LECTURE 11
HYPOTHESIS TESTING: ONE-SAMPLE *t*-TEST

PSY2002

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ONE-SAMPLE TESTS

- A one-sample (or single-sample) test is used when a single sample is collected and the researcher wants to examine if there is strong evidence that a parameter (e.g., population mean) of the population, from which the sample is extracted, is different from a hypothesized value.

TWO TYPES OF ONE-SAMPLE TEST

- Two types of one-sample tests:
  - A one sample Z-test is used when the population standard deviation (σ) is known. (We already learned this.)
  - A one sample t-test is used when the population standard deviation (σ) is NOT known. (Today's topic)

One-sample *t*-test

A PRENATAL CARE STUDY

- It is known that the birthweight for infants of women living in poverty is normally distributed with $\mu = 2800\ g$.
- Let's assume that the population standard deviation (σ) is NOT KNOWN.
- Recently, a local hospital introduced an innovative new prenatal care program to reduce the number of low birthweight babies born in the hospital. 27 mothers, all of whom live in poverty, participated in this program. The babies born to these women had an average birthweight of $\bar{X} = 2946\ g$ and a standard deviation of $s = 178\ g$.
- The question is whether this program has been effective at improving the birthweights of babies born to poor women.

http://ccnmtl.columbia.edu/projects/qmss/the_ttest/onesample_ttest.html

Population of babies born to poor mothers who did *not* participate in the prenatal care program

Sample of babies born to poor mothers who participated in the prenatal care program

$\mu = 2800$
$\sigma = ??$

$\bar{X} = 2946$
$s = 178$
$n = 27$

- Based on this sample, can we conclude that the prenatal care program is effective?

- In other words, can we conclude that this sample is not from the population with $\mu = 2800$?

BASIC STEPS OF A HYPOTHESIS TEST

- Whatever hypothesis test you use, you should always follow these five steps.
  - Step 1: State the hypotheses
  - Step 2: Set the criteria for a decision
  - Step 3: Collect data and compute test statistics
    - *t*-test: calculate *t*-statistic
  - Step 4: Make a decision
  - Step 5: State a conclusion

STEP 1

- Step 1: State the hypotheses

  - The null hypothesis (H_0)
  - The alternative hypothesis (H_1)

- The null hypothesis (H_0):
  - The prenatal care program has <u>no effect</u> on the birthweights of babies.
  - The population mean birthweight of babies born to poor mothers who participate in the program <u>is the same as</u> that of babies born to poor mothers who do not participate in the program, which is 2800 grams.
  - $\mu_{prenatal} = 2800\ g$

- The alternative hypothesis (H_1):
  - The prenatal care program <u>has an effect</u> on birthweights of babies.
  - The population mean birthweight of babies born to poor mothers who participate in the program <u>is different from</u> that of babies born to poor mothers who do not participate in the program ($2800\ g$).
  - $\mu_{prenatal} \neq 2800\ g$

STEP 2

- Step 2: Set the criteria for a decision.

  - $\alpha = 0.05$

  - The alpha level (or level of significance) indicates that the extreme 5% of the scores will be treated as the values that are too extreme to be observed when the null hypothesis is true.

  - By convention, we use α=0.05 unless otherwise specified.

STEP 3

- Step 3: Compute a test statistic.

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

We cannot calculate a *Z*-statistic because we don't know σ.

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}} = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{2946 - 2800}{178/\sqrt{27}} = \frac{146}{178/5.2} = 4.27$$

Instead, we can calculate a *t*-statistic, in which the standard error of mean is estimated.

STEP 3

- Step 3: Compute a test statistic.

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{\mathrm{X}}}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

We cannot calculate a *Z*-statistic because we don't know σ.

When the null hypothesis is true, this *Z*-statistic will follow the standard normal distribution.

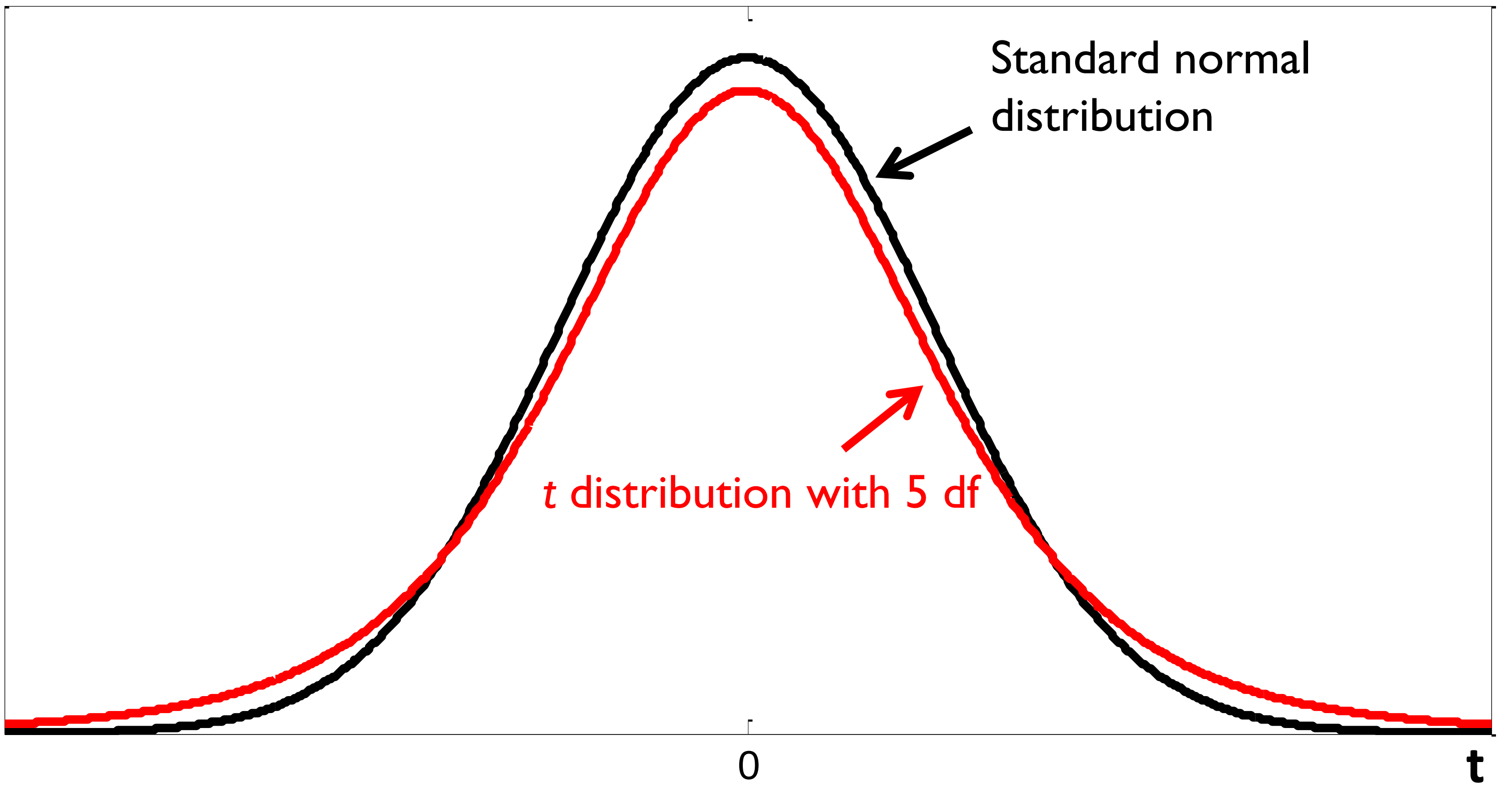
$$t = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{\mathrm{X}}}} = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{2946 - 2800}{178/\sqrt{27}} = \frac{146}{178/5.2} = 4.27$$

Instead, we can calculate a t-statistic, in which the standard error of mean is estimated using a point estimator.

When the null hypothesis is true, this *t*-statistic will follow a *t* distribution with *n* - 1 degrees of freedom (df).

t DISTRIBUTION

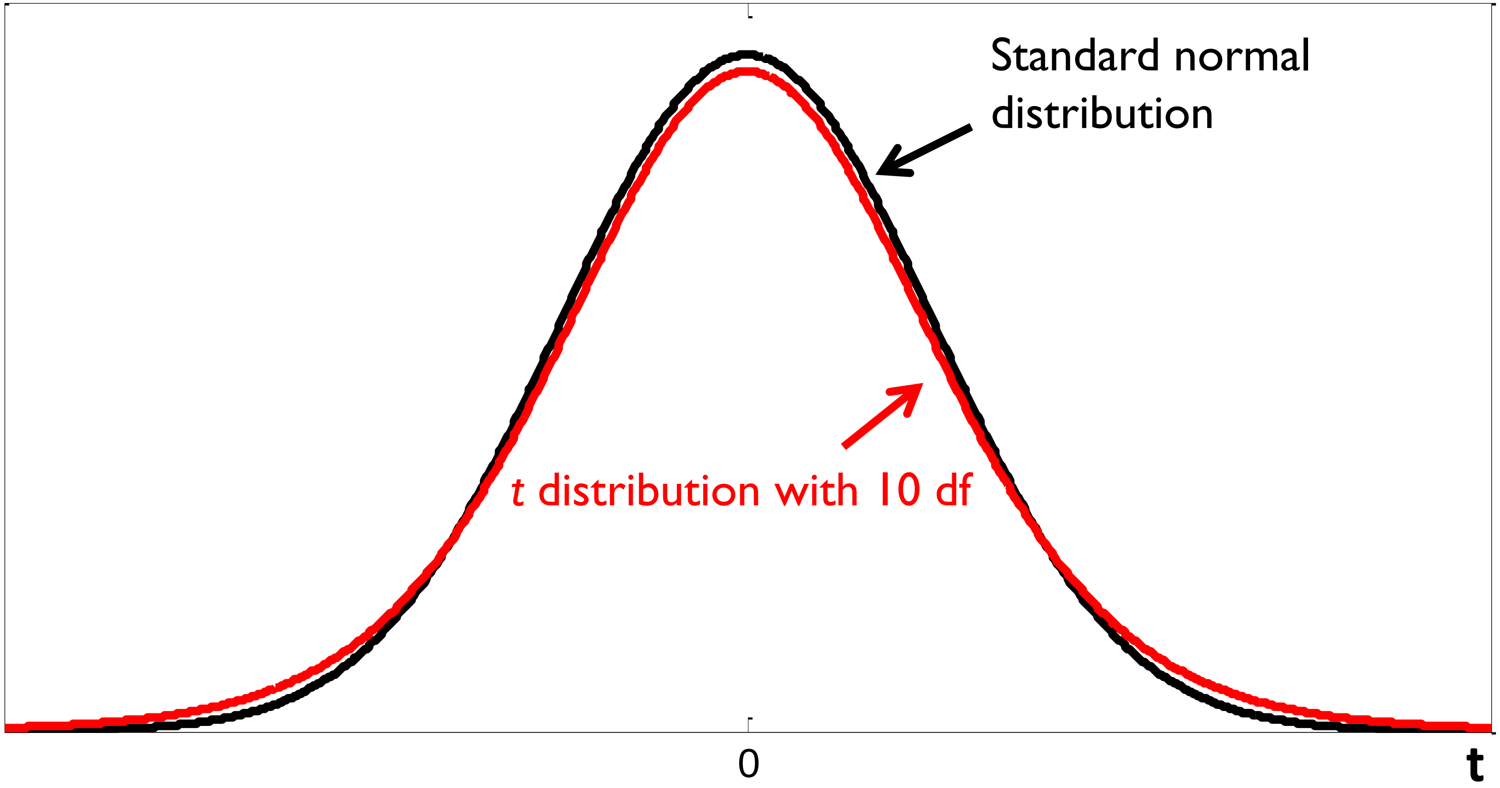
- *t* distribution



A *t*-distribution is centered at 0, bell-shaped, and symmetrical as the standard normal distribution, but it has thicker tails than the standard normal distribution. The exact shape of a *t* distribution is determined by its degrees of freedom (df).

t DISTRIBUTION

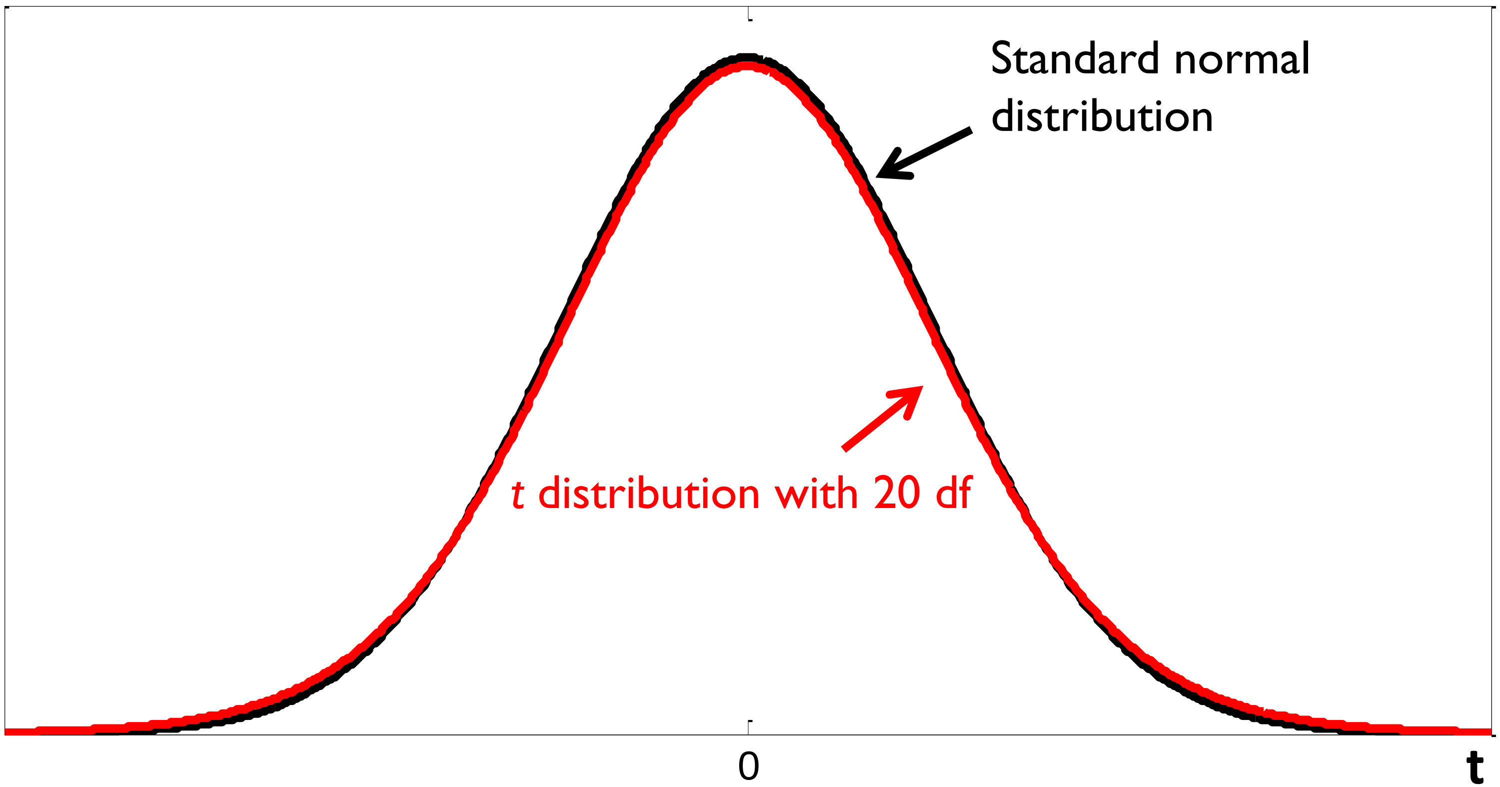
- *t* distribution



A *t*-distribution approaches to the standard normal distribution closer and closer as the df increases.

t DISTRIBUTION

- *t* distribution



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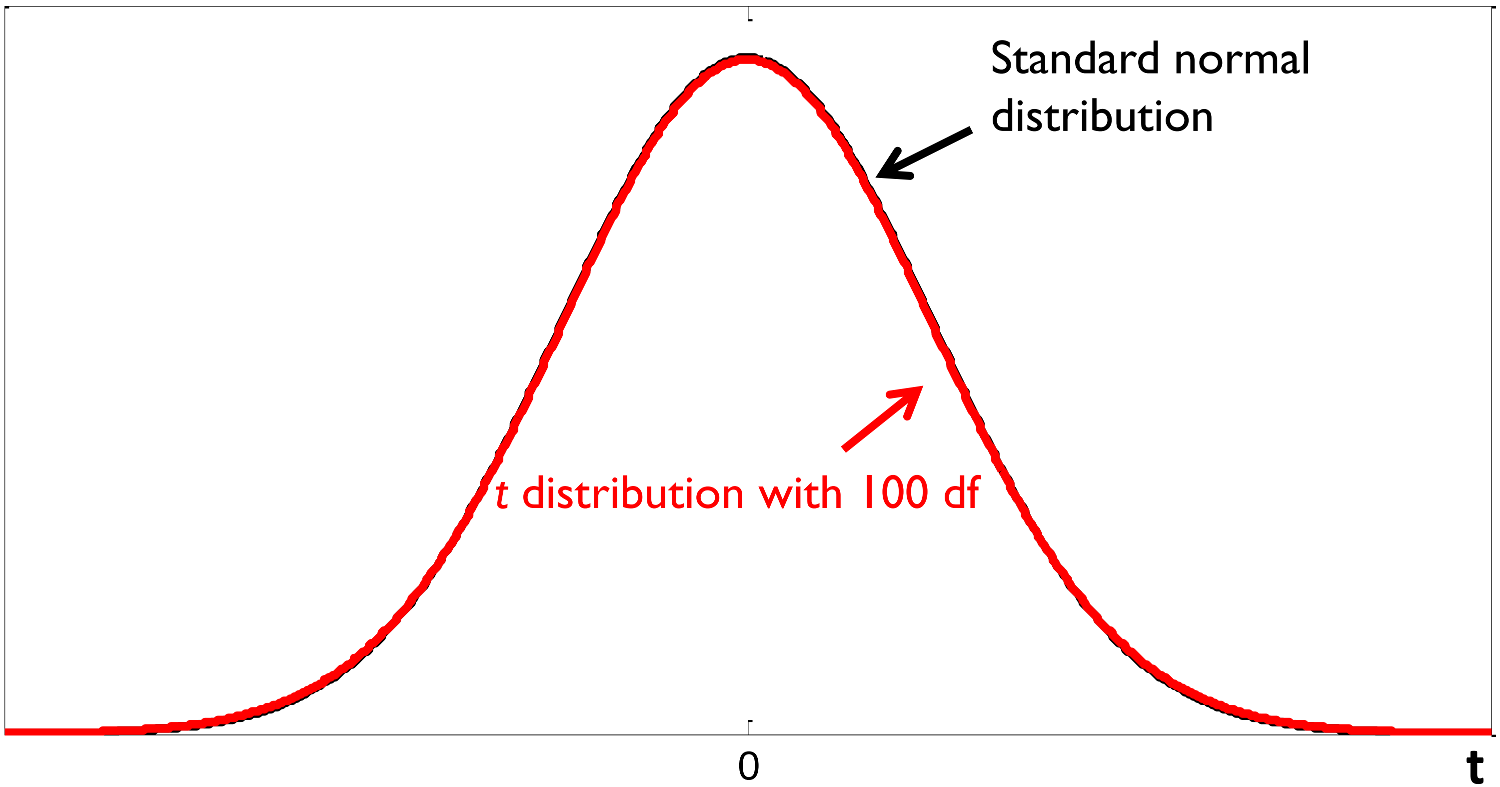
t DISTRIBUTION

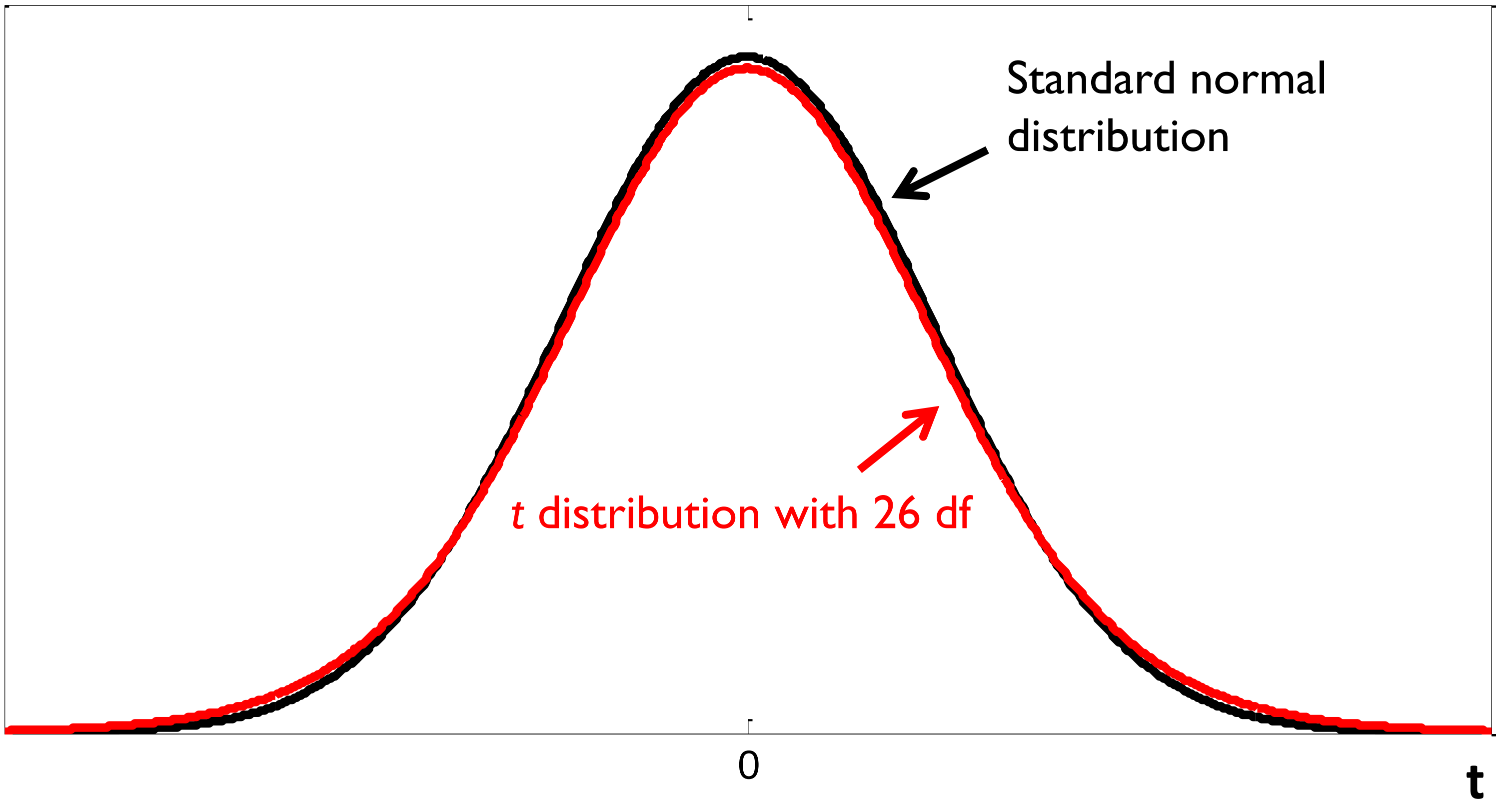
- *t* distribution



A *t*-distribution approaches to the standard normal distribution closer and closer as the df increases.

t DISTRIBUTION

- In this example, the *t*-statistic will follow a *t* distribution with $n - 1 = 27 - 1 = 26$ degrees of freedom.



WHAT IS DEGREE OF FREEDOM?

- Let's assume that the population parameters are known, say $\mu = 5$ and $\sigma = 1$. If we collect a sample of $n = 4$ scores from this population, the 4 scores can take any values.

- However, if we do not know the population standard deviation, we should estimate it by using the sample standard deviation (s).

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

In other words, we should use the sample mean in estimating the population standard deviation.

WHAT IS DEGREE OF FREEDOM?

- Estimates of parameters can be based upon different amounts of information or data. The number of independent pieces of information that go into the estimate of a parameter are called the degrees of freedom. In general, the degrees of freedom of an estimate of a parameter are equal to the number of independent scores that go into the estimate minus the number of parameters used as intermediate steps in the estimation of the parameter itself.

- df = # observations - # estimated parameters

https://en.wikipedia.org/wiki/Degrees_of_freedom_(statistics)

- Let's assume that the sample mean is $\bar{X} = 6$. If we collect a sample of $n = 4$ scores and estimate the population standard deviation, what will happen?

- The first 3 scores (out of the 4 scores in the sample) can take any values. However, the fourth score is constrained to produce the sample mean of 6.

- If the first 3 scores are 4, 8, 6, then the fourth score should be 6 so that the sample mean becomes 6.
  - $\bar{X} = \frac{4+8+6+X_4}{4} = 6$
  - $X_4 = 6$

- If the first 3 scores are 4, 8, 8, then the fourth score should be 4 so that the sample mean becomes 6.
  - $\bar{X} = \frac{4+8+8+X_4}{4} = 6$
  - $X_4 = 4$

- Therefore, only 3 scores can take any values freely. ➔ df = 3

- In general, in a one-sample *t*-test, df = n – 1.

t DISTRIBUTION

- Step 3: Compute a test statistic.

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{\mathrm{X}}}} = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{2946 - 2800}{178/\sqrt{27}} = \frac{146}{178/5.2} = 4.27$$

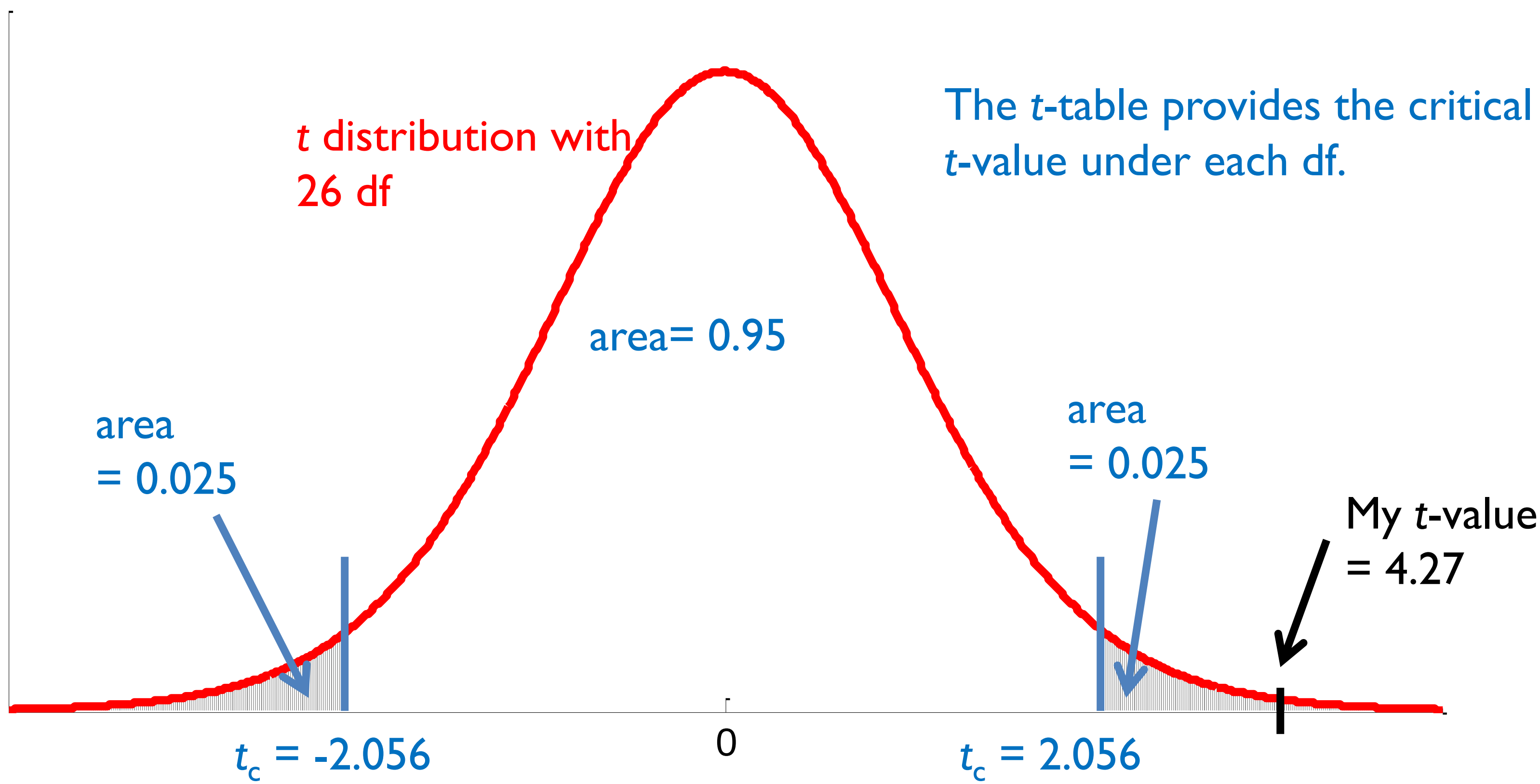
  - This *t*-statistic follows a *t*-distribution with df = n – 1 = 27 – 1 = 26.

  - $t(26) = 4.27$

STEP 4

- Step 4: Make a decision.

  $t(26) = 4.27$

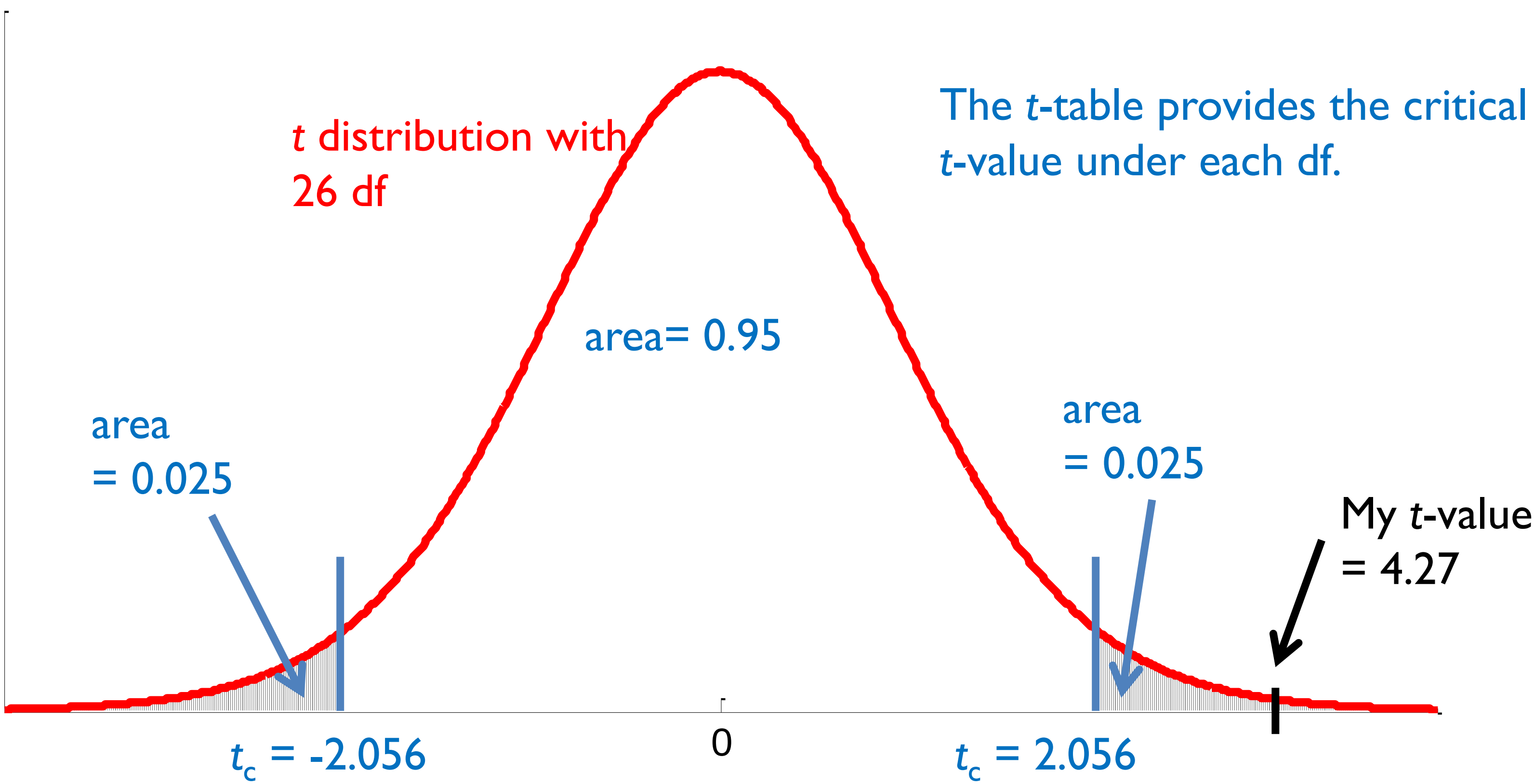


http://www.ttable.org/

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646

STEP 4

- Step 4: Make a decision.
  - My t-value is in the critical region. My t-value does provide a strong evidence against the null hypothesis. ➔ Reject H_0.



- Another equivalent way to make a decision would be to calculate the p-value and compare it to the level of significance ($\alpha = 0.05$).

- Unlike the one-sample Z-test, however, it is impossible to obtain the p-value using the t-table; the t-table provides the critical values only.

- Using SAS will provide the exact p-value (p = .0002).
  - $p < \alpha$ ➔ reject the null hypothesis.

  - If $p > \alpha$ ➔ fail to reject the null hypothesis.

STEP 5

- Step 5. State a conclusion.

  - The prenatal care has a <u>significant effect</u> on the birthweights of babies born to poor women ($t(26) = 4.27$, $p < .05$).

- If you have used SAS and knew the exact *p*-value, state a conclusion as follows:
  - The prenatal care has a <u>significant effect</u> on the birthweights of babies born to poor women ($t(26) = 4.27$, $p = .0002$).

- If we failed to reject the null hypothesis (let's assume that we obtained $t(26)=1.25$), state the conclusion as follows:

    - The prenatal care has <u>no significant effect</u> on the birthweights of babies born to poor women ($t(26) = 1.25$, $p > .05$).

- If you have used SAS and knew the exact p-value (which is .2224), state a conclusion as follows:
    - The prenatal care has <u>no significant effect</u> on the birthweights of babies born to poor women ($t(26) = 1.25$, $p = .2224$).

SAS OUTPUT

The TTEST Procedure

Variable: weight

N	Mean	Std Dev	Std Err	Minimum	Maximum
27	2945.8	177.6	34.1780	2619.0	3189.0

Mean	95% CL Mean		Std Dev	95% CL Std Dev	
2945.8	2875.5	3016.0	177.6	139.9	243.4

DF	t Value	Pr > |t|
26	4.27	0.0002

SUMMARY

- The basic five steps of hypothesis testing

  - Step 1: State the hypotheses
  - Step 2: Set the criteria for a decision
    - Set the significance level (α) and obtain critical values.
  - Step 3: Collect data and compute sample statistics
    - *t*-test: calculate *t*-statistic
  - Step 4: Make a decision
  - Step 5: State a conclusion
    - Report a verbal interpretation, *t*-statistic (with df), and *p*-value.