CSE 517: Artificial Intelligence

Chapter 3: Search and Sequential Action

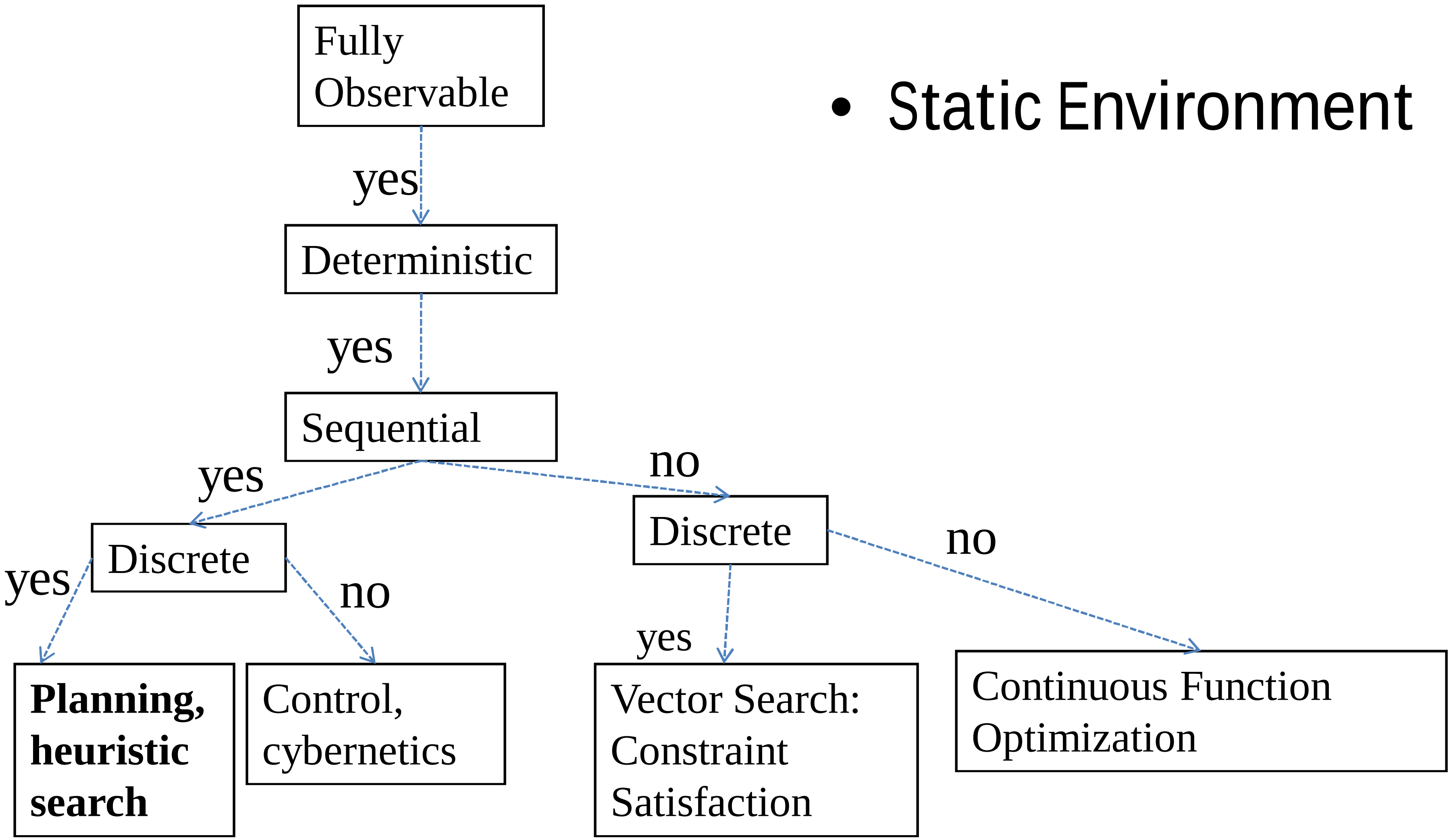
Spring 2015

Department of Computer Science and Engineering (CSE)

Outline

- Problem formulation: representing sequential problems.
- Example problems.
- Planning for solving sequential problems without uncertainty.
- Basic search algorithms

Environment Type Discussed In this Lecture



- Static Environment

Choice in a Deterministic Known Environment

- Without uncertainty, choice is trivial in principle: choose what you know to be the best option.
- Trivial if the problem is represented in a look-up table.

Option	Value
Chocolate	10
Wine	20
Book	15

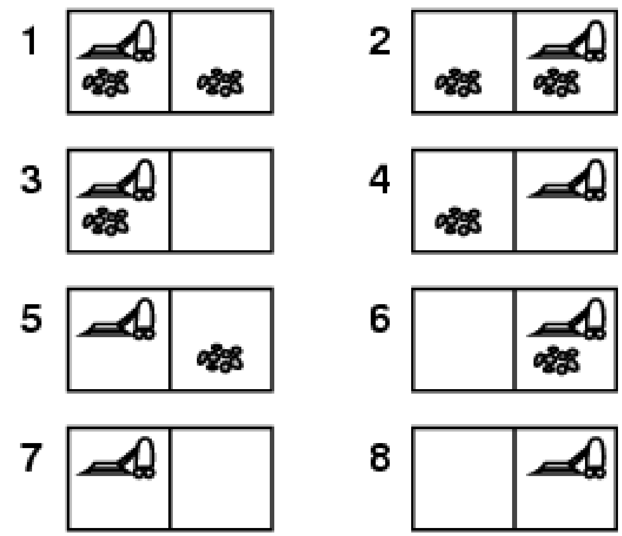
This is the standard problem representation in decision theory (economics).

Computational Choice Under Certainty

- But choice can be *computationally* hard if the problem information is represented differently.
- Options may be **structured** and the best option needs to be constructed.
  - E.g., an option may consist of a path, sequence of actions, plan, or strategy.
- The value of options may be given **implicitly** rather than explicitly.
  - E.g., cost of paths need to be computed from map.

Sequential Action Example

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
  - Vacuum world → everything observed
  - Romania → The full map is observed

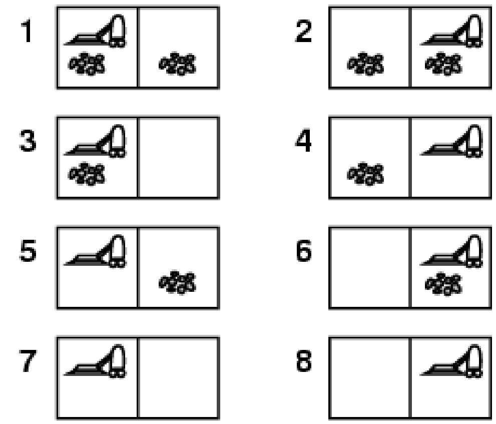


- Single-state: Start in #5. Solution??
  - [Right, Suck]

Problem types

- Non-observable → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
  - Vacuum world → No sensors
  - Romania → No map just know operators(cities you can move to)

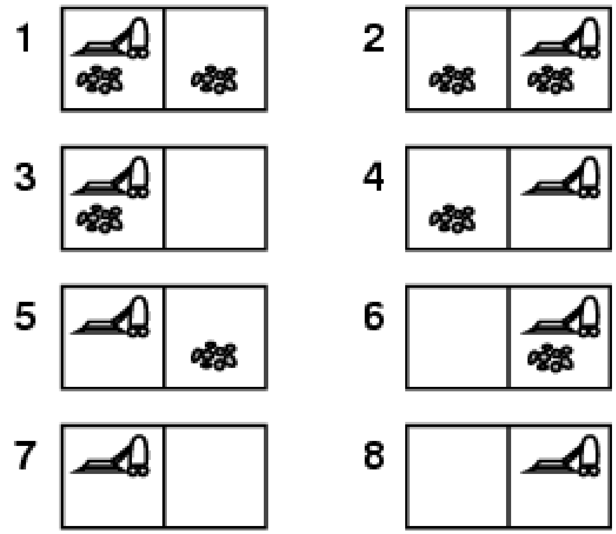
- Conformant: Start in {1, 2, 3, 4, 5, 6, 7, 8}
  - e.g., Right goes to {2, 4, 6, 8}. Solution??
  - [Right, Suck,Left, Suck]



Problem types

- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - Unknown state space → exploration problem
  - Vacuum world → know state of current location
  - Romania → know current location and neighbor cities

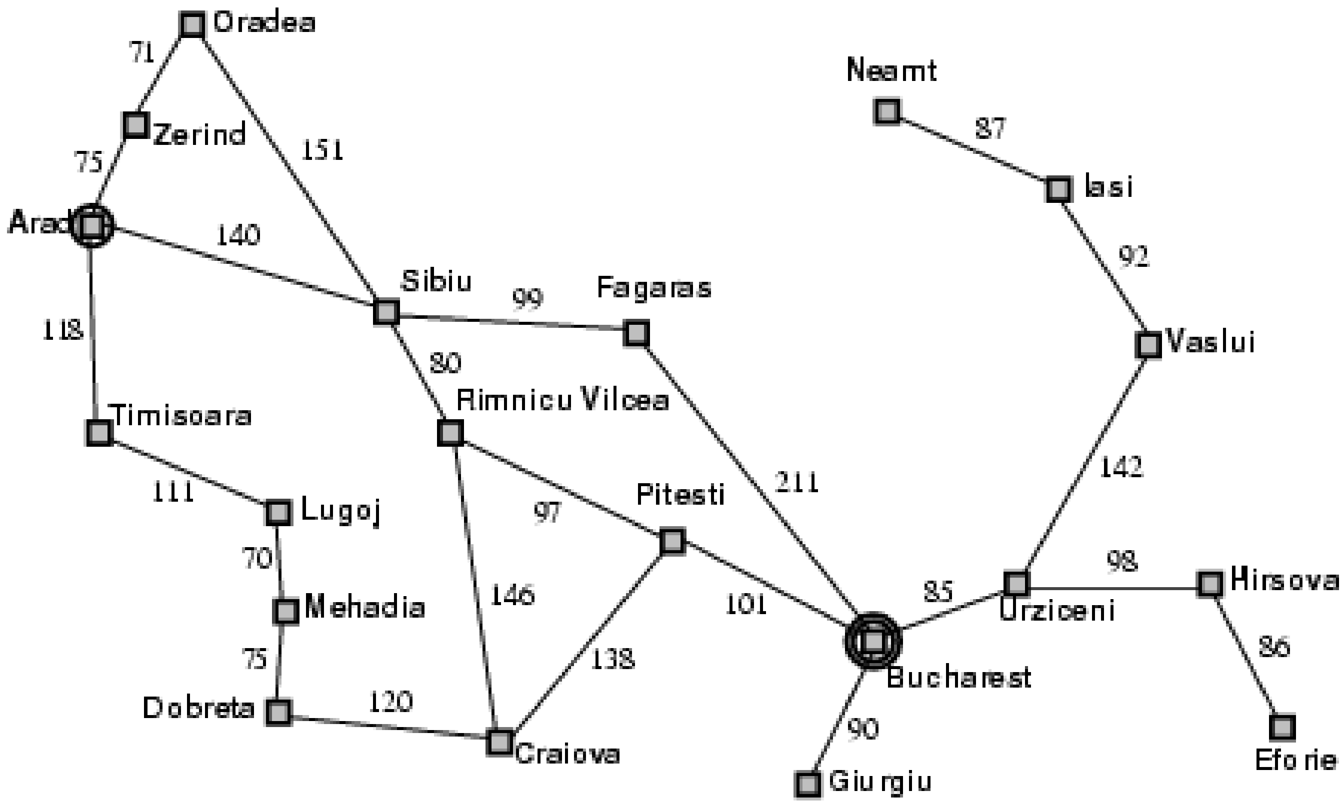
- Contingency: [L,clean]
  - Start in #5 or #7
  - Murphy's Law: Suck can dirty a clean carpet
  - Local sensing: dirt, location only.
  - Solution??
  - [Right, if dirt then Suck]



Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
-
- Formulate goal:
  - be in Bucharest
  -
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest
  -

Example: Romania



Abstraction: The process of removing details from a representation

Is the map a good representation of the problem? What is a good replacement?

Single-state problem formulation

A problem is defined by four items:

1. initial state e.g., "at Arad“

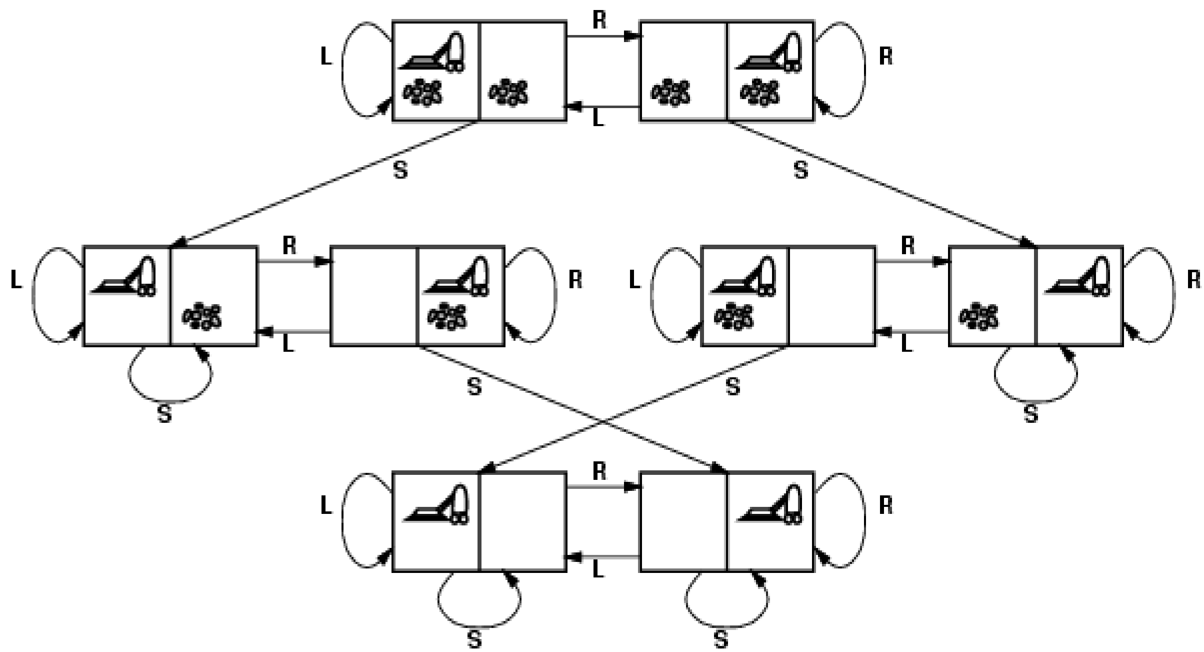
2. actions or successor function $S(x)$ = set of action–state pairs
   – e.g., $S(Arad) = \{<Arad \rightarrow Zerind, Zerind>, \ldots \}$
   –
3. goal test, can be
   – explicit, e.g., x = "at Bucharest"
   – implicit, e.g., $Checkmate(x)$
   –
4. path cost (additive)
   – e.g., sum of distances, number of actions executed, etc.
   – $c(x,a,y)$ is the step cost, assumed to be ≥ 0
   –

- A solution is
  – a sequence of actions leading from the initial state to a goal state
  –

Selecting a state space

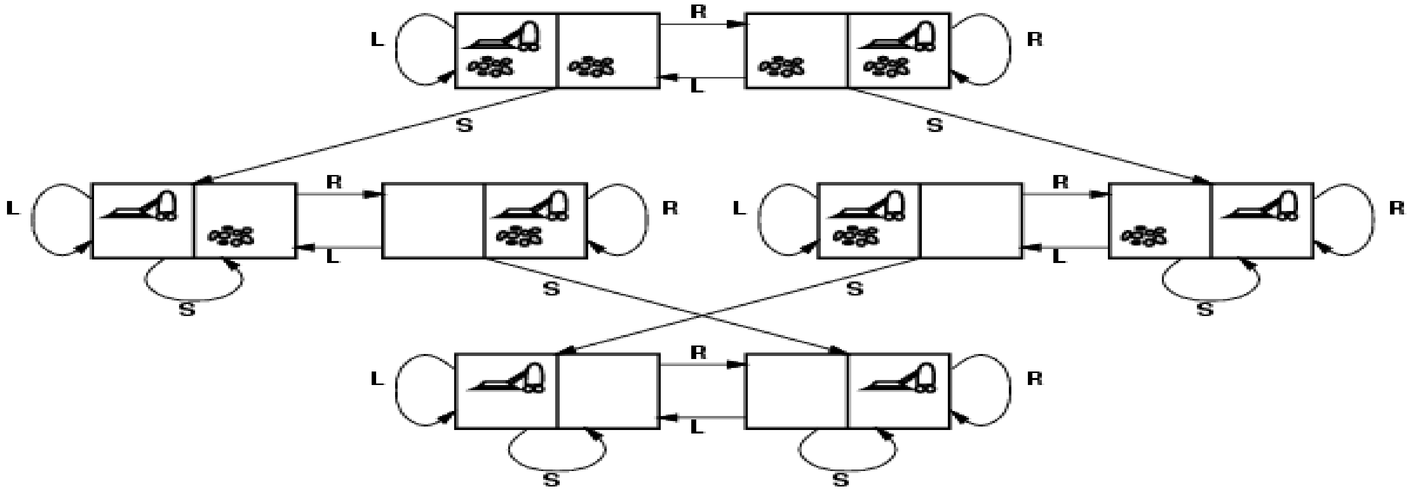
- Real world is complex
  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
-
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- (Abstract) solution =
  - set of real paths that are solutions in the real world
  -
- Each abstract action should be "easier" than the original problem
-

Vacuum world state space graph



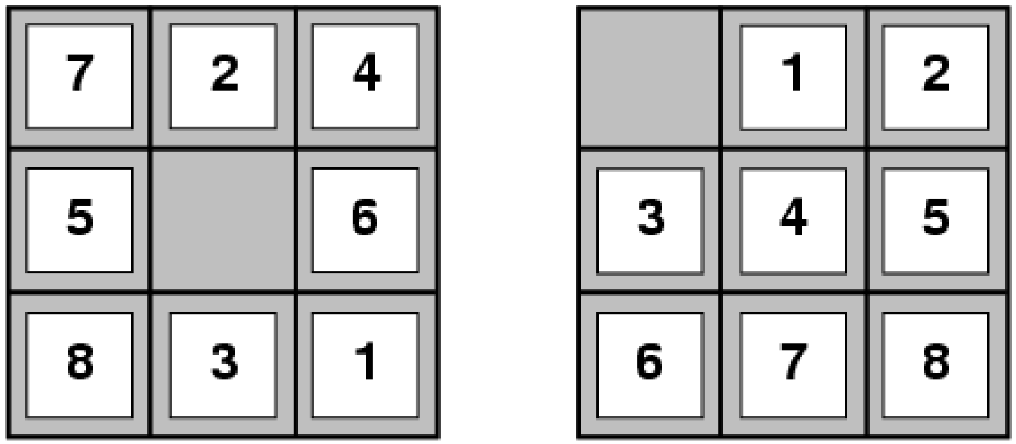
- states?
- actions?
- goal test?
- path cost?
-

Vacuum world state space graph



- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

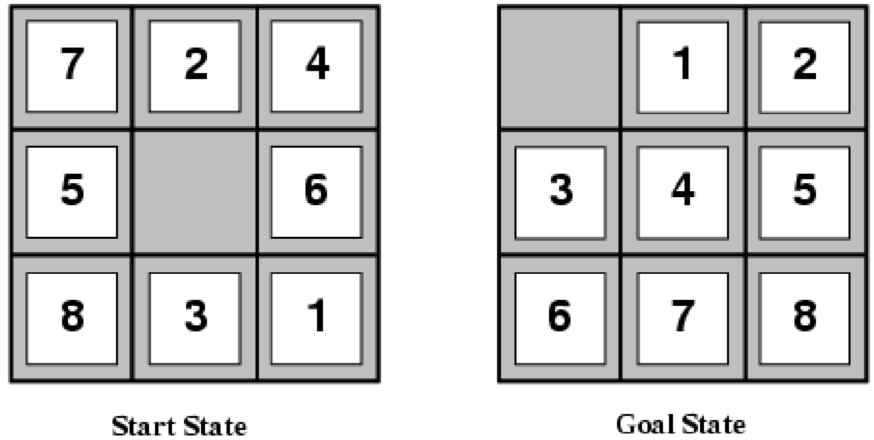
Example: The 8-puzzle



Start State Goal State

- states?
- actions?
- goal test?
- path cost?

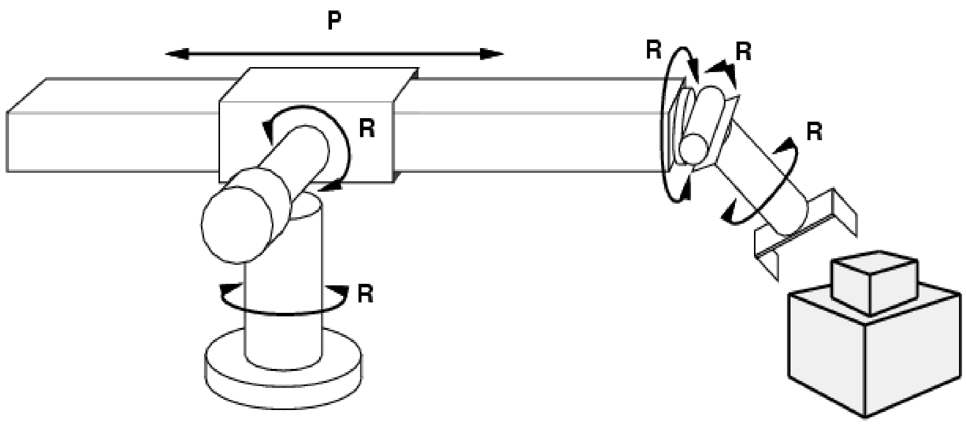
Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move

[Note: optimal solution of *n*-Puzzle family is NP-hard]

Example: robotic assembly



- states?:
  - real-valued coordinates of robot joint angles parts of the object to be assembled
  -
- actions?:
  - continuous motions of robot joints
- goal test?:
  - complete assembly
  -
- path cost?:
  - time to execute
  -

Problem-solving agents

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH(problem)
    action ← FIRST(seq)
    seq ← REST(seq)
    return action
```

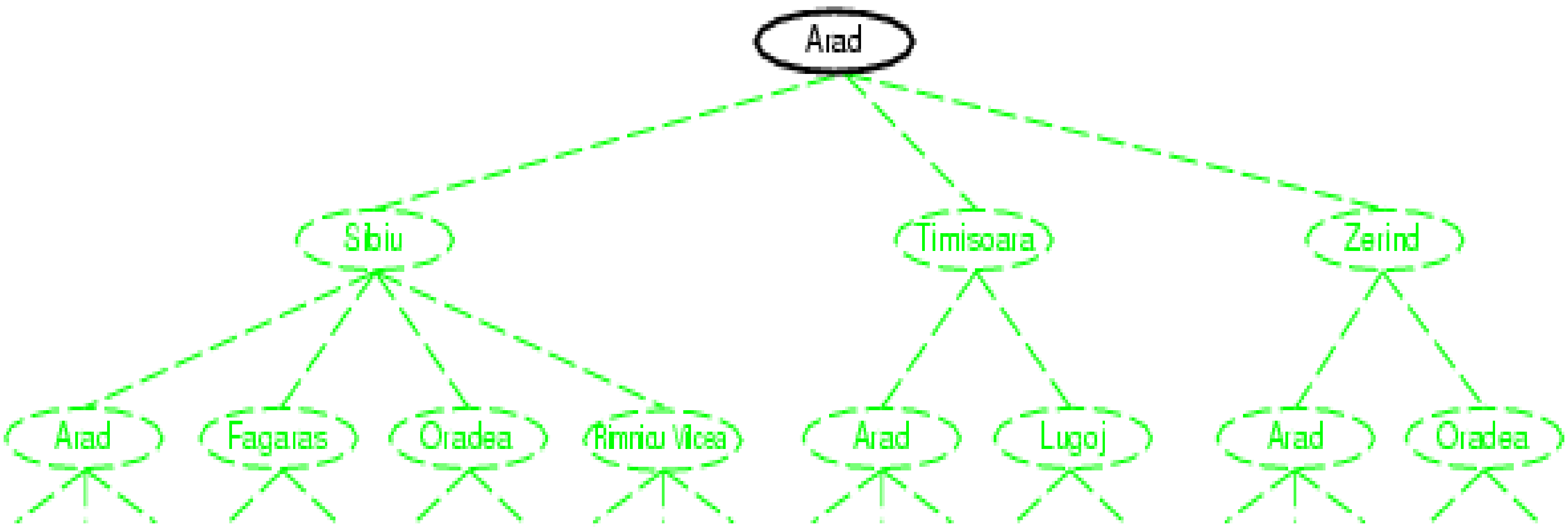
Note: this is offline problem solving; solution executed “eyes closed.”

Tree search algorithms

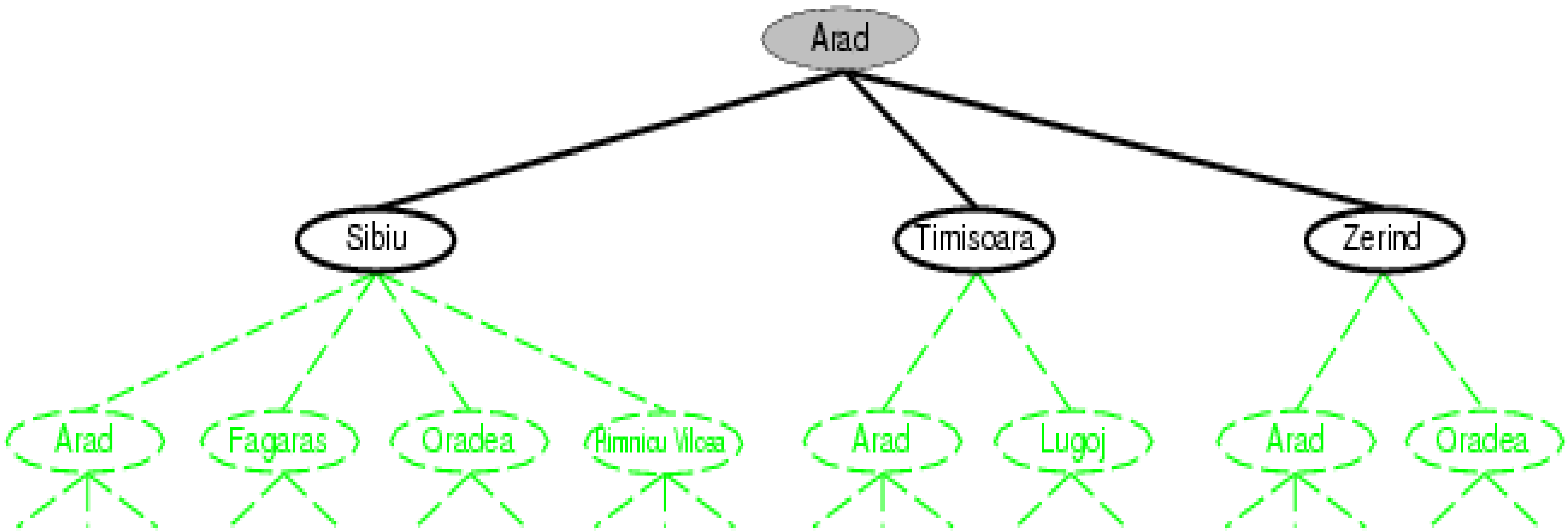
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

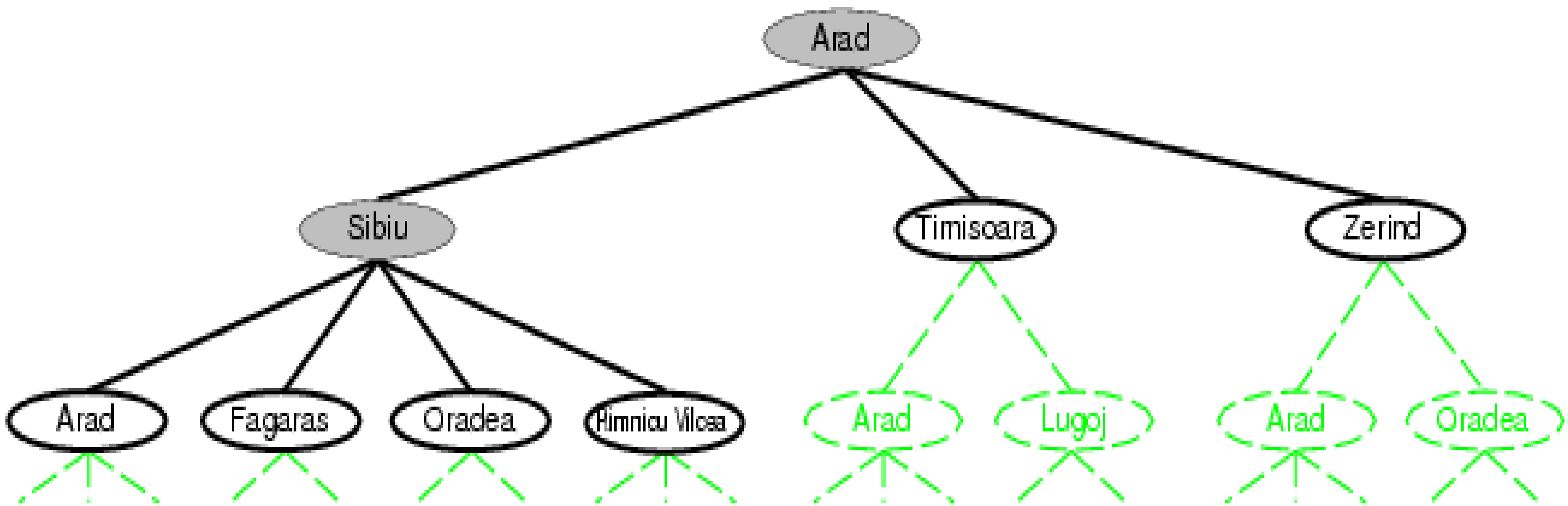
Tree search example



Tree search example



Tree search example



Search Graph vs. State Graph

- Be careful to distinguish
  - Search tree: nodes are **sequences of actions.**
  - State Graph: Nodes are states of the environment.
  - We will also consider soon **search graphs**.
- Demo: http://aispace.org/search/

Search strategies

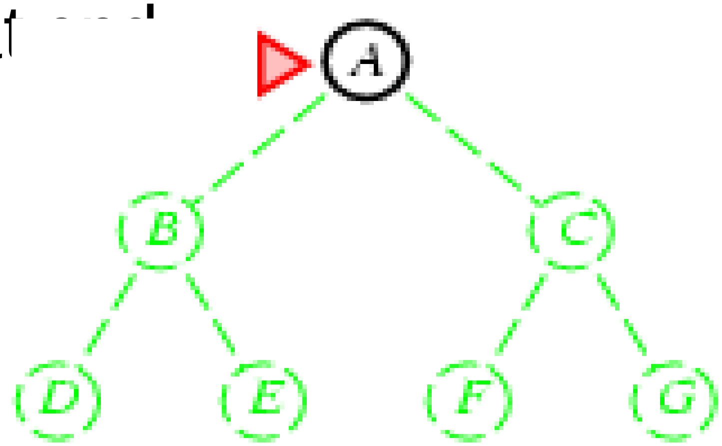
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
  -
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)
  -

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
- Breadth-first search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node
-
- Implementation:
  - *Frontier* or *fringe* is a FIFO queue, i.e., new successors go at
  -



Breadth-first search

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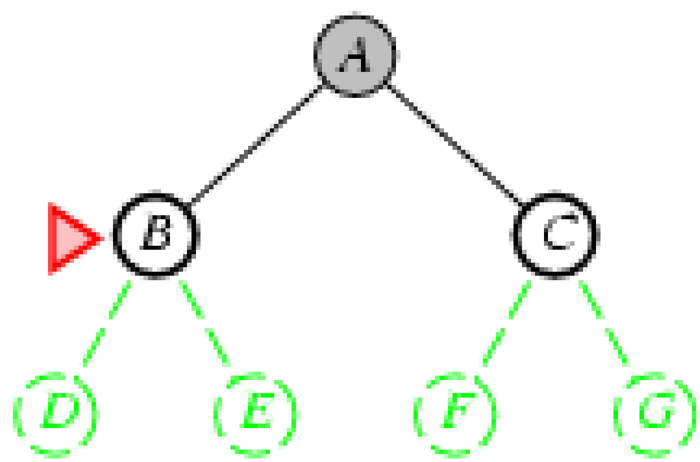

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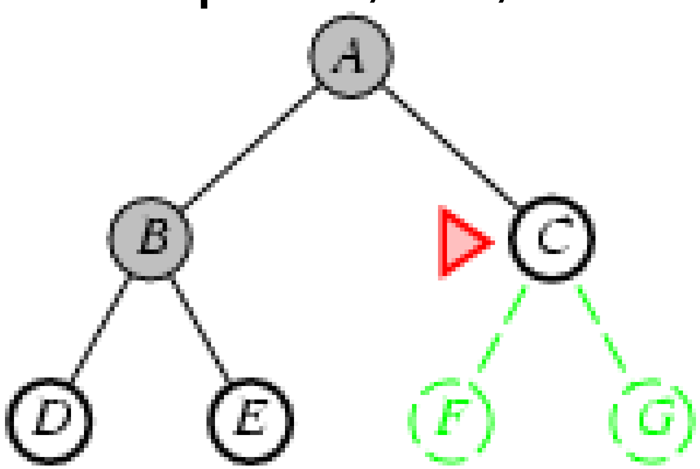

Breadth-first search

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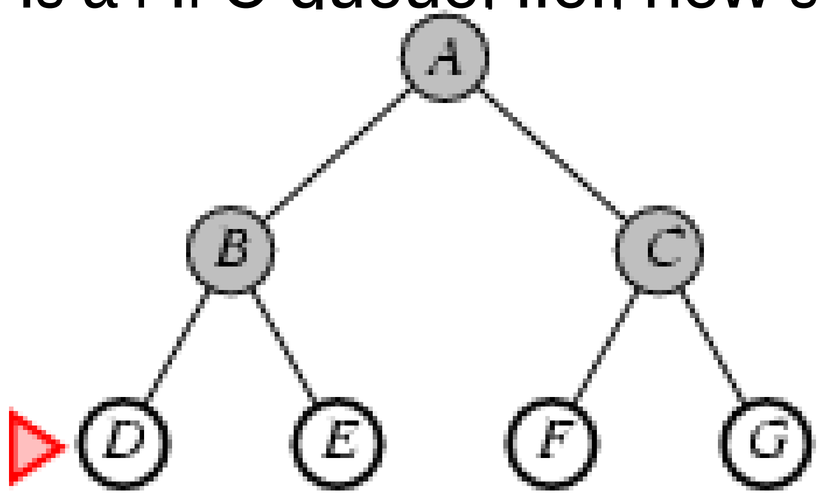
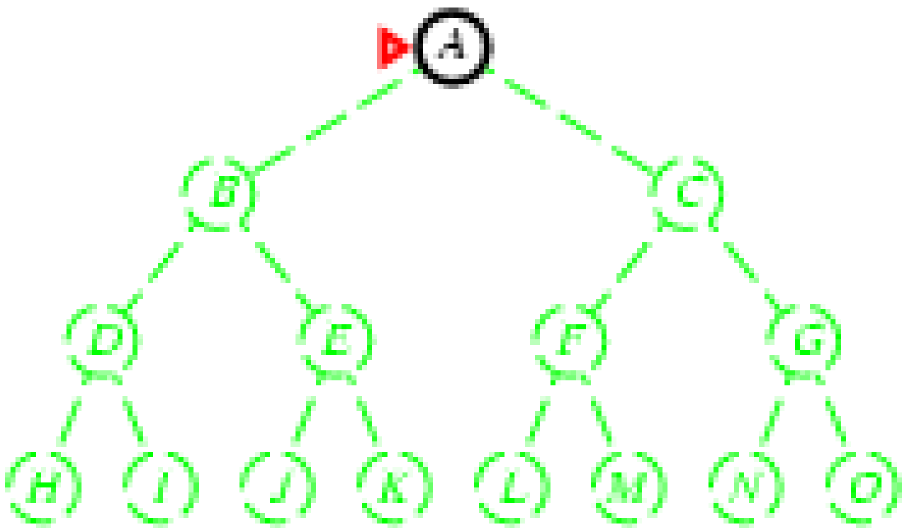


Properties of breadth-first search

- Complete? Time? Space?Optimal?
- Complete? Yes (if b is finite)
-
- Time? $1+b+b^2+b^3+\ldots +b^d + b(b^d-1) = O(b^{d+1})$
-
- Space? $O(b^{d+1})$ (keeps every node in memory)
-
- Optimal? Yes (if cost = 1 per step)
-
- Space is the bigger problem (more than time)
-

Depth-first search

- Expand deepest unexpanded node
-
- Implementation:
  - *frontier* = LIFO queue, i.e., put successors at front
  -



Depth-first search

- Expand deepest unexpanded node
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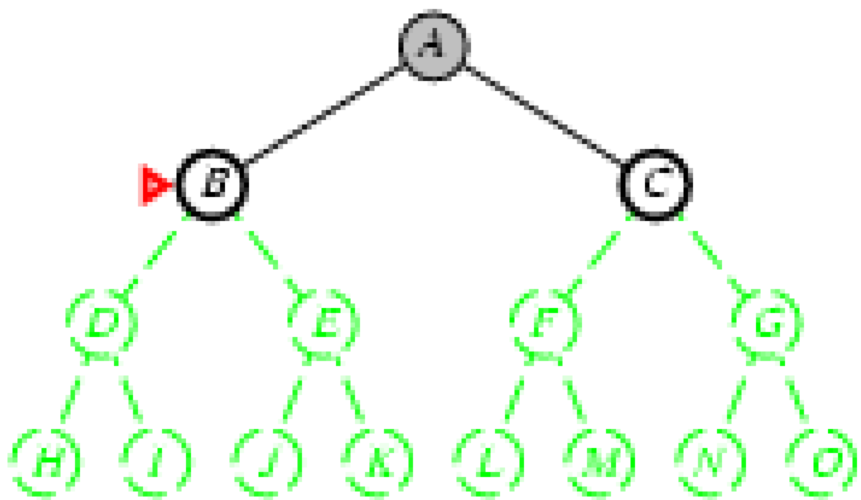

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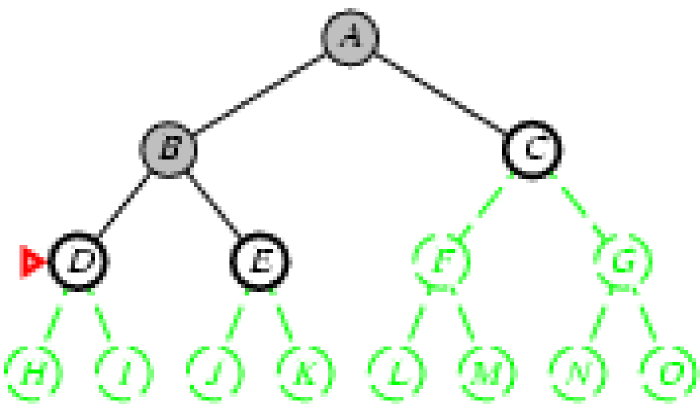

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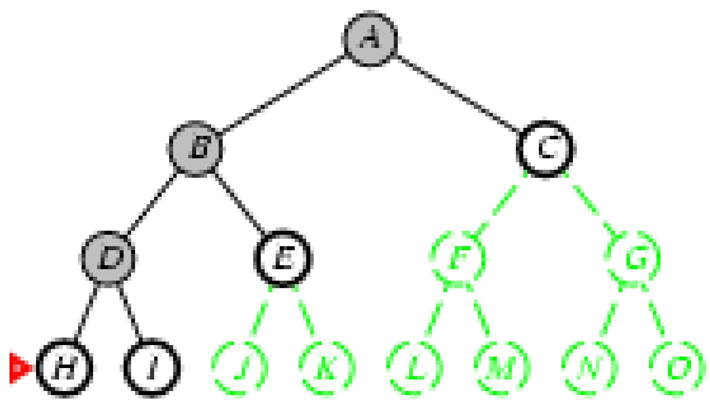

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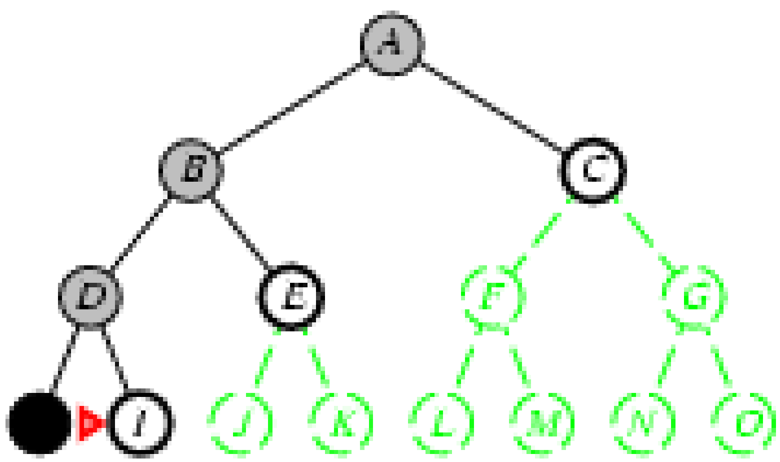

Depth-first search

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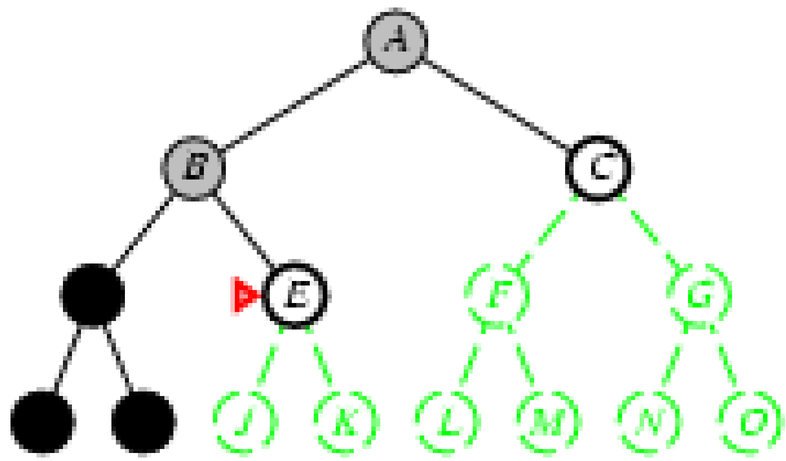

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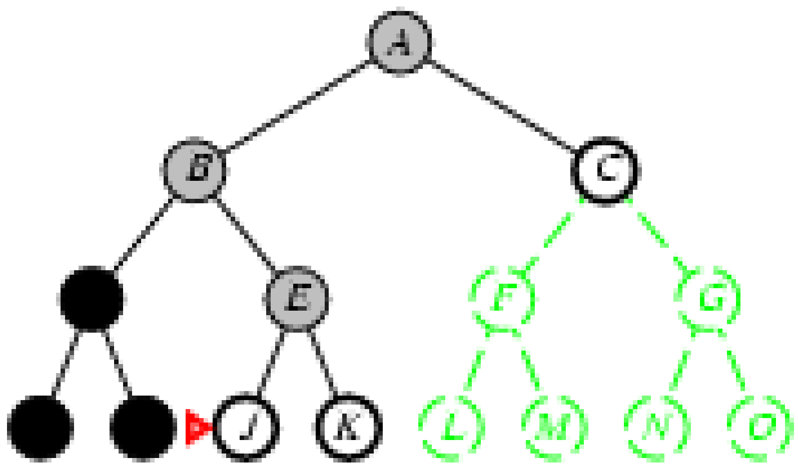

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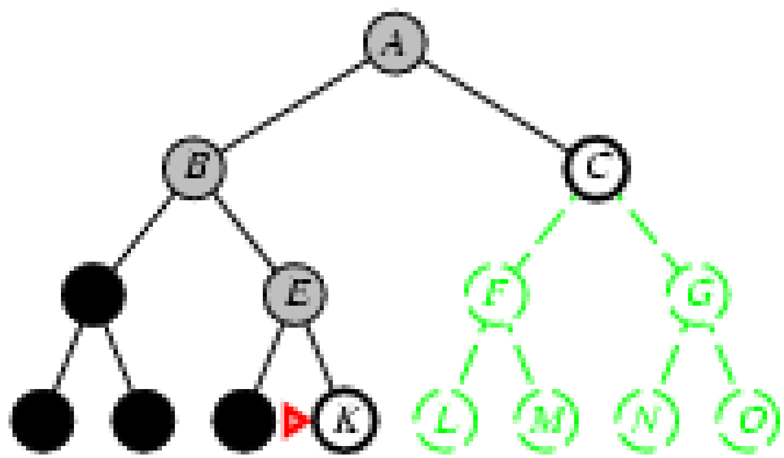

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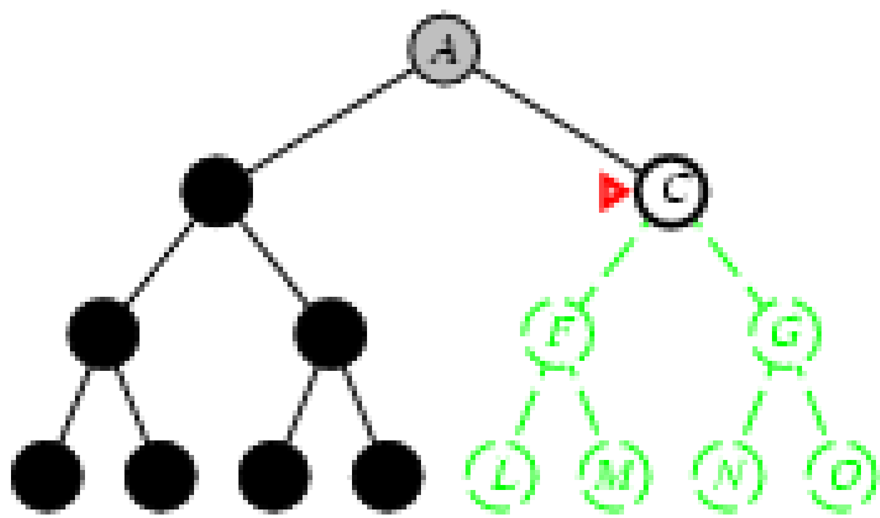

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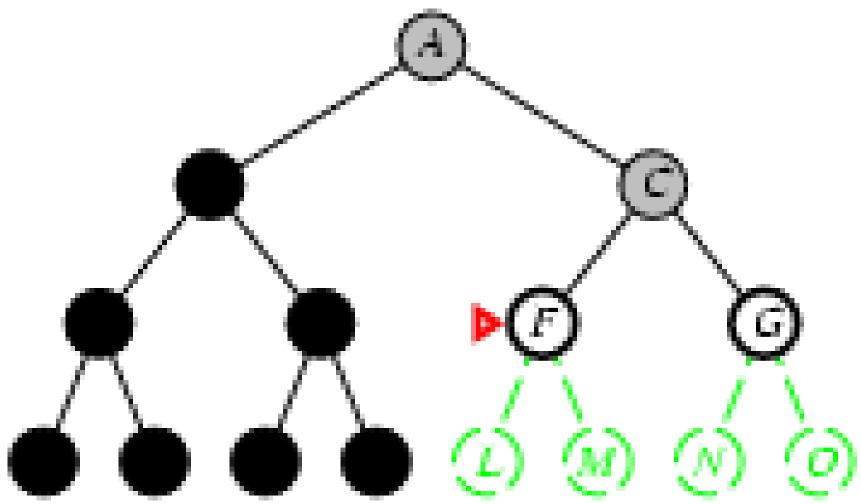

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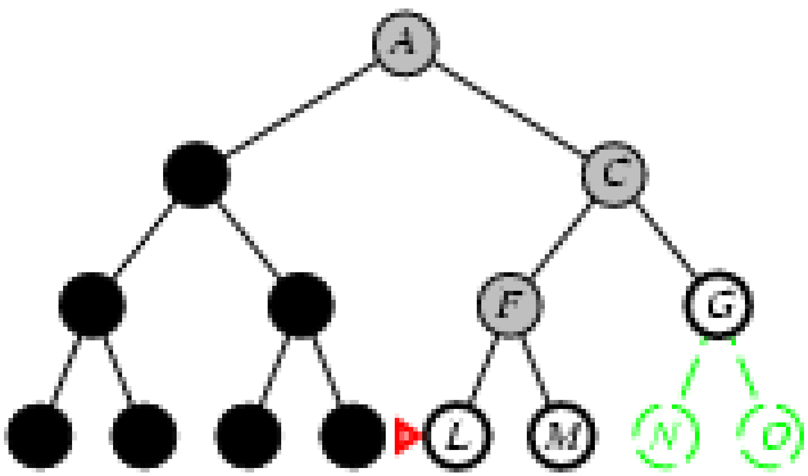

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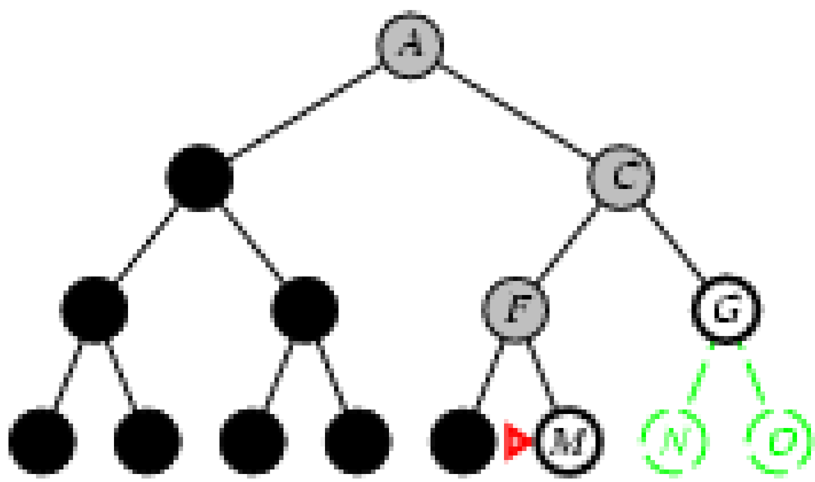


Properties of depth-first search

- Complete? Time? Space?Optimal?
- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path (graph search)
  -
    → complete in finite spaces
- Time? $O(b^m)$: terrible if maximum depth m is much larger than solution depth d
  - but if solutions are dense, may be much faster than breadth-first
  -
- Space? $O(bm)$, i.e., linear space! Store single path with unexpanded siblings.
  - Seems to be common in animals and humans.
  -
- Optimal? No.
- Important for exploration (on-line search).
-

Depth-limited search

- depth-first search with depth limit *l*,
  - i.e., nodes at depth *l* have no successors
  - Solves infinite loop problem
- Common AI strategy: let user choose search/resource bound.
- Complete? No if l < d:
- Time? $O(b^l)$:
- Space? $O(bl)$, i.e., linear space!
- Optimal? No if l > b

Iterative deepening search

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution, or failure

inputs: *problem*, a problem

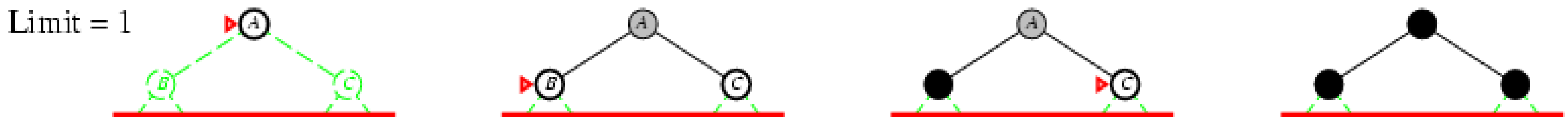
for *depth* ← 0 **to** ∞ **do**

result ← DEPTH-LIMITED-SEARCH(*problem*, *depth*)

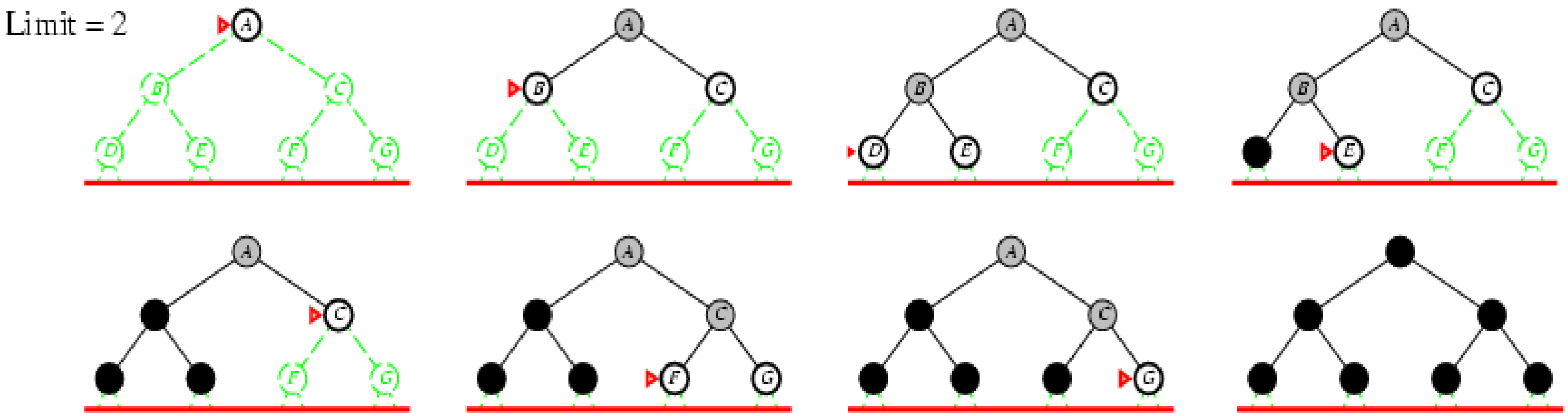
if *result* ≠ cutoff **then return** *result*

Iterative deepening search *l* =0

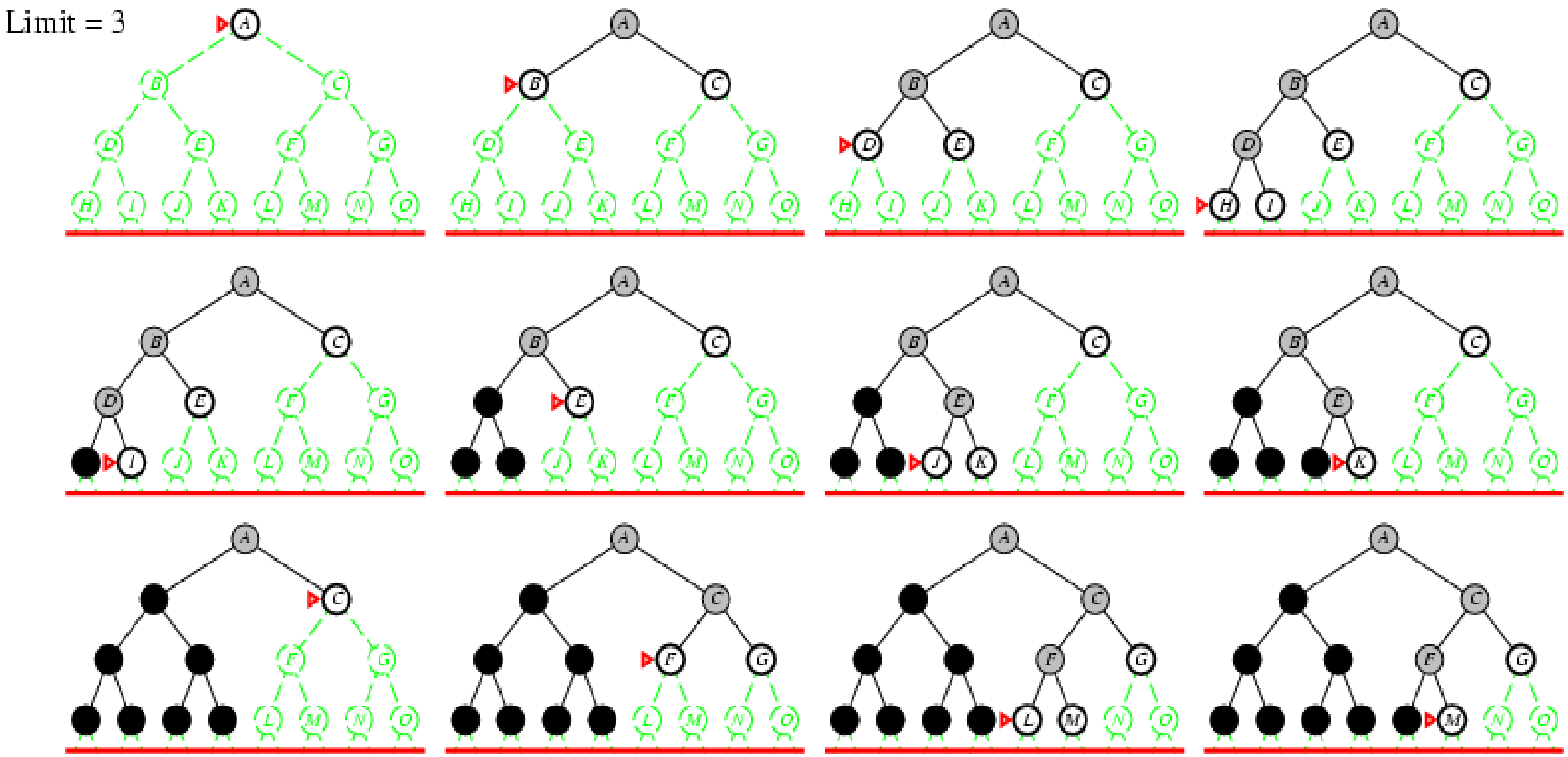
Iterative deepening search *l* =1



Iterative deepening search *l* =2



Iterative deepening search l =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\,b^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,
-
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  -
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$
  -

- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

- Complete? Yes
-
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
-
- Space? $O(bd)$
-
- Optimal? Yes, if step cost = 1

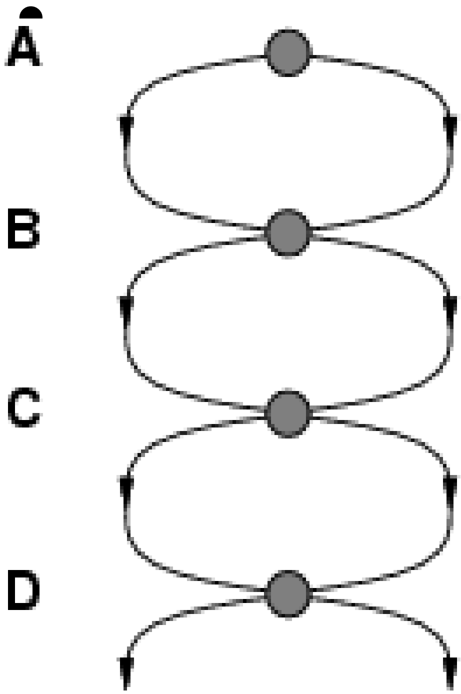
Summary of algorithms

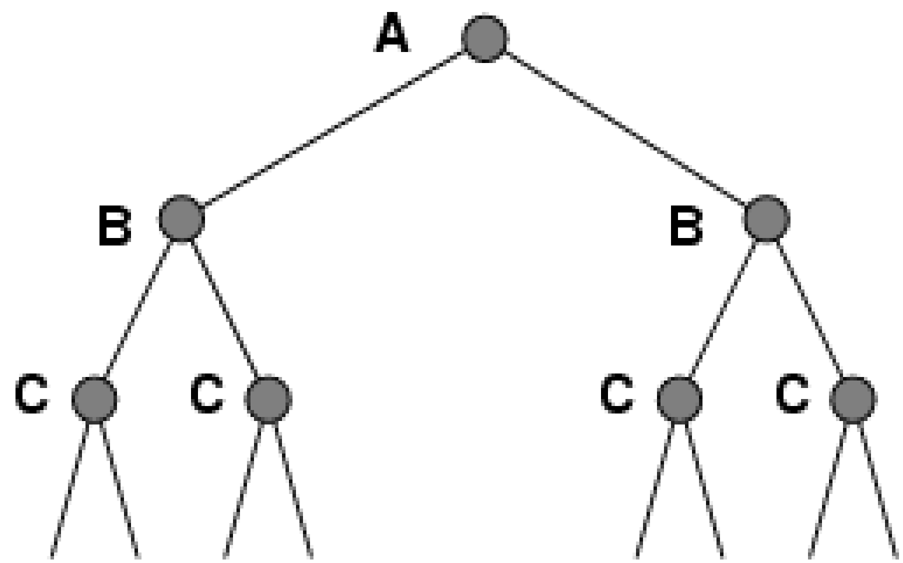
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!





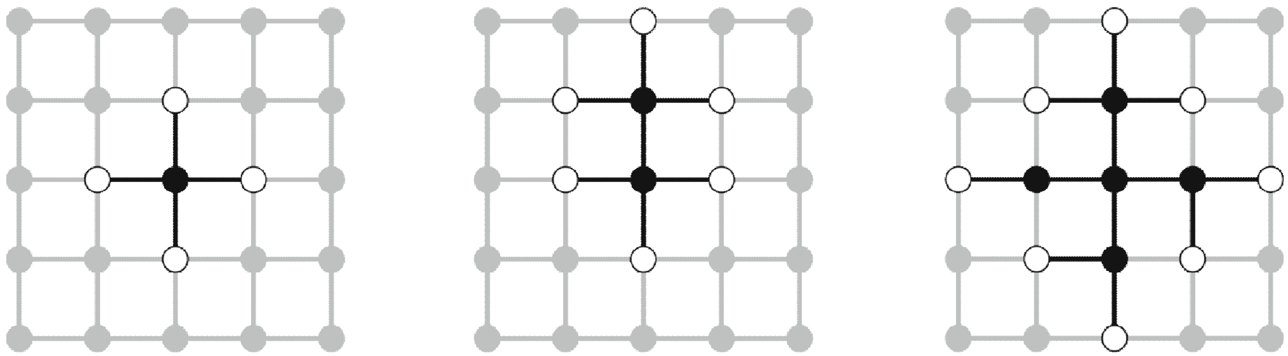
Graph search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure

   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
      if STATE[node] is not in closed then
         add STATE[node] to closed
         fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

- Simple solution: just keep track of which states you have visited.
- Usually easy to implement in modern computers.

The Separation Property of Graph Search



(a) (b) (c)

- Black: expanded nodes.
- White: frontier nodes.
- Grey: unexplored nodes.

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
-
- Variety of uninformed search strategies
-
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms
-