Coursework

Consider a double inverted pendulum system (shown in Fig. 1), where

q_1	Cart position
q_2	Angle of the lower pendulum
q_3	Angle of the upper pendulum
u	Applied force (control variable)
m_1	Mass of the cart
m_2	Mass of the lower pendulum
m_3	Mass of the upper pendulum
l_1	Lenght of the lower pendulum
l_2	Lenght of the upper pendulum
g	Gravitational acceleration

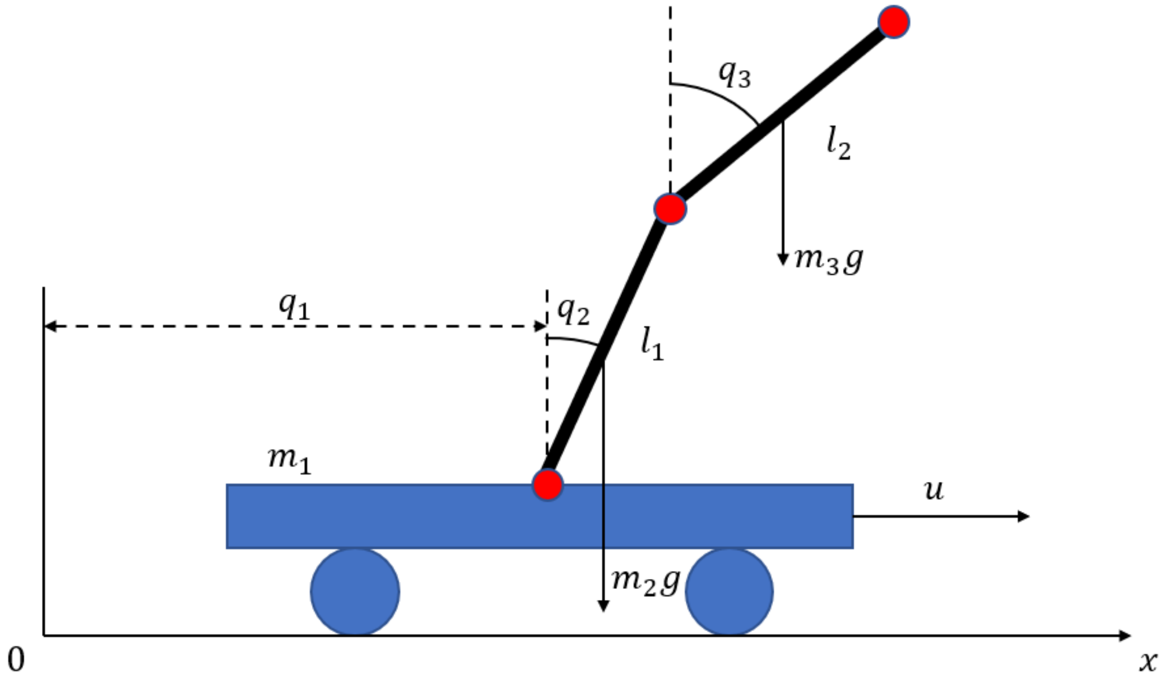

Fig. 1. Double inverted pendulum on a cart

The dynamics of this system can be described in the following standard form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Hu, \tag{1}$$

where

$q = [q_1, q_2, q_3]^T$	Generalized joint coordinates
$M(q)$	Regular mass matrix
$C(q, \dot{q})$	Centrifugal and Coriolis forces
$G(q)$	Gravity force
H	Control matrix

$$M(q) = \begin{bmatrix} a_1 & a_2 \cos q_1 & a_3 \cos q_2 \\ a_2 \cos q_1 & a_4 & a_5 \cos(q_1 - q_2) \\ a_3 \cos q_2 & a_5 \cos(q_1 - q_2) & a_6 \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & -a_2 \sin q_1 \, \dot{q}_1 & -a_3 \sin q_2 \, \dot{q}_2 \\ 0 & 0 & a_5 \sin(q_1 - q_2) \, \dot{q}_2 \\ 0 & -a_5 \sin(q_1 - q_2) \, \dot{q}_1 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} 0 \\ g \\ g_2 \end{bmatrix}, \quad H = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$a_1 = m_1 + m_2 + m_3, \qquad a_5 = \frac{1}{2} m_2 l_1 l_2,$$

$$a_2 = (\frac{1}{2} m_1 + m_2) l_1 \qquad a_6 = \frac{1}{3} m_2 l_2^2,$$

$$a_3 = \frac{1}{2} m_2 l_2, \qquad g_1 = \left(\frac{1}{2} m_1 + m_2\right) l_1 g,$$

$$a_4 = \left(\frac{1}{3} m_1 + m_2\right) l_1^2, \qquad g_2 = \frac{1}{2} m_2 l_2 g$$

Tasks:

1. Let all states be measurable. Choosing the state vector as $x = [q_1 \quad q_2 \quad q_3 \quad \dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3]^T$, represent the system in the state-space form, i.e., $\dot{x} = f(x) + h(x)u$.

Hint: Use multiplication of both sides of (1) by $M^{-1}(q)$. Note that $\dot{x}_1 = x_4$, $\dot{x}_2 = x_5$, $\dot{x}_3 = x_6$.

2. Make a simulation of the obtained model with $u = 1$ and parameters from the Table 1. You should take k equal to the last digit of your ITMO student number.

3. Linearize the system at the point $x_{eq} = 0$. Show the resulting linear state space model.

Hint: Get a model $\dot{x} = Ax + Bu$, where $A = \left.\frac{\partial f(x)}{\partial x}\right|_{x = x_{eq}}$, $B = \left.\frac{\partial h(x)}{\partial x}\right|_{x = x_{eq}}$.

4. Design a linear feedback control $u(x) = -Kx$ with 0% overshoot.

Hint: In order to obtain 0% overshoot use Newton polynomial for the desired characteristic polynomial.

5. Make a simulation of the obtained linear model with a designed control and nonzero initial conditions.

6. Analytical construct of a linear-quadratic regulator (LQR).

Hint: Solve the Riccati equation and find the LQR parameters.

7. Make a simulation of the obtained linear model with a designed LQR control and nonzero initial conditions.

8. Make a simulation of the nonlinear plant with a designed LQR control and nonzero (close to x_{eq}) initial conditions.

9. Make conclusions from your work.

Table 1. The obtained model parameters

k	m_1	m_2	m_3	l_1	l_2
0 1	$m_1 = 3$	$m_2 = 1$	$m_3 = 1.5$	$l_1 = 0.75$	$l_2 = 1$
2 3 4	$m_1 = 4$	$m_2 = 1.5$	$m_3 = 1.5$	$l_1 = 0.5$	$l_2 = 0.75$
5 6	$m_1 = 3.5$	$m_2 = 1.5$	$m_3 = 2$	$l_1 = 1$	$l_2 = 1.5$
7 8 9	$m_1 = 3$	$m_2 = 1$	$m_3 = 1$	$l_1 = 1.25$	$l_2 = 1.5$

Put all your analytical solutions into a single **doc/pdf** file. Make screenshots of your schemes and graphs, put them into that file too.

After making a report you have to submit it using the link:

https://forms.yandex.ru/u/67f528df4936397cb2c43081/