Work №1. SIMPLE EXAMPLE OF ADAPTIVE CONTROLLER DESIGN

Problem statement. Consider the scalar plant

$$\dot{x} = \theta x + u\,, \tag{1.1}$$

where x is the state variable (coincides with the output), u is the control, θ is the unknown scalar parameter.

The objective is to design a control that will compensate the influence of θ on the closed-loop system stability and ensure the limiting equality

$$\lim_{t\to\infty}\left(x_m(t) - x(t)\right) = \lim_{t\to\infty}\varepsilon(t) = 0\,, \tag{1.2}$$

where $\varepsilon = x_m - x$ is the control error, x_m is the output of the reference model

$$\dot{x}_m = -\lambda x_m + \lambda g\,, \tag{1.3}$$

with a piece-wise continuous and bounded reference signal g and the parameter $\lambda > 0$ responsible for the transient performance of closed-loop system after adaptation process.

Problem solution. Solution consists of the adjustable control

$$u = -\hat{\theta}x - \lambda x + \lambda g\,. \tag{1.4}$$

and the adaptation algorithm

$$\dot{\hat{\theta}} = -\gamma x\varepsilon \tag{1.5}$$

with adaptation gain $\gamma > 0$.

The order of the work

1. Using the parameters of the nonlinear modification of the plant (1.1) given by

$$\dot{x} = \theta f(x) + u\,,$$

where $f(x)$ is a continuous function, and the reference (1.3) given in Table 1 design the corresponding nonadaptive controller.

Make a simulation experiment, in which the plant parameter θ is increased rapidly (controller **parameters are not changed**) so that the closed-loop system looses stability. Plot separately the variable x together with x_m and variable u.

2. Design the adaptive control and repeat Experiment №1 with changing plant parameters and $\hat{\theta}(0) = 0$. Adaptation gain γ is selected experimentally based on better representation of results. Plot separately the variable x together with x_m, variable u and additionally variable $\tilde{\theta} = \theta - \hat{\theta}$.

3. Make experiment for different gains γ.

4. Make conclusions after each step of the work.

Table 1. Conditions for controllers design

№	Plant parameter θ	Nonlinear function $f(x)$	Ref. model parameter λ	Reference $g(t)$
1	27	$\lvert x \rvert$	1	5
2	33	x^2	2	$\cos 5t + 4$
3	16	$\dfrac{1}{x^2+1}$	3	$\sin 2t + 6$
4	0	$\sin x$	4	9
5	5	$\sin x + 1$	5	$sign(\cos t) + 2$
6	6	x^3	6	$3sign(\sin t) + 4$
7	7	$x^2 + x + 1$	7	$sign(\cos t) + 2$
8	8	$\cos x + 1$	8	$\cos t + \sin t$
9	9	$\sin t + \sin x$	9	1
10	30	$\ln(1 + x^2)$	10	$2 + \sin t$
11	-2	$x^2 + x$	4	$\sin(t+1) + 6$
12	1	$sign(x)$	1	$sign(\sin 0,5t) + 3$
13	14	$x \sin x$	14	$-4\sin t + 5$
14	27	$\sin^2 x$	6	$-\cos t + 2$
15	29	x^4	6	$\cos 2t \sin t + 1$
16	12	$\dfrac{x^2}{x^2+1}$	2	$\cos 2t + \sin t$
17	13	$x \cos x$	3	$sign(\sin 0,5t) + 3$
18	14	$\sin x - 1$	4	$-\sin t + 2$
19	27	x^5	5	$sign(\cos t) + 2$
20	29	$x^3 + x^2 + x + 1$	6	$2 + \sin\left(t + \dfrac{1}{4}\right)$
21	2	$\sin x^3$	1	$3\sin t$
22	3	$e^x - 1$	2	$\cos 5t + 4$
23	4	$\tanh x$	3	$\sin 2t + 6$
24	17	$\dfrac{x}{x^2+1}$	4	$\sin t + \sin 2t + 3$
25	21	$sign(0.5x) + sign(x)$	5	$sign(\cos t) + 2$
26	25	$x^3 \sin x$	6	$3sign(\sin t) + 4$
27	20	$x - x^3$	7	$3\sin t + 10$

28	15	$\dfrac{1}{2+\sin x}$	8	$2\sin t+2$
29	10	$\ln(x^2+1)$	9	$4sign(\sin t)+7$
30	21	x^x	1	$3\sin 0,5t+3$
31	1	$\sin(x^2+1)$	2	$\sin t+2\cos 2t$
32	2	x^3	3	$10\sin t$
33	3	$-x-1$	4	$3\cos t+2\sin 4t$
34	4	$\dfrac{e^x}{1+e^{-x}}$	5	$\cos t+2$
35	17	$x+\sin x$	6	$-\cos t+2$
36	21	$\ln\left(\dfrac{1}{x^2+1}\right)$	1	$4\sin t\sin 2t$
37	24	$-x^3$	2	$1-\sin 3t$
38	25	$-x^2-x-1$	3	$2+\sin 4\boldsymbol{t}$
39	22	$\cos x+1$	4	$2sign(\sin t)+3$
40	11	$\ln x^x$	5	$\sin t\cos 2t$
41	8	$\ln(2+x^2)$	1	$4sign(\sin t)+5$
42	9	x^2+x	2	$9\sin t+12$
43	5	$sign(\sin x)$	3	$\sin t+3$
44	20	$x^3\sin x$	4	$3^{\sin t}$
45	25	$\sin^4 x$	5	$7+4sign(\cos 3t)$
46	16	x^6	6	$\cos t+2$
47	17	$\dfrac{1}{\sin x+2}$	7	$sign(\cos t)+2$
48	48	$x-\cos x$	8	$\cos t+\sin t$
49	19	$\sin x^2$	9	1
50	24	$x^2\cos x$	10	$2+\sin t$