18.04 Problem Set 2, Spring 2018

Calendar
M Feb. 12: Reading: Topic 2 sections 1-5
W Feb. 14: Reading: Topic 2 sections 6-9
R Feb. 15: Recitation
F Feb. 16: Reading: Review of 18.02

Coming next

Feb. 20-23: Analytic functions; Cauchy's theorem

$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$ $\quad \frac{d\cos\theta}{d\theta} = \frac{ie^{i\theta}-ie^{-i\theta}}{2} = \frac{e^{-i\theta}-e^{i\theta}}{2i} = -\sin\theta$

Problem 1. (20: 10,10 points)
(a) Show that $\cos(z)$ is an analytic for all z, i.e. it's an entire function. Compute its derivative and show it equals $-\sin(z)$.

(b) Give the region where $\cot(z)$ is analytic. Compute its derivative.

Problem 2. (20: 10,10 points)

(a) Let $P(z) = (z - r_1)(z - r_2)\ldots(z - r_n)$. Show that $\dfrac{P'(z)}{P(z)} = \displaystyle\sum_{j=1}^{n} \frac{1}{z - r_j}$

Suggestion: try $n = 2$ and $n = 3$ first.

(b) Compute and simplify $\dfrac{d}{dz}\left(\dfrac{az+b}{cz+d}\right)$ $= \dfrac{a(cz+d) - c(az+b)}{(cz+d)^2} = \dfrac{ad-bc}{(cz+d)^2}$

What happens when $ad - bc = 0$ and why?

Problem 3. (10 points)
Why does $\log(e^z)$ not always equal z? diff. branch

Hint: This is true for any branch of log. Start with the principal branch.

Problem 4. (20: 10,10 points) $f(z) = u(x,y) + iv(x,y)$
(a) Let $f(z)$ be analytic in a D a disk centered at the origin. Show that $F_1(z) = \overline{f(\overline{z})}$ is analytic in D.

$F_1 \quad z \quad u(x,-y) - iv(x,-y)$

(b) Let $f(z)$ be as in part (a). Show that $F_2(z) = f(\overline{z})$ is not analytic unless f is constant.

Hint for both parts: Use the Cauchy-Riemann equations.

Problem 5. (10 points) $f(z) = x^2+y^2$ $\quad u_x = 2x \quad u_y = 2y \quad v_x = v_y = 0$

Let $f(z) = |z|^2$. Show the $\dfrac{df}{dz}$ exists at $z = 0$, but nowhere else.

Problem 6. (10 points)
Using the principal branch of log give a region where $\sqrt{z^2 - 1}$ is analytic.

$\sqrt{x^2-y^2+2xyi-1}$