允许的最小值(4.440892e-16)以下，才能达到积分容差要求。

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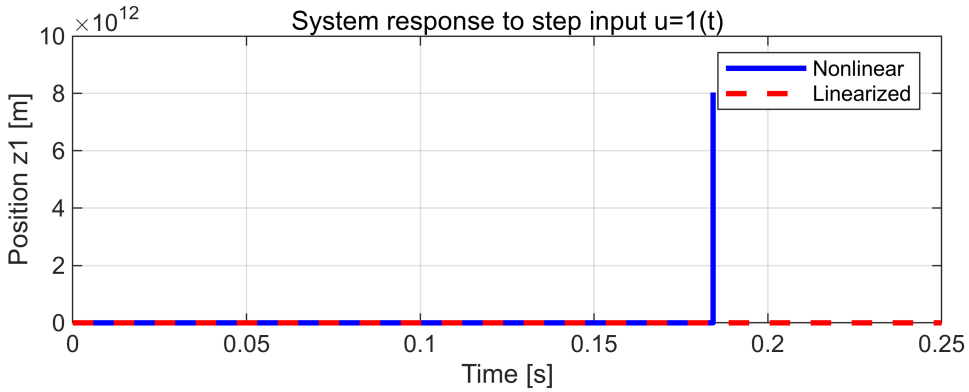

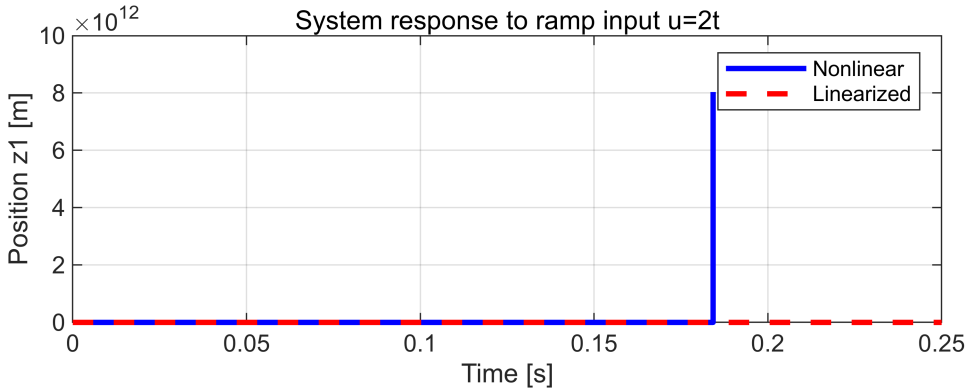


```
ans =

  包含以下字段的 struct:

    Equilibrium_point_Z1: 1
    Equilibrium_point_Z2: 0.5000
     Jacobian_matrix_J11: 0.2500
     Jacobian_matrix_J12: 0.2500
     Jacobian_matrix_J21: 0.2500
     Jacobian_matrix_J22: 0.2500
                  stable: 0.5000
             total_score: 3
```

Equilibrium_point_Z1	-0.0697
Equilibrium_point_Z2	0
Jacobian_matrix_J11	0
Jacobian_matrix_J12	1
Jacobian_matrix_J21	-1.7063
Jacobian_matrix_J22	474.5342
stable	0

```
J = 2×2
         0    1.0000
  474.5342   -1.7063
```

```
ans = 2×1
   20.9474
  -22.6536
```