LECTURE 13
HYPOTHESIS TESTING: INDEPENDENT-SAMPLES t-TEST

PSY2002

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RESEARCH DESIGNS

- There are two general research designs that can be used to obtain the two sets of data to be compared:

  - Repeated-measures design or within-subject design
    ➔ Paired-samples *t*-test

  - Independent-groups design or between-subjects design
    ➔ Independent-samples *t*-test (today's topic)

AN EXAMPLE

- Eyewitness Memory Example (Loftus & Palmer, 1974)
  - To investigate whether phrasing of a question can influence eyewitness memory of an event
  - Laboratory experiment with independent-groups design
  - 45 students; each student was randomly assigned to one of 5 experimental conditions.

AN EXAMPLE

- Procedure

  - All 45 students watched seven films of traffic accidents. https://www.youtube.com/watch?v=Rg5bBJQOL74

  - After each film, participants were given a questionnaire that embedded one critical question that was the main interest of this study.

AN EXAMPLE

- Critical question was:

  How fast were the cars going when they <u>hit</u> each other?

  - Other verbs substituted for ‘hit’
    - Smashed
    - Collided
    - Bumped
    - Contacted

  - There were 5 different conditions.

AN EXAMPLE

- Results
  - Mean speed estimates
    - Smashed 40.8 mph
    - Collided 39.3 mph
    - Bumped 38.1 mph
    - Hit 34.0 mph
    - Contacted 31.8 mph

- Conclusions
  - The wording of a question can significantly affect an eyewitness memory.

WORKING EXAMPLE

- A researcher wanted to replicate this study with a sample of today's college students. A total of 30 students participated. 15 students were assigned to 'smash' condition and the other 15 students were assigned to 'hit' condition. The results are as follows:

Condition	Sample Size (n)	Mean ($\bar{X}$)	Standard Deviation (s)
SMASH	15	43.07	6.78
HIT	15	36.96	5.10

- Can he conclude that the two groups have significantly different mean estimates of speed?

WORKING EXAMPLE

- The data of the two conditions were collected using the independent-groups design or between-subjects design.

- Therefore, an independent-samples *t*-test should be used to compare the two conditions (or groups) in terms of the population mean estimated speed.

- It will be assumed that the original scores (estimated speed) are normally distributed in both conditions.

INDEPENDENT-SAMPLES *t*-Test

- Again, follow the five steps of hypothesis testing.

  - Step 1: State the hypotheses
  - Step 2: Set the criteria for a decision
  - Step 3: Collect data and compute test statistics
    - *t*-test: calculate a *t*-statistic
  - Step 4: Make a decision
  - Step 5: State a conclusion

- Let's do the example!

STEP 1: STATE THE HYPOTHESES

- Null hypothesis (H_0)

  - The wording of a question does *not* affect the estimation of the speed.

  - The population mean of the estimated speed of the SMASH group (group 1) *is equal to* that of the HIT group (group 2).

  - $\mu_1 = \mu_2$; $\mu_1 - \mu_2 = 0$

- Alternative hypothesis (H_1)
    - The wording of a question *affects* the estimation of the speed.
    - The population mean of the estimated speed of the SMASH group (group 1) *is different from* that of the HIT group (group 2).
    - $\mu_1 \neq \mu_2$; $\mu_1 - \mu_2 \neq 0$

STEP 2: SET THE CRITERIA

- $\alpha = 0.05$

  - The alpha level (or level of significance) is a probability value that is used to define the concept of "very unlikely" in a hypothesis test.

  - By convention, we use $\alpha = 0.05$ unless otherwise specified. $\alpha = 0.05$ indicates that we will treat extreme 5% of the values as being unlikely to be observed under the null hypothesis.

STEP 3: COMPUTE TEST STATISTICS

- One-sample *t*-test

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}} = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

Expected value of mean (mean of means)

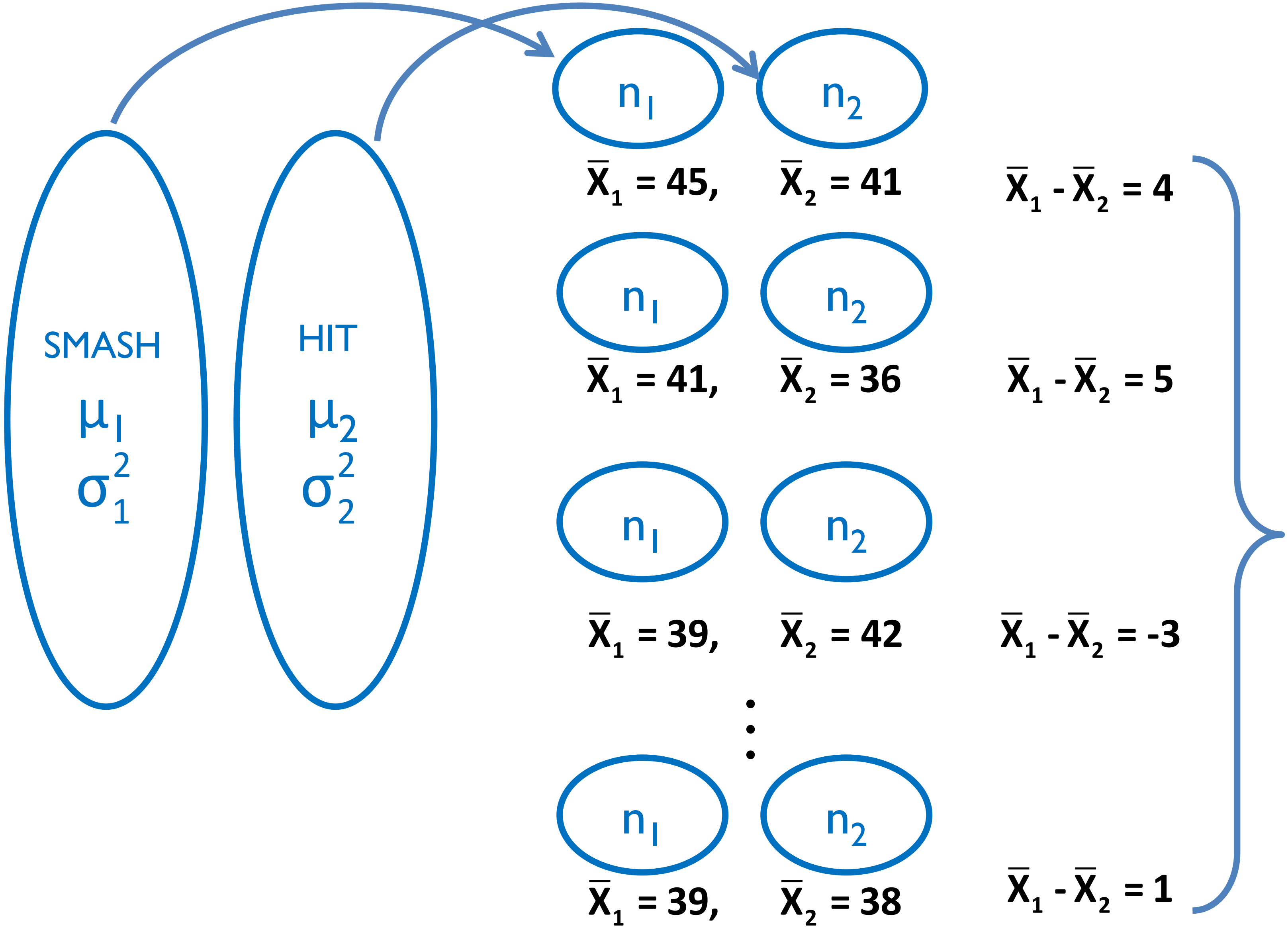
Estimated standard error of mean (estimated standard deviation of means)

- Independent-samples *t*-test

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2}}{s_{\bar{X}_1 - \bar{X}_2}}$$

Expected value of mean difference (mean of mean differences)

Estimated Standard error of mean difference (Estimated standard deviation of mean differences)



- Expected value of mean difference: mean of these mean differences
- Standard error of mean difference: standard deviation of these mean differences

SAMPLING DISTRIBUTION OF MEAN DIFFERENCE

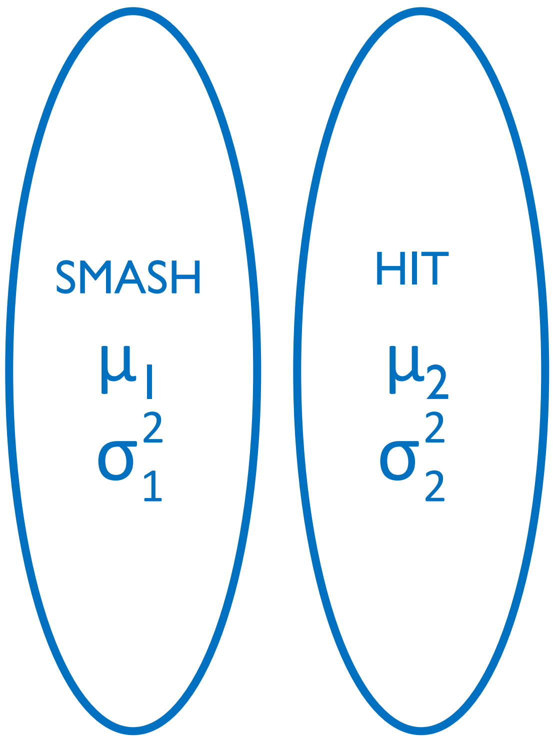
- Expected value of mean difference: $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$

- Standard error of mean difference:

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

HOMOGENEITY OF VARIANCES

- When performing an independent-samples *t*-test, we assume that the population variances of the two groups are equal.



$$\sigma_1^2 = \sigma_2^2 = \sigma^2$$

HOMOGENEITY OF VARIANCES

- This assumption should be tested. (We will test it using SAS).

- Homogeneity of variances test is performed using the following hypotheses:

  - $H_0 : \sigma_1^2 = \sigma_2^2$ (equal variances)
  - $H_1 : \sigma_1^2 \neq \sigma_2^2$ (non−equal variances)

  - If the null hypothesis is rejected, the homogeneity of variance assumption is violated.

  - If we fail to reject the null hypothesis, the homogeneity of variance assumption is not violated.

HOMOGENEITY OF VARIANCES

- If the null hypothesis is rejected, the homogeneity of variance assumption is violated.
  - In this case, the independent-samples *t*-test procedure given in the following slides will not be accurate. Therefore, some adjustment should be made on the degree of freedom to make the test more accurate.
  - SAS will do the adjustment.

- If we fail to reject the null hypothesis, the homogeneity of variance assumption is not violated.
  - In this case, the independent-sample *t*-test procedure given in the following slides will be accurate. No adjustment is required.

SAMPLING DISTRIBUTION OF MEAN DIFFERENCE UNDER HOMOGENEITY OF VARIANCES ASSUMPTION

- Expected value of mean difference: $\mu_{\bar{X}_1 - \bar{X}_2} = \mu_1 - \mu_2$

- Standard error of mean difference when the homogeneity of variance assumption is satisfied:

$$\sigma_{\bar{X}_1 - \bar{X}_2} = \sqrt{\sigma^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

- To estimate the standard error of mean difference, we need to estimate the common variance, σ^2.

POOLED VARIANCE

- The common population variance, σ^2, can be estimated by the pooled variance, s_p^2.

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{\sum_{i=1}^{n_1}(X_{1i} - \bar{X}_1)^2 + \sum_{i=1}^{n_2}(X_{2i} - \bar{X}_2)^2}{n_1 + n_2 - 2}$$

- Therefore, the *t*-statistic becomes:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2}}{s_{\bar{X}_1 - \bar{X}_2}} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

STEP 3: COMPUTE TEST STATISTICS

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - \mu_{\bar{X}_1 - \bar{X}_2}}{s_{\bar{X}_1 - \bar{X}_2}} = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

Estimated Standard error of mean difference under homogeneity of variances

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(14)(6.78^2) + (14)(5.10^2)}{15 + 15 - 2} = 35.99$$

$$t = \frac{(43.07 - 36.96) - 0}{\sqrt{(35.99)\left(\frac{1}{15} + \frac{1}{15} \right)}} = \frac{6.11}{\sqrt{4.80}} = \frac{6.11}{2.19} = 2.79$$

STEP 3: COMPUTE TEST STATISTICS

- Step 3: Compute a test statistic.

$$t = \frac{(43.07 - 36.96) - 0}{\sqrt{(35.99)\left(\frac{1}{15} + \frac{1}{15}\right)}} = \frac{6.11}{\sqrt{4.80}} = \frac{6.11}{2.19} = 2.79$$

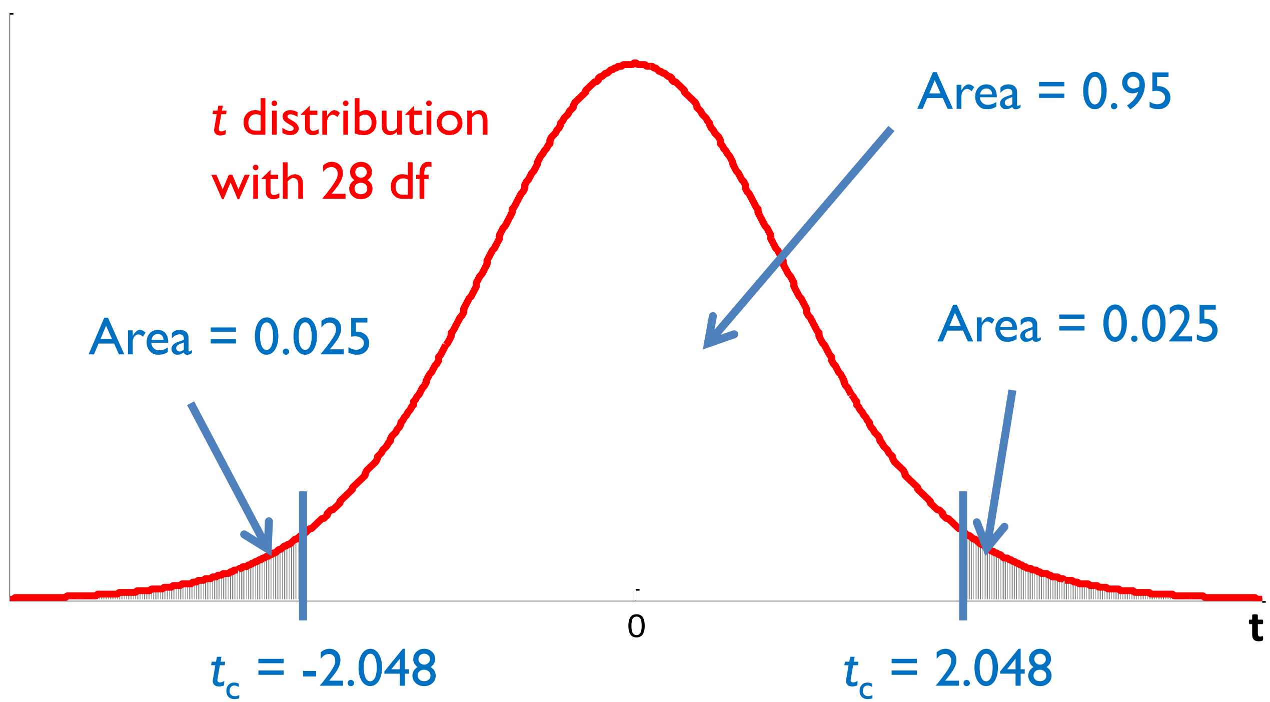
When the null hypothesis is true (and the homogeneity of variance assumption is satisfied), the *t*-statistic is known to form a *t*-distribution with df = $n_1 + n_2 - 2$.

$$df = n_1 + n_2 - 2 = 15 + 15 - 2 = 28$$

$$t(28) = 2.79$$

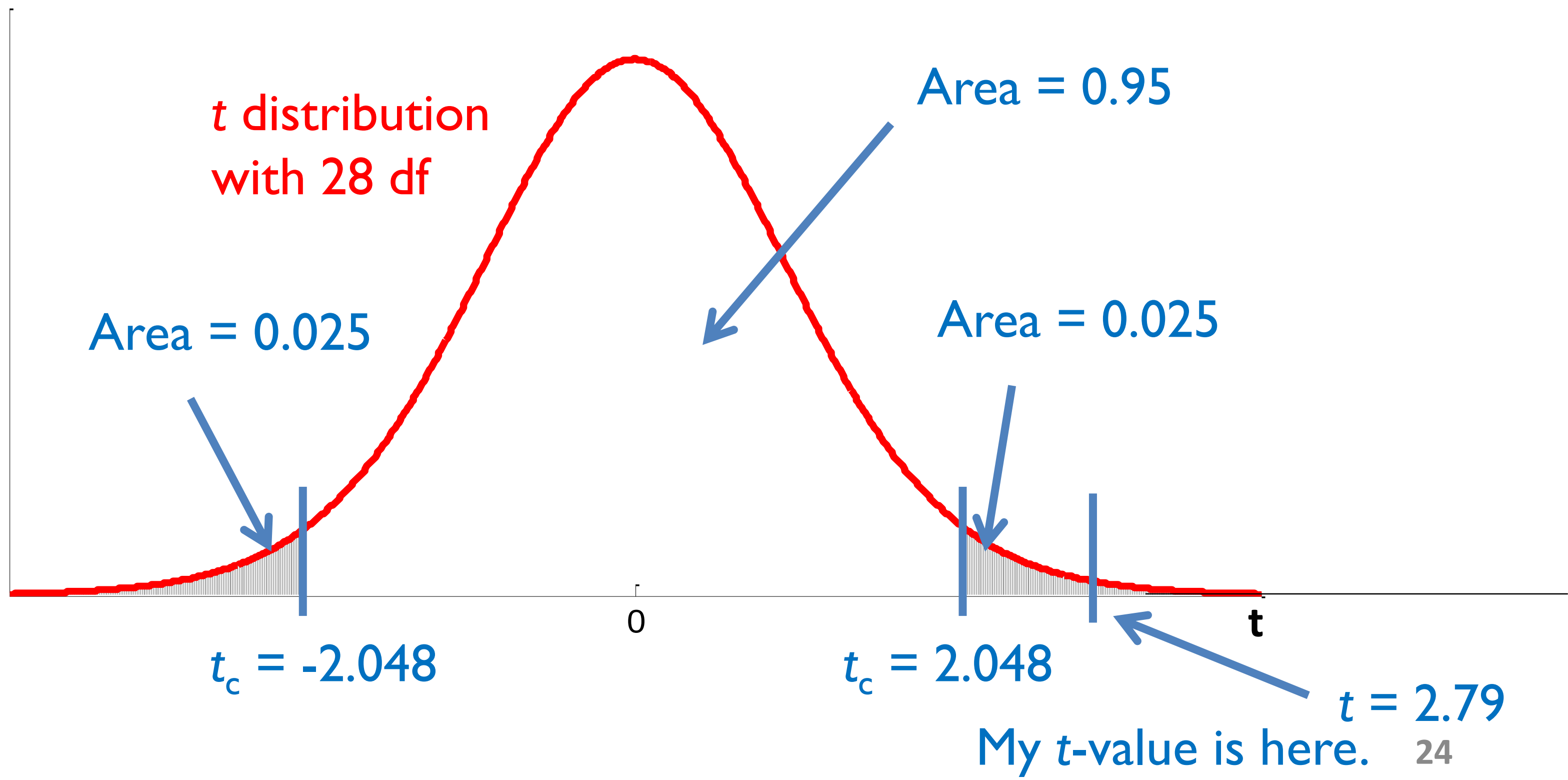
STEP 4: MAKE A DECISION

- Look up the *t*-critical value in the table.
- α = 0.05 (two-tailed), df = 28
- The *t* critical value is 2.048.



STEP 4: MAKE A DECISION

- |My t-value| > t_c; 2.79 > 2.048
- My t-value is in the extreme zone (or in the critical region). My t-value is a strong evidence against H_0. ➔ Reject H_0.



STEP 5: STATE A CONCLUSION

- The mean of the estimated speed of the SMASH group and that of the HIT group are significantly different ($t(28) = 2.79$, $p<.05$). This result implies that the wording used to ask a question can influence eyewitness's memory.

- If you obtain the exact *p*-value from SAS ($p = .0094$), report it as follows.
  - The mean of the estimated speed of the SMASH group and that of the HIT group are significantly different ($t(28) = 2.79$, $p=.0094$). This result implies that the wording used to ask a question can influence eyewitness's memory.

SAS OUTPUT

- First, find the following table to test the homogeneity of variance assumption.

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	14	14	1.77	0.2988

- Using this table, we are testing the following hypotheses.
  - $H_0 : \sigma_1^2 = \sigma_2^2$ (equal variances)
  - $H_1 : \sigma_1^2 \neq \sigma_2^2$ (non-equal variances)

- Use the *p*-value (p = .2988) to decide whether to reject the null hypothesis or not.

SAS OUTPUT

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	14	14	1.77	0.2988

- In this example, $p > \alpha$; $.2988 > .05$
- Therefore, the null hypothesis is not rejected.
- In other word, the homogeneity of variance assumption is not violated.
- In this case, the independent-samples *t*-test procedure that we learned will be accurate.
- Read the "Pooled" line in the result table.

SAS OUTPUT

- If $p < \alpha$ (which is not the case in our example), the null hypothesis is rejected.
- In other word, the homogeneity of variance assumption is violated.
- In this case, the independent-samples t-test procedure that we learned will not be accurate, and some adjustment is required.
- Read the "Satterthwaite" line in the result table. This result is obtained by adjusting the degree of freedom for significance test taking the non-equal variances into account.

SAS OUTPUT

The TTEST Procedure

Variable: speed

Descriptive statistics

group	N	Mean	Std Dev	Std Err	Minimum	Maximum
1	15	43.0707	6.7814	1.7509	28.5700	57.6800
2	15	36.9580	5.1020	1.3173	23.5400	43.7500
Diff (1-2)		6.1127	6.0007	2.1911		

group	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
1		43.0707	39.3153	46.8261	6.7814	4.9648	10.6949
2		36.9580	34.1326	39.7834	5.1020	3.7353	8.0463
Diff (1-2)	Pooled	6.1127	1.6243	10.6010	6.0007	4.7620	8.1157
Diff (1-2)	Satterthwaite	6.1127	1.6087	10.6166			

SAS OUTPUT

- When homogeneity of variance assumption is not violated, read the "Pooled" line.

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	28	2.79	0.0094
Satterthwaite	Unequal	26.003	2.79	0.0097

Homogeneity of variance assumption (equal variance assumption) is met.

df = 28

$t(28) = 2.79$

$p = .0094$

SAS OUTPUT

- When homogeneity of variance assumption is violated (this is not the case in our example), read the "Satterthwaite" line.

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	28	2.79	0.0094
Satterthwaite	Unequal	26.003	2.79	0.0097

p = .0097

Homogeneity of variance assumption (equal variance assumption) is not satisfied.

df = 26.003

$t(26.003) = 2.79$

SUMMARY

- An independent-samples *t*-test is used when two samples of data are collected using the independent-groups design or between-subject design and the researcher wants to examine the difference between the two groups.

- An independent-sample *t*-test is based on the assumption of homogeneity of variances. This assumption should be tested before performing the independent-sample *t*-test.