Practice 1

Linear Quadratic Regulator

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Variant:

k	Block mass m, kg	Viscous damping coefficient, c	Initial conditions $x(0)$
2	3	0.2	$\begin{bmatrix} \pi/2 \\ -1 \end{bmatrix}$

Simulink Model:

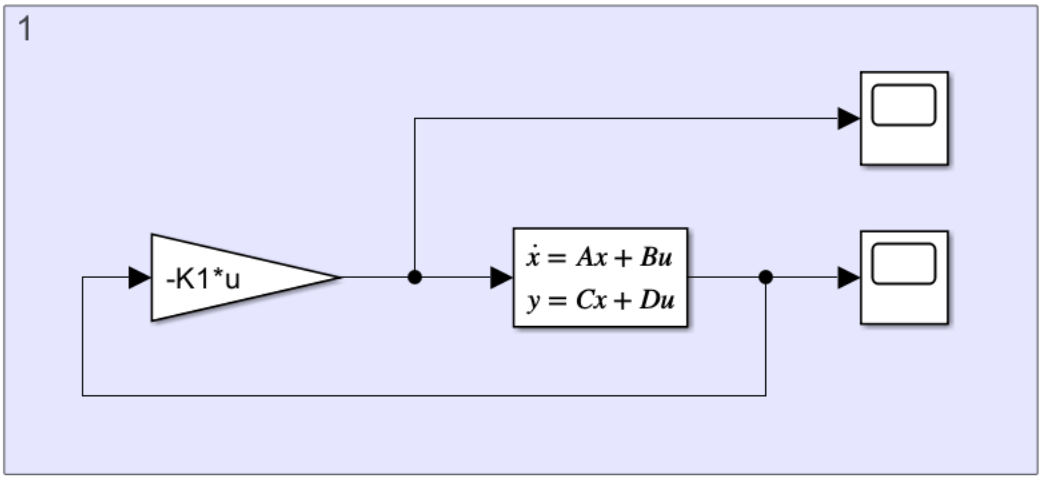

Figure 1. State-Space Model in Simulink.

1) Cheap control: $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R[0.01]$

Q1 = [1 0; 0 1];
R1 = 0.01;

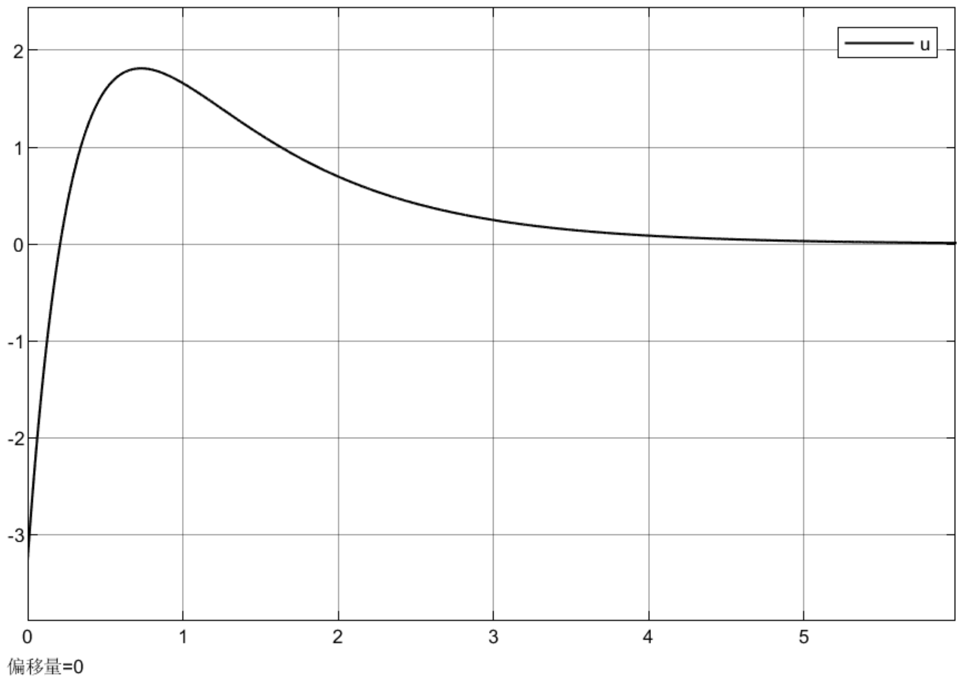

Figure 2. diagram of u(t).

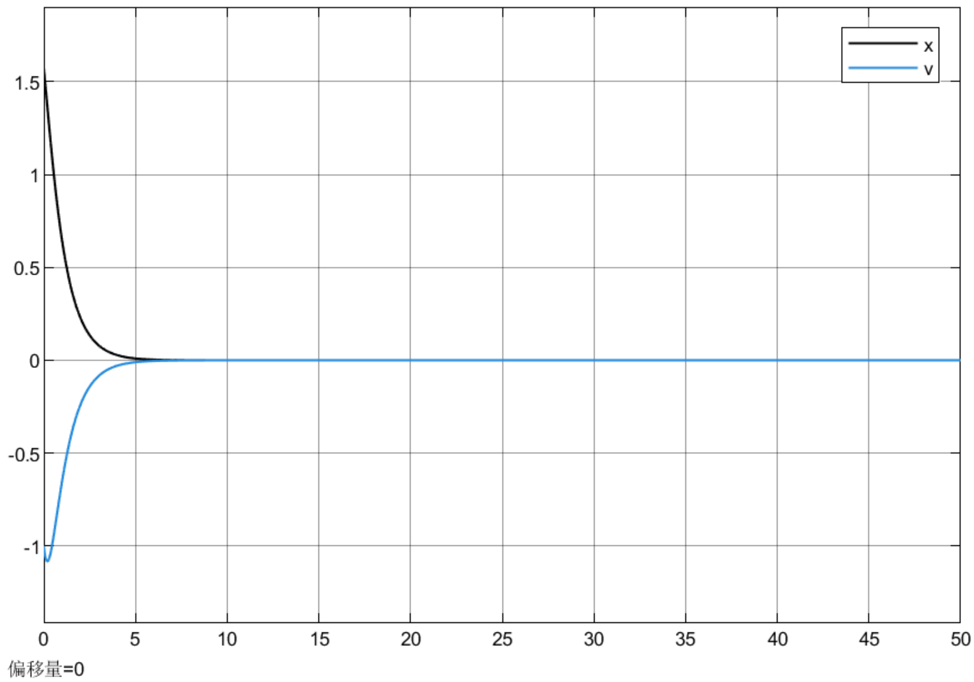

Figure 3. diagram of position and speed.

Comment:

In this case, it took about 5 seconds for u(t) to decrease to 0. And The position and speed are correct and compatible.

2) Expensive control: $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R[100]$

Q2 = [1 0; 0 1];
R2 = 100;

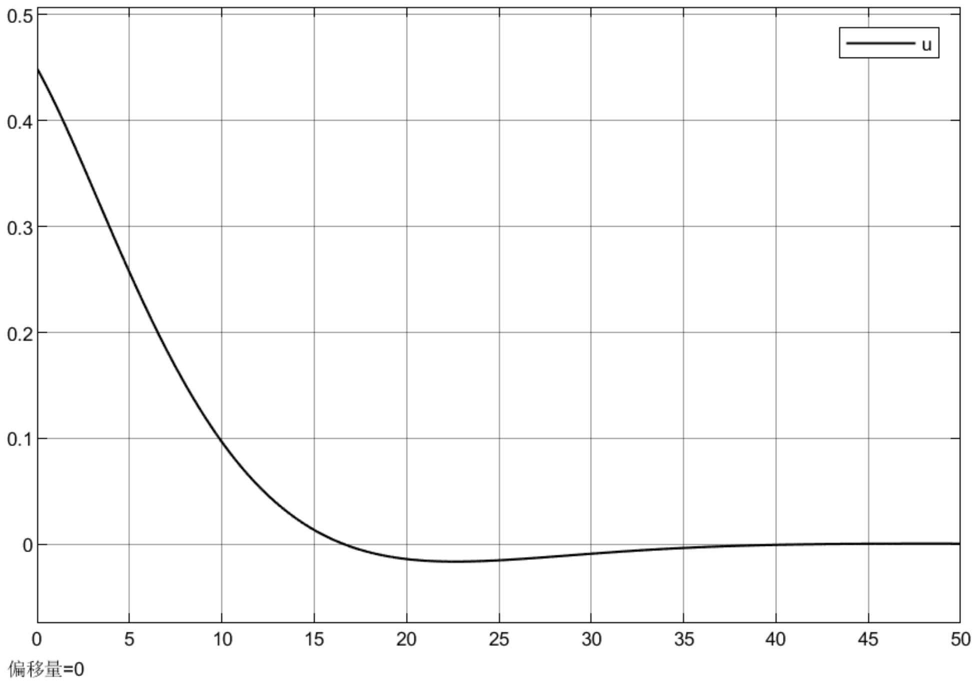

Figure 2. diagram of u(t).

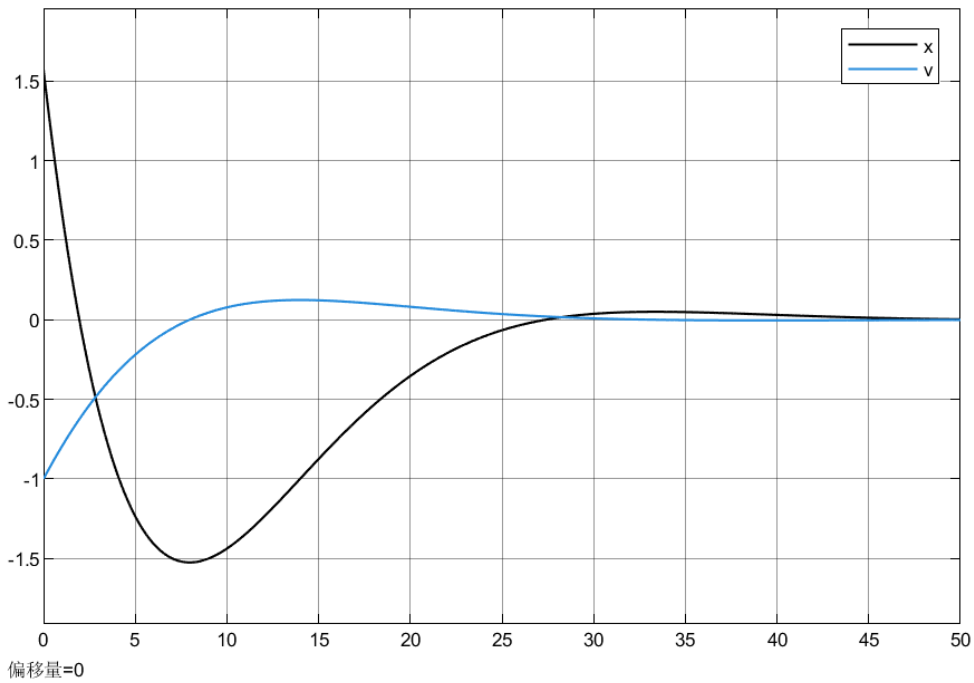

Figure 3. diagram of position and speed.

Comment:

In this case, it took about 50 seconds for u(t) to decrease to 0. And The position and speed are correct and compatible. Looks like it took longer.

3) Only penalize the velocity state: $Q = \begin{bmatrix} 0.001 & 0 \\ 0 & 10 \end{bmatrix}, R[1]$

Q3 = [0.001 0; 0 10];
R3 = 1;

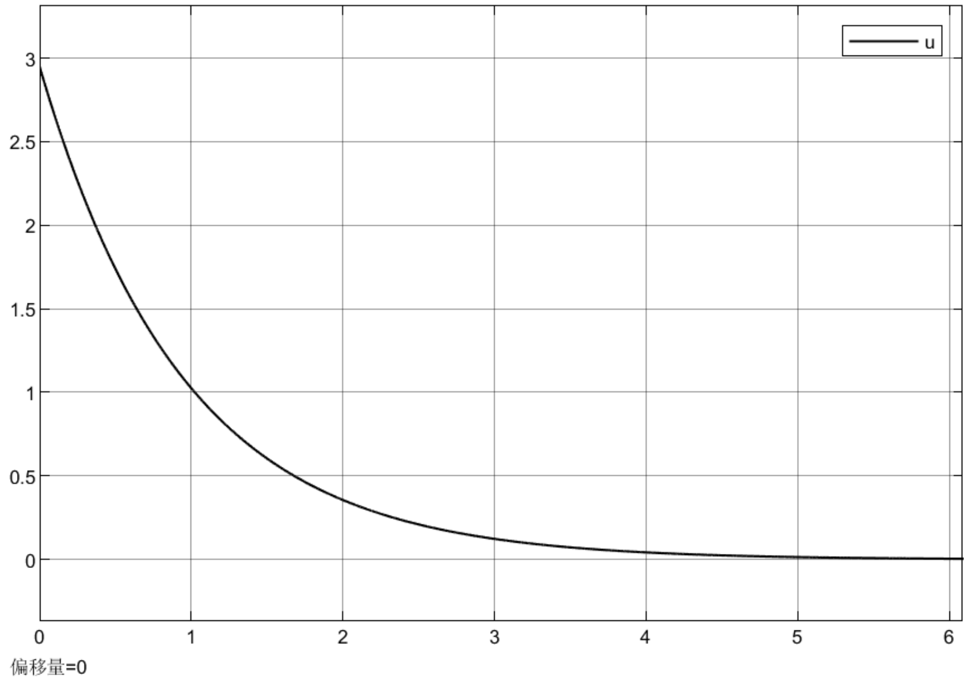

Figure 2. diagram of u(t).

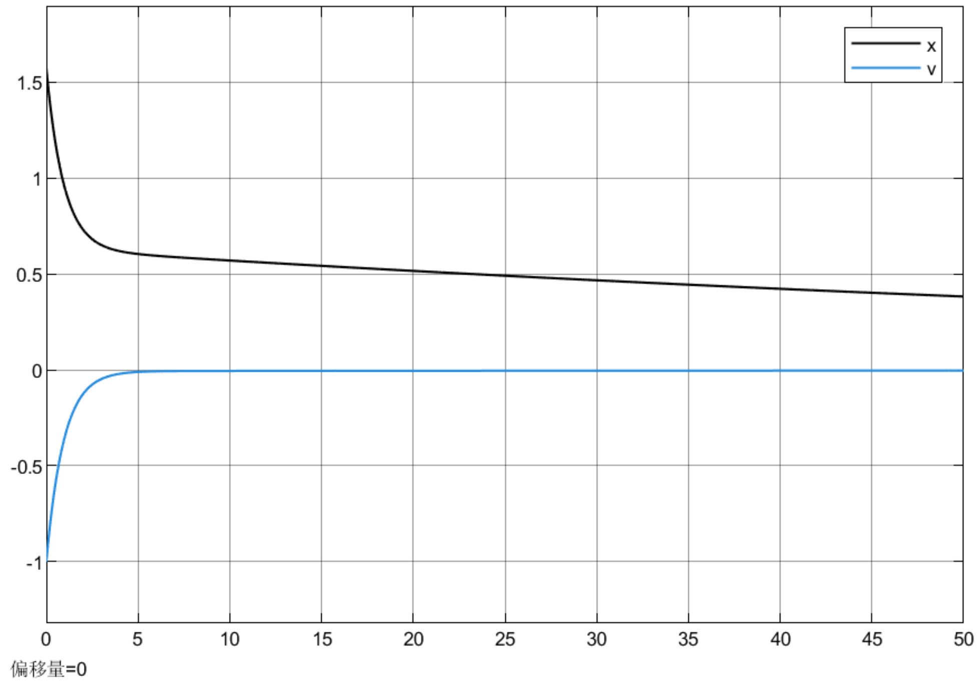

Figure 3. diagram of position and speed.

Comment:

In this case, it took about 5 seconds for u(t) to decrease to 0. And position x and velocity v do not converge to 0 at the same time. Looks like it took longer to get the position attenuation to 0.

Conclusions:

Through the three sets of experiments using different Q and R matrices in the Linear Quadratic Regulator (LQR) design, we observed how the choice of these parameters significantly affects the system's control performance. A smaller R value led to a faster response with more control effort, while a larger R caused slower response with reduced control action. Additionally, the weighting of state variables in Q influenced the convergence behavior of position and velocity. This exercise demonstrated the importance of appropriately tuning Q and R to balance performance and energy cost in optimal control systems.

Source Code:

```
m = 3;
c = 0.2;
x0 = [pi/2; -1];
```

1) Cheap control

```
A = [0 1; 0 -c/m];
B = [0; 1/m];
C = eye(2);
D = zeros(2,1);

Q1 = [1 0; 0 1];
R1 = 0.01;

[K1, P1] = lqr(A, B, Q1, R1);
```

2) Expensive control

```
Q2 = [1 0; 0 1];
R2 = 100;

[K2, P2] = lqr(A, B, Q2, R2);
```

3) Only penalize the velocity state

```
Q3 = [0.001 0; 0 10];
R3 = 1;

[K3, P3] = lqr(A, B, Q3, R3);
```