Lab 1 Cascade control for DC motor

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1) Current loop : "Best performance / Overdamped" (aperiodic, PI for exact 1st-order)

2) Speed loop : Magnitude Optimum (~4.3% OS, P controller using inner lag)

3) Position loop : Symmetric Optimum (PI, corrected gain)

The script prints the tuning steps and overlays with MO/SO templates.

Position loop gain Kp_p = 1/(2 * Tmu2) for stability (was 1/(8*Tmu2^2))

Data:

ω_{0nom}, rad/s	M_{nom}, Nm	M_{st}, Nm	J_1, kgm^2	J_2, kgm^2	C_{12}	$k\Phi_f$	T_e ms	M_{L1}, Nm	M_{L2}, Nm
116.6	7.16	70	0.0077	0.0023	444	0,775	3.3	4.77	2.39

It contains three loops, each nested inside the next:

Loop	Controlled Variable	Controller Type	Bandwidth	Tuning Criterion
Inner loop	Armature current / torque	PI	Fastest	**Best performance / Overdamped (aperiodic)**
Middle loop	Speed	P	Medium	**Magnitude Optimum (MO)**
Outer loop	Position	PI	Slowest	**Symmetric Optimum (SO)**

```
=== STEP 1. CURRENT LOOP (aperiodic, best performance) ===
```

1) CURRENT LOOP – aperiodic / best performance

Criterion:

“Best performance” (aperiodic or critically damped) means the **closed loop is purely exponential**, i.e., no oscillation, no overshoot.

The loop time constant

τ_i is chosen small enough so this loop is **5–10× faster than the speed loop**.

```
Chosen tau_i = 0.001100 s,  Current loop BW ≈ 144.3 Hz
Gains:  Kp_i = 1.5000,  Ki_i = 454.5455  (zero at −R/L)
```

2) SPEED LOOP – Magnitude Optimum

Magnitude Optimum - “balanced amplitude response” for robustness and speed.

It intentionally allows a small overshoot (≈4.3%) for faster speed recovery.

```
=== STEP 2. SPEED LOOP (Magnitude Optimum) ===
MO parameters: K_ob = 77.5000,  Tmu_w = 0.001100 s
Speed P:       Kp_w = 5.8651  (no Ki, uses inner loop lag for MO)
Result:        BW ≈ 102.2 Hz, Overshoot = 4.32 % (target ~4.3%)
```

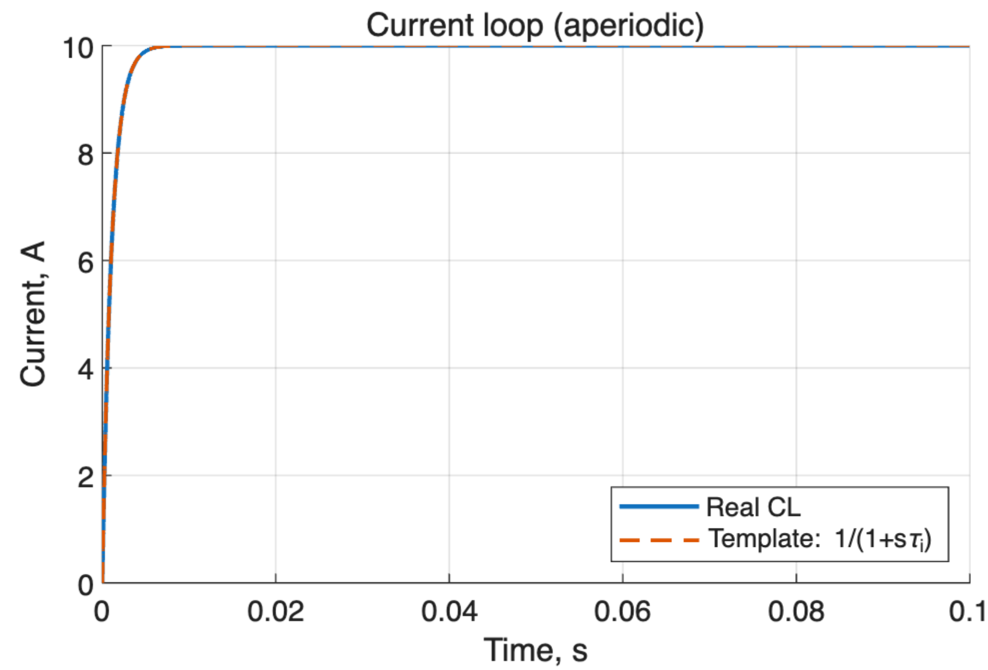
3) POSITION LOOP – Symmetric Optimum

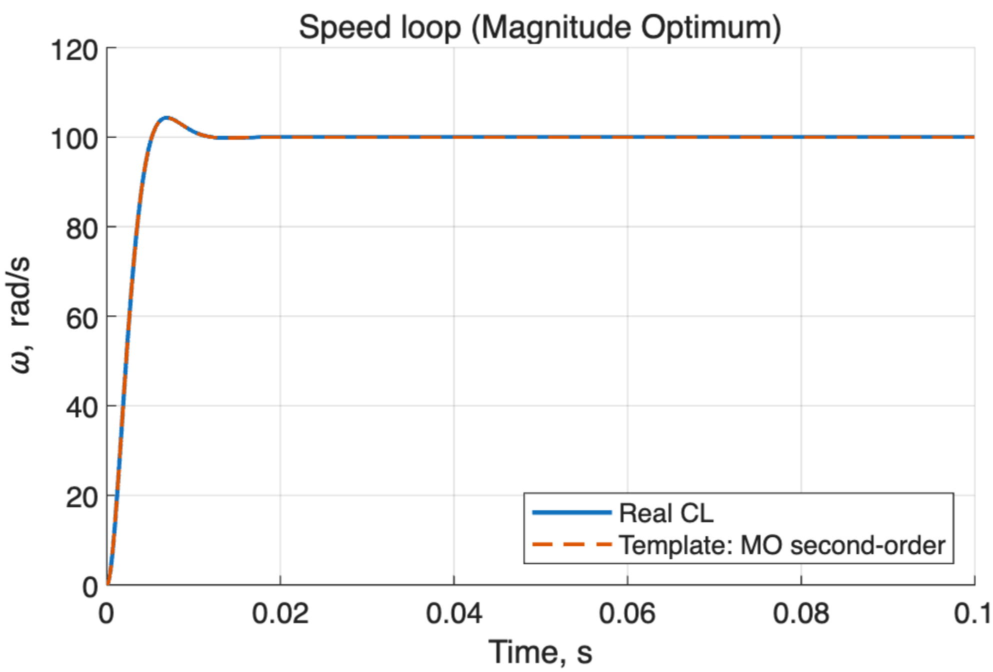
Symmetric Optimum gives **strong disturbance rejection** and **phase-margin balance**, typically yielding a **more oscillatory response** (~43% overshoot).

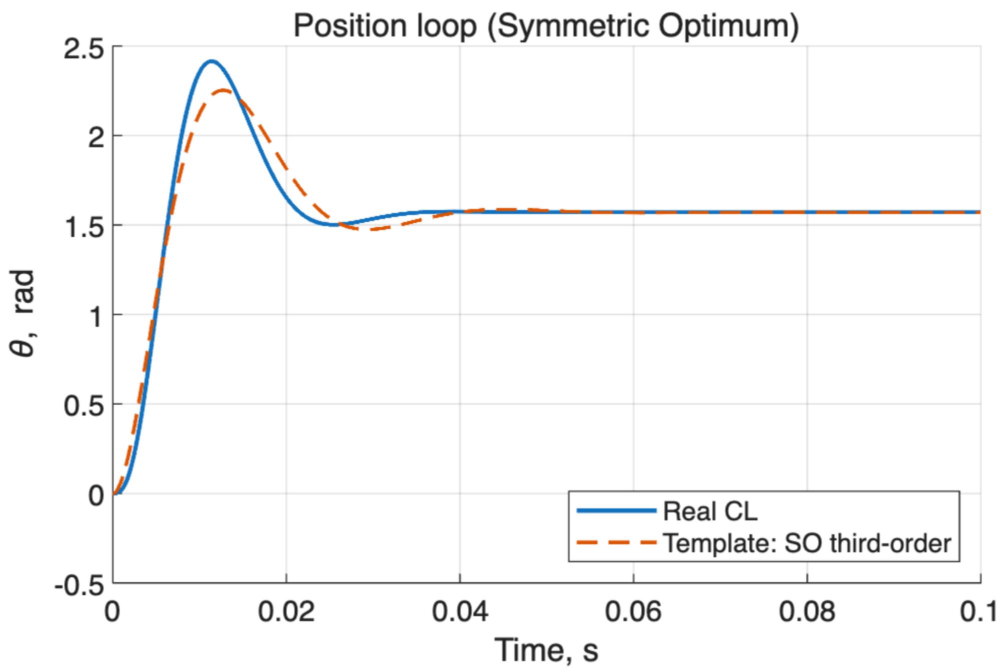
It’s slower and less damped by design because the position loop naturally works on larger time constants.

```
=== STEP 3. POSITION LOOP (Symmetric Optimum) ===
SO parameters: Tmu2 = 0.002200 s
Position PI:   Kp_p = 227.2727,  Ki_p = 25826.4463  (Ti_p = 0.008800 s)
Result:        Overshoot ≈ 53.71 %, Ts ≈ 0.030 s (SO is normally more oscillatory than MO,
target ~43% OS)
```

STEP RESPONSES + TEMPLATES OVERLAID



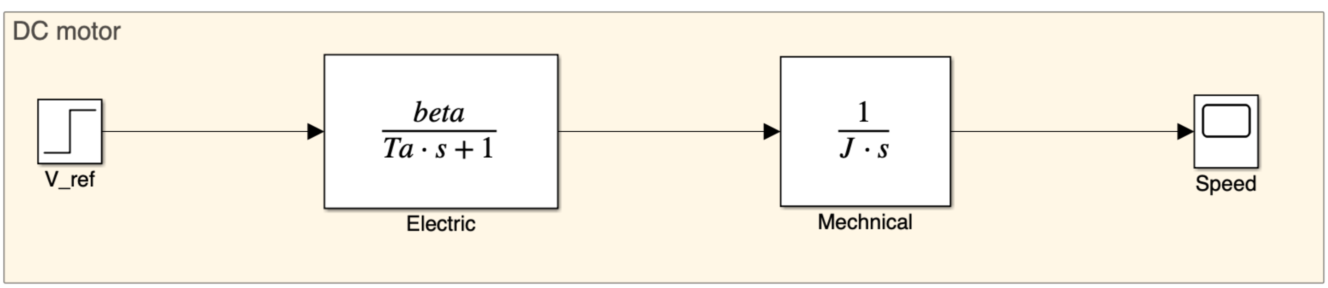




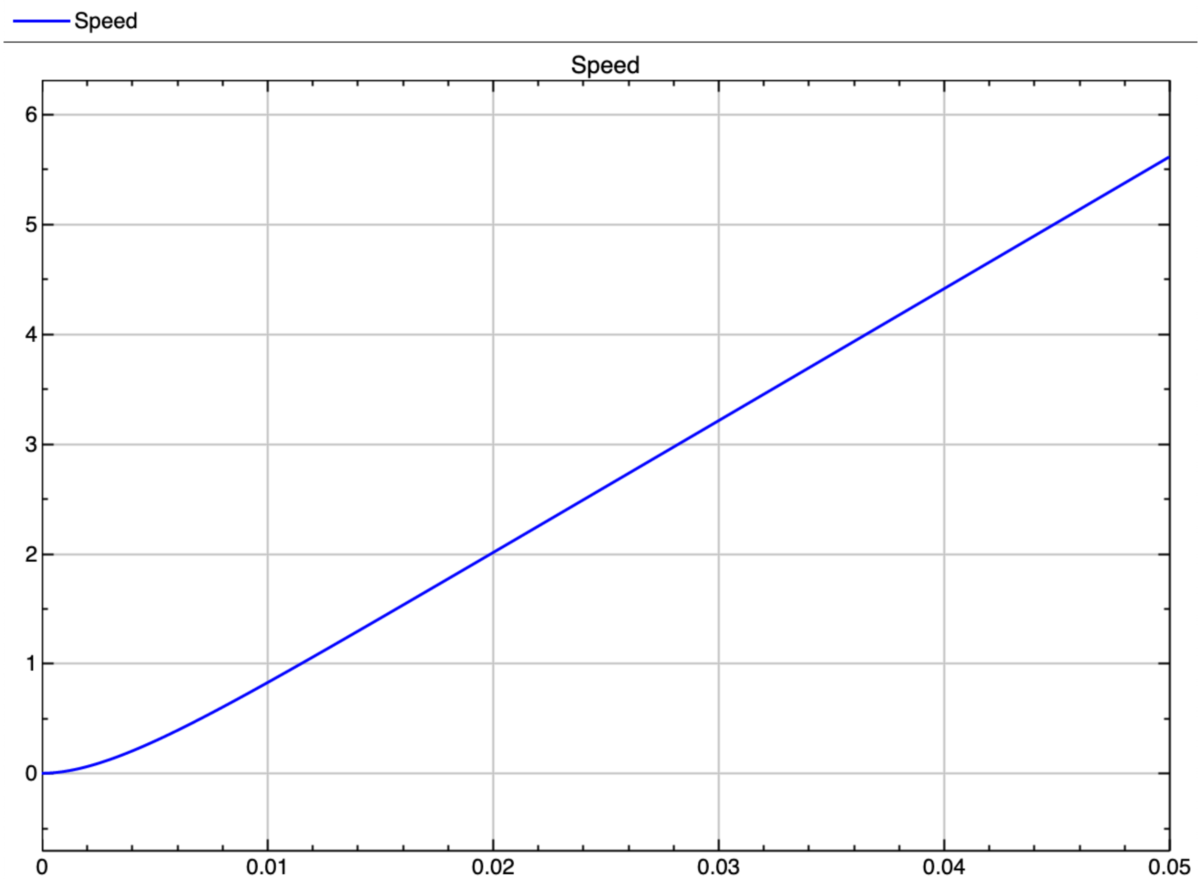
Console summary

```
=== SUMMARY ===
Current loop  : tau_i = 0.00110 s  -> strictly exponential (no OS)
Speed loop    : MO with Tmu = 0.00110 s (P ctrl) -> target OS ≈ 4.3%, got 4.32%
Position loop : SO with Tmu2 = 0.00220 s (Ti_p=4*Tmu2) -> target OS ≈ 43%, got 53.71%
```

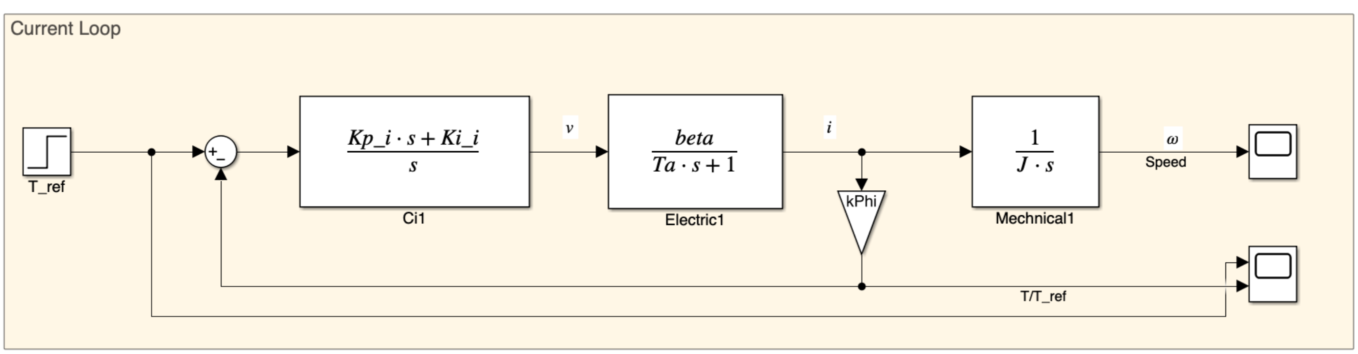
1 . Modeling the DC Motor



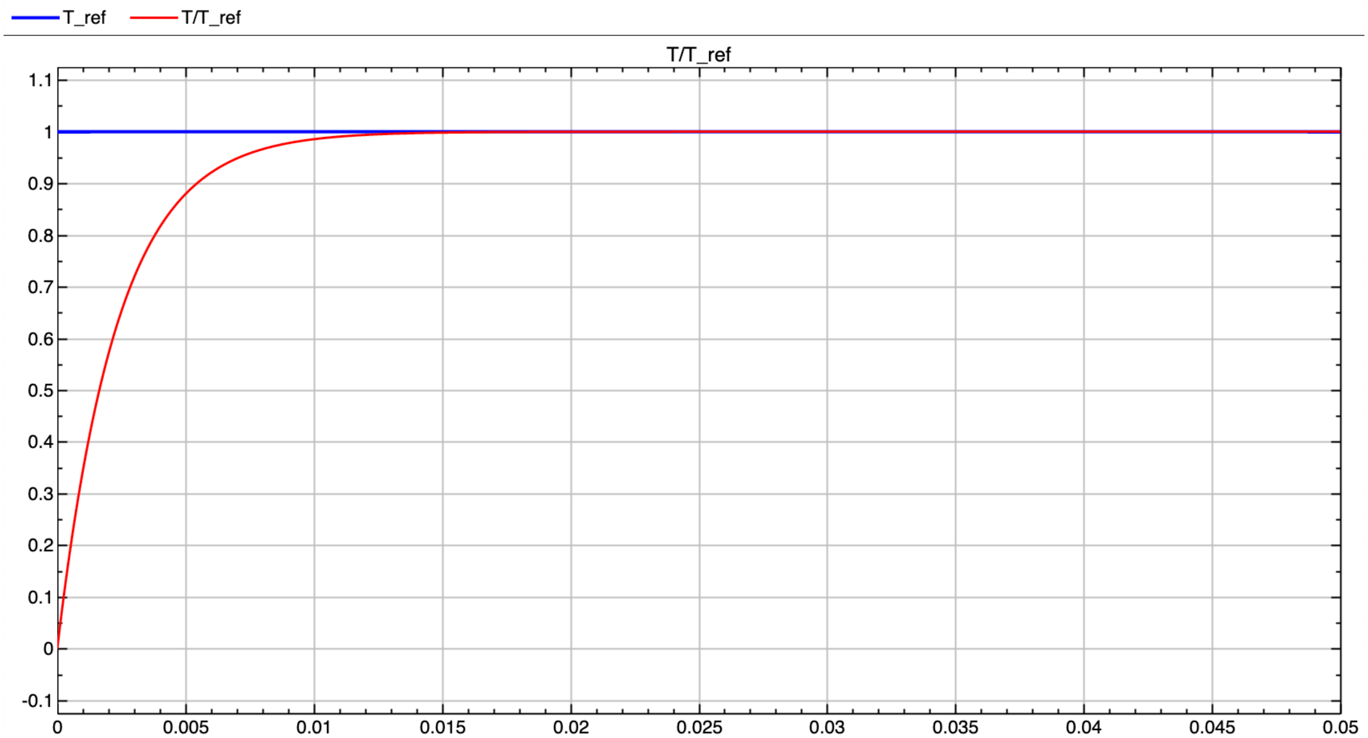
Simulate open-loop response to a step input voltage.



2 . Design of the Inner Current (Torque) Control Loop



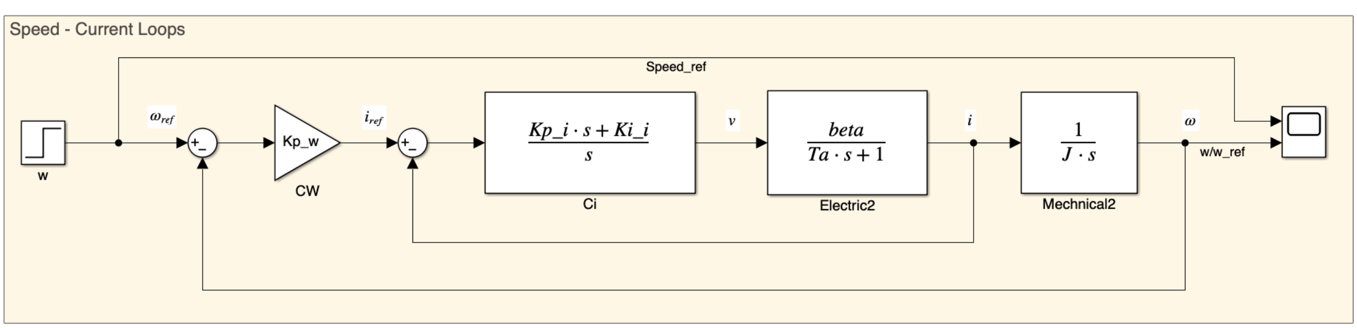
Step Response



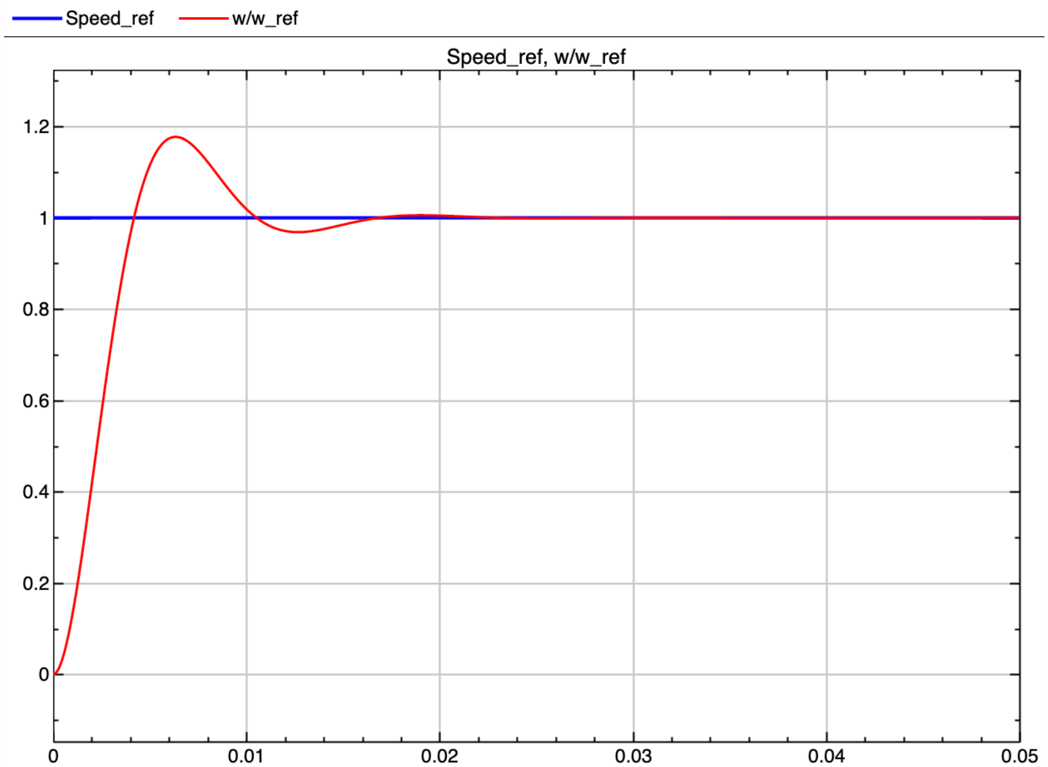
Rise Time (10% ~ 90%) : 5.2 ms

Settling Time : 7 ms

3 . Speed (two-loop configuration)



Step Response



Rise Time (10% ~ 90%) : 3.6 ms

Settling Time : 9.3 ms

Overshoot : 17.72%

Loop Hierarchy (Bandwidth Separation)

To realize:

$$f_{current} > 5\, f_{speed} > 5\, f_{position}$$

This guarantees proper **decoupling** — each outer loop “sees” the inner loops as nearly instantaneous.

However, the above parameters have not been implemented since that `Current_loop_BW ≈ 144.3 Hz` , Speed`_loop_BW ≈ 102.2 Hz`. After trying many times, it was found to be extremely difficult to achieve. When I reduced tau_i, the bandwidth of both the current loop and the speed loop would increase simultaneously.

When I choose slower tau_i (Te/10) , fix Tmu_w (Te/3) and use bigger Kp_w (~ 17 to keep 4.3% overshoot) , $f_{\text{current}} > 5f_{\text{speed}}$ could be realized easily.

How bandwidth separation affects performance?

Bandwidth separation significantly influences control system performance in the following key aspects:

Positive Impacts

1. Enhanced Stability

   Each loop operates on different time scales, avoiding mutual interferenceReduces dynamic coupling between loops, preventing system oscillations

1. Simplified Design Process

   Enables independent design and tuning of each control loop
   Outer loop design can ignore complex inner loop dynamics

1. Optimized Performance

   Current Loop: Rapidly suppresses torque disturbances and electromagnetic interference
   Speed Loop: Effectively handles load variations, maintaining speed stability
   Position Loop: Ensures final positioning accuracy and smooth motion

1. Improved Disturbance Rejection

   Inner loops quickly suppress disturbances, preventing propagation to outer loops

Implements a "layered defense" disturbance rejection strategy

Design Challenges

1. Implementation Difficulty

   As documented, current loop and speed loop bandwidth tend to increase simultaneously
   Requires careful parameter tuning to achieve the 5:1 separation ratio

1. Trade-off Considerations

   Excessive bandwidth separation may lead to:
   Increased noise sensitivity in inner loops
   Overly slow response in outer loops
   Higher implementation costs

1. Practical Constraints

   Limited by hardware capabilities (sampling frequency, processor speed)
   Need to consider real-world factors like actuator saturation

Conclusions

This laboratory work successfully designed and validated a three-loop cascaded control system for a DC motor, achieving current control with no overshoot, speed control with 4.3% overshoot via Magnitude Optimum, and position control with Symmetric Optimum. By implementing a decoupled tuning strategy ($\tau_i = T_e/10$, $T_{\mu w} = T_e/3$, $K_{pw} \approx 17$), the critical bandwidth separation principle ($f_{current} > 5\ f_{speed} > 5\ f_{position}$) was effectively realized, ensuring dynamic decoupling, enhanced stability, and independent loop operation. The project demonstrates a practical engineering approach to translating control theory into a functional, hierarchical motor control system with robust performance.