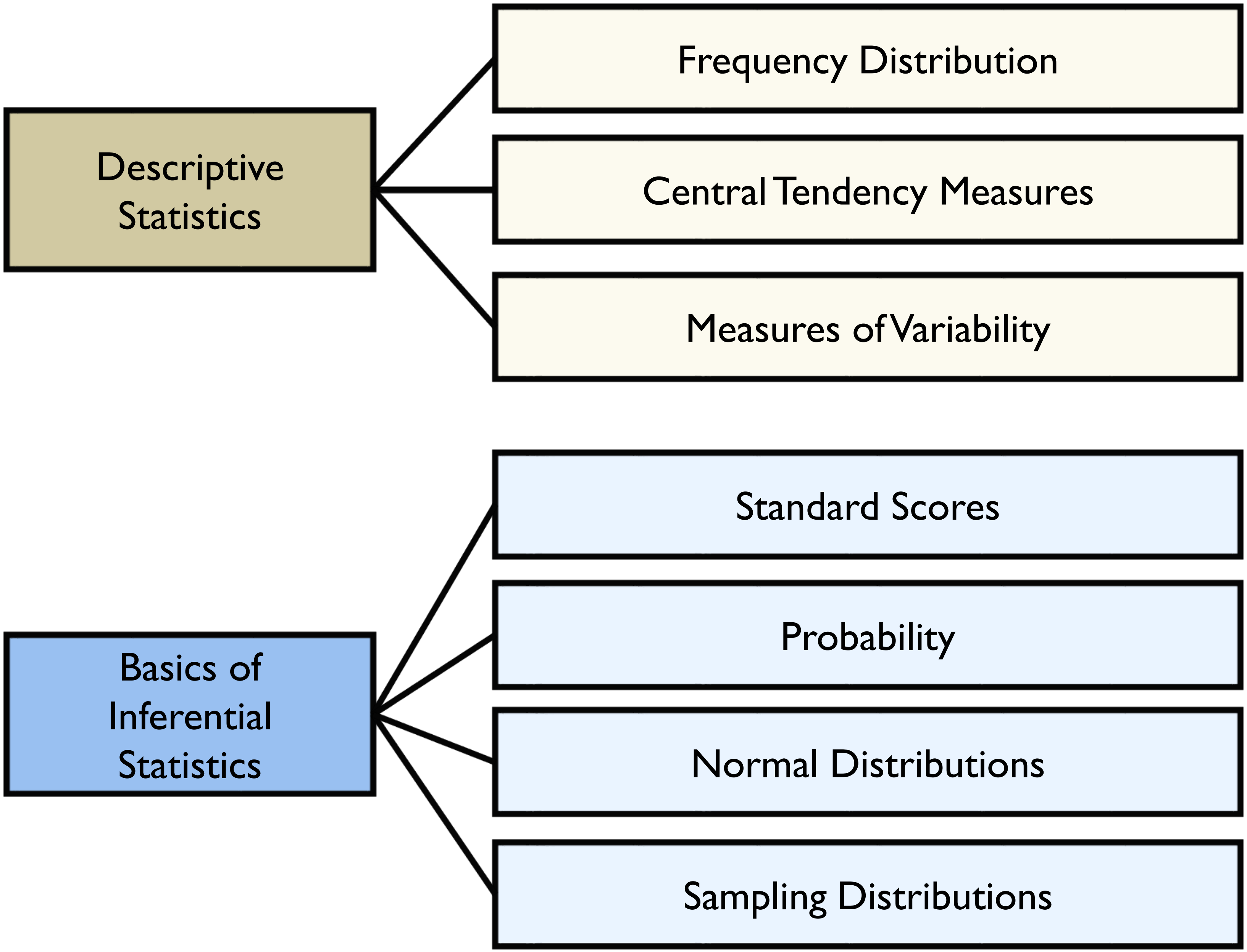
LECTURE 6
BASICS OF INFERENTIAL STATISTICS: STANDARD SCORES & PROBABILITY

PSY2002

Hye Won Suk


Descriptive Statistics
Frequency Distribution
Central Tendency Measures
Measures of Variability
Basics of Inferential Statistics
Standard Scores
Probability
Normal Distributions
Sampling Distributions

WHY DO WE STANDARDIZE SCORES?

- Example:
  - A student received a score of 30 on a math test.
  - She also received a score of 50 on a reading test.

  - Is 30 a high or low math score?
  - Is 50 a high or low reading score?

  - In educational and psychological testing, raw test scores may not be generally interpretable by themselves, but derive their meaning by comparison with other relative information, i.e., the responses of other people.

WHY DO WE STANDARDIZE SCORES?

- Example:
  - A student received a score of 30 on a math test.
  - She also received a score of 50 on a reading test.

  - Consider the following case.
    - $\mu_{MATH} = 10$
      30 is above the population mean. Her math ability is above the population average.

    - $\mu_{READING} = 65$
      50 is below the population mean. Her reading ability is below the population average.

WHY DO WE STANDARDIZE SCORES?

- Example:
  - A student received a score of 30 on a math test.
  - She also received a score of 50 on a reading test.

  - Consider the following case.
    - $\mu_{\text{MATH}} = 20$
      30 is above the population mean. Her math ability is above the population average.

    - $\mu_{\text{READING}} = 35$
      50 is above the population mean. Her reading ability is also above the population average.

WHY DO WE STANDARDIZE SCORES?

- Example:
  - A student received a score of 30 on a math test.
  - She also received a score of 50 on a reading test.

  - Consider the following case.
    - $\mu_{MATH} = 20$, $\sigma_{MATH} = 5$
      30 is above the population mean by 10. Her math ability is above the population average by 2 standard deviations.

    - $\mu_{READING} = 35$, $\sigma_{READING} = 3$
      50 is above the population mean by 15. Her reading ability is above the population average by 5 standard deviations.

WHY DO WE STANDARDIZE SCORES?

- As shown in the previous examples, raw scores are not easily interpreted often times.

- To address this issue, we can transform the scores to a new metric (standard deviation unit) that is more easily interpreted.

Z-SCORE

- In populations: $$\boxed{Z_i = \frac{X_i - \mu}{\sigma}}$$

- In samples: $$\boxed{Z_i = \frac{X_i - \bar{X}}{s}}$$

- The Z-score tells us how far the score is from the mean in standard deviation units.
- Z-scores are also called standardized scores.

EXAMPLE 1

- IQ scores are known to be distributed with $\mu = 100$ and $\sigma = 15$. If my IQ score is 130, how high is my IQ score compared to other persons? Find the Z-score for my IQ score.

EXAMPLE 2

- A student took 2 classes last semester. In class A, her total score was 25; In class B, she got a total score of 20. Given the following information, calculate her Z-score in each class and find if she performed better in class A or class B.

  - Class A: $\bar{X}$ = 27, s = 2
  - Class B: $\bar{X}$ = 18, s = 4

EXAMPLE 3

- A national university entrance exam is known to be distributed with $\mu = 100$ and $\sigma = 20$. I got a standard score of 1.5. Find my original score.

EXAMPLE 4

- In a class, students whose final scores are lower than 50 will fail. I got to know that my final score is 2 standard deviations *below* the mean. The class mean is $\bar{X} = 60$ and the class standard deviation is $s = 4.$ Am I going to fail or not?

PROPERTIES OF Z-SCORES

- If we transform each and every score in a distribution into the corresponding Z-score:

  - The mean of Z-scores is always 0.
  - The variance of Z-scores is always 1.

- The following slides explain why for population Z-scores. However, this property holds for both population Z-scores and sample Z-scores.

PROPERTIES OF Z-SCORES

- If we add a constant to each and every score, the mean is also added by the same amount.
  - Mean of $X_i = \mu$
  - Mean of $(X_i - \mu) = \mu - \mu = 0$

- If we multiply each and every score by a constant, the mean is also multiplied by the same amount.
  - Mean of $(X_i - \mu) = 0$
  - Mean of $\frac{(X_i - \mu)}{\sigma} = \frac{0}{\sigma} = 0$

- The mean of Z-scores is always 0.

PROPERTIES OF Z-SCORES

- If we add a constant to each and every score, the variance will not change.
  - Variance of $X_i = \sigma^2$
  - Variance of $(X_i - \mu) = \sigma^2$

- If we multiply each and every score by a constant, the variance will be multiplied by the squared constant.
  - Variance of $(X_i - \mu) = \sigma^2$
  - Variance of $\frac{(X_i - \mu)}{\sigma} = \frac{1}{\sigma^2} \cdot \sigma^2 = 1$

- The variance of Z-scores is always 1.
- The standard deviation of Z-scores is always 1.

PROPERTIES OF Z-SCORES

- In any distribution, if we transform each and every score into the corresponding Z-score, the mean of the Z-scores is always 0 and the variance of the Z-scores is always 1.

- Regardless of the original measurement unit,
  - Z-scores are centered at the mean (i.e., 0 indicates the mean);
  - Z-scores are in the standard deviation unit (i.e., a one-unit increase in Z-score indicates 1 standard deviation increase).

- Therefore, Z-scores are often considered unitless.

STANDARD SCORES (Z-SCORES)

- Standard scores are useful for locating a raw score compared to other scores in the distribution.

- However, we may need more information to interpret the substantial meaning of the scores.

- For example,
  - I got a blood test and my Hemoglobin level was 16g/dl. Is it normal? Or is this something that I should worry about?

  - I took an English speaking test and my score was 54. How good or bad is it? Am I qualified for teaching a class?

STANDARD SCORES (Z-SCORES)

- Example 1: Hemoglobin = 16 g/dl

  - The Hemoglobin levels of healthy persons are known to be distributed with a mean of 18.8 g/dl and a standard deviation of 1.4 g/dl.

$$\mu = 18.8, \sigma = 1.4$$

$$Z_i = \frac{X_i - \mu}{\sigma} = \frac{16 - 18.8}{1.4} = \frac{-2.8}{1.4} = -2$$

  - The Z-score for my Hemoglobin level is 2 standard deviations below the population average.
  - Okay, it is lower than the average. But, how bad is it? Am I anemic?

STANDARD SCORES (Z-SCORES)

- Example 1: $Z_i = -2$; how bad is it?

  ➔ How much proportion of the entire population has a Z-score below -2?

- If 30% of the entire population has a Z-score below -2 (or Hemoglobin level below 16 g/dl), then I might not need to worry about my low Hemoglobin level. My Hemoglobin level would be normal.

- However, if only 0.01% of the entire population manifests a Z-score below -2 (or Hemoglobin level below 16 g/dl), then this indicates I am severely anemic and need a treatment.

STANDARD SCORES (Z-SCORES)

- Example 2: English speaking test score = 54

  - The test scores are known to be distributed with a mean of 45 and a standard deviation of 6.

$$\mu = 45, \sigma = 6$$

$$Z_i = \frac{X_i - \mu}{\sigma} = \frac{54 - 45}{6} = \frac{9}{6} = 1.5$$

  - The Z-score for my English speaking test score is 1.5 standard deviations above the population average.
  - Okay, my score is above the average. But, how good is it? Am I good enough to be qualified for teaching a class?

STANDARD SCORES (Z-SCORES)

- Example 2: $Z_i = 1.5$; how good is it?

  ➔ How much proportion of the entire population has a Z-score above 1.5?

- If 45% of the entire population has a Z-score higher than 1.5 (or test score higher than 54), then my English speaking is not good enough to be qualified for teaching. I am top 45%.

- However, if only 1% of the entire population manifests a Z-score above 1.5 (or test score above 54), then this indicates I am very fluent in English and qualified for teaching. I am top 1%.

PROBABILITY

- How can we obtain these proportions?
- Let's think about simple examples first.
- In a class, there are 60 female and 40 male students. What is the proportion of female students?

  ➔ 60% or 0.60

$$\text{proportion(female)} = \frac{\text{number of female students}}{\text{total number of students}}$$

$$= \frac{60}{40+60} = \frac{60}{100} = 0.60$$

PROBABILITY

- The terms proportion and probability are often used interchangeably.

- In a class, there are 60 female and 40 male students. If we randomly choose a student from this class, what is the probability of choosing a female student?

$$\boxed{\text{probability of A} = \frac{\text{number of outcomes classified as A}}{\text{total number of possible outcomes}}}$$

$$\text{probability(female)} = \frac{60}{100} = 0.60$$

PROBABILITY

- More generally, if you are given the frequency distribution of scores, you can obtain the probability of getting a score
  - below a specific value
  - above a specific value
  - between two specific values

$$\text{probability of A} = \frac{\text{frequency of outcomes classified as A}}{\text{total frequency in the distribution}}$$

PROBABILITY

- My Hemoglobin level = 16 g/dl

Hemoglobin (g/dl)

X	f
24-25.999	1
22-23.999	5
20-21.999	57
18-19.999	80
16-17.999	45
14-15.999	10
12-13.999	2

- How much proportion of the persons in this distribution has a Hemoglobin level below 16?

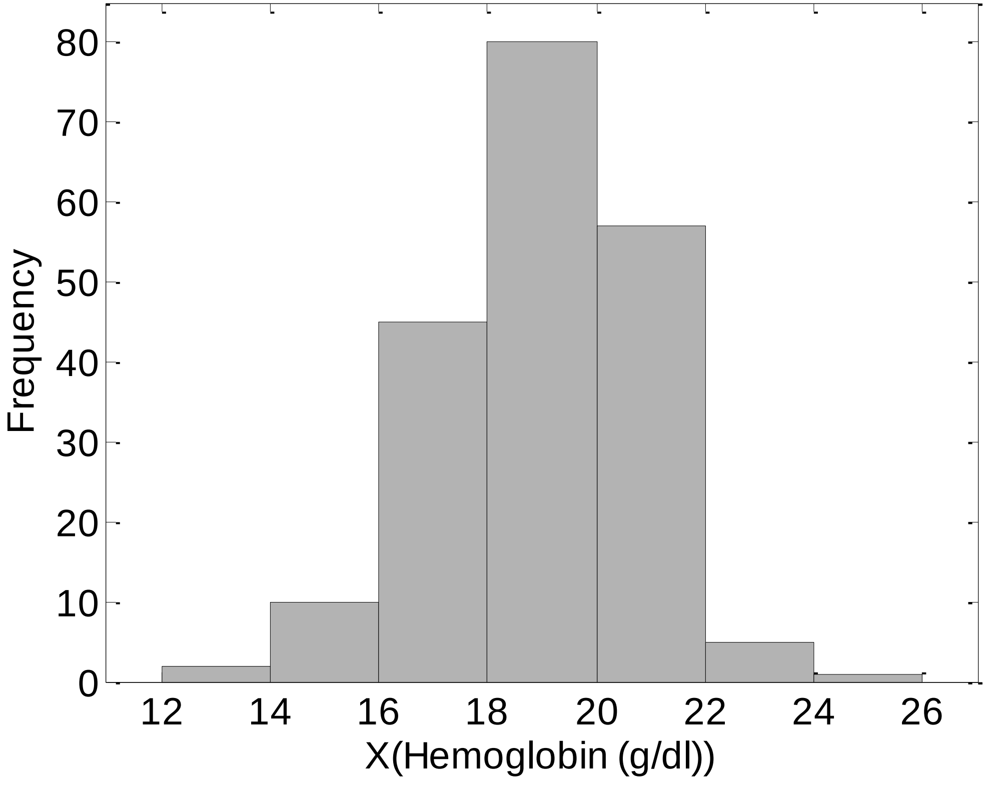
- What is the probability of choosing a person from this distribution who has a Hemoglobin level below 16?

$P(X < 16) =$

- My Hemoglobin level is in the

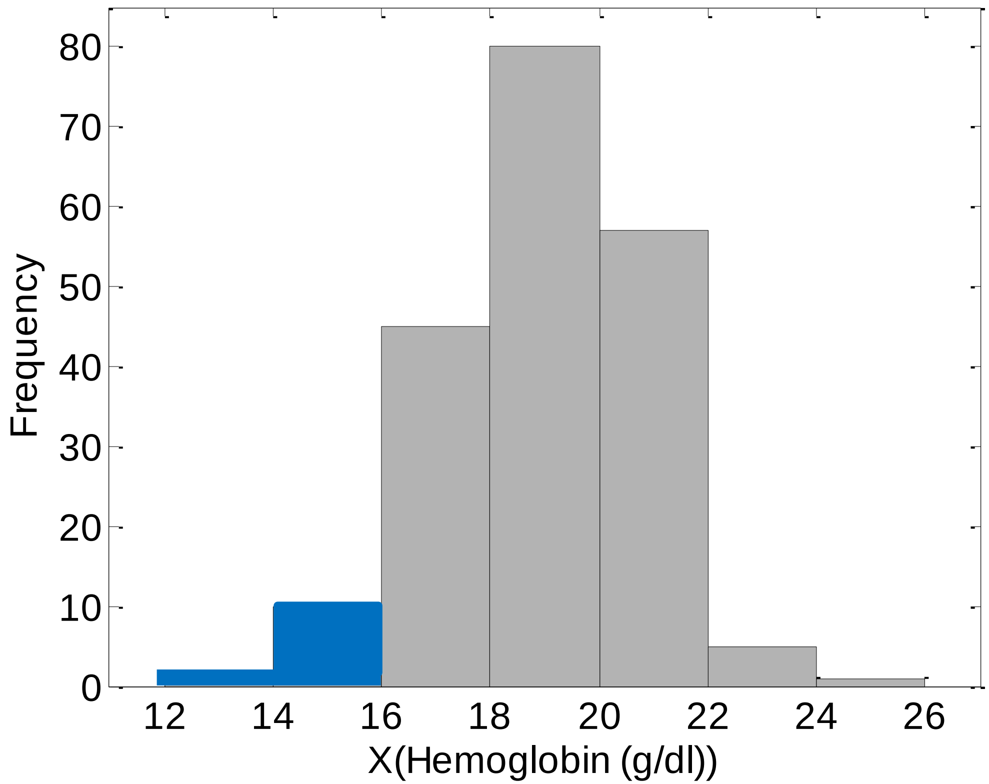
PROBABILITY

- The area under the histogram is related with probability.



PROBABILITY

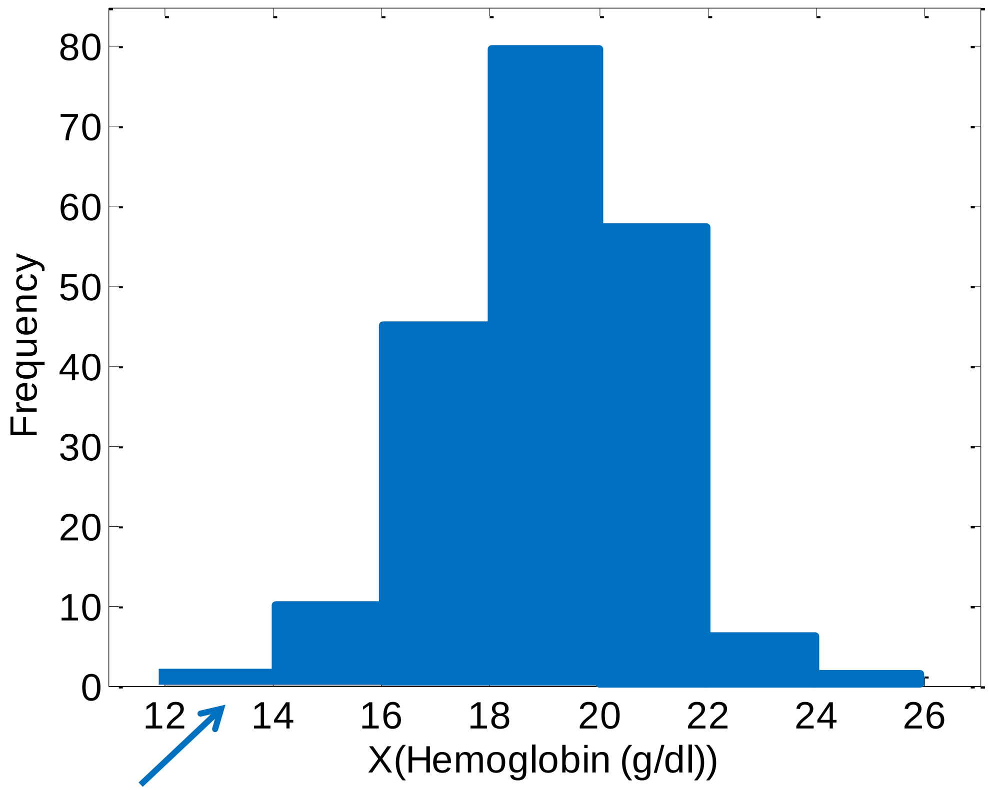
- The <u>area under the histogram</u> is related with probability.



The area under the histogram corresponding to X<16
= (2)(2)+(2)(10) = 4+20 = 24

PROBABILITY

- The <u>area under the histogram</u> is related with probability.



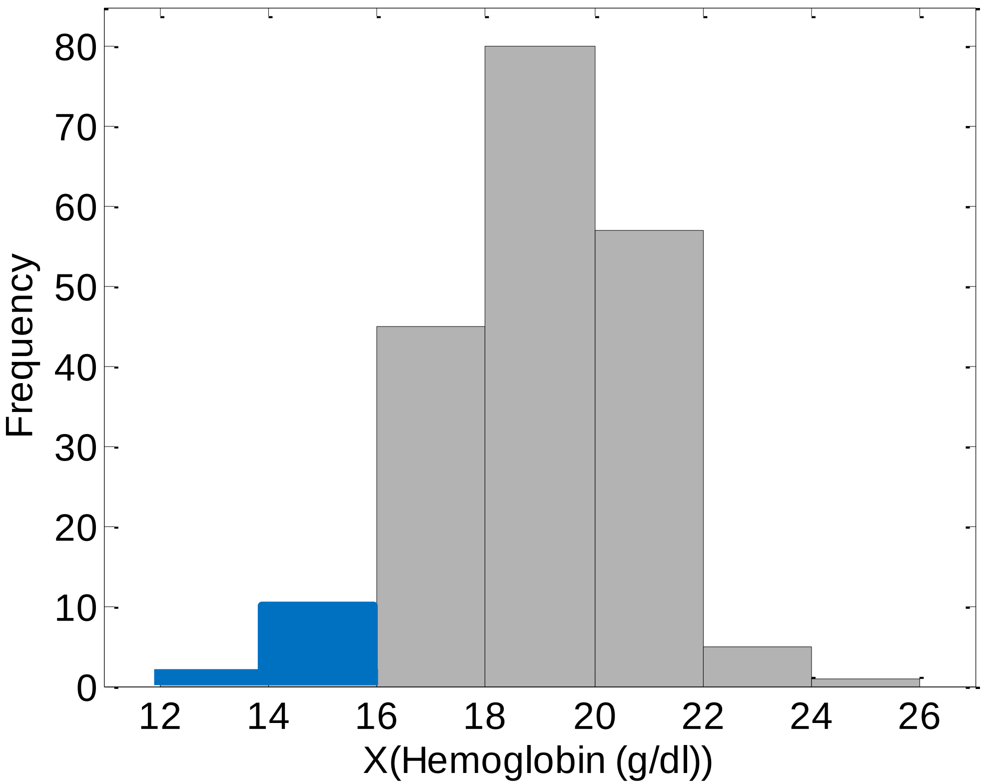
The total area under the histogram

= (2)(2)+(2)(10)+(2)(45)+(2)(80)+(2)(57)+(2)(5)+(2)(1)

= 4+20+90+160+114+10+2 = 400

PROBABILITY

- The <u>area under the histogram</u> is related with probability.



$$P(X < 16) = \frac{\text{Area}(X < 16)}{\text{Total Area}} = \frac{24}{400} = 0.06$$

SUMMARY (1)

- In populations: $$\boxed{Z_i = \frac{X_i - \mu}{\sigma}}$$

- In samples: $$\boxed{Z_i = \frac{X_i - \bar{X}}{s}}$$

- Z-scores are useful to see how far the score is from the mean in standard deviation units.

SUMMARY (2)

- Probability and proportion are used interchangeably.

- When the frequency distribution is given, we can calculate the probability of a specific event A as follows:

$$\boxed{\text{probability of } A = \frac{\text{frequency of outcomes classified as } A}{\text{total frequency in the distribution}}}$$

- The area under the frequency graph is closely related to the probability.