Linear Systems- Lab 1

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Case 4. Mass-Spring-Damper system with free base

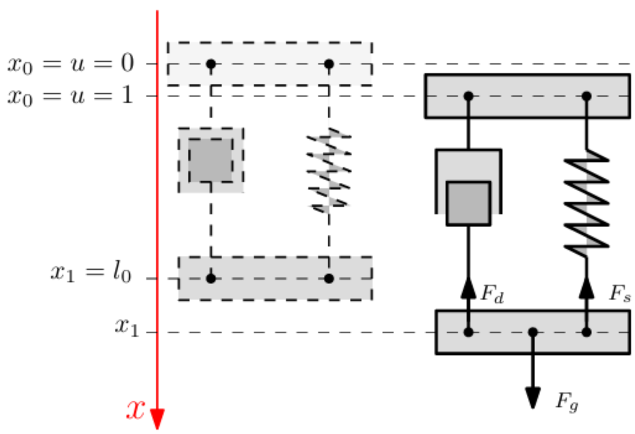

Figure 1: System scheme

System description:

- Block 0 position: $x_0 = u$
- Gravity force: $F_g = m_1 \cdot g$
- Damping force: $F_d = K_d \cdot (\dot{x}_1 - \dot{x}_0)$
- Spring force: $F_s = K_s \cdot (x_1 - x_0 - l_0)$

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Connection established
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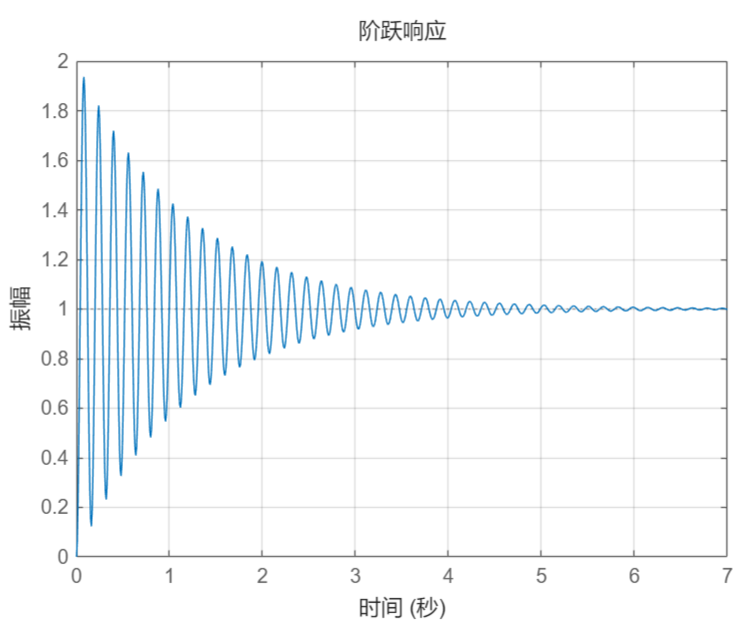
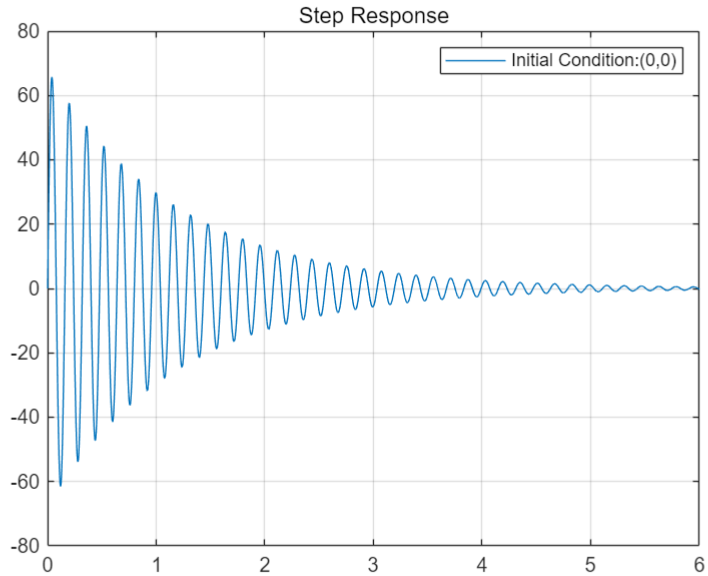
0. Loading the initial data:

1. Transfer Function:



2. Solve differential equation using zero initial conditions for u = 1.



3. Find the equilibrium point.

1. $$\frac{dx_1}{dt} = x_2 = 0$$
$$\frac{dx_2}{dt} = -\frac{K_s}{m_1}x_1 - \frac{K_d}{m_1}x_2 + \frac{1}{m_1}u = 0$$

2. $u = 0 \Rightarrow x_1 = x_2 = 0$

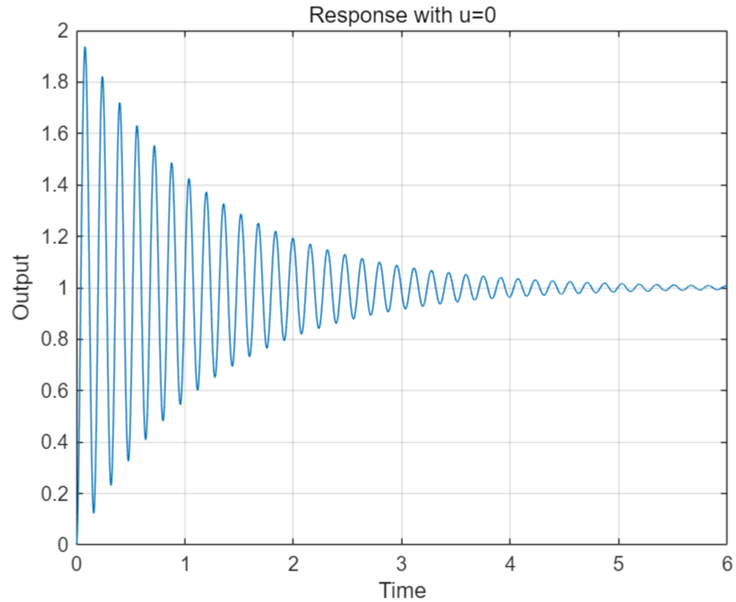
3. $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, x^* = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4. Use change of state coordinates v = x-x*

$$v = x - x^* = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

thus, it's the same with the step 3

5. New differential equationzero with initial conditions for u = 1



6. Find transfer function of the system with equilibrium point v* = 0

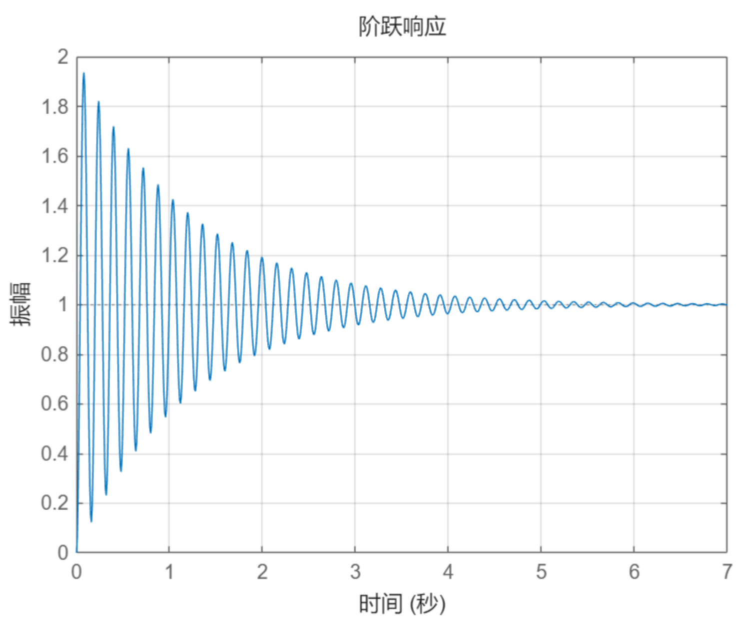
$$\frac{dx_1}{dt} = x_2 = 0$$

$$\frac{dx_2}{dt} = -\frac{K_s}{m_1}x_1 - \frac{K_d}{m_1}x_2 + \frac{1}{m_1}u = 0$$

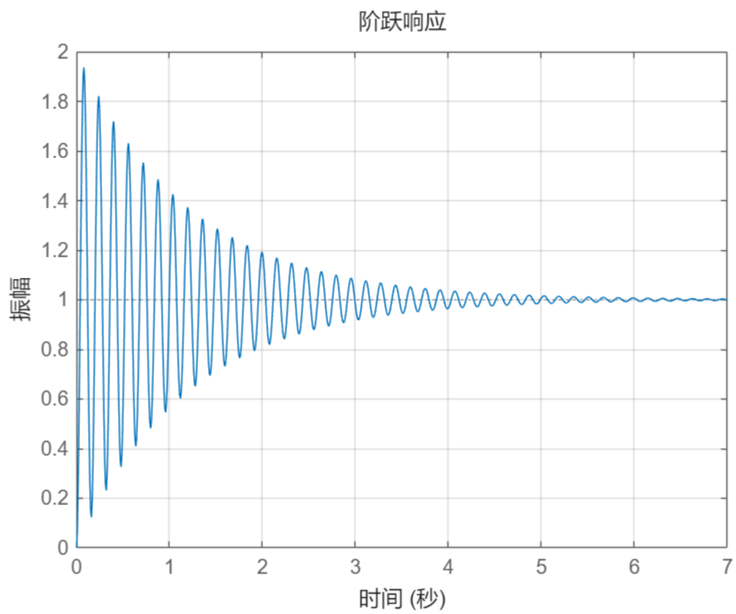
$$s \cdot x_1 = x_2$$

$$s \cdot x_2 = -\frac{K_s}{m_1}x_1 - \frac{K_d}{m_1}x_2 + \frac{1}{m_1}u$$

$$W(s) = \frac{X_1(s)}{U(s)} = \frac{K_d s + K_s}{m_1 s^2 + K_d s + K_s}$$

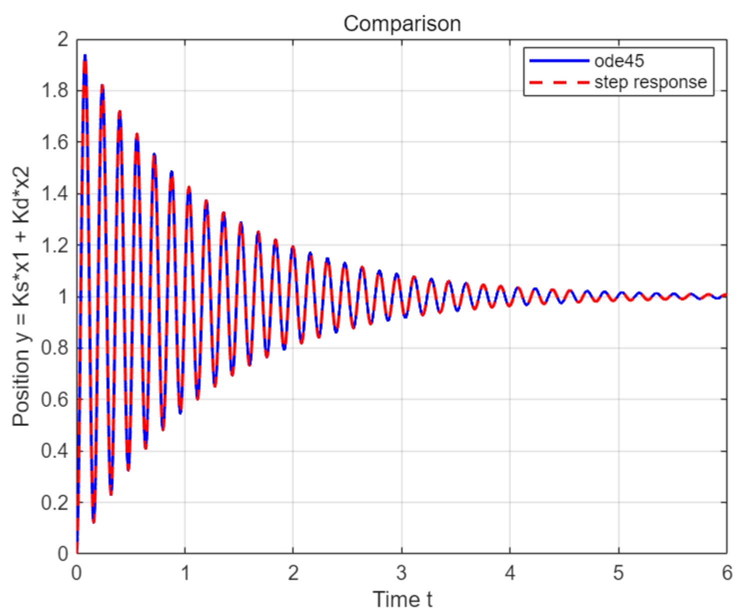


7. Construct state space model

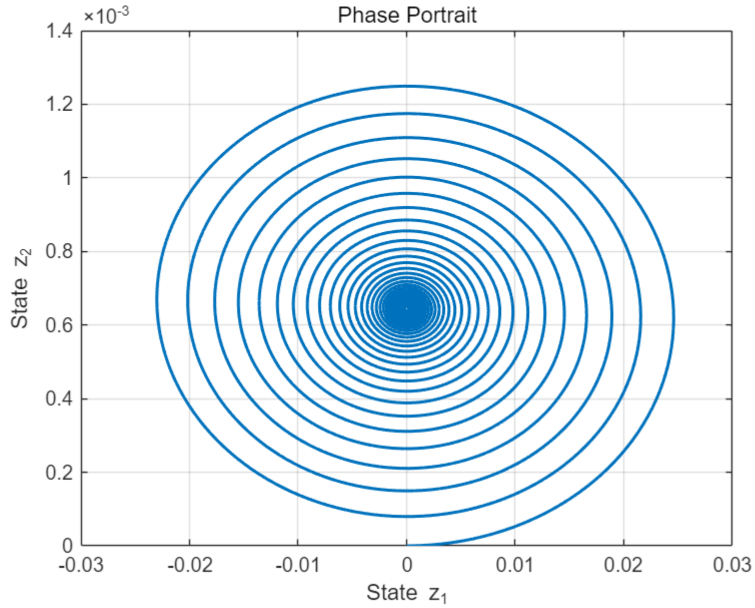


8. Plot the graphs

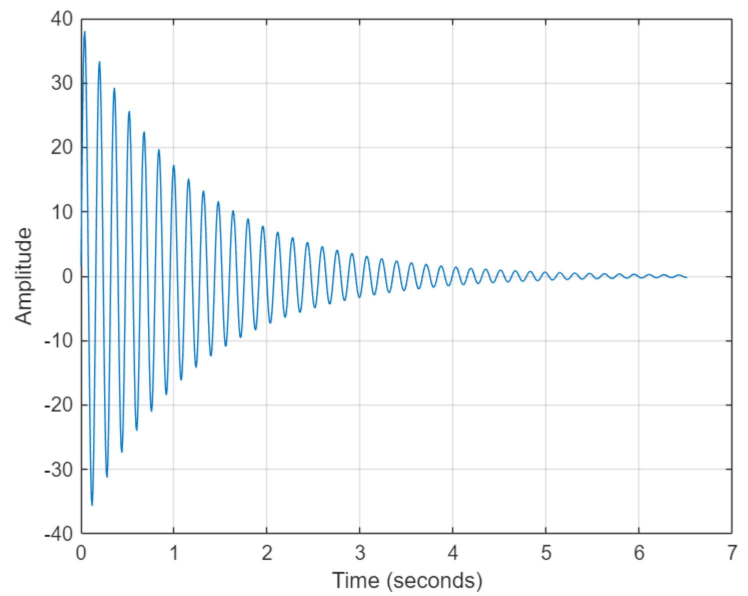
(a) Step response comparison & (b) Plot step response



(c) Plot phase portrait

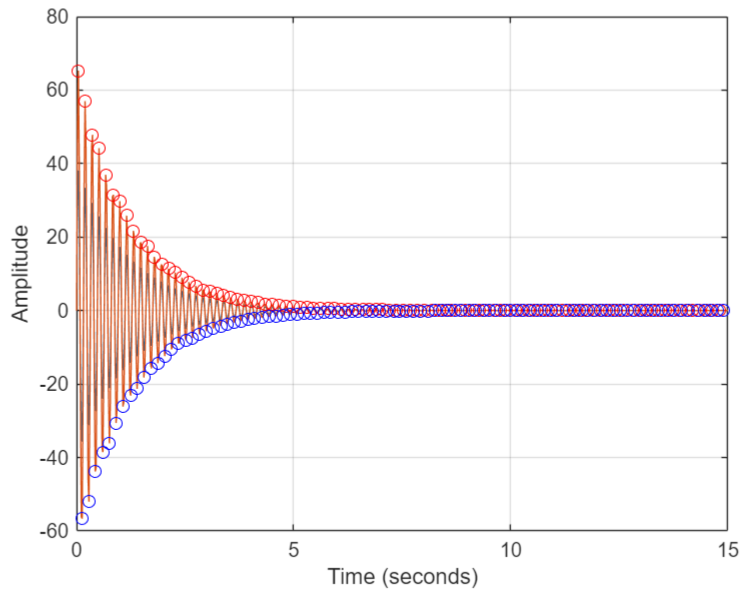


(d) Plot impulse response

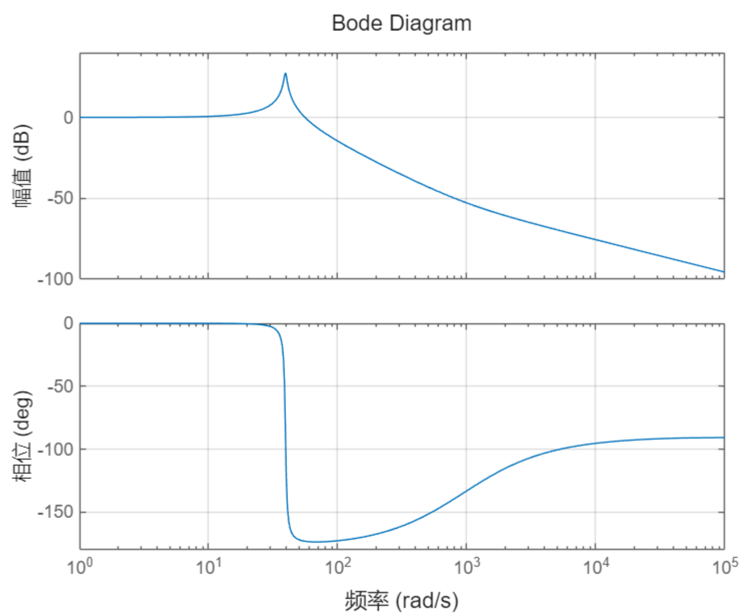


(e) For both models plot response

The amplitude of the output signal y(t) changed 188 times with respect to the input u(t) when system oscillations stabilized.



(f) Plot Magnitude and Phase data



Answers

```
ans = 包含以下字段的 struct:

        Bode_Plot_Peak_gain_of_resonance: 24
     Bode_Plot_Phase_crossover_frequency: [0 58.8000]
          Bode_Plot_Resonance_frequency: 39.4000
          Impulse_Response_SettlingTime: 5
                  Step_Response_RiseTime: 0.0267
      Bode_Plot_Gain_crossover_frequency: 0
            Impulse_Response_MaxTime: 0.0399
                      Step_Response_Peak: 1.9363
```

```
ans =

  包含以下字段的 struct:

        Bode_Plot_Peak_gain_of_resonance: 1
     Bode_Plot_Phase_crossover_frequency: 1
          Bode_Plot_Resonance_frequency: 1
          Impulse_Response_SettlingTime: 1
                  Step_Response_RiseTime: 1
                             total_score: 5
```