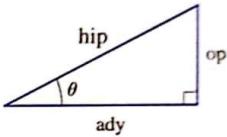


## Trigonometría de ángulo recto

$$\begin{array}{ll} \text{sen } \theta = \frac{\text{op}}{\text{hip}} & \csc \theta = \frac{\text{hip}}{\text{op}} \\ \cos \theta = \frac{\text{ady}}{\text{hip}} & \sec \theta = \frac{\text{hip}}{\text{ady}} \\ \tan \theta = \frac{\text{op}}{\text{ady}} & \cot \theta = \frac{\text{ady}}{\text{op}} \end{array}$$

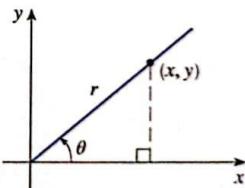


## Identidades fundamentales

$$\begin{array}{ll} \csc \theta = \frac{1}{\text{sen } \theta} & \sec \theta = \frac{1}{\cos \theta} \\ \tan \theta = \frac{\text{sen } \theta}{\cos \theta} & \cot \theta = \frac{\cos \theta}{\text{sen } \theta} \\ \cot \theta = \frac{1}{\tan \theta} & \text{sen}^2 \theta + \cos^2 \theta = 1 \\ 1 + \tan^2 \theta = \sec^2 \theta & 1 + \cot^2 \theta = \csc^2 \theta \\ \text{sen}(-\theta) = -\text{sen } \theta & \cos(-\theta) = \cos \theta \\ \tan(-\theta) = -\tan \theta & \text{sen}\left(\frac{\pi}{2} - \theta\right) = \cos \theta \\ \cos\left(\frac{\pi}{2} - \theta\right) = \text{sen } \theta & \tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \end{array}$$

## Funciones trigonométricas

$$\begin{array}{ll} \text{sen } \theta = \frac{y}{r} & \csc \theta = \frac{r}{y} \\ \cos \theta = \frac{x}{r} & \sec \theta = \frac{r}{x} \\ \tan \theta = \frac{y}{x} & \cot \theta = \frac{x}{y} \end{array}$$



## Fórmulas del ángulo doble

$$\text{sen } 2x = 2 \text{sen } x \cos x$$

$$\cos 2x = \cos^2 x - \text{sen}^2 x = 2 \cos^2 x - 1 = 1 - 2 \text{sen}^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

## Fórmulas del ángulo medio

$$\text{sen}^2 x = \frac{1 - \cos 2x}{2} \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

## Tabla de sustituciones trigonométricas

Expresión	Sustitución	Identidad
$\sqrt{a^2 - x^2}$	$x = a \text{sen } \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \text{sen}^2 \theta = \cos^2 \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2 \theta = \sec^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ o } \pi \leq \theta < \frac{3\pi}{2}$	$\sec^2 \theta - 1 = \tan^2 \theta$

## TABLA DE INTEGRALES

### Formas básicas

1.  $\int u \, dv = uv - \int v \, du$
2.  $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3.  $\int \frac{du}{u} = \ln |u| + C$
4.  $\int e^u \, du = e^u + C$
5.  $\int b^u \, du = \frac{b^u}{\ln b} + C$
6.  $\int \sin u \, du = -\cos u + C$
7.  $\int \cos u \, du = \sin u + C$
8.  $\int \sec^2 u \, du = \tan u + C$
9.  $\int \csc^2 u \, du = -\cot u + C$
10.  $\int \sec u \tan u \, du = \sec u + C$
11.  $\int \csc u \cot u \, du = -\csc u + C$
12.  $\int \tan u \, du = \ln |\sec u| + C$
13.  $\int \cot u \, du = \ln |\sin u| + C$
14.  $\int \sec u \, du = \ln |\sec u + \tan u| + C$
15.  $\int \csc u \, du = \ln |\csc u - \cot u| + C$
16.  $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$
17.  $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
18.  $\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$
19.  $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$
20.  $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

Formas que involucran  $\sqrt{a^2 + u^2}$ ,  $a > 0$

21.  $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22.  $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23.  $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24.  $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25.  $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26.  $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27.  $\int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28.  $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29.  $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

## TABLA DE INTEGRALES

INTEGRACIÓN AL PARTE MÍNIMA

Formas que involucran  $\sqrt{a^2 - u^2}$ ,  $a > 0$

$$30. \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$31. \int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$32. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$33. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$34. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$35. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$36. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$$

$$37. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \operatorname{sen}^{-1} \frac{u}{a} + C$$

$$38. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Formas que involucran  $\sqrt{u^2 - a^2}$ ,  $a > 0$

$$39. \int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

$$40. \int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

$$41. \int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

$$42. \int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

$$43. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

$$44. \int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

$$45. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$46. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

## TABLA DE INTEGRALES

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Formas que involucran  $a + bu$

$$47. \int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$$

$$48. \int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$$

$$49. \int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$$

$$50. \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$51. \int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$$

$$52. \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$$

$$53. \int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left( a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$$

$$54. \int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$$

$$55. \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$$

$$56. \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu) \sqrt{a + bu} + C$$

$$57. \int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{si } a > 0$$

$$= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{si } a < 0$$

$$58. \int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u \sqrt{a + bu}}$$

$$59. \int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u \sqrt{a + bu}}$$

$$60. \int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[ u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$$

$$61. \int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$$

$$62. \int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$$

## TABLA DE INTEGRALES

Conte aquí y guarda para consulta.

### Formas trigonométricas

$$63. \int \operatorname{sen}^2 u \, du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$$

$$64. \int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$$

$$65. \int \tan^2 u \, du = \tan u - u + C$$

$$66. \int \cot^2 u \, du = -\cot u - u + C$$

$$67. \int \operatorname{sen}^3 u \, du = -\frac{1}{3}(2 + \operatorname{sen}^2 u) \cos u + C$$

$$68. \int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \operatorname{sen} u + C$$

$$69. \int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$$

$$70. \int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\operatorname{sen} u| + C$$

$$71. \int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$72. \int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$73. \int \operatorname{sen}^n u \, du = -\frac{1}{n} \operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n} \int \operatorname{sen}^{n-2} u \, du$$

$$74. \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$75. \int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$$

$$76. \int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$$

$$77. \int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

$$78. \int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

$$79. \int \operatorname{sen} au \operatorname{sen} bu \, du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$80. \int \cos au \cos bu \, du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$81. \int \operatorname{sen} au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$82. \int u \operatorname{sen} u \, du = \operatorname{sen} u - u \cos u + C$$

$$83. \int u \cos u \, du = \cos u + u \operatorname{sen} u + C$$

$$84. \int u^n \operatorname{sen} u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

$$85. \int u^n \cos u \, du = u^n \operatorname{sen} u - n \int u^{n-1} \operatorname{sen} u \, du$$

$$86. \int \operatorname{sen}^n u \cos^m u \, du = -\frac{\operatorname{sen}^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \operatorname{sen}^{n-2} u \cos^m u \, du \\ = \frac{\operatorname{sen}^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \operatorname{sen}^n u \cos^{m-2} u \, du$$

### Formas trigonométricas inversas

$$87. \int \operatorname{sen}^{-1} u \, du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$$

$$88. \int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$89. \int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$$

$$90. \int u \operatorname{sen}^{-1} u \, du = \frac{2u^2-1}{4} \operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$91. \int u \cos^{-1} u \, du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$92. \int u \tan^{-1} u \, du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$$

$$93. \int u^n \operatorname{sen}^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \operatorname{sen}^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

$$94. \int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$$

$$95. \int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$$

(continúa)

## TABLA DE INTEGRALES

### Formas exponenciales y logarítmicas

96.  $\int ue^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$

97.  $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$

98.  $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$

99.  $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$

100.  $\int \ln u du = u \ln u - u + C$

101.  $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$

102.  $\int \frac{1}{u \ln u} du = \ln |\ln u| + C$

### Formas hiperbólicas

103.  $\int \sinh u du = \cosh u + C$

104.  $\int \cosh u du = \sinh u + C$

105.  $\int \tanh u du = \ln \cosh u + C$

106.  $\int \coth u du = \ln |\sinh u| + C$

107.  $\int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$

108.  $\int \operatorname{csch} u du = \ln |\tanh \frac{1}{2} u| + C$

109.  $\int \operatorname{sech}^2 u du = \tanh u + C$

110.  $\int \operatorname{csch}^2 u du = -\coth u + C$

111.  $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$

112.  $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

### Formas que involucran $\sqrt{2au - u^2}$ , $a > 0$

113.  $\int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + C$

114.  $\int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + C$

115.  $\int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a-u}{a}\right) + C$

116.  $\int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$

117.  $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$

118.  $\int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a-u}{a}\right) + C$

119.  $\int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u+3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1}\left(\frac{a-u}{a}\right) + C$

120.  $\int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$