18.04 Recitation 2
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1.1. Show directly using the definition of the complex derivative that $\bar{z}$ is not complex differentiable.

1.2. What are the values that $\lim_{z\to 0} \frac{\bar{z}}{z}$ can attain? $\frac{x+iy}{x-iy} = \pm 1$

2.1. What are the real and imaginary parts of $\cos(z)$? Of $\sin(z)$? $\cos z = \frac{e^{iz}+e^{-iz}}{2}$

2.2. What are these real and imaginary parts for $z = x + i0$? What about for $z = 0 + iy$? $= \frac{e^{ix}\cdot e^{-y} + e^{-ix}\cdot e^{y}}{2}$

2.3. Is it true that $\cos(z)$ and $\sin(z)$ are bounded functions?

2.4. Is it true that $\cos^2 z + \sin^2 z = 1$? $\frac{e^{2iz}+e^{-2iz}+2}{4} + \frac{e^{2iz}+e^{-2iz}-2}{-4} = 1$

3.1 Show that e^z is continuous as a function of z. $e^z = e^x(\cos y + i\sin y)$

3.2. Use this to show that $\cos(z)$ and $\sin(z)$ are continuous as functions of z.

3.3. Is $\bar{z}$ continuous as a function of z? Is this consistent with Problem 1?

(x_0, y_0) $|(x-iy)-(x_0-iy_0)| \le$

4.1. Express the following functions in the form $f(z) = u(x, y) + iv(x, y)$: e^z, z^2, $\cos(z)$ and $\sin(z)$ (see also Problem 2) $z^2 = (x+iy)^2 = (x^2-y^2) + i\cdot 2xy$

4.2. Compute the partial derivatives u_x, u_y, v_x, v_y for each of these functions. Do they satisfy the Cauchy-Riemann equations? $u_x = 2x$ $u_y = -2y$ $v_x = 2y$ $v_y = 2x$

4.3. What is $f'(z)$ in each of these cases?

4.3. Repeat this for $\bar{z}$. Is this consistent with Problem 1?