Practice 6

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1. Calculation

System 1:

$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_2 = 2{x_1}^3 x_2 + x_1 - u$$

linearize the system:

$$u = 2{x_1}^3 x_2 + 2x_1 - x_2 + v$$
$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = v$$
$$v = -k_1 z_1 - k_2 z_2$$

System 2:

$$\dot{x}_1 = -x_1 + x_2 - x_3$$
$$\dot{x}_2 = -x_1 x_3 - x_2 + u$$
$$\dot{x}_3 = -x_1 + u$$

linearize the system:

$$u = -\frac{-v - 3x_1 + 4x_2 - 2x_3 + 3x_1 x_3 - x_2 x_3 + x_3^2 + x_1^2}{1 + x_1}$$
$$z = T(x) := \begin{bmatrix} x_1 \\ -x_1 + x_2 - x_3 \\ 2x_1 - 2x_2 + x_3 - x_1 x_3 \end{bmatrix}$$
$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = z_3$$
$$\dot{z}_3 = v$$
$$v = -k_1 z_1 - k_2 z_2 - k_3 z_3$$

2. Simulation

System 1

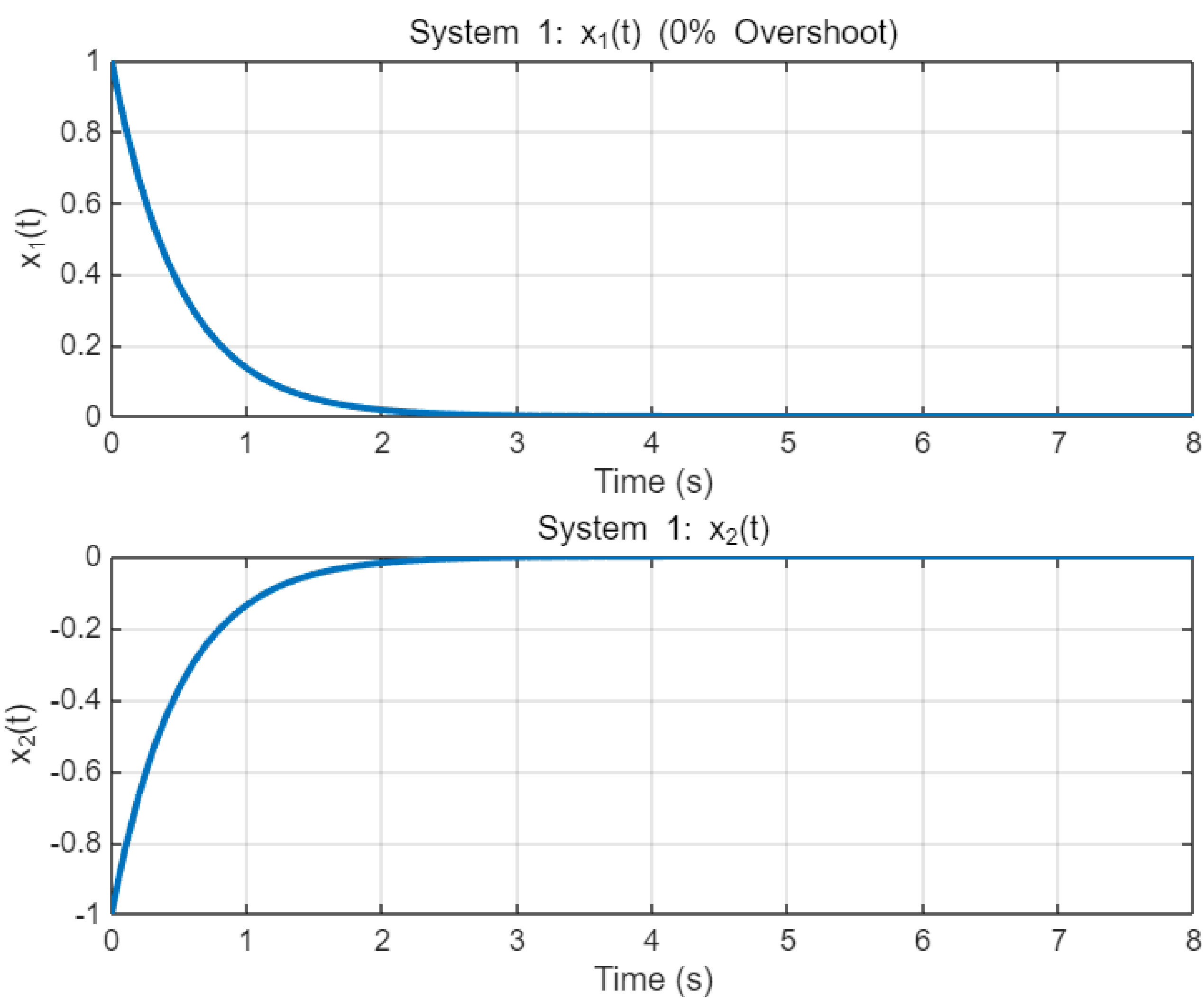
k1 = k2 = 2

initial condition:(1, -1)

```
clc; clear; close all;
% Initial conditions
x0 = [1; -1];
tspan = [0 8];
a = 2;
k1 = a^2;
k2 = 2*a;

odefun = @(t, x) [
    -x(1) + x(2);
    2*x(1)^3 * x(2) + x(1) - (2*x(1) + 2*x(1)^3*x(2) - x(2) - (-k1*x(1) - k2*(-x(1) +
x(2)))) % dx2/dt (with control u)
];

[t, x] = ode45(odefun, tspan, x0);
```



System 2

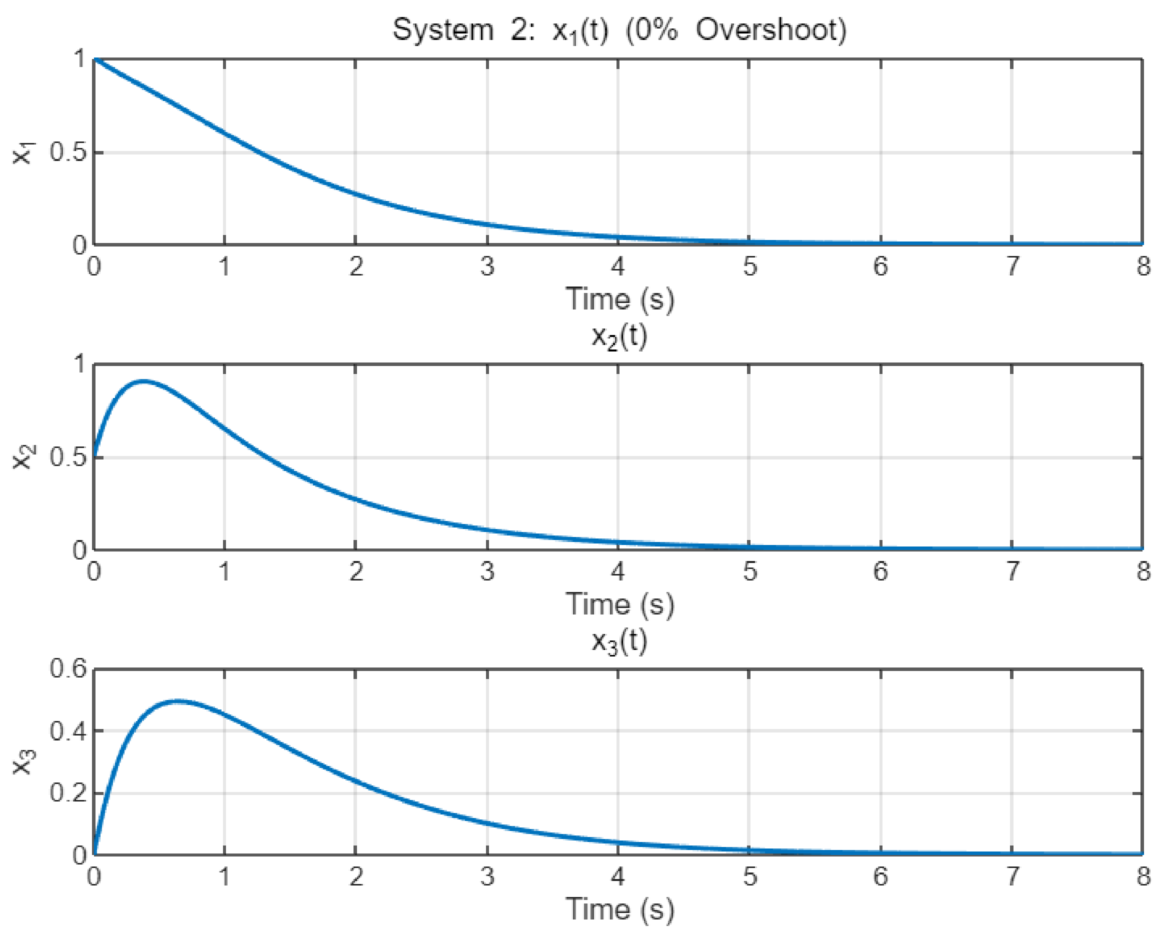
k1 = 6; k2 = 11; k3 = 6

initial condition:(1, 0.5, 0)

```
clear; clc;
poles = [-1, -2, -3];
k1 = abs(poles(1)*poles(2)*poles(3)); % 6
k2 = abs(poles(1)*poles(2) + poles(2)*poles(3) + poles(1)*poles(3)); % 11
k3 = abs(sum(poles)); % 6
tspan = [0 8];
x0 = [1; 0.5; 0];

v = @(x) -k1*x(1) - k2*(-x(1) + x(2) - x(3)) - k3*(2*x(1) - 2*x(2) + x(3) - x(1)*x(3));
u = @(x) (-v(x) - 3*x(1) + 4*x(2) - 2*x(3) + 3*x(1)*x(3) - x(2)*x(3) + x(3)^2 + x(1)^2) /
max(1e-3, 1 + x(1));

odefun = @(t, x) [
    -x(1) + x(2) - x(3);
    -x(1)*x(3) - x(2) + u(x);
    -x(1) + u(x)
];
options = odeset('RelTol', 1e-6);
[t, x] = ode45(odefun, tspan, x0, options);
```



Conclusion

Both feedback linearization and inearization at a point (Jacobian linearization) are techniques used to control nonlinear systems, but they differ significantly in their approaches, advantages, and limitations.

Feedback Linearization **Advantages**:

- Exact Linearization:
  - Transforms the entire nonlinear system into a linear one through exact state transformations and feedback, rather than approximation.
  - Works globally (across the entire state space) if the system is feedback-linearizable.
- Preserves Nonlinearity:
  - Unlike Jacobian linearization, it does not discard nonlinear terms—instead, it cancels them out via control.
  - Useful for systems with strong nonlinearities that cannot be ignored.
- Better Performance:
  - Can achieve precise tracking and stabilization since the linearization is exact.
  - Allows for pole placement in the transformed coordinates.

Disadvantages:

- Requires Precise Model Knowledge:
  - The exact cancellation of nonlinearities depends on perfect knowledge of system dynamics.
  - Model uncertainties or disturbances can degrade performance.
- Complexity:
  - Requires coordinate transformations and may involve solving partial differential equations (PDEs) to find the right transformation.
  - Not all systems are feedback-linearizable (must satisfy involutivity conditions).