

### **Announcement on the Midterm Exam**

The midterm exam will be **Thursday, April 25**. The exam will **start at 10:30am and end at 11:45am**. Come to class by 10:25am so that we can start the exam on time. Students will be divided into **two classrooms (D209A, D402)** for the exam. Check out the table in the following page for the exam room assignment.

1. The exam will cover the materials in lecture notes from Lecture 1 to Lecture 7. How to use SAS will NOT be covered in the exam. There will no questions about derivation or proof.
2. The exam is a closed-book test. You will not be allowed to look at any course materials during the exam.
3. The exam will have true/false questions, short answer questions, and calculation questions.
4. The standard normal distribution table and the formulas that are needed for the calculation questions will be given during the exam. The exceptions include  $(n + 1)/2$  for finding the location of the median and the formulas for calculating standard scores (Z-scores). You should memorize these formulas.
5. Bring your **pencil, eraser, and calculator**. You will not be allowed to use your cellphone, tablet PC, or laptop for calculation. Bring a simple calculator.
6. All exam questions will be written in English. However, you may write your answers in English, Korean, or mixture of them.
7. **Cheating will NOT be tolerated**. Anyone found cheating shall be immediately disqualified for the exam and get a score of 0. **NO EXCEPTIONS**.
8. If you have to miss the exam due to an unavoidable situation, contact the instructor at [hsuk2@sogang.ac.kr](mailto:hsuk2@sogang.ac.kr) as soon as you can so as to arrange your exam at another time. If you miss the exam due to an unexpected event on the exam day, you should contact the instructor within 7 days to arrange a makeup exam.



## Review Questions

1. A researcher was interested in examining the relationship between the amount of daily sleeping hours of college students and their life satisfaction. The researcher recruited 380 college students at Sogang University and asked them to report their amount of daily sleeping hours (measured in hours) and their life satisfaction using a 5-point scale (1: very satisfied, 2: somewhat satisfied, 3: neutral, 4: not very satisfied, 5: not at all satisfied). Answer the following TRUE/FALSE questions.

(1) The population of this study is the 380 college students who participated in this study. (TRUE/FALSE)

(2) The amount of daily sleeping hours is a ratio variable. (TRUE/FALSE)

(3) The life satisfaction level is a ratio variable. (TRUE/FALSE)

(4) The researcher calculated the variance of daily sleeping hours of the 380 college students and obtained a value of -1.25. This indicates that she made a calculation error. (TRUE/FALSE)

(5) The researcher calculated the deviation scores of daily sleeping hours of the 380 college students and obtained the sum of the deviation scores being zero. This indicates that she made a calculation error. (TRUE/FALSE)

2. Answer the following TRUE/FALSE questions.

(1) It is known that the scores on a math test are distributed with a mean of 40. My Z-score for this test is -1.3. This indicates that my original math test score is below 40. (TRUE/FALSE)

(2) We can transform a score in a distribution into the corresponding Z-score if we know the mean and the standard deviation of the distribution. (TRUE/FALSE)

(3) If we transform each and every score in a distribution into the corresponding Z-score, the mean of the Z-scores will be 0 regardless of the mean of the original scores. (TRUE/FALSE)

(4) If we transform each and every score in a distribution into the corresponding Z-score, the standard deviation of the Z-scores will be 0 regardless of the standard deviation of the original scores. (TRUE/FALSE)

(5) For a test, my Z-score is 2.4. This indicates that my score is within the middle 95% of the distribution. (TRUE/FALSE)    False

(6) In the standard normal distribution, the following two probabilities,  $P(Z > 2)$  and  $P(Z < -2)$ , will add up to 1. (TRUE/FALSE)

3. Compute the following expressions for the scores given in the table.

$X$	$Y$
3	10
8	12
7	17

$$(1) \frac{\sum_{i=1}^3 X_i}{3} = \frac{3+8+7}{3} = \frac{18}{3} = 6$$

$$(2) \sum_{i=1}^3 X_i^2 = 3^2 + 8^2 + 7^2 = 9 + 64 + 49 = 122$$

$$(3) \sum_{i=1}^3 (X_i - 6) \sum_{i=1}^3 (Y_i - 13) = 0$$

$$(4) \sum_{i=1}^3 (X_i - 6)(Y_i - 13) = \begin{aligned} &(3-6)(10-13) + (8-6)(12-13) + (7-6)(17-13) \\ &= 9 + (-2) + 4 \\ &= 11 \end{aligned}$$

4. A professor wants to examine the relationship between midterm scores and final scores. The following table shows the *sample* data obtained from 4 students.

$X(\text{midterm})$	$Y(\text{final})$
88	74
99	98
91	82
86	86

(1) Find the median of  $X$ .

$$\begin{aligned} &86, 88, 91, 99 \\ &n+1/2 = 4+1/2 = 2.5 (\text{second \& third}) \\ &88+91/2 = 89.5 \end{aligned}$$

(2) Find the mean of  $Y$ .

$$Y = (\text{formula-mean-sample}) = \frac{74+98+82+86}{4} = \frac{340}{4} = 85$$

(3) Find the standard deviation of  $Y$  and interpret it.

standard deviation:

$$\begin{aligned} S &= (\text{formula-sd-sample}) = \sqrt{\frac{(74-85)^2 + (98-85)^2 + (82-85)^2 + (86-85)^2}{3}} \\ &= \sqrt{\frac{121 + 169 + 9 + 1}{3}} \\ &= \sqrt{\frac{300}{3}} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

interpretation:

The final scores deviate from the mean by 10, on average.

5. In a class, the mean of the midterm scores was  $\bar{X} = 75$  and the variance was  $s^2 = 5$ . Answer the following questions.

(1) A constant of 10 is subtracted from each and every student's midterm score. Find the mean and the variance of the newly obtained scores.

$$\text{Mean} = 75 - 10 = 65$$

$$\text{Variance} = 5 \text{ (Adding a constant does not affect variance)}$$

(2) Each and every student's score is multiplied by a constant of 0.2. Find the mean and the variance of the newly obtained scores.

$$\text{Mean} = 75 \cdot 0.2 = 15$$

$$\text{Variance} = 5 \cdot (0.2)^2 = 5 \cdot 0.04 = 0.2$$

5. It is known that the scores on a self-esteem scale are distributed with a mean of  $\mu = 40$  and a standard deviation of  $\sigma = 12$ . My self-esteem score is 1.6 standard deviations below the mean. Find my original self-esteem score.

$$\begin{aligned} Z &= (X - \mu) / \sigma \\ 1.6 &= (x - 40) / 12 \\ -19.2 &= x - 40 \\ 40 - 19.2 &= x \\ 20.8 &= x \end{aligned}$$

6. Find the Z-score that separate the bottom 24% from the rest.

$$\begin{aligned} & \text{(According to the graph:)} \\ & 1 - 0.24 = 0.76 \\ & \text{(According to the table:)} \\ & z' = 0.71 \\ & = -0.71 \end{aligned}$$

7. A national study of healthy males and females showed that their body temperatures are normally distributed with a mean of  $37^{\circ}\text{C}$  and a standard deviation of  $0.17^{\circ}\text{C}$ . I got a cold and my body temperature is  $38.1^{\circ}\text{C}$ . What is the probability of a healthy person having his/her body temperature higher than  $38.1^{\circ}\text{C}$ ?

$$P(X > 38.1)$$

$$Z = \frac{X - \mu}{\sigma} = \frac{38.1 - 37}{0.17} = \frac{1.1}{0.17} = 6.47$$

8. In a college, the entrance exam scores of the freshmen form a normal distribution with mean of  $\mu = 100$  and standard deviation of  $\sigma = 20$ .

(1) A professor wants to select the freshmen for an interview whose entrance exam scores belong to the middle 60%. What Z-scores should he use to separate the middle 60% from the rest?

$$Z_1 = -.84$$

$$Z_2 = .84$$

(2) Find the original exam scores corresponding to the Z-scores found in (1).

$$Z = \frac{X - \mu}{\sigma}$$

$$20(-.84) = \frac{X_1 - 100}{20} \cdot 20$$

$$-16.8 = X_1 - 100$$

$$X_1 = 100 - 16.8$$

$$= 83.2$$

$$Z = \frac{X - \mu}{\sigma}$$

$$20(.84) = \frac{X_2 - 100}{20} \cdot 20$$

$$16.8 = X_2 - 100$$

$$X_2 = 100 + 16.8$$

$$= 116.8$$

### Formula sheet

	Population	Sample
Mean	$\mu = \frac{\sum_{i=1}^N X_i}{N}$	$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$
Variance	$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N}$	$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}$
Standard deviation	$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}}$	$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$

## Standard Normal Probabilities

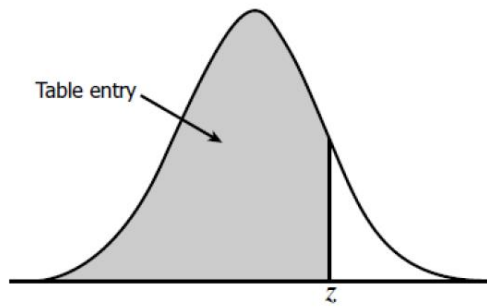


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998