Problem Set 0

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Problem 0-1.

(a) 6,12

(b) 7

(c) 3

Problem 0-2.

(a) 1.5

(b) 12.25

(c) 13.75

Problem 0-3.

(a) True

(b) False

(c) False

Problem 0-4.

Base Case: For $n = 1$, we have

$$\sum_{i=1}^{1} i^3 = 1^3 = 1, \quad \text{and} \quad \left(\frac{1(1+1)}{2}\right)^2 = 1.$$

Thus, the formula holds for $n = 1$.

Inductive Hypothesis: Assume that the formula holds for some $n = k$:

$$\sum_{i=1}^{k} i^3 = \left(\frac{k(k+1)}{2}\right)^2.$$

Inductive Step: We must show that the formula also holds for $n = k + 1$:

$$\sum_{i=1}^{k+1} i^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2.$$

Starting from the left-hand side:

$$\sum_{i=1}^{k+1} i^3 = \left(\sum_{i=1}^{k} i^3\right) + (k+1)^3.$$

Apply the inductive hypothesis:

$$= \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3.$$

Factor out $(k+1)^2$:

$$= (k+1)^2 \left(\frac{k^2}{4} + (k+1)\right) = (k+1)^2 \left(\frac{k^2+4k+4}{4}\right) = (k+1)^2 \left(\frac{(k+2)^2}{4}\right).$$

Thus,

$$\sum_{i=1}^{k+1} i^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2.$$

Conclusion: By the principle of mathematical induction, the formula

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

holds for all positive integers n. $\square$

Problem 0-5. **Base Case:** For $|V| = 1$,the graph is trivially acyclic.

Assuming the claim is true for all graphs with $|V| \leq k$, we will show it holds for $|V| = k + 1$.

G is connected,so the aveerage degree of vertices is $\frac{2k}{k+1} < 2$.so there exists a vertex v with degree 1 connected to u. removing v and the edge (u, v), we get a graph G' with $|V| = k$,which is acyclic by the inductive hypothesis.Vertex v cannot be part of any cycle in G because it has degree 1, so G is acyclic.

Problem 0-6. Submit your implementation to `alg.mit.edu`.

```
def count_long_subarray(A):
    '''
    Input:  A     | Python Tuple of positive integers
    Output: count | number of longest increasing subarrays of A
    '''
    count = 0
    ##################
    see .python file
    ##################
    return count
```