

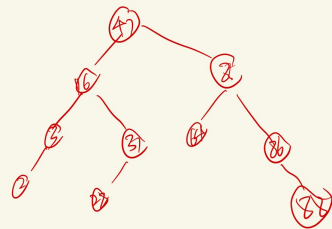
## Problem Set 4

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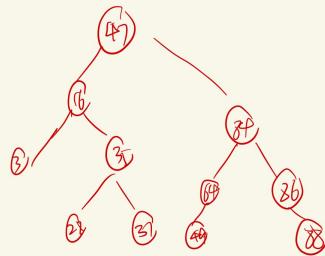
**Collaborators:** None

### Problem 4-1.

(a) 16(skew=2), 37(skew=-2)



(b)



(c)

**Problem 4-2.**

- (a) Min-heap
- (b) Mx-heap
- (c) [0,2,2,8,9,13]
- (d) Min-heap

**Problem 4-3.**

(a) Build a max-heap from  $A$  keyed on  $s_i$ , mapping to  $r_i$ , then use *delete\_max*  $k$  times to extract the  $k$  largest elements.

(b)

```
greater = []
def greater_than_x(A, i=0, x):
    assert len(A) > 0
    while i < len(A):
        if A[i] <= x:
            greater_than_x(A, (i+1)//2, x)
        else:
            greater.append(A[i])
            greater_than_x(A, 2*i+1, x)
            greater_than_x(A, 2*i+2, x)
    return greater
```

**Problem 4-4.** 1. Priority Queue P on the solar farms(a Max-Heap), storing for each solar farm its address  $s_i$ , capacity  $c_i$ , and its available capacity  $a_i$  (initially  $a_i = c_i$ ), keyed on available capacity;

2. Set data structure B(Hash-Table) mapping each building's name  $b_j$  to the address of the solar-powered building's name  $b_j$  to the address of the solar farm  $s_i$  that it is connected to and its demand  $d_j$ ;

3. Set data structure F(Hash-Table) mapping the address of each solar farm  $s_i$  to: (1) its own Set data structure  $B_i$ (Hash-Table) containing the buildings associated with that farm, and (2) a pointer to the location of  $s_i$  in P .

the operations:

initialize(S): build Set data structures P and then F from S, and initialize B as empty.

power\_on( $b_j, d_j$ ): First, find a solar farm to connect by deleting a solar farm  $s_i$  from P having largest available capacity  $c_i$  (delete\_max) and checking whether it's capacity is at least  $d_j$  . There are two cases:

a.  $d_j > c_i$ , so reinsert the solar farm back into P (relinking a pointer from F to a location in P ) and return that no solar farm can currently support the building.

b.  $d_j \leq c_i$ , so subtract  $d_j$  from  $c_i$  and reinsert it back into P (relinking a pointer). Then, add  $b_j$  to B mapping to  $s_i$ , and then find the  $B_i$  in F associated with  $s_i$  and add  $b_j$  to  $B_i$ .

power off( $b_j$ ): Lookup the  $s_i$  and  $d_j$  associated with  $b_j$  in B, lookup  $B_i$  in F using  $s_i$ , and remove  $b_j$  from  $B_i$ . Lastly, go to  $s_i$ 's location in P and remove  $s_i$  from P , increase  $c_i$  by  $d_j$ ,and reinsert  $s_i$  into P.

customers( $s_i$ ): lookup  $B_i$  in F using  $s_i$ , and return all names stored in  $B_i$ .

**Problem 4-5.** Store the matrices in a Sequence AVL tree  $T$ , where every node is augmented with the following information:

1. the size of the subtree of node;
2. the ordered product of all matrices in the subtree rooted of node;

operations:

`initialize( $M$ )`: build the AVL tree  $T$  from  $M$

`update_joint( $k, M$ )`: find the  $k$ th node  $v$  in  $T$ , replace the  $v$ .matrix with  $M$ , and update the ordered product of all matrices in the subtree rooted at  $v$ .

`full_transformation()`: return the ordered product of root in  $T$ .

**Problem 4-6.**

- (a) find the maximum
- (b)
- (c)
- (d) Submit your implementation to `alg.mit.edu`.