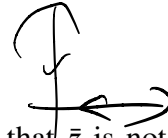


**18.04 Recitation 2**  
**Vishesh Jain**



1.1. Show directly using the definition of the complex derivative that  $\bar{z}$  is not complex differentiable.

1.2. What are the values that  $\lim_{z \rightarrow 0} \frac{\bar{z}}{z}$  can attain?  $\frac{x+iy}{x-iy} = \pm 1$

2.1. What are the real and imaginary parts of  $\cos(z)$ ? Of  $\sin(z)$ ?  $\cos z = \frac{e^{iz} + e^{-iz}}{2}$

2.2. What are these real and imaginary parts for  $z = x + i0$ ? What about for  $z = 0 + iy$ ?  $e^{ix} \cdot e^{-iy} + e^{-ix} \cdot e^{iy}$

2.3. Is it true that  $\cos(z)$  and  $\sin(z)$  are bounded functions?  $\frac{e^{iz} + e^{-iz}}{2}$

2.4. Is it true that  $\cos^2 z + \sin^2 z = 1$ ?  $\frac{e^{2iz} + e^{-2iz} + 2}{4} + \frac{e^{2iz} - e^{-2iz}}{4} = 1$

3.1 Show that  $e^z$  is continuous as a function of  $z$ .  $e^z = e^{x+iy} = e^x (\cos y + i \sin y)$

3.2. Use this to show that  $\cos(z)$  and  $\sin(z)$  are continuous as functions of  $z$ .

3.3. Is  $\bar{z}$  continuous as a function of  $z$ ? Is this consistent with Problem 1?

$$(x_0, y_0) \rightarrow (x - iy) - (x_0 - iy_0) \leq$$

4.1. Express the following functions in the form  $f(z) = u(x, y) + iv(x, y)$ :  $e^z$ ,  $z^2$ ,  $\cos(z)$  and  $\sin(z)$  (see also Problem 2)  $z^2 = (x+iy)^2 = (x^2 - y^2) + i \cdot 2xy$

4.2. Compute the partial derivatives  $u_x, u_y, v_x, v_y$  for each of these functions. Do they satisfy the Cauchy-Riemann equations?  $u_x = 2x, u_y = -y, v_x = 2y, v_y = 2x$

4.3. What is  $f'(z)$  in each of these cases?

4.3. Repeat this for  $\bar{z}$ . Is this consistent with Problem 1?

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## 18.04 Complex Variables with Applications

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