Typical dynamics

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Laplace transforms and LTI systems

Transfer functions are derived by computing the **Laplace transform** of **linear time-invariant** (**LTI**) dynamic systems.

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Laplace transform

Definition

The Laplace transform of a locally integrable function f is

$$F(s) = \mathscr{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\ dt$$

The corresponding inverse Laplace transform is denoted as

$$f(t) = \mathscr{L}^{-1}\{F(s)\}$$

Inverse Laplace transforms (and forward Laplace transforms, for that matter) are usually found by appealing to a transform table. The formal definition of the inverse Laplace transform is rarely used and so is not discussed here.

Example.

(a) Compute the Laplace transform of $f(t) = H(t-a)$ by hand, where H is the Heaviside step function with $a > 0$.

(b) Compute the analytic Laplace transform of f

Solution.

(a)

$$F(s) = \int_0^{\infty} H(t-a)e^{-st}\ dt = \int_a^{\infty} e^{-st}\ dt = -\frac{1}{s}e^{-st}\ \Big|_a^{\infty} = \frac{1}{s}e^{-as}$$

(b) The `laplace` function computes the analytic transform of a symbolic expression `f`. You can use the syntax:

```
Fs = laplace(f,var,transformVar)
```

1. Start by declaring symbolic variables.

```
syms t s
syms a positive
```

2. Then, define the function `f`. We use list of mathematical functions present in the Symbolic Math Toolbox

```
f = heaviside(t-a)
```

`f =` $\mathrm{heaviside}(t-a)$

3. Finally, compute the Laplace transform with the input variable `t` and transform variable `s`.

```
Fs = laplace(f,t,s)
```

`Fs =`

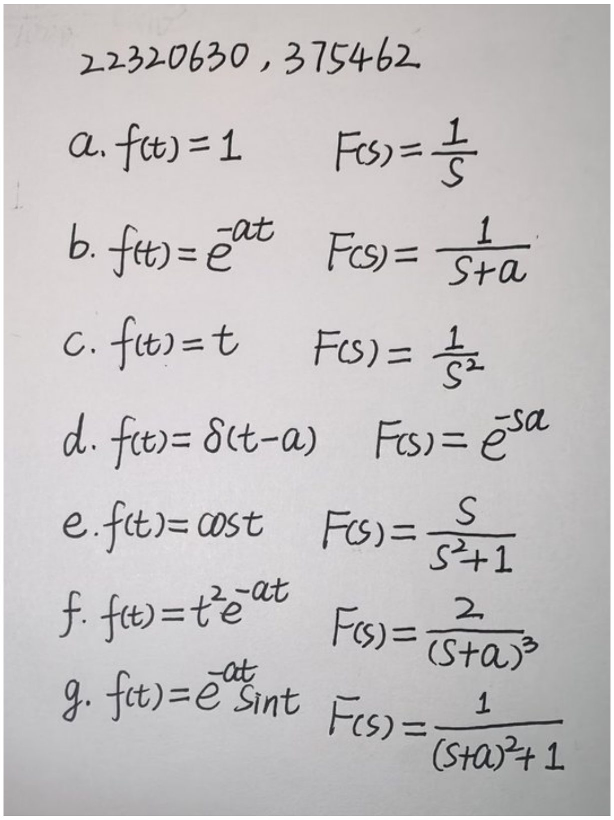
$$\frac{\mathrm{e}^{-a\,s}}{s}$$

Exercise 1.

1. Compute the Laplace transform of some standard functions listed below on paper. Assume a is real and $a > 0$.

a. $f(t) = 1$ **b.** $f(t) = e^{-at}$ **c.** $f(t) = t$ **d.** $f(t) = \delta(t-a)$

e. $f(t) = \cos(t)$ **f.** $f(t) = t^2e^{-at}$ **g.** $f(t) = e^{-at}\sin(t)$



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a. $f(t)=1$ $\quad F(s)=\frac{1}{s}$

b. $f(t)=e^{-at}$ $\quad F(s)=\frac{1}{s+a}$

c. $f(t)=t$ $\quad F(s)=\frac{1}{s^2}$

d. $f(t)=\delta(t-a)$ $\quad F(s)=e^{-sa}$

e. $f(t)=\cos t$ $\quad F(s)=\frac{s}{s^2+1}$

f. $f(t)=t^2e^{-at}$ $\quad F(s)=\frac{2}{(s+a)^3}$

g. $f(t)=e^{-at}\sin t$ $\quad F(s)=\frac{1}{(s+a)^2+1}$

Hints: a/b. substitution, **c.** integration by parts, **d.** `dirac` (for the symbolic computation), **e/g.** integrate by parts and rearrange terms, **f.** multiple integrations by parts,

2. Verify your answers by computing the Laplace transforms in the space provided below using symbolic math.

```
% Symbolic variable declarations
syms t s
syms a positive

f = 1
```

```
f = 1
```

```
Fs = laplace(f,t,s)
```

```
Fs =
```

$$\frac{1}{s}$$

Visualize Laplace transforms

Exercise 2. Use the controls below to visualize common Laplace transforms.

```
syms t real
syms a b positive
```

Set constants m, a, and b:

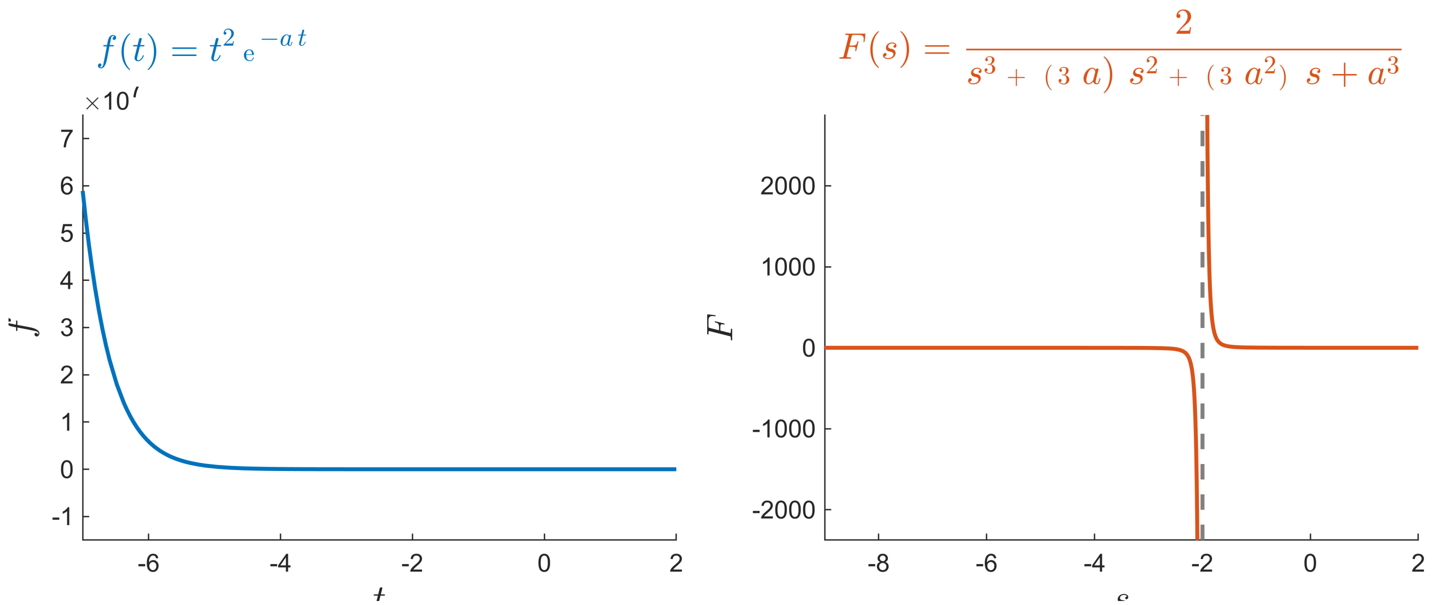
```
m = 2; % Positive integer
anum = 2; % Positive constant
bnum = 3; % Positive constant
```

Select a function f:

```
f = t^m*exp(-a*t);
```

Define axis ranges:

```
trange = [-7, 2];
srange = [-9, 2];

generateSinglePlot(f,anum,bnum,trange,srange); % This local function generates the plots
```



Exercise.

Write your answer

- What class of functions is most commonly observed in the Laplace transform?

Rational Function: *F*(*s*)=*Q*(*s*)/*P*(*s*)

P(*s*) is the numerator polynomial of degree *m*.

Q(s) is the denominator polynomial of degree n.

- How do the poles of the Laplace transform reflect the behavior of the time domain function $f(t)$?

The location of the poles in the complex ss-plane provides insights into the characteristics of the time-domain response, such as stability, growth, decay, and oscillatory behavior.

Laplace transforms and their inverses are also commonly found using a table, like this one.

Laplace transform properties

Laplace transforms have several important properties that can be derived from the definition. A few essential properties are reviewed below.

Name	**f(t)**	**F(s)**
Time derivative	$\dot{f}(t)$	$sF(s) - f(0)$
Time integral	$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
Frequency shift	$e^{at}f(t)$	$F(s-a)$
Time shift	$f(t-a)H(t-a)$	$e^{-as}F(s)$
Time scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time domain convolution	$(f * g)(t)$	$F(s)G(s)$

For example, the Laplace transform of a time derivative can be computed through integration by parts:

$$\begin{aligned}\mathscr{L}\{f'(t)\} &= \int_0^\infty \dot{f}(t)e^{-st}\ dt \\ &= f(t)e^{-st}\ \Big|_0^\infty - \int_0^\infty f(t)\left(-se^{-st}\right)\ dt \\ &= -f(0) + sF(s)\end{aligned}$$

Example. The symbolic derivative of f is defined below by declaring a symbolic function using the syntax

```
syms f(t)
```

The derivative is then computed using the `diff` function.

```
syms t f(t)
dfdt = diff(f)
```

```
dfdt(t) =
```

$$\frac{\partial}{\partial t}\, f(t)$$

Use the `laplace` function to compute the Laplace transform of `dfdt` in the space below.

```
Fs = laplace(dfdt)
```

Fs = $s\,\mathrm{laplace}(f(t), t, s) - f(0)$

Example.

Find the formula for the Laplace transform of the second time derivative $\ddot{f}(t)$ by computing the Laplace transform using symbolic math.

To compute the second derivative, use the `diff` function with the syntax: `diff(f,n)`, where `n` is the order of the derivative.

```
syms t f(t) % Definitions of the symbolic variables
% Perform your symbolic computations here
d2fdt2 = diff(f,2)
```

```
d2fdt2(t) =
```

$$\frac{\partial^2}{\partial t^2}\, f(t)$$

```
Fs = laplace(d2fdt2)
```

```
Fs =
```

$$s^2\,\mathrm{laplace}(f(t), t, s) - s\,f(0) - \left(\left(\frac{\partial}{\partial t}\, f(t)\right)\Big|_{t=0}\right)$$

Visualize Laplace transform properties

Exercise 3. Use the controls below to visualize the properties of Laplace transforms.

```
syms t real
syms a real
```

Set constants m and a:

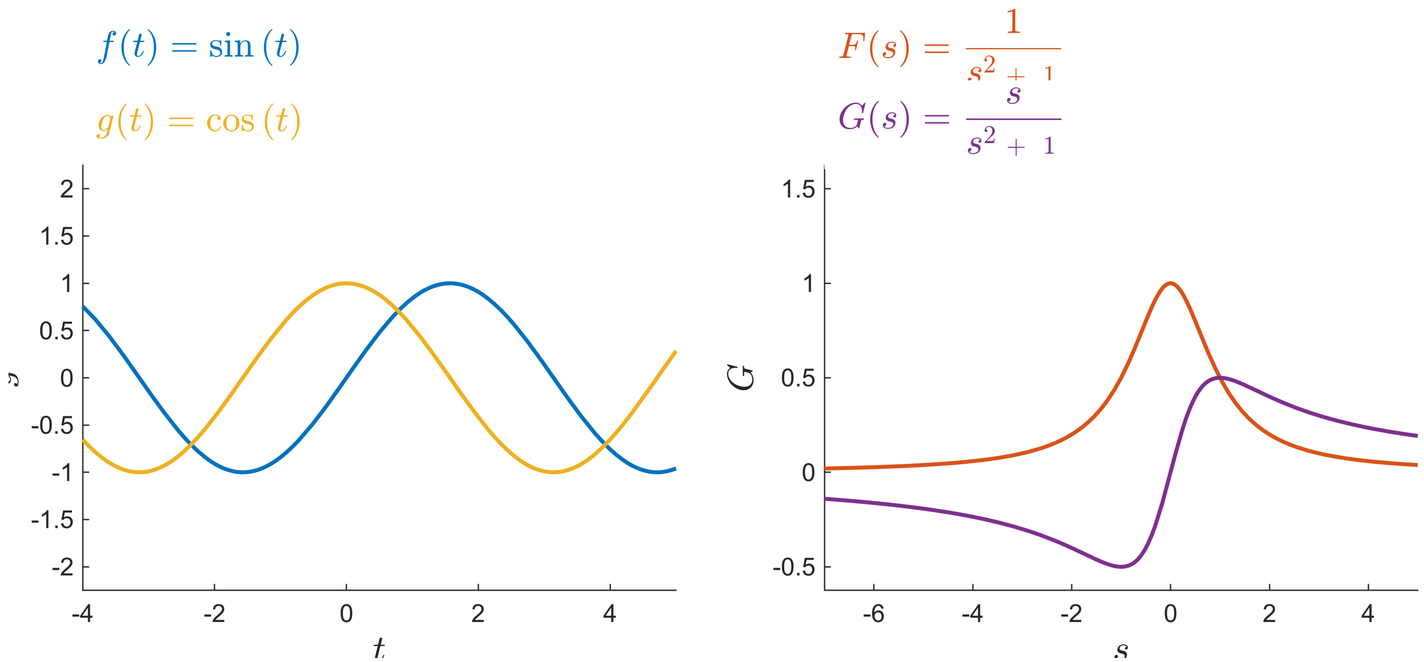
```
m = 1; % Positive integer
anum = -2.75; % Constant
```

Select a function f and a property to create g.

```
f = sin(t);
g = diff(f);
```

Define axis ranges:

```
trange = [-4, 5];
srange = [-7, 5];

generateDoublePlot(f,g,anum,0,trange,srange) % This local function generates the plots
```



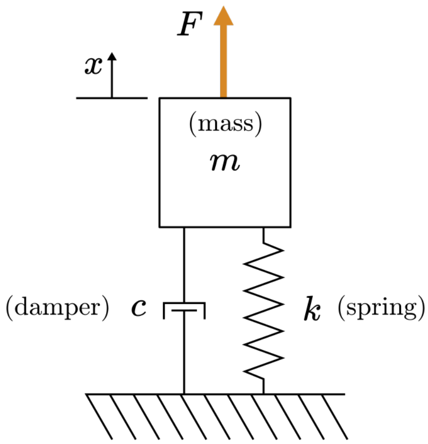
Solving differential equations using the Laplace transform

You can solve initial value problems analytically using Laplace transforms. In general, this is accomplished by

1. taking the Laplace transform,
2. solving for the solution variable in the Laplace domain ($X(s)$), and
3. taking the inverse Laplace transform by referring to a Laplace transform table.

Example. Use the Laplace transform to solve for the dynamics of the mass-spring-damper with

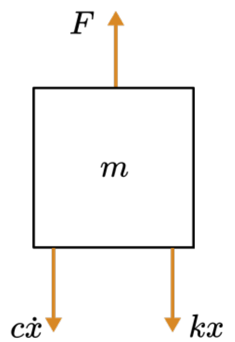
- constant forcing $F = 10$ N
- physical parameters: $m = 1$ kg, $c = 2$ N·s/m, and $k = 10$ N/m
- zero initial conditions: $x(0) = 0$ and $x'(0) = 0$



Solution.

1. Derive the equations of motion. You can draw a free body diagram and apply Newton's second law to derive the equations of motion.

Free Body Diagram



Newton's 2nd law application

$$m\ddot{x} = \sum \text{Forces}$$

$$m\ddot{x} = F - kx - c\dot{x}$$

or

$$m\ddot{x} + c\dot{x} + kx = F$$

2. Compute the Laplace transform of the dynamic system ODE and solve for X. Note: $\mathscr{L}\{x(t)\} = X(s)$.

$$\mathscr{L}\{m\ddot{x} + c\dot{x} + kx = F\} \rightarrow m[s^2X - sx(0) - x'(0)] + c[sX - x(0)] + kX = \frac{F}{s}$$

Applying the zero initial conditions and the values of the physical parameters implies

$$s^2X + 2sX + 10X = \frac{10}{s}$$

Solving for X yields

$$X = \frac{10}{s^3 + 2s^2 + 10s}$$

3. Use partial fraction decomposition to separate the expression for X **into terms where the inverse Laplace transform is known**

Try performing the partial fraction decomposition on paper and comparing your result to the symbolic solution found below.

```
% Define X(s)
```

```
syms s
X = 10/(s^3+2*s^2)
```

X =

$$\frac{10}{s^3 + 2\,s^2}$$

```
% Compute the partial fraction decomposition
Xdecomp = 1/s - (s+2)/(s^2+2*s+10)
```

Xdecomp =

$$\frac{1}{s} - \frac{s+2}{s^2 + 2\,s + 10}$$

4. Take the inverse Laplace transform

You can take the inverse Laplace transform by rewriting X as a sum of terms in forms found on a transform table.

$$X(s) = \frac{1}{s} - \frac{s+2}{s^2+2\ \ s+10} = \frac{1}{s} - \frac{s+2}{(s+1)^2+9} = \frac{1}{s} - \frac{s+1}{(s+1)^2+3^2} - \frac{1}{3}\frac{3}{(s+1)^2+3^2}$$

The solution is then constructed by taking the inverse transform:

$$\begin{aligned} x(t) \quad &= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s+1}{(s+1)^2+3^2} - \frac{1}{3}\frac{3}{(s+1)^2+3^2}\right\} \\ &= H(t) - H(t)e^{-t}\cos(3t) - \frac{1}{3}H(t)e^{-t}\sin(3t) \\ &= H(t)(1 - e^{-t}\cos(3t) - \frac{1}{3}e^{-t}\sin(3t)) \end{aligned}$$

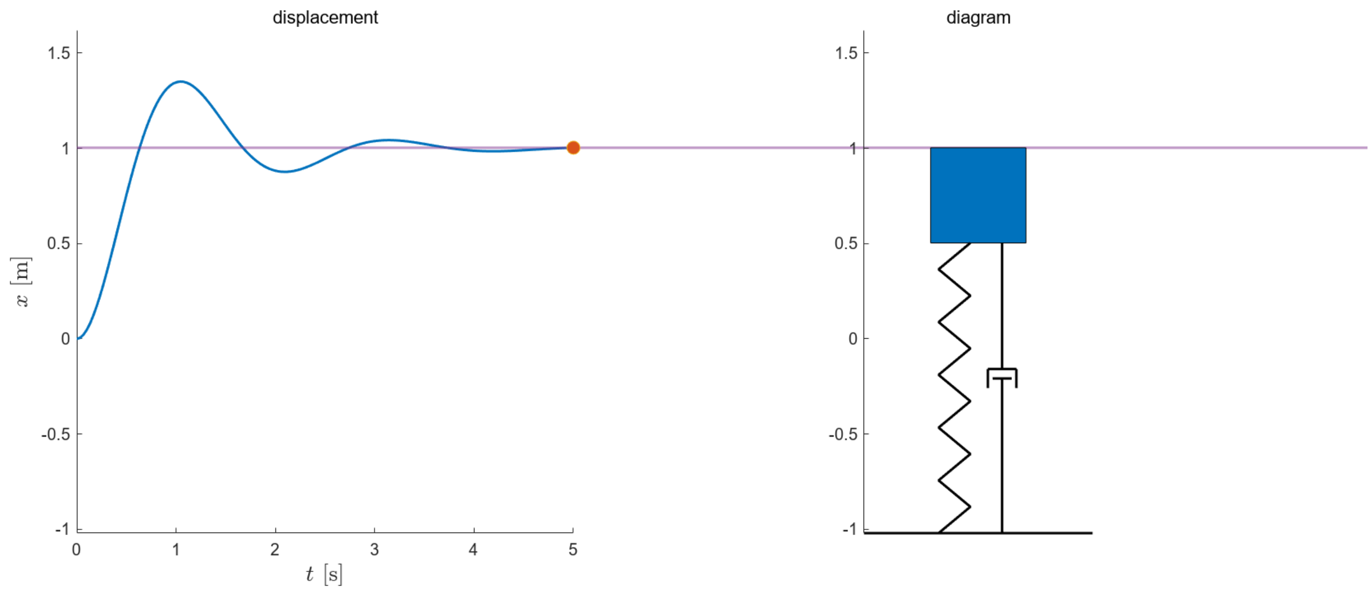
Alternatively, you can use the symbolic function `ilaplace` to find the inverse Laplace transform of $X(s)$.

```
syms x
x = ilaplace(Xdecomp, s, t); % ADD The inverse Laplace transform of X(s)
```

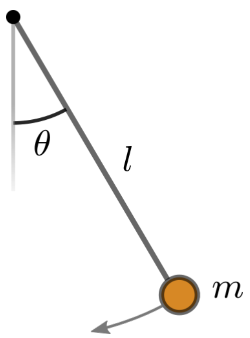
5. Plot the solution

Click the checkbox to plot the solution.

```
plotSoln = true;

% Create solution array
t = linspace(0,5,150);
x = 1-exp(-t).*(cos(3*t) + 1/3*sin(3*t));
% This generates a plot (do not edit)
if(plotSoln)
    animateSingleMSD(t,x,0.5)
end
```



Exercise. In this exercise, you will solve for the dynamics of the simple pendulum using the Laplace transform.



(a) Draw a free-body diagram for the simple pendulum shown above and derive the equation of motion for a pendulum with length $l = 0.5$ m and a gravitational constant of 9.8 m/s. Linearize the equation near $\theta = 0$ and show that it is equivalent to

$$\ddot{\theta} + 19.6\theta = 0$$

(b) Use the Laplace transform to solve the linearized equations of motion:

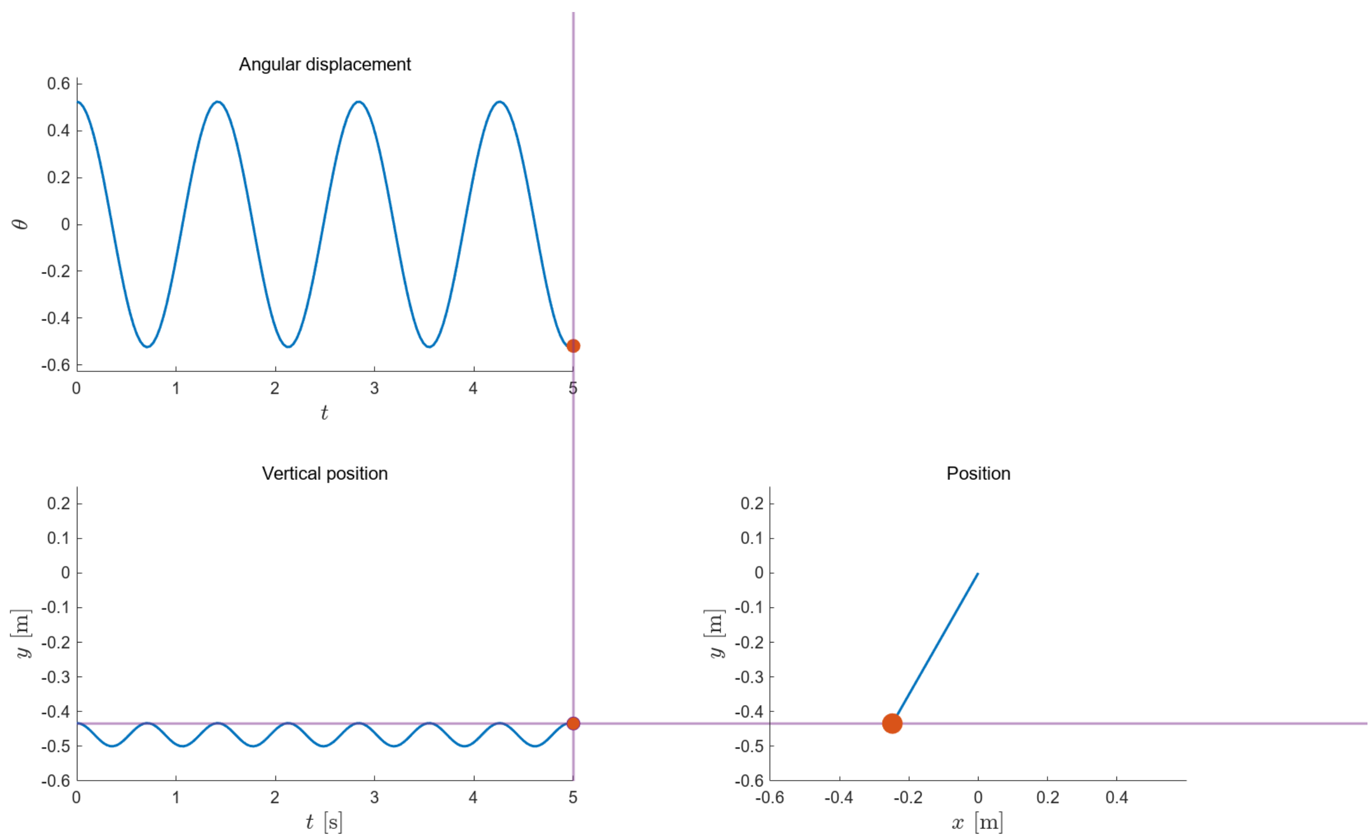
$$\ddot{\theta} + 19.6\theta = 0$$

with initial conditions $\theta(0) = \pi/6,\ \dot{\theta}(0) = 0$.

Write your solution in the variable `theta` in terms of the symbolic variable `t`. Then click the checkbox to plot your solution.

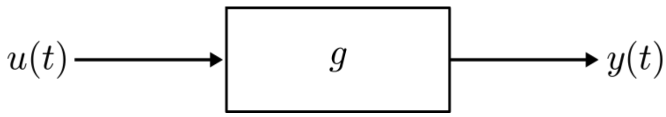
```
%syms t
```

```
syms t s theta

%Define the Laplace transform of theta
Theta_s = s^2+19.6;

%Define the initial condition in Laplace domain
Theta_0=pi/6*s;

%Apply the initial condition for theha(0);
Theta_s_with_IC = Theta_0/Theta_s;
% Theta_s_with_IC = pi/6*s/(s^2+19.6);

%Inverse Laplase transform to obtain the solution in time domain
theta=ilaplace(Theta_s_with_IC, s, t);

%
%add your code here
theta_t = pi/6*cos(4.427*t);


plotSoln = true;
% This generates a plot (do not edit)
if(plotSoln)
    plotPendulum(theta)
end
```



Linear time-invariant systems

Linear time-invariant (LTI) systems are characterized by the two properties stated in the name: linearity and time-invariance.



Consider an operator g that maps an input $u(t)$ to an output $y(t)$.

1. Linearity

The operator g is linear if it has two properties:

- Superposition: $g[u_1(t) + u_2(t)] = y_1(t) + y_2(t)$
- Homogeneity: $g[au(t)] = ay(t)$

Often these two properties are written together as

$g[au_1(t) + bu_2(t)] = ay_1(t) + by_2(t)$

2. Time invariance

The operator g is time-invariant if a time-shifted input produces an output with the same time shift:

- $g[u(t - \tau)] = y(t - \tau)$

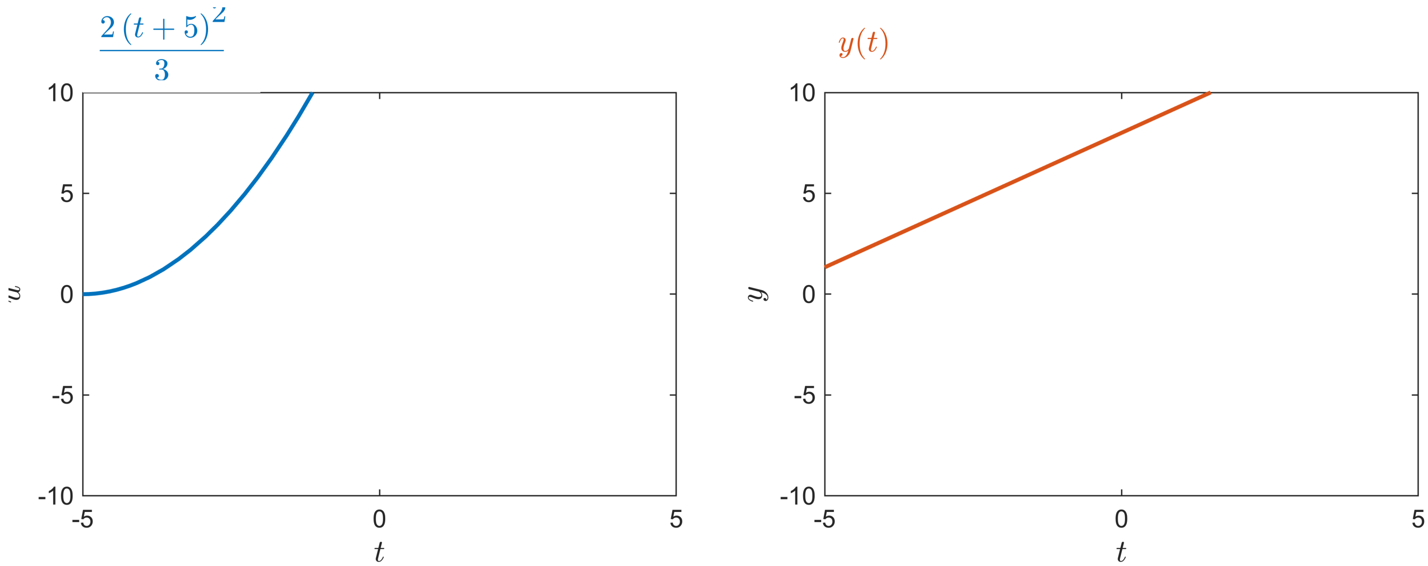
Exercise 4. In this exercise, you will identify if several unknown operators are linear and time-invariant by examining their outputs.

a. Identify if the operators g, h, i, and j are **time-invariant** by examining the graphs of their inputs and outputs.

- Use the dropdown to change the operator.
- Adjust `tau` to change the time-shift of the input.

My Answer: h,g,i are time-invariantand j is not.

```
syms t
operator = "g"; % Select an operator
u = t^2/6; % Input function
tau = -5;

plotInputsOutputs(u,tau,4,operator)
```

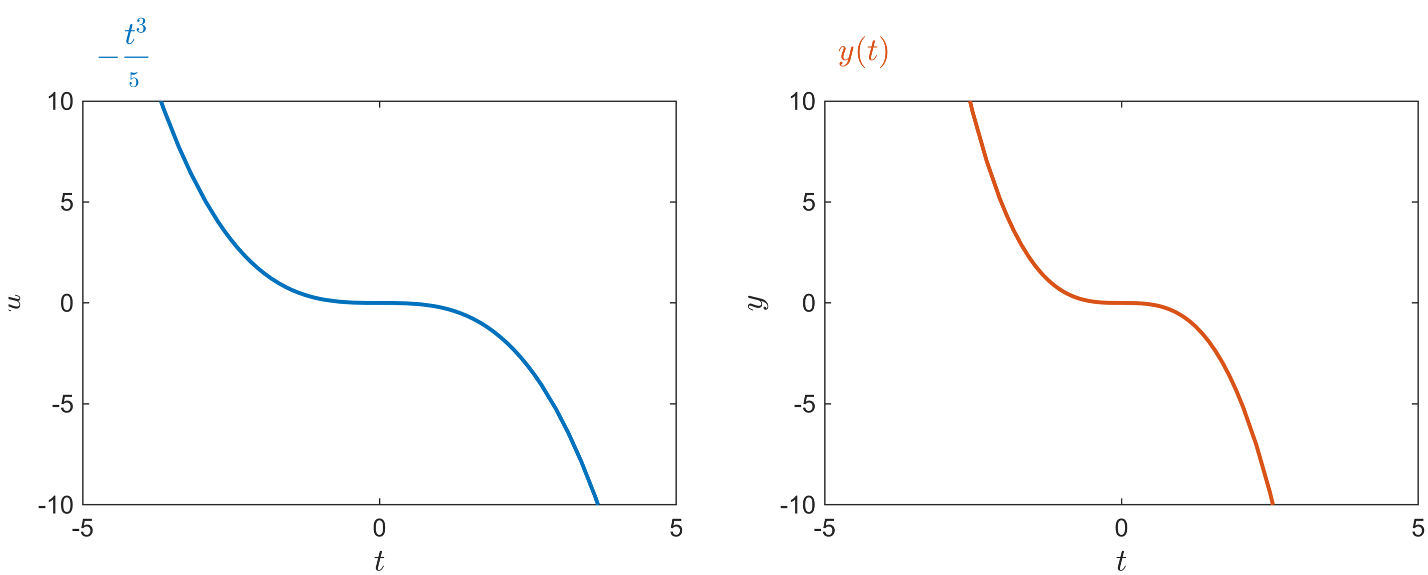


b. Identify if the operators g, h, i, and j satisfy the **homogeneity** condition ($g[au(t)] = ay(t)$) by examining the graphs of the inputs and outputs.

- Use the dropdown to change the operator.
- Adjust a to change the scaling of the input.

<u>My Answer: g,i,j are homogeneity, but h is not.</u>

```
operator = "j";
syms t
u = t^3/10; % Input function
a = -2;

plotInputsOutputs(u,0,a,operator)
```

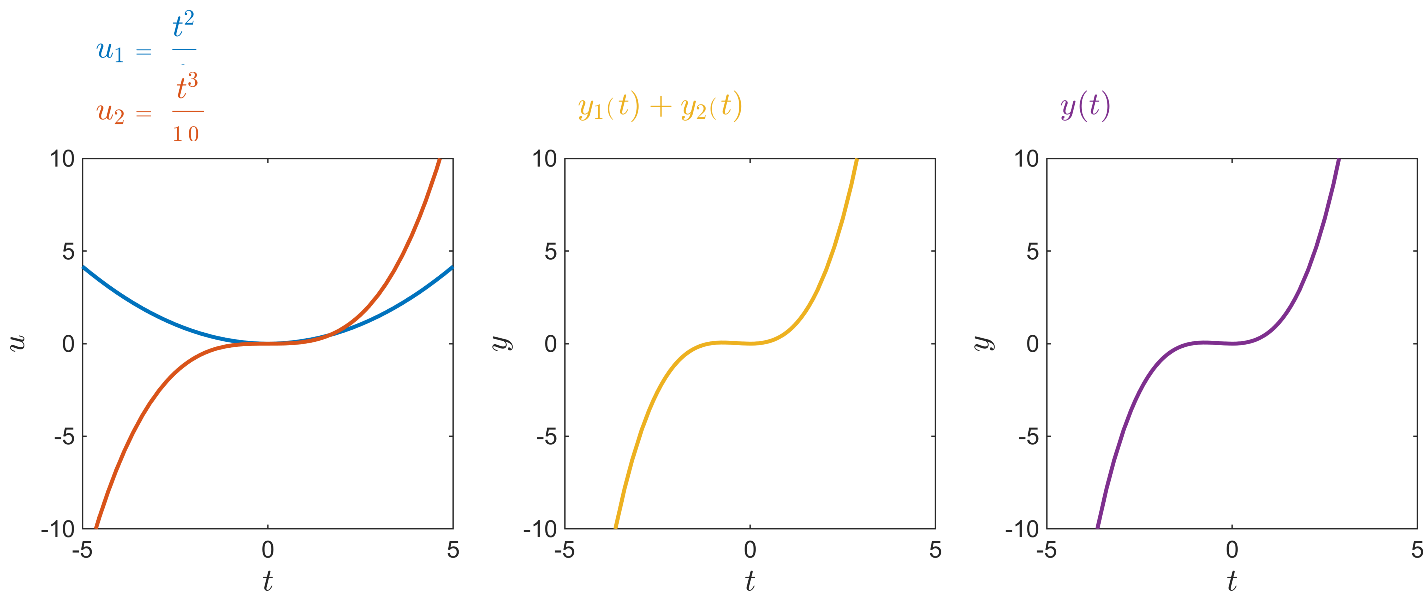


c. Identify if the operators g, h, i, and j satisfy **superposition**: $g[u_1(t) + u_2(t)] = y_1(t) + y_2(t)$ by examining the graphs of the inputs and outputs.

- Use the dropdown to change the operator.
- Try different functions for u_1 and u_2.

<u>My Answer: g,i,j are superposition, but h is not.</u>

```
operator = "j";
syms t
u1 = t^2/6; % Input function 1
u2 = t^3/10; % Input function 2

plotAddition(u1,u2,operator)
```



Helper functions

Plots

```
function generateSinglePlot(f,anum,bnum,trange,srange)
% This function plots a single transform pair
    colors = lines(2);
    figure("position",[0 0 1100 450])
    plotLaplace(f,anum,bnum,trange,srange,colors(1,:),colors(2,:),0.05,0.55,0.85,0.7)
end

function generateDoublePlot(f,g,anum,bnum,trange,srange)
% This function plots two transform pairs together
    colors = lines(4);
    figure("position",[0 0 1100 500])
    plotLaplace(f,anum,bnum,trange,srange,colors(1,:),colors(2,:),0.05,0.55,0.85,0.63) % This
```

```
    plotLaplace(g,anum,bnum,trange,srange,colors(3,:),colors(4,:),0.05,0.55,0.74,0.63) % This I
end

function plotLaplace(func,anum,bnum,trange,srange,c1,c2,p1,p2,py,vpos)
% This function plots a transform pair
    fs = 14; % fontsize
    funcName = inputname(1);

    % Compute the Laplace transform and generate functions
    syms s
    syms t real
    syms a b positive
    Fs = laplace(sym(func),t,s);
    Fs = collect(Fs);

    % Generate functions for plotting
    ffunc = matlabFunction(sym(func),"vars",[t a b]);
    Fsfunc = matlabFunction(Fs,"vars",[s a b]);
    fplotfunc = @(t)ffunc(t,anum,bnum);
    Fplotfunc = @(s)Fsfunc(s,anum,bnum);

    % Plot f(t) and label it with a latex function
    subplot("position",[0.05 0.1 0.4 vpos])
    hold on
    if(func == dirac(t-a))
        hold on
        plot([1,1]*anum,[0,1],"color",c1,"linewidth",1.5)
        plot(anum,1,"^","color",c1,"linewidth",1.5,"MarkerFaceColor",c1)
        plot(trange,0*trange,"linewidth",1.5,"color",c1)
        hold off
    elseif(func == 1)
        plot(trange,0*trange,"linewidth",1.5,"color",c1)
    else
        fplot(fplotfunc,trange,"linewidth",1.5,"color",c1)
    end
    hold off
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    frange = get(gca,'ylim');
    frange = [frange(1) - diff(frange)/4, frange(2) + diff(frange)/4 ];
    axis([trange frange])
    ylabel("$"+funcName+"$","Interpreter","latex","fontsize",fs)
    textVal = "$$"+funcName+"(t) = "+latex(sym(func))+"$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c1,"VerticalAlignment",
        "fontsize",fs,"Position",[p1 py 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Backgr

    % Plot F(s) and label it with a latex function
    subplot("position",[0.55 0.1 0.4 vpos])
    hold on
    if( Fs == 1 )
        plot(srange,0*srange,"linewidth",1.5,"color",c2)
```

```
    else
        fplot(Fplotfunc,srange,"linewidth",1.5,"color",c2)
    end
    hold off
    xlabel("$s$","Interpreter","latex","fontsize",fs)
    ylabel("$"+upper(funcName)+"$","Interpreter","latex","fontsize",fs)
    Frange = get(gca,'ylim');
    Frange = [Frange(1) - diff(Frange)/4, Frange(2) + diff(Frange)/4];
    axis([srange Frange])
    textVal = "$$"+upper(funcName)+"(s) = "+latex(sym(Fs))+"$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c2,"VerticalAlignment",
        "fontsize",fs,"Position",[p2 py 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Backgr

end

function plotInputsOutputs(u,tau,a,operator)
% Plots operator input/output and annotates the plots with latex
    figure("position",[0 0 900 350])
    tlims = [-5 5];
    ylims = [-10 10];
    colors = lines(2);
    c1 = colors(1,:);
    c2 = colors(2,:);
    p1 = 0.05;
    p2 = 0.55;
    py = 0.85;
    vpos = 0.70;

    fs = 12; % fontsize

    % Apply operator and generate functions
    syms t
    u = a*subs(u,t,t-tau);
    uFunc = matlabFunction(u,"var",t);
    y = applyOperator(u,operator);
    yFunc = matlabFunction(y,"var",t);

    % Plot u(t) and label it with a latex function
    subplot("position",[0.05 0.12 0.4 vpos])
    fplot(uFunc,tlims,"linewidth",1.5,"color",c1)
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    axis([tlims ylims])
    ylabel("$u$","Interpreter","latex","fontsize",fs)
    textVal = "$$"+latex(sym(u))+"$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c1,"VerticalAlignment",
        "fontsize",fs,"Position",[p1 py 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Backgr

    % Plot y(t) and label it with a latex function
    subplot("position",[0.55 0.12 0.4 vpos])
    fplot(yFunc,tlims,"linewidth",1.5,"color",c2)
```

```
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    ylabel("$y$","Interpreter","latex","fontsize",fs)
    axis([tlims ylims])
    textVal = "$$y(t)$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c2,"VerticalAlignment",
        "fontsize",fs,"Position",[p2 py 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Backgr
end

function plotAddition(u1,u2,operator)

    figure("position",[0 0 1200 500])
    tlims = [-5 5];
    ylims = [-10 10];
    colors = lines(5);
    c1 = colors(1,:);
    c2 = colors(2,:);
    c3 = colors(3,:);
    c4 = colors(4,:);
    px1 = 0.05;
    px2 = 0.375;
    px3 = 0.7;
    py1 = 0.85;
    py2 = 0.75;
    vpos = 0.60;

    fs = 12; % fontsize

    % Apply operator and generate functions
    syms t
    u = u1 + u2;
    uFunc = matlabFunction(u,"var",t);
    u1Func = matlabFunction(u1,"var",t);
    u2Func = matlabFunction(u2,"var",t);

    y = applyOperator(u,operator);
    y1 = applyOperator(u1,operator);
    y2 = applyOperator(u2,operator);
    yFunc = matlabFunction(y,"var",t);
    y12Func = matlabFunction(y1+y2,"var",t);

    % Plot u(t) and label it with a latex function
    subplot("position",[px1 0.12 0.25 vpos])
    fplot(u1Func,tlims,"linewidth",1.5,"color",c1)
    hold on
    fplot(u2Func,tlims,"linewidth",1.5,"color",c2)
    hold off
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    axis([tlims ylims])
    ylabel("$u$","Interpreter","latex","fontsize",fs)
    textVal = "$$u_1 = "+latex(sym(u1))+"$$";
```

```
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c1,"VerticalAlignment",
        "fontsize",fs,"Position",[px1 py1 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Back
    textVal = "$$u_2 = "+latex(sym(u2))+"$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c2,"VerticalAlignment",
        "fontsize",fs,"Position",[px1 py2 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Back

    % Plot y(t) and label it with a latex function
    subplot("position",[px2 0.12 0.25 vpos])
    fplot(y12Func,tlims,"linewidth",1.5,"color",c3)
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    ylabel("$y$","Interpreter","latex","fontsize",fs)
    axis([tlims ylims])
    textVal = "$$y_1(t) + y_2(t)$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c3,"VerticalAlignment",
        "fontsize",fs,"Position",[px2 py2 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Back

    % Plot F(s) and label it with a latex function
    subplot("position",[px3 0.12 0.25 vpos])
    fplot(yFunc,tlims,"linewidth",1.5,"color",c4)
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    ylabel("$y$","Interpreter","latex","fontsize",fs)
    axis([tlims ylims])
    textVal = "$$y(t)$$";
    annotation("textbox","String",textVal,"Interpreter","latex","Color",c4,"VerticalAlignment",
        "fontsize",fs,"Position",[px3 py2 0.1 0.1],"FitBoxToText","on","EdgeColor","none","Back

end

function y = applyOperator(u,operator)
    syms t
    if(operator == "g")
        y = diff(u,1) + diff(u,2);
    elseif(operator == "h")
        y = 4*sin(u).^2;
    elseif(operator == "i")
        y = int(3*u,t,t-1,t+1);
    else
        y = t*diff(u,1);
    end
end

function plotPendulum(thetaSym)
    try
        % This generates a plot (do not edit)
        t = linspace(0,5,150);
        thetaFunc = matlabFunction(thetaSym);
        thetaArray = thetaFunc(t);
        generatePendulumPlot(t,thetaArray,0.5)
    catch ME
        warning("Plotting failed with error: " + ME.message)
```

```
    end
end

function generatePendulumPlot(t,theta,l)
% Generates an animation of a single pendulum
% t: time array
% theta: angle array
% l: length of pendulum

    colors = lines(6);
    ms = 8;
    fs = 14;

    f = figure("position",[0,0,1200,700]);
    % Generate solution variables
    tlim = [min(t),max(t)];
    y = -l*cos(theta);
    x = l*sin(theta);

    % Setup the figure
    buff = 1.2;
    ymax = max([l/2,max(y)*buff]);
    axisLim0 = [tlim(1),tlim(2),min(theta)*buff,max(theta)*buff];
    axisLim1 = [tlim(1),tlim(2),-l*buff,ymax];
    axisLim2 = [-l*buff,l*buff,-l*buff,ymax];

    k = 1;
    % Create plot
    sp0 = subplot(2,2,1);
    xlabel("$t$","Interpreter","latex","fontsize",fs)
    ylabel("$\theta$","Interpreter","latex","fontsize",fs)
    hold on
    plot(t,theta,"color",colors(1,:),"linewidth",1.5);
    b = plot(t(k),theta(k),"o","markerfacecolor",colors(2,:),"markersize",ms);
    hold off
    axis(axisLim0)
    title("Angular displacement")
    box off

    sp1 = subplot(2,2,3);
    set(gca, "Clipping","off","Color","none");
    hold on
    plot(t,y,"k-","color",colors(1,:),"linewidth",1.5)
    e = plot([0, tlim(2)+(tlim(2)-tlim(1))*4], [y(k),y(k)],"-","color",[colors(4,:),0.5],"linew
    f = plot([t(k), t(k)], [-l*buff,ymax+(ymax+l*buff)*4],"-","color",[colors(4,:),0.5],"linewi
    g = plot(t(k),y(k),"o","markerfacecolor",colors(2,:),"markersize",ms);
    hold off
    axis(axisLim1)
    box off
    xlabel("$t$ [s]","Interpreter","latex","FontSize",fs)
```

```
    ylabel("$y$ [m]","Interpreter","latex","FontSize",fs)
    title("Vertical position")

    sp2 = subplot(2,2,4);
    set(gca, "Clipping","off","Color","none");
    hold on
    d = plot([0,x(k)],[0,y(k)],"color",colors(1,:),"linewidth",1.5);
    c = plot(x(k),y(k),"o","markerfacecolor",colors(2,:),"markersize",ms*1.5);
    hold off
    axis equal
    axis(axisLim2)
    box off
    xlabel("$x$ [m]","Interpreter","latex","FontSize",fs)
    ylabel("$y$ [m]","Interpreter","latex","FontSize",fs)
    title("Position")

    % Create animation
    for k = 1:length(t)
        b.XData = t(k);
        b.YData = theta(k);
        c.XData = x(k);
        c.YData = y(k);
        d.XData = [0,x(k)];
        d.YData = [0,y(k)];
        e.YData = [y(k),y(k)];
        f.XData = [t(k),t(k)];
        g.XData = t(k);
        g.YData = y(k);
        pause(0)
    end
    drawnow
    close all
end


function animateSingleMSD(t,x,w)
% Generates an animation of a single mass/spring/damper
% t: time array
% x: displacement array
% w: width of the mass

    colors = lines(6);
    fs = 14;

    % Create plot
    k = 1;
    f = figure("position",[0 0 1200 500]);

    % Setup the figure
    buffer = 1.2;
```

```
    xrange = max(x) - min(x);
    xmax = min(x) + xrange*buffer ;
    xmin = max(x) - xrange*buffer - w*1.5;
    tlim = [t(1) t(end)];
    axisLim1 = [tlim(1),tlim(2),xmin,xmax];
    axisLim2 = [-w*buffer,w*buffer,xmin,xmax];
    xground = xmin;

    sp1 = subplot(1,2,1);
    set(gca, "Clipping","off","Color","none");
    hold on
    plot(t,x,"k-","color",colors(1,:),"linewidth",1.5);
    a = plot([0, tlim(2)+(tlim(2)-tlim(1))*4], [x(k),x(k)],"-","color",[colors(4,:),0.5],"linew
    b = plot(t(k),x(k),"o","markerfacecolor",colors(2,:),"MarkerSize",8);
    hold off
    axis(axisLim1)
    box off
    xlabel("$t$ [s]","Interpreter","latex","FontSize",fs)
    ylabel("$x$ [m]","Interpreter","latex","FontSize",fs)
    title("displacement")

    sp2 = subplot(1,2,2);
    set(gca, "Clipping","off","Color","none");
    hold on
    plot([-w*buffer w*buffer],[xground xground],'k-',"linewidth",1.5);

    % Plot mass
    x1 = x(k) - w/2;
    c = rectangle("Position",[-w/2 x1-w w w],"FaceColor",colors(1,:));

    % Plot spring
    xextension = x(k)-w;
    xs = linspace(xground,xextension,12);
    ys = w/6*(-1).^(1:numel(xs)) - w/4;
    d = plot(ys,xs,"k","linewidth",1.5);
    % Plot damper
    ydamp = w/4;

    xdamp1 = xground + (xextension-xground)/2 + w*0.1;
    xdamp2 = xground + (xextension-xground)/2;
    xdamp3 = xground + (xextension-xground)/2 + w*0.2;
    e = plot([ydamp,ydamp,NaN,ydamp-w/10,ydamp+w/10],[xground,xdamp1,NaN,xdamp1,xdamp1],"k","li
    f = plot([ydamp-w*0.15,ydamp-w*0.15,NaN,ydamp+w*0.15,ydamp+w*0.15,NaN,ydamp-w*0.15,ydamp+w*
        [xdamp2,xdamp3,NaN,xdamp2,xdamp3,NaN,xdamp3,xdamp3,NaN,xdamp3,xextension],"k","linewidt

    hold off
    axis equal
    axis(axisLim2)
    ax = gca;
    ax.XAxis.Visible = 'off';
```

```
    title("diagram")

    % Create animation
    for k = 1:length(t)
        % displacement
        a.YData = [x(k),x(k)];
        b.XData = t(k);
        b.YData = x(k);
        % mass
        c.Position = [-w/2,x(k)-w,w,w];
        xextension = x(k)-w;
        % spring
        d.YData = linspace(xground,xextension,12);
        % damper
        xdamp1 = xground + (xextension-xground)/2 + w*0.1;
        xdamp2 = xground + (xextension-xground)/2;
        xdamp3 = xground + (xextension-xground)/2 + w*0.2;
        e.YData = [xground xdamp1 NaN xdamp1 xdamp1];
        f.YData = [xdamp2 xdamp3 NaN xdamp2 xdamp3 NaN xdamp3 xdamp3 NaN xdamp3 xextension];
        pause(0)
    end
    drawnow
    close all

end


% Suppress unused suggestions
%#ok<*NASGU>
```