

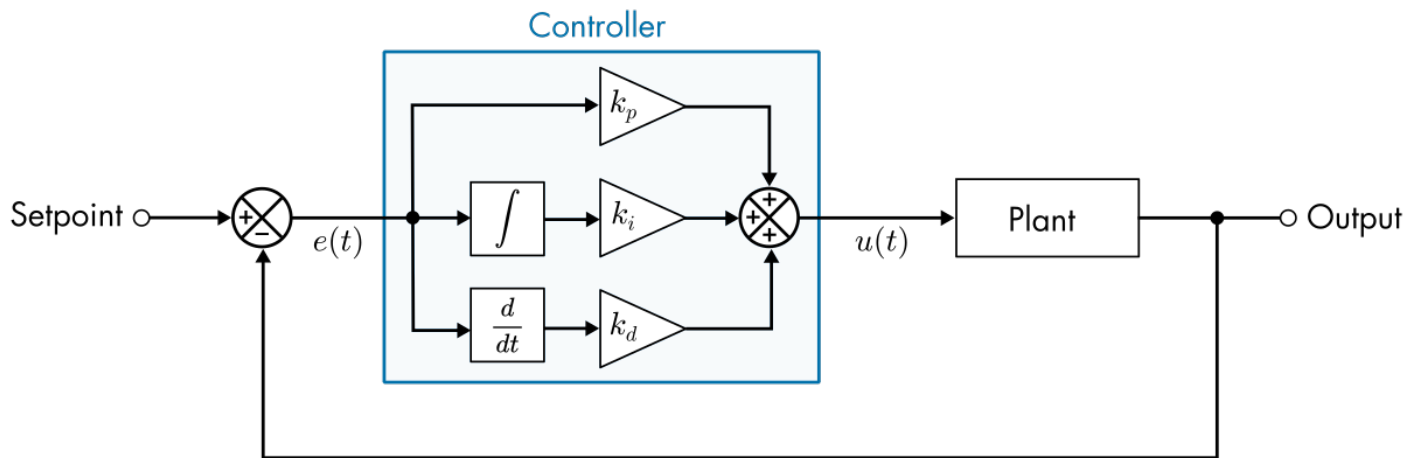
Position Control

Part I: DC motor position control implementation

In this part of the lab, you will implement the PID controller.

PID Control

A PID controller uses a feedback loop with proportional, derivative, and integral terms to calculate the control signal. Together, the controller, plant, and feedback loop are often referred to as a *closed-loop system*.



Block diagram of a PID controller. The control signal $u(t)$ is the sum of the P, I, and D terms.

Each of the PID terms is computed from the error, $e(t)$. The error is the difference between the setpoint and the current output value.

- The **P**roportional term, $P = k_p e$, is proportional to the error.
- The **I**ntegral term, $I = k_i \int_0^{\tau} e(\tau) d\tau$, is proportional to the integral of the error.
- The **D**erivative term, $D = k_d \frac{de}{dt}$, is proportional to the derivative of the error.

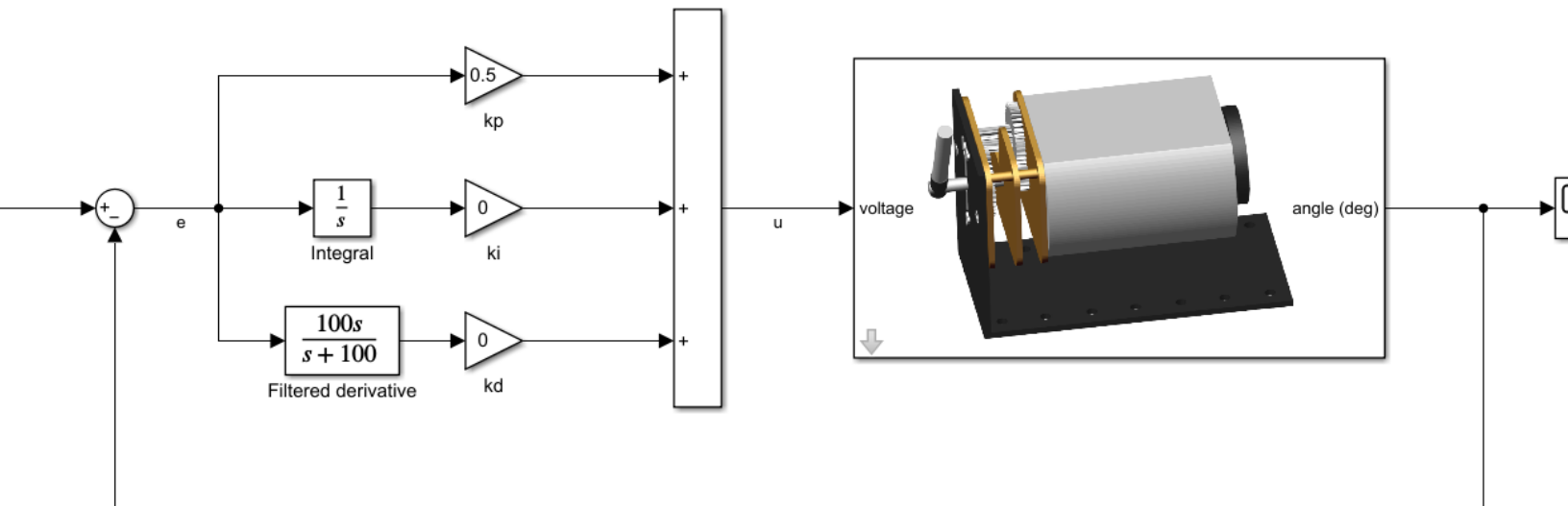
The control signal $u(t)$ is computed as the sum of all three terms:

$$u(t) = k_p e + k_i \int_0^{\tau} e(\tau) d\tau + k_d \frac{de}{dt}$$

The constants k_p , k_i , and k_d determine the relative influence of each term. Appropriately selecting these constants is essential to any PID control application.

Simulink Implementation

Task 1. To implement the PID controller in Simulink, open **DCMotor.slx** and add the controller to the model as shown in the diagram. For additional details, see the step-by-step instructions below.



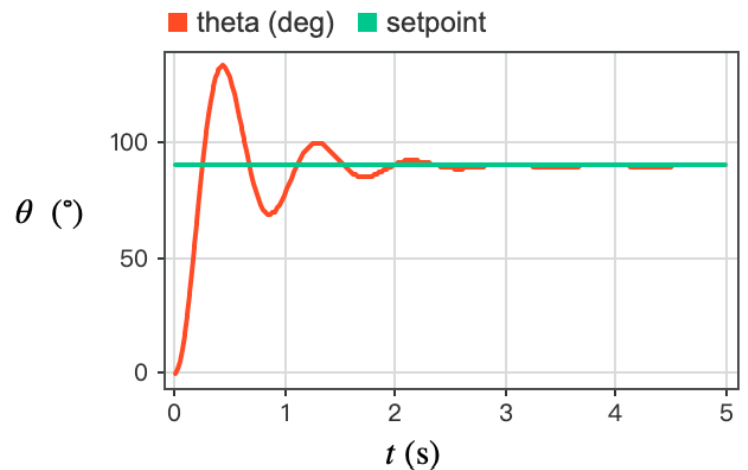
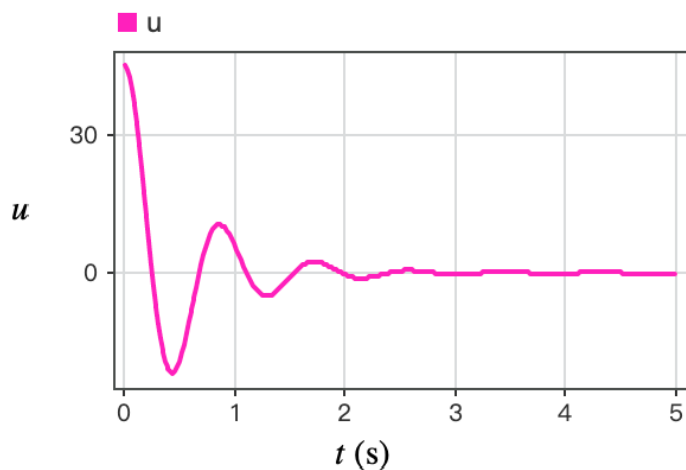
PID controller implemented using basic Simulink blocks. The integral is computed using an Integrator block and the filtered derivative is computed using a Transfer Fcn block.

Step-by-step instructions

1. Add a Constant block with a value of 90 (this represents the setpoint of 90°).
2. Add a Sum block. Double-click the Sum block to access the dialog. In the block dialog, set the **Icon shape** to **round** and the **List of signs** to **|+-**, so that the block appears as shown.
3. Add an Integrator block and connect it to the output of the Sum block. Notice that the block is labeled $1/s$, the Laplace domain representation of an integral.
4. Add a Transfer Fcn block. Double-click the Transfer Fcn block to access the dialog. In the dialog, set the numerator to $[100 \ 0]$ and the denominator to $[1 \ 100]$. This block computes and filters the derivative.
5. Add three Gain blocks for k_p , k_i , and k_d . Set the gains to $k_p = 0.5$ and $k_i = k_d = 0$.
6. Add an Add block and stretch it vertically by clicking and dragging one of the corners of the Add block. Then, connect all three gains to the Add block. To create a third plus sign, drag the third signal to the block. This will cause an additional input to automatically be created.
7. Add a Scope block to the plant output so that you can observe the angle.

Task 2. Test the controller.

- Double-click the motor image to show the plant parameters.
- Set the virtual DC motor parameters to **$B = 0.028$ N·m/(rad/s)**, **$K = 0.05$ V/(rad/s)**, **$L = 0.1$ H**, **$R = 0.25$ Ohm**, **Pendulum mass = 0.15 Kg**, **Pendulum length = 0.015 m**, and **Sensor noise gain = 0**. Also, turn voltage saturation off by unchecking the **Use voltage saturation** checkbox.
- Set the simulation **Stop Time** to **5**.
- Run the model. If correct, the output in the scope should appear as shown below. Notice how the arm oscillates back and forth around the setpoint before settling down in the Mechanics Explorer.



Result of running the PID controlled DC motor model with $k_p = 0.5$, $k_i = k_d = 0$. This plot characterizes the response of the closed-loop feedback system to a step input.

Part 2: Manual PID tuning

Time response characteristics

In the previous example, the motor overshoot the setpoint by more than 40° and took around 2 seconds to settle down to the target position of 90° . This happened because the PID parameters are not optimal for the system. The PID parameters can be tuned to improve the controller response. With a manual tuning approach, the controller gains are adjusted to improve several standard time-response characteristics. The essential quantities are:

- **Rise time:** Time it takes for the response curve $y(t)$ to rise from 10% to 90% of the steady-state value y_∞ .
- **Settling time:** Time it takes for the difference between the response curve $y(t)$ and the steady-state value y_∞ to fall below 2%.
- **Overshoot:** How far the peak value y_{peak} overshoots y_∞ . This is often expressed as a percentage:

$$100\% \cdot (y_{peak} - y_\infty)/y_\infty$$
- **Peak time:** Time at which the peak value occurs.

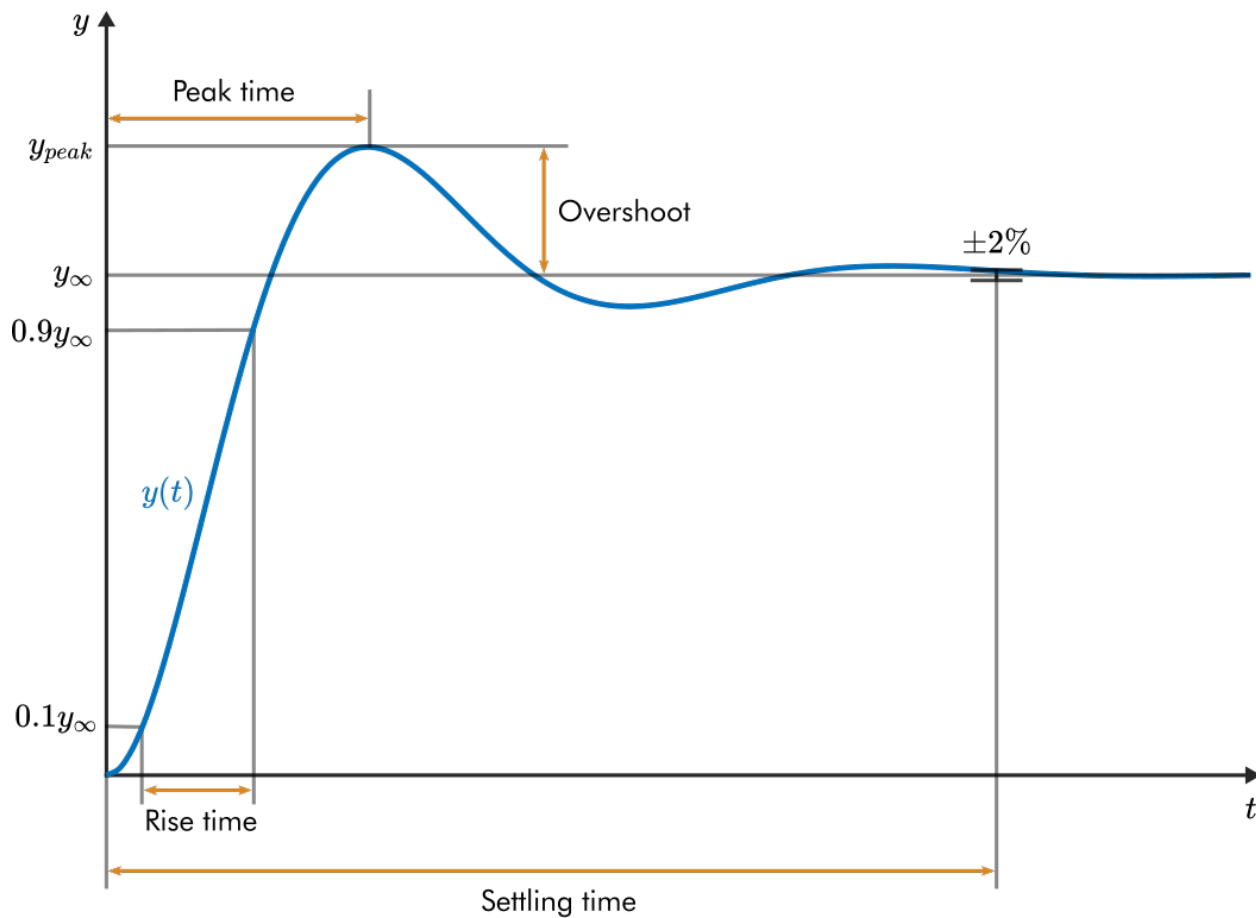


Illustration of the Time response characteristics of a second-order response.

Task 3. Estimate the step response characteristics of the controlled system you created in Simulink in the previous section. Continue to use the PID parameters $k_p = 0.5$, $k_i = k_d = 0$ and the model parameters $\mathbf{B} = 0.028 \text{ Nm}/(\text{rad/s})$, $\mathbf{K} = 0.05 \text{ V}/(\text{rad/s})$, $\mathbf{L} = 0.1 \text{ H}$, $\mathbf{R} = 0.25 \text{ Ohm}$, **Pendulum mass = 0.15 Kg**, **Pendulum length = 0.015 m**, and **Sensor noise gain = 0**. Also, turn voltage saturation off by unchecking the **Use voltage saturation** checkbox.

You can measure values in the scope using **Cursor Measurements**. From the Scope menu, select **Tools > Measurements > Cursor measurements**. Select and drag the vertical lines to change the measurement locations.

a. What is the peak value y_{peak} (round to the nearest degree)?

```
% Replace NaN with your answer
task3a = 133
```

```
task3a =
133
```

b. What is the settling value y_{∞} (round to the nearest degree)?

```
task3b = 90
```

```
task3b =  
90
```

c. What is the rise time in seconds (round to the nearest hundredth)?

```
task3c = 0.16
```

```
task3c =  
0.1600
```

d. What is the settling time in seconds (round to the nearest hundredth)?

```
task3d = 2.23
```

```
task3d =  
2.2300
```

e. What is the percent overshoot?

```
task3e = 0.4778
```

```
task3e =  
0.4778
```

f. What is the peak time in seconds (round to the nearest hundredth)?

```
task3f = 0.43
```

```
task3f =  
0.4300
```

Qualitative tuning of the PID parameters

In this section, you will manually tune the PID parameters to improve the position control response. Independent manual adjustment of each PID parameter leads to predictable changes in the response of the closed-loop system.

	Rise time	Overshoot	Settling time
Increase k_p	↓	↑	↑
Increase k_i	↓	↑	↑
Increase k_d	↓	↓	↓

Typical effects of independently adjusting the PID parameters on the response of the closed-loop system

The parameters have additional effects besides those listed. For example, increasing the integral gain reduces the steady-state error. So, in systems that are prone to steady-state error, it is often useful to increase k_i .

Task 4. Run the tuning model.

- Open the completed DC motor control model, **DCMotorControl.slx**.
- Double-click the motor image to show the parameters. Set the virtual DC motor parameters to **$B = 0.028$ Nm/(rad/s), $K = 0.05$ V/(rad/s), $L = 0.1$ H, $R = 0.25$ Ohm, Pendulum mass = 0.15 Kg, Pendulum length**

= 0.015 m, and **Sensor noise gain = 0**. Also, turn voltage saturation off by unchecking the **Use voltage saturation** checkbox.

- Run the model . Adjust the PID parameters three times using the dashboard controls.
- For each parameter set, flip the setpoint back and forth between 90° and 270° to observe the alterations to the step response in both the graphs and the physical model displayed in the Mechanics Explorer.

Task 5. In this task, you will tune the PID parameters to obtain a good response. Because the controller's goal is to reach the setpoint as quickly as possible, a good response will have short rise and settling times and relatively little overshoot. Run the model to get started. You can leave the model running for the duration of this task.

a. Start by setting all three PID parameters to zero. Then, gradually increase the proportional gain, k_p . Start with very small values of k_p (e.g., $k_p = 0.05$). What happens to the time-domain characteristics of the step response as k_p increases?

Hint: To observe the step response, start with a setpoint of 90° . Then, switch the setpoint dial to 270° . You can flip the setpoint back to 90° to observe another step response.

```
task5a = "As  $k_p$  gradually increases from 0.05, the response speed of the system will accelerate (the rise time will decrease), but overshoot will occur, and the amount of overshoot will increase with the increase of  $k_p$ . When  $k_p$  is small, the system response is slow but stable. When  $k_p$  is too large, the system will show obvious oscillation and overshoot."
```

```
task5a =
```

```
"As  $k_p$  gradually increases from 0.05, the response speed of the system will accelerate (the rise time will
```

b. Set k_p to 0.4 and then gradually increase the integral gain, k_i . Does this improve the performance of the controller? Why or why not?

```
task5b = "Adding  $k_i$  on the basis of  $k_p=0.4$  can improve control performance. Because the integration term can eliminate the steady-state error and enable the system to ultimately reach the set point precisely. However, excessive  $k_i$  will increase the overshoot amount and adjustment time, and it needs to be adjusted moderately."
```

```
task5b =
```

```
"Adding ki on the basis of kp=0.4 can improve control performance. Because the integration term can elimin
```

- c. Set k_p to 1 and set k_i to zero. Then, gradually increase the derivative gain, k_d . Does this improve the performance of the controller? Why or why not?

```
task5c = "Adding kd on the basis of kp=1 and ki=0 can significantly improve  
the control performance. The differential term has a damping effect, which  
can suppress overshoot and oscillation, reduce the adjustment time, and  
make the system response smoother and more stable."
```

```
task5c =
```

```
"Adding kd on the basis of kp=1 and ki=0 can significantly improve the control performance. The differentt
```

- d. With $k_p = 1$ and $k_i = 0$, what is a good value (i.e., one that has a short rise time without overshoot) of the derivative term gain, k_d ?

```
task5d = 0.1
```

```
task5d =  
0.1000
```

- e. Based on your observations, which controller appears to work best for this system?

- P controller (proportional control only)
- PI controller (proportional and integral terms)
- PD controller (proportional and derivative terms)

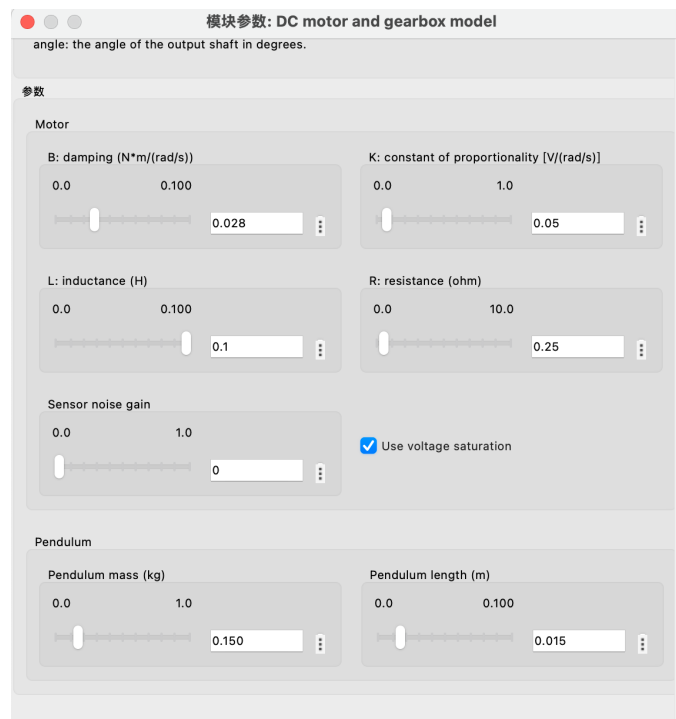
```
task5e = "PD"
```

```
task5e =  
"PD"
```

Task 6. Because the input motor voltage was unlimited in the last exercise, the motor was able to respond very quickly. In practice, the input voltage should be limited to the motor's nominal voltage (for example, 6V). There may also be noise in the position sensor readings.

1. Turn on the voltage saturation (double-click the motor image and check **Use voltage saturation**) and try retuning the PID parameters. Does it appear that the best parameters have changed?

slower response



2. Set the **Sensor noise gain** to 0.5 (double-click the motor image to access the dialog box). Does introducing sensor noise lead to noticeable issues?

Yes, introducing sensor noise can lead to significant problems. The main issues include: 1) High-frequency jitter and oscillation in the control output; 2) Increased wear and tear on the actuator (motor); 3) Reduced system stability; 4) Decreased measurement accuracy. Particularly, the differential term (kd) is highly sensitive to noise and amplifies high-frequency noise.