

# GRE

## 数学-张斯乐

学GRE  
就上新东方在线

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新东方在线

# 1. 关于 GRE 数学

## 1.1 GRE 数学考试介绍&课程学习说明

- 数量推理 VS 数学

The Quantitative Reasoning measure of the GRE revised General Test assesses your:

- basic mathematical skills
- understanding of elementary mathematical concepts
- ability to reason quantitatively and to model and solve problems with quantitative methods

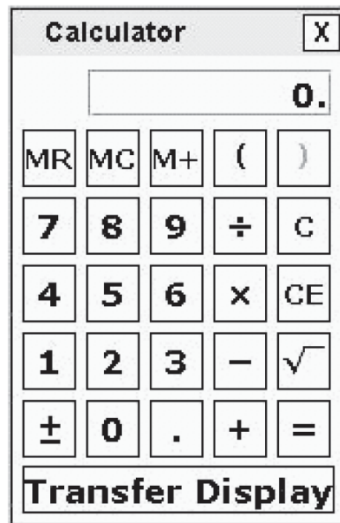
四大知识点：算数，代数，几何，数据分析

- 考试时间&题量

不算加试，2 个 section，每个 section 共 20 题，35 分钟，共 40 题，满分 170

自适应考试，第一个 section 难度中等，第二个 section 可能变难

- 关于计算器



- 课程学习方法&注意事项（写下适合自己考试时间的规划）



## 1.2 GRE 数学题型介绍

- 数量推理 VS 数学

The Quantitative Reasoning measure has four types of questions:

- Quantitative Comparison questions
- Multiple-choice questions-Select One Answer Choice
- Multiple-choice questions-Select One or More Answer Choices
- Numeric Entry questions

- 数量比较 (8-9/section)

1. Quantity A: The least prime number greater than 24

Quantity B: The greatest prime number less than 28

A. Quantity A is greater.

- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2. Lionel is younger than Maria.

Quantity A: Twice Lionel's age

Quantity B: Maria's age

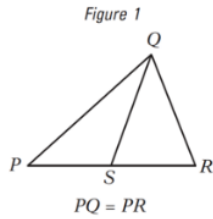
- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3. Quantity A: 54% of 360

Quantity B: 150

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

4.



Quantity A: PS

Quantity B: SR

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

5. The following sample questions focus on simplifying the comparison.

$$y > 4$$

Quantity A:  $\frac{3y+2}{5}$

Quantity B:  $y$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

6. Quantity A:  $x^2 + 1$

Quantity B:  $2x-1$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

- 数量比较需要注意事项

Tips for Answering

- Become familiar with the answer choices. Quantitative Comparison questions always have the same answer choices, so get to know them, especially the last choice, "The relationship cannot be determined from the information given. Never select this last choice if it is clear that the values of the two quantities can be determined by computation. Also, if you determine that one quantity is greater than the other, make sure you carefully select the corresponding choice so as not to reverse the first two choices.
- Avoid unnecessary computations. Don't waste time performing needless computations in order to compare the two quantities. Simplify, transform, or estimate one or both of the given quantities only as much as is necessary to compare them.

- Remember that geometric figures are not necessarily drawn to scale. If any aspect of a given geometric figure is not fully determined, try to redraw the figure, keeping those aspects that are completely determined by the given information fixed but changing the aspects of the figure that are not determined. Examine the results. What variations are possible in the relative lengths of line segments or measures of angles?
- Plug in numbers. If one or both of the quantities are algebraic expressions, you can substitute easy numbers for the variables and compare the resulting quantities in your analysis. Consider all kinds of appropriate numbers before you give an answer: e.g., zero, positive and negative numbers, small and large numbers, fractions and decimals. If you see that Quantity A is greater than Quantity B in one case and Quantity B is greater than Quantity A in another case, choose "The relationship cannot be determined from the information given."
- Simplify the comparison. If both quantities are algebraic or arithmetic expressions and you cannot easily see a relationship between them, you can try to simplify the comparison. Try a step-by-step simplification that is similar to the steps involved when you solve the equation  $5-4x+3$  for  $x$ , or similar to the steps involved when you determined that the inequality  $\frac{3y+2}{5} < y$  is equivalent to the simpler inequality  $1 < y$ . Begin by setting up a comparison involving the two quantities, as follows:

Quantity A ? Quantity B

Where ? is a “placeholder” that could represent the relationship greater than ( $>$ ), less than ( $<$ ), or equal to ( $=$ ) or could represent the fact that the relationship cannot be determined from the information given. Then try to simplify the comparison, step by step, until you can determine a relationship between simplified quantities. For example, you may conclude after the last step that ? represents equal to ( $=$ ). Based on this conclusion, you may be able to compare Quantities A and B. To understand this strategy more fully, see sample questions 6 to 9.

● 单选题 (8-9/section)

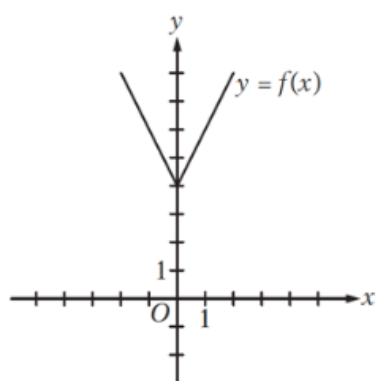


Figure 5

1. The figure above shows the graph of the function  $f$  defined by  $f(x) = |2x| + 4$  for all numbers  $x$ . For which of the following functions  $g$ , defined for all numbers  $x$ , does the graph of  $g$  intersect the graph of  $f$ ?
- A.  $g(x) = x - 2$
  - B.  $g(x) = x + 3$
  - C.  $g(x) = 2x - 2$
  - D.  $g(x) = 2x + 3$
  - E.  $g(x) = 3x - 2$

2. A certain jar contains 60 jelly beans – 22 white, 18 green, 11 yellow, 5 red, and 4 purple. If a jelly bean is to be chosen at random, what is the probability that the jelly bean will be neither red nor purple?
- A. 0.09
  - B. 0.15
  - C. 0.54
  - D. 0.85
  - E. 0.91

● 单选题需要注意事项

Tips for Answering

- Use the fact that the answer is there. If your answer is not one of the five answer choices given, you should assume that your answer is incorrect and do the following:
  - Reread the question carefully-you may have missed an important detail or misinterpreted some information.
  - Check your computations-you may have made a mistake, such as miskeying a number on the calculator.
  - Reevaluate your solution method-you may have a flaw in your reasoning.
- Examine the answer choices. In some questions you are asked explicitly

which of the choices has a certain property. You may have to consider each choice separately, or you may be able to see a relationship between the choices that will help you find the answer more quickly. In other questions, it may be helpful to work backward from the choices, say, by substituting the choices in an equation or inequality to see which one works. However, be careful, as that method may take more time than using reasoning.

- For questions that require approximations, scan the answer choices to see how close an approximation is needed. In other questions, too, it may be helpful to scan the choices briefly before solving the problem to get a better sense of what the question is asking. If computations are involved in the solution, it may be necessary to carry out all computation exactly and round only your final answer in order to get the required degree of accuracy. In other questions, you may find that estimation is sufficient and will help you avoid spending time on long computations.
- 不定选择题 (1-2/section)
  1. Which two of the following numbers have a product that is between -1 and 0?  
Indicate both of the numbers.  
  
A. -20  
  
B. -10  
  
C.  $2^{-4}$



D.  $3^{-2}$

2. Each employee of a certain company is in either Department X or Department Y, and there are more than twice as many employees in Department X as in Department Y. The average (arithmetic mean) salary is \$25,000 for the employees in Department X and \$35,000 for the employees in Department Y. Which of the following amounts could be the average salary for all of the employees of the company?

Indicate all such amounts.

- A. \$26,000
- B. \$28,000
- C. \$29,000
- D. \$30,000
- E. \$31,000
- F. \$32,000
- G. \$34,000

3. Which of the following could be the units digit of  $57^n$ , where  $n$  is a positive integer?

Indicate all such digits.

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4
- F. 5
- G. 6
- H. 7
- I. 8
- J. 9

- 不定选择题需要注意事项

Tips for Answering

- Note whether you are asked to indicate a specific number of answer choices or all choices that apply. In the latter case, be sure to consider all of the choices, determine which ones are correct, and select all of those and only those choices. Note that there may be only one correct choice.
- In some questions that involve conditions that limit the possible values of numerical answer choices, it may be efficient to determine the least and/or the greatest possible value. Knowing the least and/or greatest possible value may enable you to quickly determine all of the choices that are correct.

- Avoid lengthy calculations by recognizing and continuing numerical patterns.

● 填空题 (1-2/section)

1. Rectangle R has length 30 and width 10, and square S has length 5. The perimeter of S is what fraction of the perimeter of R?

2.

**RESULTS OF A USED-CAR AUCTION**

	<u>Small Cars</u>	<u>Large Cars</u>
Number of cars offered	32	23
Number of cars sold	16	20
Projected sales total for cars offered (in thousands)	\$70	\$150
Actual sales total (in thousands)	\$41	\$120

*Figure 7*

For the large cars sold at an auction that is summarized in the table above, what was the average sale price per car?

3. A merchant made a profit of \$5 on the sale of a sweater that cost the merchant

\$15. What is the profit expressed as a percent of the merchant's cost?

Give your answer to the nearest whole percent.

4. Working alone at its constant rate, machine A produces  $k$  car parts in 10 minutes. Working alone at its constant rate, machine B produces  $k$  car parts in 15 minutes. How many minutes does it take machine A and B, working simultaneously at their respective constant rate, to produce  $k$  car parts?

- 填空题需要注意事项

Tips for Answering

- Make sure your answer the question that is asked. Since there are no answer choices to guide you, read the question carefully and make sure you provide the type of answer required. Sometimes there will be labels before or after the answer box to indicate the appropriate type of answer. Pay special attention to units such as feet or miles, to orders of magnitude such as millions or billions, and to percents as compared with decimals.
- If you are asked to round your answer, make sure you round to the required degree of accuracy. For example, if an answer of 46.7 is to be rounded to the nearest integer, you need to enter the number 47. If your solution strategy involves intermediate computations, you should carry out all computations exactly and round only your final answer in order to get the required degree of accuracy. If no rounding instructions are given, enter

the exact answer.

- Examine your answer to see if it is reasonable with respect to the information given. You may want to use estimation or another solution path to double-check your answer.

- 数据分析题 (第 14-16 题)

Questions 1 to 3 are based on the following data.

**ANNUAL PERCENT CHANGE IN DOLLAR AMOUNT OF SALES  
AT FIVE RETAIL STORES FROM 2006 TO 2008**

Store	Percent Change from 2006 to 2007	Percent Change from 2007 to 2008
<i>P</i>	10	-10
<i>Q</i>	-20	9
<i>R</i>	5	12
<i>S</i>	-7	-15
<i>T</i>	17	-8

Figure 8

1. If the dollar amount of sales at Store P was \$800,000 for 2006, what was the dollar amount of sales at that store for 2008?
  - A. \$727,000
  - B. \$792,000
  - C. \$800,000
  - D. \$880,000
  - E. \$968,000
2. At Store T, the dollar amount of sales for 2007 was what percent of the dollar amount of sales for 2008?

Give your answer to the nearest 0.1 percent.

3. Based on the information given, which of the following statements must be true?

Indicate all such statements.

- A. For 2008 the dollar amount of sales at Store R was greater than that at each of the other four stores.
- B. The dollar amount of sales at Store S for 2008 was 22 percent less than that for 2006.
- C. The dollar amount of sales at Store R for 2008 was more than 17 percent greater than that for 2006.

● 数据分析题注意事项

Tips for Answering

- Scan the data presentation briefly to see what it is about, but do not spend time studying all of the information in detail. Focus on those aspects of the data that are necessary to answer the questions. Pay attention to the axes and scales of graphs; to the units of measurement or orders of magnitude (such as billions) that are given in the titles, labels, and legends; and to any notes that clarify the data.
- Bar graphs and circle graphs, as well as other graphical displays of data, are drawn to scale, so you can read or estimate data visually from such graphs.

For example, you can use the relative sizes of bars or sectors to compare

the quantities that they represent, but be aware of broken scales and of bars that do not start at 0.

- The questions are to be answered only on the basis of the data presented, everyday facts (such as the number of days in a year), and your knowledge of mathematics. Do not make use of specialized information you may recall from other sources about the particular context on which the questions are based unless the information can be derived from the data presented.

## 2. 算数 (Arithmetic)

### 2.1 整数

#### 2.1.1 整数的概念

1. Natural Numbers (自然数) : 大于零的正整数。如: 1, 2, 3, .....其中 1 为最小的自然数。
2. Odd Numbers (奇数) : 不能被 2 所整除的整数。如: 1, -1, 3, -3.....
3. Even Numbers (偶数) : 能够被 2 所整除的整数。如: 0, 2, -2, 4, -4.....
4. Prime Numbers (质数) : 除了 1 和它本身之外, 不能被其他正整数所整除的自然数, 如: 2, 3, 5, 7, 11.....其中 2 是最小的质数。
5. Composite Numbers (合数) : 除了 1 和它本身之外, 还有其他因子的自然数, 如: 4, 6, 8, 9, 10.....其中 4 是最小的合数。(注: 质数和合数都不能为负数, 0 和 1 既不是质数也不是合数。)

6. Mutual Prime Numbers (互质数) : 如果两个数的最大公约数为 1, 那么这两个数叫做互质数, 例如: 13 和 15, 19 和 23 等。
7. Multiple and Divisions (倍数和约数) : 当整数  $a$  能被另一个整数  $b$  所整除时,  $a$  称为  $b$  的倍数,  $b$  称为  $a$  的约数和因数, 例如: 10 是 5 的倍数, 5 是 10 的约数。
8. Common Multiple (公倍数) : 如果一个数同时是几个数的倍数, 则称这个数为它们的公倍数; 公倍数中最小的称为最小公倍数 (least 或 lowest common multiple) 。例如: 12, 24, 36 等都是 2, 4, 6, 12 的公倍数, 其中 12 是它们的最小公倍数。
9. Common Factor or Divisor (公约数或公因数) : 如果一个数同时是几个数的约数, 则称这个数为它们的公约数或公因数; 公约数中最大的被称为最大公约数 (公因数) (greatest common factor or divisor) 。例如: 2, 7, 14 都是 28, 42, 70 的公约数, 14 是它们的最大公约数。
10. Perfect Square (完全平方数) : 若一个整数开平方后还是整数, 则这个数被称之为完全平方数。例如: 4, 9, 16, 25.....完全平方数均为自然数。
11. Perfect Cube (完全立方数) : 若一个整数开三次方后还是整数, 则这数称之为完全立方数。例如: -27, -8, 0, 8, 27.....
12. Quotients and Remainder (商和余数) : 当一个正整数除以另一个正整数其商不为整数时就存在商和余数问题。余数和商为大于或等于零的整数, 余数总小于除数。例如 15 除以 7 时, 其商为 2, 余数为 1。
13. Consecutive Integers (连续整数) : 按从小到大的顺序相连的几个整数称为连续整数。例如: -2, -1, 0, 1, 2 是五个连续的整数。连续正整数的算术平均值是首项和末项的算术平均值。



## 2.1.2 整数的性质

- 奇偶性
  1.  $n$  是整数, 则  $2n$  为偶数,  $2n+1$  为奇数。
  2. 奇数个奇数相加减其结果必为奇数。
  3. 偶数个奇数相加减其结果必为偶数。
  4. 奇数和偶数相加减, 其结果必为奇数。
  5. 任意多个偶数相加减, 其结果必为偶数。
  6. 若  $n$  ( $n$  为大于 1 的自然数) 个整数连乘其结果为奇数, 则这  $n$  个整数必然都是奇数。
  7. 若  $n$  ( $n$  为大于 1 的自然数) 个整数连乘其结果为偶数, 则这  $n$  个整数中至少有一个为偶数。
  8. 若  $n$  ( $n$  为大于 1 的自然数) 个连续整数相加等于零, 则  $n$  必为奇数。
  9. 若  $n$  ( $n$  为大于 1 的自然数) 个连续奇数相加等于零, 则  $n$  必为偶数。

10. 若  $n$  ( $n$  为大于 1 的自然数) 个连续偶数相加等于零, 则  $n$  必为奇数。
11. 自然数间相加或相乘必然还是自然数。
12. 自然数间相减必然为整数 (可正可负)。
13. 奇数个连续整数的算术平均值等于这奇数个数中中间大小那个数的值。
14. 偶数个连续整数的算术平均值等于这偶数个数中中间两个数的算术平均值。
15. 任何一个大于 2 的偶数都可以表示为两个质数的和。

例: 下面哪个数不能表达为两个质数的和?

A.21    B.14    C.18    D.28    E.23

● 最大公约数和最小公倍数

1. 如果整数  $a$  能被整数  $b$  整除, 则  $a$  能被  $b$  的因数 (或约数) 所整除。
2. 如果  $a$  为质数,  $n$  为非负整数 (non-negative integers or whole numbers), 则  $a^n$  的因数为  $n+1$  个 (包括 1 和  $a^n$ )。
3. 0 为任何一个非 0 整数的倍数, 1 为任何一个整数的约数, 任何一个质数有且只有 1 和它本身两个约数。
4. 最小公倍数的求解步骤:
  - ① 所有的数分别表示为各自的质因数的乘积;
  - ② 如果所有的乘积中有公因数, 则将式子中相同的质因子都提出来, 且只保留指数较大的一个因子作为公因数, 除去其他乘积中指数较小的公因数;
  - ③ 将剩下的乘积中的所有因数乘起来, 就得到最小公倍数。
5. 最大公约数的求解步骤:
  - ① 将所有的数表示成自己的质因数乘积的形式;

- ② 将式子中相同的质因子都提出来，并取幂指数较小的一个作为其相应的公因数；
- ③ 将取出的公因数相乘，就得到了最大公约数。

例：求 84 和 90 的最小公倍数和最大公约数。

### ● 因子数量

1. 因子个数求法：将数  $n$  分解成为质因子相乘的形式，然后将每个质因子的幂指数分别加 1 后连续相乘所得的结果就是  $n$  的因子个数，

$$n = a^x * b^y * c^z (a, b, c \text{ 为质数})$$

$$\text{因子数} = (x+1)(y+1)(z+1)$$

例：求 252 因子个数。

2. 任何一个自然数若有奇数个因子，则此自然数必为完全平方数，若有偶数个因子，则必不为完全平方数
3. 只有一个因子的自然数只有 1 个，为 1
4. 只有 2 个因子的自然数都是质数
5. 有 2 个以上（不包括 2 个）因子的数都是合数

- 余数算法

例：若自然数  $n$  被 3 除余 2，被 4 除余 1，问  $n$  被 12 除余几？

例：2001 年的元旦是星期六，问 2002 年的元旦是星期几？

- 自然数  $n$  次幂尾数特征

1. 尾数为 2 的数的幂的个位数一定以 2, 4, 8, 6 循环
2. 尾数为 3 的数的幂的个位数一定以 3, 9, 7, 1 循环
3. 尾数为 4 的数的幂的个位数一定以 4, 6 循环
4. 尾数为 6 的数的幂的个位数一定以 6 循环
5. 尾数为 7 的数的幂的个位数一定以 7, 9, 3, 1 循环
6. 尾数为 8 的数的幂的个位数一定以 8, 4, 2, 6 循环
7. 尾数为 9 的数的幂的个位数一定以 9, 1 循环

例： $3^{321}$  和  $7^{123}$  的个位哪个大？

## 2.1.3 练习

1. How many positive whole numbers less than 81 are NOT equal squares of whole numbers?  
A. 9

- B. 70
- C. 71
- D. 72
- E. 73

2. A printer numbered consecutively the pages of a book, beginning with 1 on the first page. In numbering the pages, he printed a total of 189 digits.

Quantity A: The number of pages in the book

Quantity B: 100

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3.  $n = 7 \cdot 19^3$

Quantity A: The number of distinct positive factors of  $n$

Quantity B: 10

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

4. Seven is equal to how many thirds of seven?

- A.  $\frac{1}{3}$
- B. 1
- C. 3
- D. 7
- E. 21

5. How many positive integers less than 20 are equal to the sum of a positive multiple of 3 and a positive multiple of 4?

- A. Two
- B. Five
- C. Seven
- D. Ten
- E. Nineteen

6. What is the remainder when  $6^3$  is divided by 8?

- A. 5
- B. 3
- C. 2
- D. 1

- E. 0
7. For which of the following pairs of integers is the least common multiple of the integers minus their greatest common divisor the greatest?
- A. 3,12
- B. 5,6
- C. 10,20
- D. 11,12
- E. 15,30
8. If  $p$  is a prime number greater than 11, and  $p$  is the sum of the two prime numbers  $x$  and  $y$ , then  $x$  could be which of the following?
- A. 2
- B. 5
- C. 7
- D. 9
- E. 13
9. If  $x$ ,  $y$  and  $z$  are consecutive integers and  $x < y < z$ , which of the following must be true?
- I.  $xyz$  is even
- II.  $x+y+z$  is even.
- III.  $(x+y)(y+z)$  is odd.

- A. None
- B. I only
- C. II only
- D. I and III only
- E. I, II and III

10. When a certain number is divided by 7, the remainder is 0. If the remainder is not 0 when the number is divided by 14, then the remainder must be

- A. 1
- B. 2
- C. 4
- D. 6
- E. 7

11. Quantity A: The number of different positive divisors of 12

Quantity B: The number of different positive divisors of 50

- A. Quantity A is greater.
- B. Quantity B is greater.



- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.
12. Which of the following numbers is NOT the sum of three consecutive odd integers?
- A. 15
- B. 75
- C. 123
- D. 297
- E. 313
13. The number  $10^{30}$  is divisible by all of the following EXCEPT
- A. 250
- B. 125
- C. 32
- D. 16
- E. 6
14.  $x$  is the sum of the first 25 positive even integers.  $y$  is the sum of the first 25 positive odd integers.

Quantity A:  $x$

Quantity B:  $y+25$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

15. When the even integer  $n$  is divided by 7, the remainder is 3.

Quantity A: The remainder when  $n$  is divided by 14

Quantity B: 10

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

16. If the sum of five consecutive even integers is 70, what is the value of the greatest of the five integers.

- A. 12
- B. 14
- C. 18
- D. 20
- E. 22

## 2.2 分数，小数和百分比

### 2.2.1 分数和小数

- 小数的性质

In the decimal system, the position of the period or decimal point determines the place value of the digits. For example, the digits in the number 8,796.435 have the following place values:

8--thousands' digit(千位数字)

7--hundreds' digit(百位数字)

9--tens' digit(十位数字)

6--ones or units' digit(个位数字)

4--tenths' digit(十分位数字)

3--hundredths' digit(百分位数字)

5--thousandths' digit(千分位数字)

注意: digit 是“数字”, 即 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 这是个阿拉伯数字;

number 是“数”, 如 898 是一个数, 由 3 个 digit 组成。

\* reciprocal 为倒数, 如 the reciprocal of 2 is  $\frac{1}{2}$

- 比例问题

一个比率(ratio)可以表示成许多方式, 例如:the ratio of 3 to 4 可以被表达为 3 to 4,

3: 4 或  $\frac{3}{4}$ 。注意比率中项的顺序是重要的, 即 3 to 4 和 4 to 3 不同。

例: Among registered voter in a certain district, the ratio of men to women is 3:5.

Of the district currently includes 24,000 registered voters, how many additional men must register to make the ratio 4:5?

- A. 2000
- B. 3000
- C. 4000
- D. 5000
- E. 6000

- 无限循环小数

当一个分数的分子和分母都是整数的时候, 这个分数写成小数的时候可能是有限小数

(terminating decimal), 也可能是循环小数 (repeating decimal), 如

$$\frac{1}{4} = 0.25$$

$$\frac{1}{3} = 0.333 \dots = 0.\bar{3}$$

$$\frac{1}{7} = 0.\overline{142857}$$

## 2.2.2 百分比

例：How many fifths are in 280%

- A. 1.4
- B. 2.8
- C. 14
- D. 28
- E. 56

## 2.2.3 投资，税收，打折问题

### 一、基本概念

1. Discount(折扣)：商品按原定价格扣除百分之几出售。If a price is discounted by  $n$  percent, the price becomes  $(100-n)$  percent of the original price.
2. Interest(利息)：借款人支付给贷款人的报酬。利息可分单利(simple interest)和复利(compound interest)两种计算方法。
3. Simple Interest(单利)：计算利息的一种方法。不管期限长短，仅按本金(principal)计算利息，其所生利息不再加入本金重复计算利息。

4. Compound Interest(复利): 单利的对称。进过一定的期限, 将所生利息加入本金再计利息, 逐期滚算, 俗称“利上滚利”。
5. Rate or Percent of Interest(利率): 亦称“利息率”, 指一定时期内利息额同贷出金额的比率, 有年利率、月利率和日利率。
6. Profit(利润): Gross profit is equal to revenues minus expenses, or selling price minus cost.

## 二、基本性质

1. Selling Price(销售价)=Cost(原价或价值) $\pm$ Gain 或 Loss(盈或亏)
2. Discount(折扣)=Cost(原价) $\times$ Discount Rate(折扣率)  
Discount Price(折扣价)=原价-折扣
3. Interest(利息)
  1. Simple Interest(单利)=Principal(本金) $\times$ Interest Rate(利率) $\times$ Time(时间), 式中时间单位与利率的时间单位应一致。以单利计算的本金利息和  
= $p(1+n\cdot r)$ , 其中,  $p$  为本金,  $n$  为时间,  $r$  为利率。
  2. Compound Interest(复利):  $A=P(1+r)^n$ , 式中:  $A$  为本利和(principal + interest),  $P$  为本金,  $r$  为利率(rate or percent of interest),  $n$  为期数。

注意: 单利与复利计算时, 一定要注意单位换算, 如是以半年为单位计算复利, 还是以三个月末单位计算复利

## 2.2.4练习

1.  $3 \times 10^4$  is greater than  $4 \times 10^3$  by what percent?

- A. 25%
- B. 75%
- C.  $133\frac{1}{3}\%$
- D. 650%
- E. 750%

2.  $s$ ,  $t$ , and  $u$  are integers, and  $10 \leq s < t < u \leq 20$ .

Quantity A:  $s + \frac{t}{u}$

Quantity B: 11

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3. Of the students in a certain group, 22 percent are juniors and 26 percent are seniors.

Quantity A: The ratio of the number of juniors in the group to the number of seniors in the group.

Quantity B:  $\frac{4}{5}$

- A. Quantity A is greater.
- B. Quantity B is greater.

- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.
4. In year Y, the population of Colorado was approximately half that of New Jersey, and the land area of Colorado was approximately 14 times that of New Jersey. The population density (number of persons per unit of land area) of Colorado in year Y was approximately how many times the population density of New Jersey?
- A.  $\frac{1}{28}$
- B.  $\frac{1}{14}$
- C.  $\frac{1}{7}$
- D.  $\frac{1}{4}$
- E.  $\frac{1}{2}$
5. If  $0 < a < 1 < b$ , which of the following is true about the reciprocals of a and b?
- A.  $1 < \frac{1}{a} < \frac{1}{b}$
- B.  $\frac{1}{a} < 1 < \frac{1}{b}$
- C.  $\frac{1}{a} < \frac{1}{b} < 1$
- D.  $\frac{1}{b} < 1 < \frac{1}{a}$
- E.  $\frac{1}{b} < \frac{1}{a} < 1$
6. Runner A ran  $\frac{4}{5}$  kilometer and Runner B ran 800 meters.



Quantity A: The distance that A ran

Quantity B: The distance that B ran

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

7. Which of the following operations carried out on both the numerator and the denominator of a fraction will always produce an equivalent fraction?

Indicate all such operations.

- A. Adding 2
- B. Multiplying 5
- C. Dividing by 100

8.  $k$  is a digit in the decimal  $1.3k5$ , and  $1.3k5$  is less than  $1.33$ .

Quantity A:  $k$

Quantity B:  $l$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

9.  $\overline{b}$  represents the decimal in which the digit  $b$  is repeated without end.

Quantity A:  $\overline{0.3+0.7}$

Quantity B: 1.0

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

10.  $D$  is the decimal form of the fraction  $\frac{4}{11}$ .

Quantity A: The 25<sup>th</sup> digit to the right of the decimal point in  $D$

Quantity B: 4

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

11. On a certain map, 1 centimeter represents 5 kilometers. On the map, region  $X$  has an area of 6.4 square centimeters.

Quantity A: The actual area of region  $X$

Quantity B: 150 square kilometers

- A. Quantity A is greater.
- B. Quantity B is greater.

- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

12. Aisha's income in 2004 was 20 percent greater than her income in 2003.

What is the ratio of Aisha's income in 2004 to her income in 2003?

- A. 1 to 5
  - B. 5 to 6
  - C. 6 to 5
  - D. 5 to 1
  - E. 20 to 1
13. A car dealer received a shipment of cars, half of which were black, with the remainder consisting of equal numbers of blue, silver, and white cars. During the next month, 70 percent of the black cars, 80 percent of the blue cars, 30 percent of the silver cars, and 40 percent of the white cars were sold. What percent of the cars in the shipment were sold during that month?
- A. 36%
  - B. 50%
  - C. 55%
  - D. 60%
  - E. 72%

14. By weight, liquid A makes up 8 percent of solution R and 18 percent of solution S. If 3 grams of solution R are mixed with 7 grams of solution S, then liquid A accounts for what percent of the weight of the resulting solution?
- A. 10%
- B. 13%
- C. 15%
- D. 19%
- E. 26%
15. Alice earns  $d$  dollars and has  $t$  percent of what she earns deducted for taxes. How much of what she earns does Alice have left after taxes?
- A.  $d(1-100t)$  dollars
- B.  $d(1-10t)$  dollars
- C.  $d(1-t)$  dollars
- D.  $d(1-0.1t)$  dollars
- E.  $d(1-0.01t)$  dollars
16. During a one-year study, biologists observed the number of fish in a certain pond as well as the percent of the fish that were catfish. At the beginning of the year, there were 300 fish in the pond, of which 15 percent were catfish; and at the end of the year, there were 400 fish in the pond, of which 10

percent were catfish. From the beginning of the year to the end of the year, the number of catfish in the pond

- A. decreased by more than 5%
- B. decreased by 5%
- C. did not change
- D. increased by 5%
- E. increased by more than 5%

17. Geoff used \$630 to buy a new guitar. This amount was 15 percent of his earnings last summer.

Quantity A: The amount of Geoff's earnings last summer not used to buy the new guitar

Quantity B: \$3,570

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

18. In 2009 the property tax on each home in Town X was  $p$  percent of the assessed value of the home, where  $p$  is a constant. The property tax in 2009 on a home in Town X that had an assessed value of \$125,000 was \$2,500.

Quantity A: The property tax in 2009 on a home in Town X that had an assessed value of \$160,000

Quantity B: \$3,000

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

19. Emma spent \$75 buying a used bicycle and \$27 repairing it. Then she sold the bicycle for 40 percent more than the total amount she spent buying and repairing it.

Quantity A: The price at which Emma sold the bicycle.

Quantity B: \$140

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

20. If  $x > 0$ , which of the following is equal to 1.25 percent of  $x$ ?

- A.  $\frac{x}{80}$
- B.  $\frac{x}{8}$
- C.  $\frac{x}{4}$

D.  $\frac{5x}{8}$

E.  $\frac{3x}{4}$

## 2.3 幂运算和根

### 2.3.1 幂相关概念

幂指数(exponents)用来表示一个数字重复相乘。例  $4^3=4 \times 4 \times 4=64$ 。而  $5^3=5 \times 5 \times 5=125$ 。

在  $5^3$  这个表达式中，5 被称为底数(base)，3 被称为幂指数。当幂指数为 2 的时候，我们一般称为平方(process squaring)，例如  $6^2=6 \times 6=36$ 。

负数的偶数次幂(even power)是整数；负数的奇数次幂(odd power)是负数。例如  $(-3)^2=9$ ,  $(-3)^3=-27$ 。另外注意， $-3^2=-9 \neq (-3)^2$ ， $-3^3=(-3)^3=-27$ 。

一些重要的性质：

- 对于任何非零数字  $a$ ， $a^0=1$ ， $0^0$  是没有意义的。
- 对于任何非零数字  $a$ ， $a^{-1}=\frac{1}{a}$ ， $a^{-2}=\frac{1}{a^2}$ ， $a^{-3}=\frac{1}{a^3}$ ，以此类推。 $a \times a^{-1}=a \times \frac{1}{a}=1$ 。
- 所有正数都有两个平方根，一个正的一个负的。

- 0 的唯一平方根是 0。

例如  $\sqrt{100}=10$ ,  $-\sqrt{100}=-10$ ,  $\sqrt{0}=0$ 。

下面是一些重要的平方根运算法则，其中  $a>0$ ,  $b>0$ 。

- $(\sqrt{a})^2=a$
- $\sqrt{a^2}=a$
- $\sqrt{a}\sqrt{b}=\sqrt{ab}$
- $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

例如： $(\sqrt{3})^2=3$ ,  $\sqrt{3^2}=3$ ,  $\sqrt{3}\sqrt{4}=\sqrt{12}$ ,  $\frac{\sqrt{3}}{\sqrt{4}}=\sqrt{\frac{3}{4}}$

$\sqrt[n]{n}$  表示  $n$  的平方根，类似有  $\sqrt[3]{n}$  表示  $n$  的立方根(cube root),  $\sqrt[4]{n}$  表示  $n$  的四次方根(fourth root)。

- 负数有且仅有一个奇数次方根。例如  $\sqrt[3]{-8}=-2$ 。
- 负数没有偶次方根。例如  $\sqrt[2]{-8}$  在实数范围内是没有意义的。
- 整数有且只有一个技术次方根。例如  $\sqrt[3]{8}=2$ 。
- 正数有两个偶数次方根。例如  $\sqrt[2]{4}=2$ ,  $-\sqrt[2]{4}=-2$ , 因为  $(\pm 2)^2=4$ 。

## 2.3.2 幂运算的性质

Rules of operation

1.  $a^n \cdot a^m = a^{n+m}$
2.  $(a^n)^m = a^{nm}$
3.  $\frac{a^n}{a^m} = a^{n-m}$
4.  $(ab)^n = a^n b^n$
5.  $a^{-n} = \frac{1}{a^n}$
6.  $\sqrt[n]{a} = a^{\frac{1}{n}}$



例: If  $3^x = 81$  and,  $2^{x+y} = 64$  then  $\frac{x}{y} =$

- A. 1
- B.  $\frac{3}{2}$
- C. 2
- D.  $\frac{5}{2}$

### 2.3.3练习

1.  $R = 2^{16} \times 5^{34} \times N^{50}$

N is a positive integer

Quantity A:  $\sqrt{R}$

Quantity B:  $\frac{R}{10}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2. Which of the following equals to  $(8)(72)^{-5}$ ?

- A.  $8^{-4}$

- B.  $8^{-5}$
- C.  $\frac{(72)^{-4}}{9}$
- D.  $\frac{(72)^{-5}}{8}$
- E.  $\frac{(72)^{-6}}{9}$

3. If  $n$  is a positive odd integer and  $k=n^3+2n$ , what is the value of  $(-1)^k - (-1)^{k+1}$ ?

- A. -2
- B. -1
- C. 0
- D. 1
- E. 2

4. Quantity A:  $27^{-8}$

Quantity B:  $81^{-6}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

5. Quantity A:  $\frac{3^{-1}}{4^{-1}}$

Quantity B:  $\frac{4}{3}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

6.  $N=824^x$ , where  $x$  is a positive integer.

Quantity A: the number of possible values the units digit of  $N$

Quantity B: 4

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

7.  $m=10^{32}+2$ , when  $m$  is divided by 11, the remainder is  $r$ .

Quantity A:  $r$

Quantity B: 3

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

8. Quantity A:  $\sqrt[3]{270}-\sqrt[3]{10}$

Quantity B:  $\sqrt[3]{80}$

- A. Quantity A is greater.
- B. Quantity B is greater.

- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

9.  $s$  and  $t$  are positive integers, and  $32^s = 2^t$

Quantity A:  $\frac{s}{t}$

Quantity B:  $\frac{1}{5}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

10. If  $n = 2^3$ , then  $n^n =$

- A.  $2^6$
- B.  $2^{11}$
- C.  $2^{18}$
- D.  $2^{24}$
- E.  $2^{27}$

11.  $x$  and  $m$  are positive numbers, and  $m$  is a multiple of 3.

Quantity A:  $\frac{x^m}{x^3}$

Quantity B:  $x^{\frac{m}{3}}$

- A. Quantity A is greater.

- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

12. Which of the following is equivalent to  $\frac{x(x^2)^3}{x^2}$ ?

- A.  $x^2$
- B.  $x^3$
- C.  $x^4$
- D.  $x^5$
- E.  $x^6$

13. Which of the following is equal to  $\frac{2^{x-y}}{2^{x+y}}$  for all integers  $x$  and  $y$ ?

- A.  $4^{-x}$
- B.  $4^{-y}$
- C.  $4^{xy}$
- D.  $4^x$
- E.  $4^y$

14. If  $10^x$  equals 0.1 percent of  $10^y$ , where  $x$  and  $y$  are integers, which of the following must be true?

- A.  $y=x+2$
- B.  $y=x+3$

- C.  $x=y+3$
- D.  $y=1,000x$
- E.  $x=1,000y$

15. If  $t$  is an integer and  $8m=16^t$ , which of the following expresses  $m$  in terms of  $t$ ?

- A.  $2^4$
- B.  $2^{t-3}$
- C.  $2^{3(t-3)}$
- D.  $2^{4t-3}$
- E.  $2^{4(t-3)}$

16.  $x>0$  and  $x\neq 1$

Quantity A:  $(2x^{-4})\cdot 3x^2$

Quantity B:  $\frac{24x}{4x^2}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

## 3. 代数 (Algebra)

### 3.1 直线方程

#### 3.1.1 直线方程定义

Equation of a line:  $y = kx + b$  where  $k$  is the slope,  $b$  is the y-intercept.

Slope/Gradient/Average rate of change:

$$k = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The constant  $k$  tells for each unit change in  $x$  how much  $y$  increases or decreases.

The constant  $b$  represents some starting value for  $y$  or some initial condition.

### 3.1.2练习

1. A total of 1,500 boxes are stored in four warehouses. The number of boxes stored in the individual warehouses are  $x, y, z$  and  $w$ , respectively, where  $w=2x$  and  $z=2y$ .

Quantity A:  $x+y$

Quantity B: 500

- A. Quantity A is greater.
  - B. Quantity B is greater.
  - C. The two quantities are equal.
  - D. The relationship cannot be determined from the information given.
2. If  $x$  is 4 more than half of  $y$  and if  $y$  is 10 more than half of  $x$ , what is the value of  $x$ ?
  3. The system of equations has how many solutions?  

$$\begin{cases} 3x - 6y = 9 \\ 2y - x - 3 = 0 \end{cases}$$
    - A. None
    - B. Exactly 1
    - C. Exactly 2
    - D. Exactly 3
    - E. Infinitely many
  4. In the  $xy$ -plane, the equation of line  $k$  is  $3x-2y=0$ .



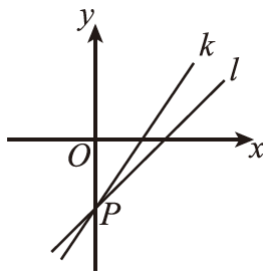
Quantity A: The  $x$ -intercept of line  $k$

Quantity B: The  $y$ -intercept of line  $k$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

5. Quantity A: The slope of line  $k$

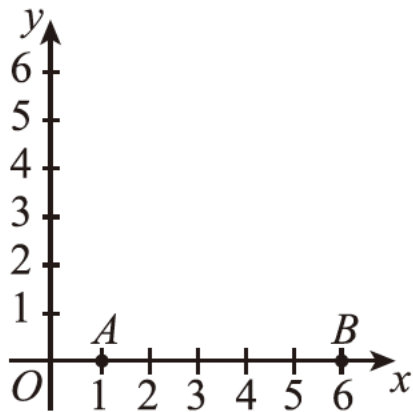
Quantity B: The slope of line  $l$



- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

6. Points A and B are shown in the  $xy$ -plane below. Point C (not shown) is above the  $x$ -axis so that the area of triangle ABC is 10. Which of the following could be the coordinates of C?

Indicate all such coordinates.



- A. (0,4)
- B. (1,3)
- C. (2,5)
- D. (3,4)
- E. (4,5)

7. In the  $xy$ -plane, line  $k$  is a line that does not pass through the origin.

Which of the following statements individually provide(s) sufficient additional information to determine whether the slope of line  $k$  is negative?

Indicate all such statements.

- A. The  $x$ -intercept of line  $k$  is twice the  $y$ -intercept of line  $k$ .
- B. The product of the  $x$ -intercept and the  $y$ -intercept of line  $k$  is positive.
- C. Line  $k$  passes through the points  $(a,b)$  and  $(r,s)$ , where  $(a-r)(b-s) < 0$ .

## 3.2 直线不等式

### 3.2.1 绝对值不等式的基本性质

$$1. |ab| = |a||b|$$

$$2. \left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

$$3. |x| \leq a \leftrightarrow -a \leq x \leq a, a > 0$$

$$4. |x| \geq a \leftrightarrow x \geq a \text{ 或 } x \leq -a, a > 0$$

$$5. a \leq |x| \leq b \leftrightarrow a \leq x \leq b \text{ 或 } -b \leq x \leq -a, \text{ 其中 } 0 < a < b$$

$$6. ||a| - |b|| \leq |a + b| \leq |a| + |b|$$

$$7. ||a| - |b|| \leq |a - b| \leq |a| + |b|$$

### 3.2.2 不等式求解注意事项

1. 若不等式两边同乘以负号，不等号要改变方向；

2.对于绝对值不等式，当把绝对值符号展开时，要写清不等式的范围；

例：  $|x - 4| < 3$ ,  $|x - 4| > 3$

### 3.2.3练习

1.  $x + 2y = 12$  and  $2y > 7$

Quantity A:  $x$

Quantity B:  $y$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2. If  $x < y$ , which of the following must be true?

- A.  $2x < y$
- B.  $2x > y$
- C.  $x^2 < y^2$

D.  $2x - y < y$

E.  $2x - y < 2xy$

3.  $x < y - 2$

Quantity A: The average (arithmetic mean) of  $x$  and  $y$

Quantity B:  $y - 1$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

4. Two different points on a number line are both 3 units from the point with coordinate  $-4$ . The solution to which of the following equations gives the coordinates of both points?

A.  $|x + 4| = 3$

B.  $|x - 4| = 3$

C.  $|x + 3| = 4$

D.  $|x - 3| = 4$

## 3.3 二次方程

### 3.3.1 二次方程特性

#### 1. 标准式 (Standard form)

$$y = ax^2 + bx + c$$

$a > 0$ , 开口向上, 有最低点 (minimum value)

$a < 0$ , 开口向下, 有最高点 (maximum value)

$a$  越大, 开口越小, 向上增长越迅速

$a$  越小, 开口越大, 向上增长越缓慢

#### 2. Factorization (十字相乘)

$$x^2 - 3x + 2 = 0$$

### 3. Quadratic Formula (万能公式)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### 4. Number of solutions (解的数量) :

$$\Delta = b^2 - 4ac$$

$\Delta > 0$  *two distinct real solutions* (方程有两个解)

$\Delta = 0$  *one distinct real solution* (方程有一个解)

$\Delta < 0$  *no real solution* (方程没有解)

### 5. Vieta theorem 韦达定理

$$ax^2 + bx + c = 0$$

The sum of the solutions of is  $-\frac{b}{a}$

The product of the solutions of is  $\frac{c}{a}$

## 3.3.2练习

1. The equation  $ax^2 = bx^2 + 1$ , where  $a$  and  $b$  are constants, has two real solutions.

Quantity A:  $a$

Quantity B:  $b$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2. If  $a$  and  $b$  are the two solutions of the equation  $x^2 - 5x + 4 = 0$ , what is the value of  $\frac{1+a}{a} \cdot \frac{1+b}{b}$ ?

Give your answer as a fraction.

3.  $\frac{x(x-2)}{(x+3)(x-4)} = 0$

Quantity A:  $x$

Quantity B:  $-2$

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.  
D. The relationship cannot be determined from the information given.

## 3.4 应用题

### 3.4.1 平均数

平均数基本公式  $A = \frac{a+b+c+\dots}{n}$  ( $a, b, c, \dots$  代表每个数据,  $n$  代表数字个数)。换句话说表达就是

平均数 =  $\frac{\text{总数}}{\text{数字个数}}$ , GRE 数学题喜欢在“总数”上做文章, 增加题目难度。

例: 一个俱乐部有 25 个男性, 35 个女性, 男性的平均年龄是 27.6 岁, 女性的平均年龄是 25.4 岁, 求整个俱乐部成员的平均年龄。

### 3.4.2 运动问题

运动问题的核心关系式: 距离(distance) = 速度(velocity) × 时间(time)。运动问题又细分为: 反向运动和同向运动。



反向运动有两种情况，一种是两个物体同时同地向相反的方向运动，第二种是两个物体同时但不同的向相反的方向运动。在两种情况下， $d_1 + d_2 = d$ ，这里的  $d_1$  和  $d_2$  分别表示第一个物体和第二个物体在一定时间里移动的距离， $d$  表示两个物体移动的距离之和。

同向运动也称为“追赶运动”，两个物体同时同地以不同速度向相同的方向运动，被称为同向运动。

例：A certain train travels 150 miles in  $h$  hours at the average rate of  $m$  miles per hour.

Quantity A: The number of hours required for the train to travel 320 miles at the average rate of  $2m$  miles per hour.

Quantity B:  $h$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

### 3.4.3 工作问题

工作问题主要是研究工人或机器完成一定工作量所需时间问题。在工作问题中完成某项工作所用的时间与参加该项工作的人数成反比，也就是说劳动者（工人或机器）越多，工作就完成得越快。下面是解决工作问题的通用公式：

$$\frac{A}{x} + \frac{A}{y} = 1$$

在这个公式中， $x$  和  $y$  分别代表两个劳动者单独完成这个工作所需时间， $A$  代表的是两个劳动者共同完成这工作所需时间。例如，工人甲单独完成某一项工作所需时间是 4 天，工人乙单独完成同一项工作所需时间是 12 天，则甲每天完成总工作量的  $\frac{1}{4}$ ，乙每天完成总工作量的  $\frac{1}{12}$ ，所以  $A=3$ ，因为  $\frac{3}{4} + \frac{3}{12} = 1$ 。

在工作问题中一般要出现三个量：工作总量、工作时间（完成工作总量所需的时间）和工作效率（单位时间内完成的工作量）。这三个量之间有下列一些关系：

- 工作效率  $\times$  工作时间 = 工作总量
- 工作总量  $\div$  工作时间 = 工作效率
- 工作总量  $\div$  工作效率 = 工作时间

例：Machine A, working alone at its constant rate, produces  $x$  pounds of peanut in 12 minutes. Machine B, working alone at its constant rate, produces  $x$  pounds of peanut in 18 minutes. How many minutes will it take machine A and B, working simultaneously at their respective constant rates, to produce  $x$  pounds of peanut?

### 3.4.4混合物问题

在混合物问题中，我们会把一些有着不同性质的物质混合在一起，形成一个特定的混合物。混合物问题包括以下两种分类：

1.湿混合物(wet mixtures)包括混合液体，气体，颗粒，这些是用重量和体积进行混合，不是用个数混合。

2.干混合物(dry mixtures)包括混合一些固体，例如硬币，石头，球，这些东西的混合以数量，体积，重量都可以衡量。

湿混合问题通常要涉及浓度的百分比，而干混合物的问题则通常涉及原始数目和数量。但是无论用哪种混合，解决它们的思路都是一样的，即要牢牢抓住混合前后的不变量。

例：How many quarts of pure alcohol must you add to 15 quarts of solution that is 40% alcohol to strengthen it to a solution that is 60% alcohol?

- A. 5
- B. 7.5
- C. 10
- D. 12.5
- E. 15

### 3.4.5利息问题

Interest(利息)

1. Simple Interest(单利)=Principal(本金) $\times$ Interest Rate(利率) $\times$ Time(时间),式中时间单位与利率的时间单位应一致。以单利计算的本金利息和= $p(1+n\cdot r)$ , 其中,  $p$  为本金,  $n$  为时间,  $r$  为利率。

2. Compound Interest(复利):  $A=P(1+r)^n$ , 式中:  $A$  为本利和(principal + interest),  $P$  为本金,  $r$  为利率(rate or percent of interest),  $n$  为期数。

注意: 单利与复利计算时, 一定要注意单位换算, 如是以半年为单位计算复利, 还是以三个月末单位计算复利

### 3.4.6练习

- From 2011 to 2012, Jack' s annual salary increased by 10 percent and Arnie' s annual salary decreased by 5 percent. If their annual salaries were equal in 2012, then Arnie' s annual salary in 2011 was what percent greater than Jack' s annual salary in 2011?

Give your answer to the nearest 0.1 percent.

2. The population of Country X for 1980 was  $p$ . The population of Country X increased by 3.8 percent in each of the next two years.

Quantity A: The population of Country X for 1982.

Quantity B:  $1.076p$

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.  
D. The relationship cannot be determined from the information given.

3. The function  $f$  is defined by  $f\left(\frac{x+3}{2}\right) = 3x^2 - x + 5$  for all  $x$ .

Quantity A:  $f(4)$

Quantity B: 75

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.  
D. The relationship cannot be determined from the information given.

4. The function  $f$  has the property that  $f(x) = f(x+1)$  for all numbers  $x$ . If  $f(4) = 17$ , what is the value of  $f(8)$ ?

5. At a certain elementary school, 10 percent of the fifth-grade students are members of the school band. If 12 percent of the fifth-grade boys and 8

percent of the fifth-grade girls are members of the band, what percent of the fifth-grade students at the school are boys?

- A. 10%
  - B. 12%
  - C. 20%
  - D. 30%
  - E. 50%
6. According to a tax rate formula for a certain year, the amount of tax owed by an individual whose annual income was between \$31,850 and \$77,100 was equal to a base tax of \$4,386 plus 24 percent of the annual income that exceeded \$31,850. According to this formula, what was the amount of tax owed by an individual whose annual income that year was \$42,000?
7. For each of the last 5 years, the population of a colony of beetles increased by 8 percent of the preceding year's population. If  $P$  represents the current population of the colony, which of the following best represents the population 5 years ago, in terms of  $P$ ?
- A.  $A.5 \times 1.08P^{-1}$
  - B.  $B.1.08P^{-5} \cdot P^{-1}$
  - C.  $C.1.08P^{-5}$
  - D.  $D.1.08^{-5}P$

E. E.1.08-5.P<sup>5</sup>

8. To obtain an FHA mortgage for \$50,000 or more, the home buyer must have a down payment equal to 4 percent of the first \$25,000 of the mortgage amount and 5 percent of the portion in excess of \$25,000. At settlement the buyers pays a mortgage-insurance premium equal to 3 percent of the mortgage amount. What is the maximum FHA mortgage, if any, a buyer can obtain if the buyer has only \$6,000 available for the down payment and insurance premium?
- A. \$62,500
- B. B.\$71,875
- C. C.\$78,125
- D. D.\$125,000
- E. The home buyer cannot obtain an FHA mortgage.

答案: C

9. The 20 people at a party are divided into  $n$  mutually exclusive groups in such a way that the number of people in any group does not exceed the number in any other group by more than 1.

Quantity A: The value of  $n$  if at least one of the groups consists of 3 people

Quantity B: 6

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

答案: D

10. Last year Leo bought two paintings. This year he sold them for \$2,000 each.

On one, he made a 25% profit, and on the other he had a 25% loss. What was his net loss or profit?

- A. He broke even.
- B. He lost less than \$100.
- C. He lost more than \$100.
- D. He earned less than \$100.
- E. He earned more than \$100.

答案: C

11. A manufacturing company has plants in three locations: Indonesia, Mexico, and Pakistan. The company has 6,000 employees, and each of the employee works at only one of the plants. If  $\frac{3}{8}$  of the employee work at the plant in Indonesia and if twice as many employees work at the plant in Mexico as work at the plant in Pakistan, how many employees work at the plant in Mexico?



答案: 2500

12. The fabric needed to make 3 curtains sells for \$8.00 per yard and can be purchased only by the full yard. If the length of fabric required for each curtain is 1.6 yards and all of the fabric is purchased as a single length, what is the total cost of the fabric that needs to be purchased for the 3 curtains?

- A. \$40.00
- B. \$38.40
- C. \$24.00
- D. \$16.00
- E. \$12.80

13. The total amount that Mary paid for a book was equal to the price of the book plus a sales tax that was 4 percent of the price of the book. Mary paid for the book with a \$10 bill and received the correct change, which was less than \$3.00. Which of the following statements must be true?

Indicate all such statements.

- A. The price of the book was less than \$9.50.
- B. The price of the book was greater than \$6.90.
- C. The sales tax was less than \$0.45.

14. Machine R, working alone at a constant rate, produces  $x$  units of a product in 30 minutes, and machine S, working alone at a constant rate, produces  $x$  units of the product in 48 minutes, where  $x$  is a positive integer.

Quantity A: The number of units of the product that machine R, working alone at its constant rate, produces in 3 hours

Quantity B: The number of units of the product that machine S, working alone at its constant rate, produces in 4 hours

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.
15. If an investment of  $P$  dollars is made today and the value of the investment doubles every 7 year, what will be the value of the investment, in dollars, 28 years from today?
- A.  $8P^4$
- B.  $P^4$
- C.  $16P$
- D.  $8P$
- E.  $4P$

16. A certain money market account that had a balance of \$48,000 during all of last month earned \$360 in interest for the month. At what simple annual interest rate did the account earn interest last month?
- A. 7%
  - B. 7.50%
  - C. 8%
  - D. 8.50%
  - E. 9%

## 4. 几何 (Geometry)

### 4.1 平面几何

#### 4.1.1 直线和角

Vertical Angle (对角)：两条直线相交形成的角成为对顶角，两个角相等，180 度的角称为平角 (straight angle)，小于 90 度的角称为锐角 (acute angle)，大于 90 度小于 180 度的角称为钝角 (obtuse angle)，等于 90 度的角称为直角 (right angle)。

## 4.1.2 三角形的角和边

### 三角形基本性质

1. 在三角形中，任一边的长度小于其他两条边长度的和。

推论：三角形中两边之差小于第三边。

2. 三角形中，大角对大边，小角对小边。
3. 三角形的一个外角等于其不相邻两个内角的和。

## 4.1.3 特殊三角形

### 直角三角形勾股定理

1. 等腰直角三角形 (1:1: $\sqrt{2}$ )
2. 30 度直角三角形 (1:2: $\sqrt{3}$ )
3. 其他比例 (3:4:5/5:12:13)

### 其他特殊三角形

1. Isosceles Triangles (等腰三角形)
2. Equilateral Triangles (等边三角形)

$$\text{面积} = A = \frac{s^2\sqrt{3}}{4}$$

## 4.1.4 多边形

1. The Square (正方形)
2. Rectangles (矩形)
3. Parallelograms (平行四边形)

#### 4. 多边形内角和公式

内角和=  $(n-2) \times 180$  (n 为边数)

### 4.1.5 圆

1. Radius (半径)
2. Diameter (直径)
3. Circumference (周长)
4. Arc (弧长)
5. Sector (扇形)

公式

圆方程:  $(x - a)^2 + (y - b)^2 = r^2$ , where the center is (a,b) and radius is r.

圆面积:  $A = \pi r^2$

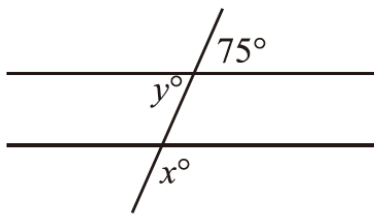
圆周长:  $C = \pi d = 2\pi r$

在圆中:  $\frac{\text{弧长}}{\text{周长}} = \frac{\text{弧长所对应的角度}}{360^\circ}$

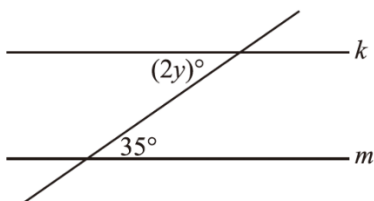
### 4.1.6 练习

1. Quantity A: x

Quantity B: y



- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.
2. P, Q, and R are three points in a plane, and R does not lie on line PQ. Which of the following is true about the set of all points in the plane that are the same distance from all three points?
- A. It contains no points.
- B. It contains one point.
- C. It contains two points.
- D. It is a line.
- E. It is a circle.
3. In the figure below, line  $k$  is parallel to line  $m$ . What is the value of  $y$ ?

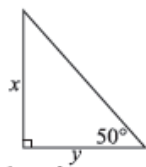


4. Quantity A: The length of a side of a regular pentagon with a perimeter of 12.5

Quantity B: The length of a side of a regular hexagon with a perimeter of 15

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.  
D. The relationship cannot be determined from the information given.

5.



$$A = \frac{x}{y}, B = 1.$$

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.  
D. The relationship cannot be determined from the information given.

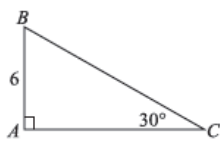
6. If the lengths of two sides of a triangle are 5 and 9, respectively, which of the following could be the length of the third side of the triangle?

Indicate all such lengths.

- A. 3

- B. 5
- C. 8
- D. 15

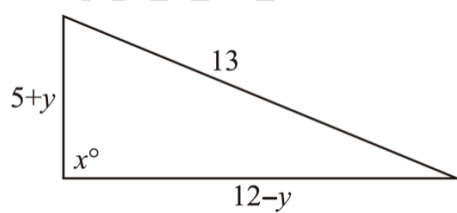
7. What is the area of triangle ABC shown below?



- A. 20
- B. 18
- C.  $12\sqrt{3}$
- D.  $18\sqrt{3}$
- E. 36

8. Quantity A:  $x$

Quantity B: 90



- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.



9. What is the perimeter, in meters, of a rectangular playground 24 meter wide that has the same area as a rectangular playground 64 meters long and 48 meters wide?

- A. 112
- B. 152
- C. 224
- D. 256
- E. 304

10. In the  $xy$ -plane, a quadrilateral has vertices at  $(-1, 4)$ ,  $(7, 4)$ ,  $(7, -5)$ , and  $(-1, -5)$ .

What is the perimeter of the quadrilateral?

- A. 17
- B. 18
- C. 19
- D. 32
- E. 34

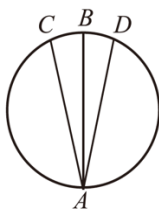
11. The length of each side of rectangle  $R$  is an integer, and the area of  $R$  is 36.

Quantity A: The number of possible values of the perimeter of  $R$

Quantity B: 6

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

12. AB is a diameter of the circle below



Quantity A: The length of AB

Quantity B: The average (arithmetic mean) of the lengths of AC and AD

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

13. The relationship between the area  $A$  of a circle and its circumference  $C$  is

given by the formula  $A = kC^2$ , where  $k$  is a constant. What is the value of  $k$ ?

- A.  $\frac{1}{4\pi}$
- B.  $\frac{1}{2\pi}$

- C.  $\frac{1}{4}$
- D.  $2\pi$
- E.  $4\pi^2$

14. O is the center of the circle below.



A=x, B=5

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

## 4.2 立体几何

### 4.2.1 长方体

Volume=length\*width\*height= $l*w*h$

Surface Area= $2lh+2lw+2hw=2(lh+lw+hw)$

### 4.2.2 立方体

Volume= $a^3$

Surface Area= $6a^2$

### 4.2.3圆柱体

$$\text{Volume} = \pi r^2 h$$

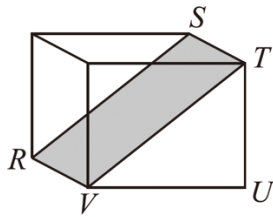
$$\text{Surface Area} = 2\pi r^2 + 2\pi r h$$

### 4.2.4练习

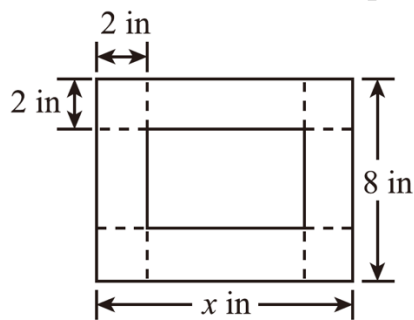
1. A rectangular solid P has height  $2c$  and a base of width  $a$  and length  $b$ . Two other rectangular solids, Q and R, each have height  $c$  and bases of width  $a$  and length  $b$ . Which of the following represents the amount by which the sum of the surface areas of Q and R exceeds the surface areas of P?
- A.  $2ab$
- B.  $4ab$
- C.  $2ab+2bc$
- D.  $2ab+4ac$

E.  $2ab+4ac+4bc$

2. In the rectangular solid below,  $TU=3$ ,  $UV=4$ , and  $VR=2$ . What is the area of the shaded rectangular region?



3. The thin rectangular sheet of metal shown in the figure is 8 inches wide and  $x$  inches long. An open box is to be made by cutting a 2 inch square from each corner of the sheet of metal and then folding up the sides. If the volume of the box is to be 48 cubic inches, what is the value of  $x$ ?



- A. 6  
B. 8  
C. 10  
D. 12  
E. 14

4. The volume  $V$  of a right circular cylinder is  $V = \pi \cdot r^2 h$ , where  $r$  is a radius of the base and  $h$  is the height of the cylinder. If the volume of a right circular cylinder is  $45\pi$  and its height is 5, what is the circumference of its base?
- A. 3  
B. 9  
C.  $3\pi$   
D.  $6\pi$   
E.  $9\pi$
5. The interior dimensions of a rectangular tank are as follows: length 110 centimeters, width 90 centimeters, and height 270 centimeters. The tank rests on level ground. Based on the assumption that the volume of water increases by 10 percent when it freezes, which of the following is closest to the maximum height, in centimeters, to which the tank can be filled with water so that when the water freezes, the ice would not rise above the top of the tank?
- A. 230  
B. 235  
C. 240  
D. 245  
E. 250

## 5. 数据分析 (Data analysis)

### 5.1 统计

#### 5.1.1 数据分布

##### 1. 平均数 (Mean)

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

##### 2. 中位数 (Median)

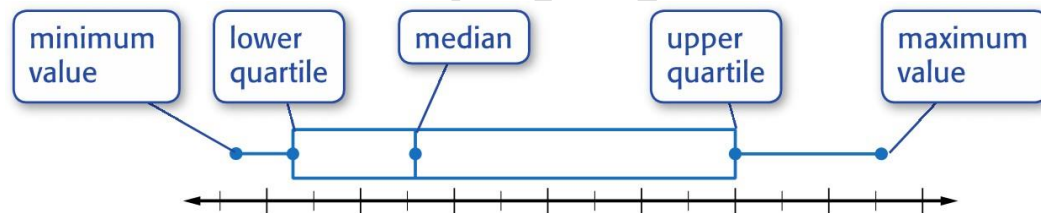
In statistics, a *median* is the number separating the higher half of a data sample, a population, or a probability distribution, from the lower half. The *median* of a finite list of numbers can be found by arranging all the observations from lowest value to highest value and picking the middle one. For example, the median of {3, 3, 5, 9, 11} is 5; the median of {3, 5, 7, 9} is  $(5 + 7) / 2 = 6$ .

### 3. 众数 (Mode)

The mode of a sample is the element that occurs most often in the collection.

### 4. 箱形图 (Box Plot)

对于 Box Plot 而言，需要掌握图像中几根线表示的含义即可。（从小到大，前后两个点表示最小值，最大值；第二根竖线表示整体数据的中位数，将所有数据分为两部分；第一根为前面一半数据的中位数；最后一根为后面一半数据的中位数）



### 5. 极差 (Range)

The range of a sample is the difference between the biggest element and the smallest element.

### 6. 方差&标准差 (Variance & Standard Deviation)

$$s^2 = \frac{1}{n} [(x_1 - x)^2 + (x_2 - x)^2 + \cdots + (x_n - x)^2]$$

其中 n 为样本容量，x 为样本平均值



\*标准差  $s$  是方差的算术平方根。同样用来衡量数据的离散程度。数据越集中，标准差和方差越小；越离散，标准差和方差越大

例 1:

The numbers in data set  $S$  have a standard deviation of 5. If a new data set is formed by adding 3 to each number in  $S$ , what is the standard deviation of the numbers in the new data set?

- A. 2
- B. 3
- C. 5
- D. 8
- E. 15

例 2:

A list of numbers has a mean of 8 and a standard deviation of 2.5. If  $x$  is a number in the list that is 2 standard deviation above the mean, what is the value of  $x$ ?

## 7. 离群值 (Outlier)

The outlier of a data set may have a few values that are much larger or smaller than the rest of the values in the set.

离群值是指在一组数据中有几个数值与其他数值相比相差更大或更小的数值在 23, 15, 35,

66, 45, 29, 158 数列当中，158 是 outlier

### 5.1.2练习

1. The first term in a certain sequence is 1, the 2<sup>nd</sup> term in the sequence is 2, and, for all integers  $n \geq 3$ , the  $n$ th term in the sequence is the average (arithmetic mean) of the first  $n-1$  terms in the sequence. What is the value of the 6<sup>th</sup> term in the sequence?

Give your answer as a fraction.

2.  $x$  is an integer and  $23 < x < 27$ .

Quantity A: The median of the five integers 23, 24, 26, 27, and  $x$ .

Quantity B: 25

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

### 3. DISTRIBUTION OF THE HEIGHTS OF 80 STUDENTS

Height (centimeters)	Number of Students
140-144	6
145-149	26
150-154	32
155-159	12
160-164	4
Total	80

The table above shows the frequency distribution of the heights of 80 students.

What is the least possible range of the heights of the 80 students?

- A. 15
- B. 16
- C. 20
- D. 24

E. 28

4. For a certain distribution, the measurement 12.1 is 1.5 standard deviations below the mean, and the measurement 17.5 is 3.0 standard deviations above the mean. What is the mean of the distribution?

- A. 13.8  
B. 13.9  
C. 14.0  
D. 14.1  
E. 14.2

5. Each of the following linear equations defines  $y$  as a function of  $x$  for all integers  $x$  from 1 to 100. For which of the following equations is the standard deviation of the  $y$ -values corresponding to all the  $x$ -values the greatest?

- A.  $y = \frac{x}{3}$   
B.  $y = \frac{x}{2} + 40$   
C.  $y = x$   
D.  $y = 2x + 50$   
E.  $y = 3x - 20$

6. Quantity A: The sum of the first 7 positive integers

Quantity B: 7 times the median of the first 7 positive integers

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

7. AGE DISTRIBUTION OF EMPLOYEES OF A BUSINESS

Age Interval	Number of Employees
15-24	17
25-34	25
35-44	26
45-54	21
55-64	18
Total	106

Quantity A: The range of the ages of the 20 oldest employees of the business

Quantity B: 11 years

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

8. In a quality-control test, 50 boxes—each containing 30 machine parts—were examined for defective parts. The number of defective parts was recorded for each box, and the average (arithmetic mean) of the 50 recorded numbers of defective parts per box was 1.12. Only one error was made in recording the 50 numbers: “1” defective part in a certain box was incorrectly recorded as “10” .

Quantity A: The actual average number of defective parts per box

Quantity B: 0.94

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

9. Frequency Distribution for List X

Number	1	2	3	5
Frequency	10	20	18	12

Frequency Distribution for List Y

Number	6	7	8	9
Frequency	24	17	10	9

List X and List Y each contain 60 numbers. Frequency distributions for each list are given above. The average (arithmetic mean) of the numbers in list X is 2.7, and the average of the numbers in list Y is 7.1. List Z contains 120 numbers: the 60 numbers in list X and the 60 numbers in list Y.

Quantity A: The average of the 120 numbers in list Z

Quantity B: The median of the 120 numbers in list Z

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

10.  $b-3$ ,  $b-1$ ,  $b+2$ ,  $b+3$ ,  $b+4$

The median of the five terms listed above is 5, where  $b$  is a constant. What is the average (arithmetic mean) of the five terms?

- A. 3
- B. 4

- C. 5
- D. 6
- E. 7

## 5.2 描述数据的图形

### 5.2.1 表格

Questions 1-4 are based on the following data.

SIGHTINGS OF SELECTED BIRD SPECIES IN PARK H IN 1999, BY SEASON

Speices	Number of Sightings			
	Winter	Spring	Summer	Fall
Cardinal	30	18	11	20
Goldfinch	5	12	6	9
Junco	12	0	0	6
Nuthatch	8	2	0	4
Robin	6	12	28	18
Sparrow	20	19	23	22
Wren	0	18	30	12



1. In the winter,  $\frac{2}{3}$  of the cardinal sighting,  $\frac{1}{2}$  of the junco sightings, and  $\frac{1}{4}$  of the sparrow sightings were in January. What fraction of the total number of sightings of these three bird species in the winter were in January?
- A.  $\frac{1}{4}$   
B.  $\frac{1}{3}$   
C.  $\frac{1}{2}$   
D.  $\frac{2}{3}$   
E.  $\frac{2}{5}$
2. For which of the following bird species is the standard deviation of the numbers of sightings shown for the four seasons least?
- A. Cardinal  
B. Junco  
C. Robin  
D. Sparrow  
E. Wren
3. Which of the following is closest to the average (arithmetic mean) number of cardinal sightings for the 4 seasons?

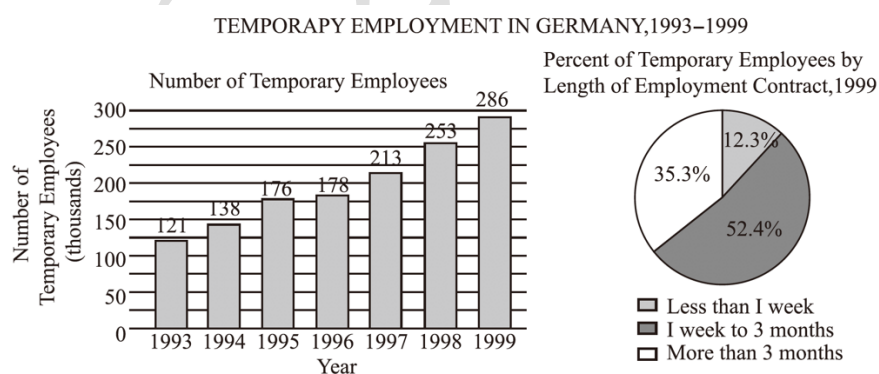
- A. 12
- B. 14
- C. 16
- D. 18
- E. 20

4. By what percent did the number of wren sightings increase from spring to summer?

Give your answer to the nearest whole percent.

## 5.2.2柱状图+饼图

Questions 5-7 are based on the following data.



5. Which of the following is closest to the percent increase in the number of temporary employees from 1993 to 1999?

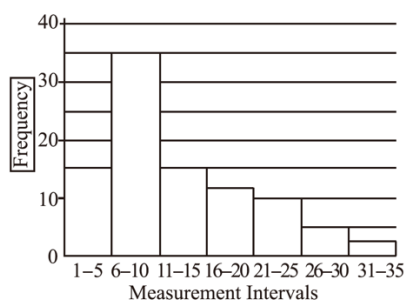
- A. 36%

- B. 58%
- C. 136%
- D. 158%
- E. 236%
6. In 1999 approximately how many of the temporary employees had an employment contract with a length of at most 3 months?
- A. 185,000
- B. 150,000
- C. 101,000
- D. 35,000
- E. 19,000
7. In 1998 the ratio of the number of female temporary employees to the number of male temporary employees was 1 to  $x$ , where  $x > 0$ . In terms of  $x$ , what was the number, in thousands, of female temporary employees in 1998?
- A.  $253(x-1)$
- B.  $253(x+1)$
- C.  $\frac{253}{x}$
- D.  $\frac{253}{x-1}$

E.  $\frac{253}{x+1}$

### 5.2.3直方图

6.



In the course of an experiment, 95 measurements were recorded, and all of the measurements were integers. The 95 measurements were then grouped into 7 measurement intervals. The graph above shows the frequency distribution of the 95 measurements by measurement interval.

Quantity A: The average (arithmetic mean) of the 95 measurements

Quantity B: The median of the 95 measurements

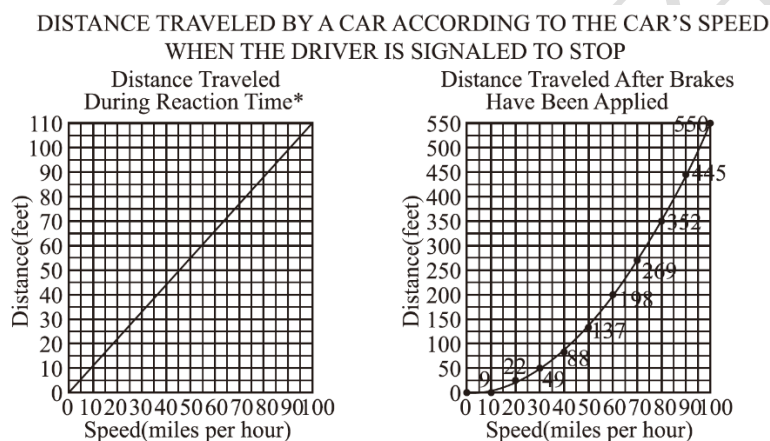
A. Quantity A is greater.

B. Quantity B is greater.

- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

## 5.2.4 散点图

Questions 10-13 are based on the following data.



\*Reaction time is the time period that begins when the driver is signaled to stop and ends when the driver applies the brakes.

Note: Total stopping distance is the sum of the distance traveled during reaction time and the distance traveled after brakes have been applied.

7. The speed, in miles per hour, at which the car travels a distance of 52 feet during reaction time is closest to which of the following?

A. 43

B. 47

C. 51

D. 55

E. 59

8. Approximately what is the total stopping distance, in feet, if the car is traveling at a speed of 40 miles per hour when the driver is signaled to stop?

A. 130

B. 110

C. 90

D. 70

E. 40

9. Of the following, which is the greatest speed, in miles per hour, at which the car can travel and stop with a total stopping distance of less than 200 feet?

A. 50

B. 55

C. 60

D. 65

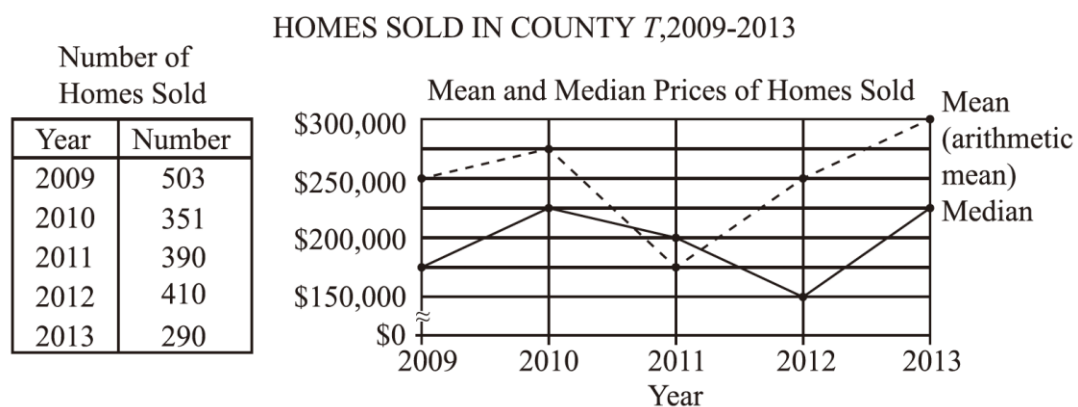
E. 70

10. The total stopping distance for the car traveling at 60 miles per hour is approximately what percent greater than the total stopping distance for the car traveling at 50 miles per hour?

- A. 22%
- B. 30%
- C. 38%
- D. 45%
- E. 52%

## 5.2.5时距图

Questions 14-17 are based on the following data.



11. Which of the following is closest to the mean of the prices of the 700 homes sold in 2012 and 2013 combined?

- A. \$265,000
- B. \$270,000
- C. \$275,000
- D. \$280,000
- E. \$285,000

12. By approximately what percent did the median price of homes sold in County T decrease from 2011 to 2012?

- A. 10%
- B. 15%
- C. 25%
- D. 33%
- E. 50

13. Based on the information given, which of the following statements about the sum of the prices of all the homes sold in a given year must be true?

Indicate all such statements.

- A. The sum of the prices for 2010 was greater than the sum for 2009.
- B. The sum of the prices for 2010 was greater than the sum for 2011.



C. The sum of the prices for 2009 was greater than the sum for 2011

14. County T collected a tax equal to 3 percent of the price of each home sold in the county in 2009. Approximately how much did County T collect in taxes from all homes sold in 2009?

- A. \$38,000
- B. \$260,000
- C. \$380,000
- D. \$2,600,000
- E. \$3,800,000

## 5.3 概率

### 5.3.1 独立/互斥事件概率

Probability (概率)：亦称“或然率”、“几率”，某一类事件在相同的条件下可能发生也可能不发生，这类事件成为随机事件 (random occurrence)。概率就是用来表示随机事件发生的可能性大小的一个量。很自然地把必然发生的事件的概率定为 1，并把不可能发生的事件的概率定为 0，而一般随机事件的概率是介于 0 和 1 之间的一个数。

(1) 等可能性事件的概率：如果一次试验中共有  $n$  种等可能出现的结果，其中事件  $A$  包含的结果有  $m$  种，那么事件  $A$  的概率  $P(A) = \frac{m}{n}$ 。

例：有 7 个奇数，5 个偶数，从这 12 个数中任取一个是奇数的概率？

答案：7/12

(2) 互斥事件发生的概率：如果事件  $A_1, A_2, \dots, A_n$  彼此互斥，那么事件  $A_1,$

$A_2, \dots, A_n$  中有一个发生的概率为这  $n$  个事件分别发生的概率的和，即

$P(A_1 + A_2 + \dots + A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$ ，也即用“or，或”表达。（注：所谓

互斥是指任两个之间都不可能同时发生）。

例：在 12 个球中，8 个是一等品，3 个是二等品，1 个是三等品，求任取一个球是一等品或是二等品的概率？

答案：11/12

(3) 相互独立事件同时发生的概率：如果时间相互独立，那么  $n$  个事件同时发生的概率等于每个事件发生的概率的积，即  $P(A_1, A_2, \dots, A_n) = P(A_1)P(A_2)\dots P(A_n)$ ，也即用“且”或“and”来表达。

例：A 坛中有 7 个白球，有 3 个黑球，B 坛中有 4 个白球，5 个黑球，问从这两个坛中分别摸出一个都是白球的概率？

(4) 独立重复试验发生的概率：如果在一次试验中某事件发生的概率是  $P$ ，那么在  $n$  次独立重复试验中这个事件恰好发生  $K$  次的概率为  $P_n(K) = C_n^k \cdot P^k(1-P)^{n-k}$ 。

例：某气象站天气预报准确率为 80%，求 5 次预报中有 4 次准确的概率？

### 5.3.2 加法原则和乘法原则

加法原则：做一件事，完成它可以有  $n$  类办法，在第一类办法中有  $m_1$  种不同的方法，在第二类方法中有  $m_2$  种不同的方法……，在第  $n$  类方法中有  $m_n$  种不同的方法，那么完成这件事共有  $N = m_1 + m_2 + \dots + m_n$  种不同的方法（在表达中用“或，or”时即为加法原则）。

例：某天从 A 地到 B 地，可乘汽车，也可乘火车，还可乘飞机，一天中，汽车有 5 班，火车有 4 班，飞机有 2 班，问一天中 A 地到 B 地共有多少种走法？

乘法原则：做一件事，完成它需要  $n$  个步骤，做第一步有  $m_1$  种方法，做第二步有  $m_2$  种方法...，做第  $n$  步有  $m_n$  种方法，则完成这件事共有  $N = m_1 * m_2 * \dots * m_n$

例：由 A 到 B 有 3 条路，由 B 到 C 有 4 条路，问由 A 经 B 到 C 有多少种不同的走法？

### 5.3.3 概率练习

- From the 5 points A, B, C, D, and E on the number line below, 3 different points are to be randomly selected. What is the probability that the coordinates of the 3 points selected will all be positive?



- $\frac{1}{10}$
  - $\frac{1}{5}$
  - $\frac{3}{10}$
  - $\frac{2}{5}$
  - $\frac{3}{5}$
- A and B are independent events, and the probability that both events occur is  $\frac{1}{2}$ . Which of the following could be the probability that event A occurs?  
Indicate all such probabilities.

- 0
- $\frac{1}{4}$

C.  $\frac{1}{2}$

D.  $\frac{3}{4}$

E. 1

3. If one number is chosen at random from the first 1,000 positive integers, what is the probability that the number chosen is a multiple of both 2 and 8?

A.  $\frac{1}{125}$

B.  $\frac{1}{8}$

C.  $\frac{1}{2}$

D.  $\frac{9}{16}$

E.  $\frac{5}{8}$

4. A box contains 10 balls numbered from 1 to 10 inclusive. If Ann removes a ball at random and replaces it, and then Jane removes a ball at random, what is the probability that both women removed the same ball?

A.  $\frac{1}{100}$

B.  $\frac{1}{90}$

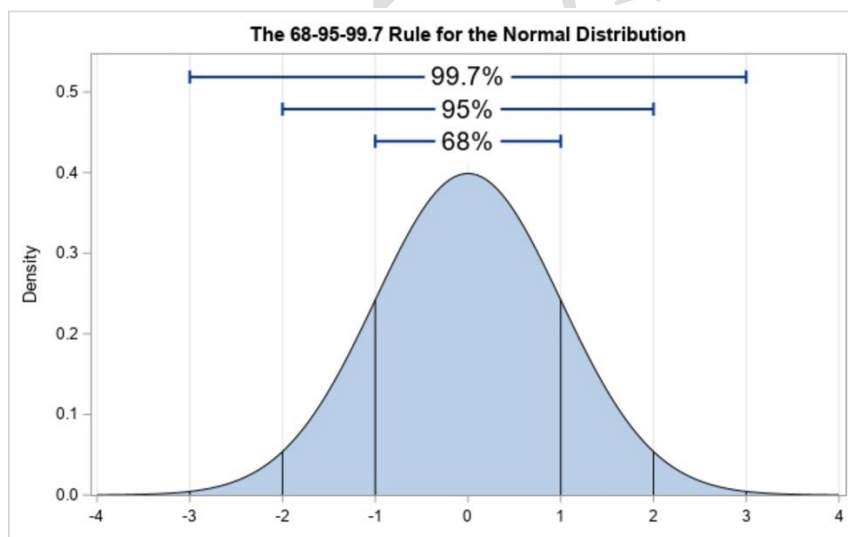
C.  $\frac{1}{45}$

D.  $\frac{1}{10}$

E.  $\frac{41}{45}$

### 5.3.4 正态分布

为什么叫“正态分布”，也有地方叫“常态分布”，这两个名字都不太直观，但如果我们各取一字变为“正常分布”，就很白话了，而这正是“正态分布”的本质含义，Normal Distribution。它太常见了，基本上能描述所有常见的事物和现象：正常人群的身高、体重、考试成绩、家庭收入等等。这里的描述是什么意思呢？就是说这些指标背后的数据都会呈现一种中间密集、两边稀疏的特征。以身高为例，服从正态分布意味着大部分人的身高都会在人群的平均身高上下波动，特别矮和特别高的都比较少见。



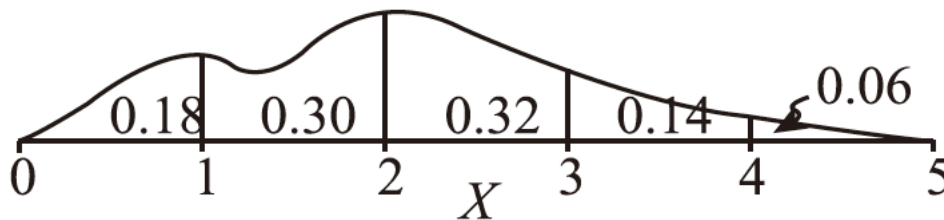
这样的数据成为近似正态分布 (approximately normally distributed) 而且有以下性质。

- 平均值、中位数和众数大致相等。
- 平均值近似地把数据分为对称的两组。

- 大约三分之二的数数据偏离平均值不超过 1 个标准差。
- 几乎所有的数据偏离平均值不超过 2 个标准差。

### 5.3.5 正态分布练习

- The figure below shows the probability distribution of a continuous random variable  $X$ . For each of the five intervals shown, the figure gives the probability that the value of  $X$  is in that interval. What is the probability that  $1 < X < 4$ ?



答案: 0.76

- The distribution of the numbers of hours that students at a certain college studies for final exams has a mean of 12 hours and a standard deviation of 3 hours. Which of the following numbers of hours are within 2 standard deviations of the mean of the distribution?

Indicate all such numbers.

- 2
- 5

- C. 10
- D. 14
- E. 16
- F. 20

3. A random variable  $Y$  is normally distributed with a mean of 200 and a standard deviation of 10.

Quantity A: The probability of the event that the value of  $Y$  is greater than 220

Quantity B:  $\frac{1}{6}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

4. The random variable  $X$  is normally distributed. The values 650 and 850 are at the 60<sup>th</sup> and 90<sup>th</sup> percentiles of the distribution of  $X$ , respectively.

Quantity A: The value at the 75<sup>th</sup> percentile of the distribution of  $X$

Quantity B: 750

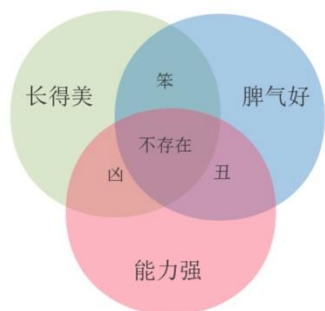
- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.



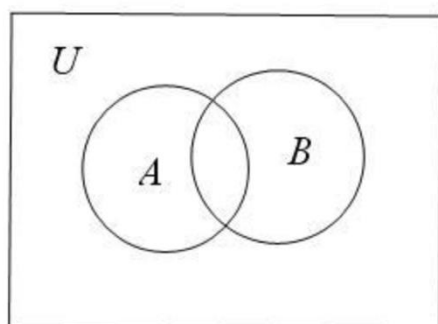
D. The relationship cannot be determined from the information given.

## 5.4 集合

### 5.4.1 韦恩图和集合

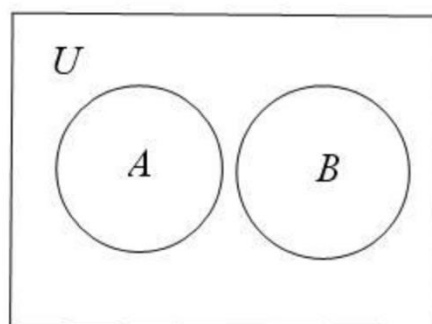


维恩图 (Venn diagram) 是一种能够有效地展示两个或三个集合以及它们的交集的方式。在维恩图中，我们用圆形区域表示集合。



独立但不互斥

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



互斥

$$P(A \cup B) = P(A) + P(B)$$

集合的分类及定义：

A set is simply a group of two or more numbers or other terms.

具有某种属性的事物的全体称为集合，它一般由一组数或其他符号构成。组成集合的每个事物称为该集合的元素 (element)。如果  $S$  是一个有限数量的集合，那么  $|S|$  被定义为元素的数目。例如： $S = \{2, 7, 17, 25\}$ ，则  $|S| = 4$ 。

### Relationship between Sets (集合之间的关系)

1. Union (并集) : the union of set A and set B 是指两个或多个集合中的所有元素，对两个集合 A, B 可表示为  $A \cup B$ 。
2. Intersection (交集) : the intersection of set A and set B 是指两个或多个集合中的所有共同元素，对两个集合 A, B 可表示为： $A \cap B$ 。
3. Disjoint or Mutually Exclusive: 指两个集合中没有共同元素。
4. 全集: 将各个子集中所有元素非重复地都加起来就是全集，用  $I$  表示。
5. 非集: 非某集合元素组成的集合，称为这集合的非，对单集合 A 可记为  $\bar{A}$ 。

### 集合的一般公式

1.  $\overline{A \cap B} = \bar{A} \cup \bar{B}$ ，即 A 交 B 的非等于 A 非并上 B 非
2.  $\overline{A \cup B} = \bar{A} \cap \bar{B}$ ，即 A 并 B 的非等于 A 非交上 B 非
3.  $|A \cup B| = |A| + |B| - |A \cap B|$  (对于两个集合而言)
4.  $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$  (对于三个集合而言)

### 5.4.2 练习

1. A total of 40 tourists went on a trip. If 18 of the tourists visited site X, 18 visited site Y, and 8 visited both site X and site Y, how many visited neither site X nor site Y?
  - A. 4
  - B. 10
  - C. 12
  - D. 22
  - E. 28
  
2. In a class of 120 students, 60 percent can speak French and the rest can speak only English. If 25 percent of those in the class who can speak French can also speak English, how many of the students in the class can speak English?
  - A. 54
  - B. 60
  - C. 66
  - D. 84
  - E. 90

3. In a graduating class of 236 students, 142 took algebra and 121 took chemistry. What is the greatest number of students that could have taken both algebra and chemistry?
- A. 21  
B. 27  
C. 37  
D. 121  
E. 141

## 5.5 排列和组合

### 5.5.1 排列组合相关概念

#### 1. Factorial Notation (阶乘) :

假设把 1, 2, 3 三个数字组成三位数, 组合的可能性有: 百位数 3 种, 十位数 2 种, 个位数 1 种,  $3 \times 2 \times 1 = 3!$

$n$  个自然数 1, 2, 3, ...,  $n$  的乘积成为  $n$  的“阶乘”, 记作  $n!$ , ( $4! = 4 \times 3 \times 2 \times 1$ ), 零的阶乘规定为 1, 即  $0! = 1! = 1$ 。

$$*n! = n (n-1) !$$

#### 2. Permutation (排列) :

排列分为两种,

可重复排列：

从  $n$  个不同的元素  $a_1, a_2, a_3, \dots, a_n$  中，有放回任取  $m$  次，每次取一个，得到不同的序列共有多少种？这种排列共有  $n^m$  种。

不可重复排列简称为排列问题，假设把 1, 2, 3, 4, 5 五个数字组成三位数，根据前面的讲解分析一共有  $5 \times 4 \times 3 = 60$  种情况。

现在我们来考虑一般的情况，从  $n$  个对象中选出  $m$  个对象 ( $m \leq n$ )，然后对  $m$  个对象按顺序排列，排第一的有  $n$  种可能，第二的有  $n-1$  种，第  $m$  的有  $(n-m+1)$  种，共有

$$n(n-1)(n-2) \dots (n-m+1)$$

$$n(n-1)(n-2) \dots (n-m+1) \frac{(n-m)!}{(n-m)!} = \frac{n!}{(n-m)!}$$

这样的排列总数计为  $P_n^m$ ，

$$P_n^m = \frac{n!}{(n-m)!} (1 \leq m \leq n)$$

### 3. Recombination (组合)：

把 1, 2, 3, 4, 5 五个数字组成三位数，改成 ABCDE 中选出三个字母，但不要求进行排列（不需要排序），一共有多少种？

ABC ABD ABE ACD ACE ADE BCD BCE BDE CDE, 共 10 种。

不需要排序时, 五个对象选出三个有 10 种不同的情况, 对三个对象排序的方法有  $3! = 6$

种, 两个结果相乘就是排列的结果 (从  $n$  个对象中选出  $m$  个对象 ( $m \leq n$ ), 选择但不排序

的总数计为  $C_n^m$ )

选且排序=选但不排序\*只排序

$$P_n^m = C_n^m * m!$$

$$C_n^m = \frac{P_n^m}{m!} = \frac{n!}{m!(n-m)!} (1 \leq m \leq n)$$

$$C_n^m = C_n^{n-m} (1 \leq m \leq n)$$

$$0! = 1, C_n^0 = P_n^0 = 1, C_n^1 = P_n^1 = n (n \geq 1)$$

例:

From the 5 points A, B, C, D, and E on the number line below, 3 different points are to be randomly selected. What is the probability that the coordinates of the 3 points selected will all be positive?



- A.  $\frac{1}{10}$
- B.  $\frac{1}{5}$
- C.  $\frac{3}{10}$
- D.  $\frac{2}{5}$
- E.  $\frac{3}{5}$

4. 独立重复事件发生的概率：如果一次试验中某件事发生的概率为  $P$ ，那么在  $n$  次独立

重复事件中这件事恰好发生的  $K$  次的概率为  $P_n(K) = C_n^K * P^K(1 - P)^{n-K}$

例：某气象站天气预报准确率为 80%，求 5 次预报中有 4 次准确的概率？

## 5.5.2 练习

1. A linen shop has a certain table cloth that is available in 8 sizes and 10 colors.

What is the maximum possible number of different combinations of size and color available?

- A. 9
- B. 18
- C. 40
- D. 80
- E. 90

2. A gardener wishes to plant 5 bushes in a straight row. Each bush has flowers of a different solid color (white, yellow, pink, red, and purple). How many ways can the bushes be arranged so that the middle bush is the one with red flowers?

- A. 24
- B. 30
- C. 60
- D. 96
- E. 120

3. What is the total number of different 5-digit numbers that contain all of the digits 2, 3, 4, 7 and 9 and in which none of the odd digits occur next to each other?

- A. 12
- B. 10
- C. 8
- D. 6
- E. 1

4. In a series of races, 10 toy cars are raced, 2 cars at a time. If each car must race each of the other cars exactly twice, how many races must be held?

- A. 40
- B. 90
- C. 100



- D. 180
- E. 200
5. Three red marbles and two white marbles are placed in an empty box. One marble at a time is to be selected randomly and removed from the box until all 5 marbles have been removed. What is the probability that each of the first 3 marbles removed will be red?
- A.  $\frac{1}{32}$
- B.  $\frac{1}{20}$
- C.  $\frac{1}{10}$
- D.  $\frac{1}{2}$
- E.  $\frac{3}{5}$
6. The buyer of a certain mechanical toy must choose 2 of 4 optional motions and 4 of 5 optional accessories. How many different combinations of motions and accessories are available to the buyer?
- A. 8
- B. 11
- C. 15
- D. 20

E. 30

7. In a soccer league, if there were 10 teams, and each team played each of the other teams 16 times, how many games did each team play?

A. 144

B. 140

C. 134

D. 125

E. 106

新东方在线

## 6. 综合练习

### 综合练习 1-Section1

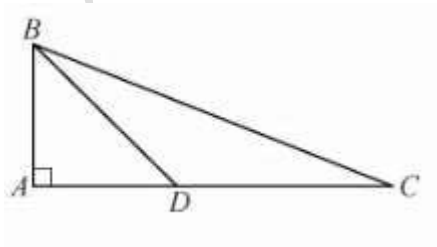
1. A certain brand of dishwashing liquid was sold in two different bottle sizes. The small bottle was sold with  $\frac{2}{5}$  as many ounces of liquid as the large bottle and was sold at a price that was  $\frac{1}{2}$  the price of the large bottle.

Quantity A: The price per ounce of the liquid in the small bottle

Quantity B: The price per ounce of the liquid in the large bottle

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal  
D. The relationship cannot be determined from the information given.

2.



$AB=12$ ,  $AC=30$ , and  $AD=\frac{2}{5}(AC)$ .

Quantity A: The measure of angle BDC    Quantity B:  $120^\circ$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

3. Set T consists of the integers from 11 through 100, inclusive.

Quantity A: 4 times the number of integers in set T that are multiples of 4

Quantity B: 5 times the number of integers in set T that are multiples of 5

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

4.  $x^2 + 6x = 7$

Quantity A:  $(x+3)^2$

Quantity B: 16

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.
5. Quantity A: The number of different prime factors of 500 Quantity B: The number of different prime factors of 360.
- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.
6. Quantity A: The area of a triangular region with perimeter 8  
Quantity B: 8
- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.
7. List L consists of 7 numbers. The range of the numbers in list L is 0.

Quantity A: The average (arithmetic mean) of the numbers in list L.

Quantity B: 0

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

8.  $s = |t - 2|$

Quantity A:  $s + 2$

Quantity B:  $|t|$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

9. A jar contains exactly 10 dimes and  $x$  quarters and no other coins. If a coin is randomly selected from the jar, the probability that a quarter is selected is 0.6. What is the value of  $x$ .

- A. 5
- B. 6
- C. 8
- D. 12
- E. 15

10. In the rectangular coordinate system, the point (3,1) is on the circle with center (0,- 3). What is the area of the circle?

- A.  $5\pi$
- B.  $7\pi$
- C.  $10\pi$
- D.  $25\pi$
- E.  $\pi\sqrt{7}$

11.  $(2x+1)^2 - (2x-1)^2 =$

- A. 2
- B.  $8x$
- C.  $4x-1$
- D.  $4x+1$
- E.  $8x+2$

12. Which of the following is an equation of a line that does NOT contain any points in the xy-plane for which both coordinates are integers?

- A.  $y=4$
- B.  $y=\frac{1}{2}x$



C.  $y = x + 3$

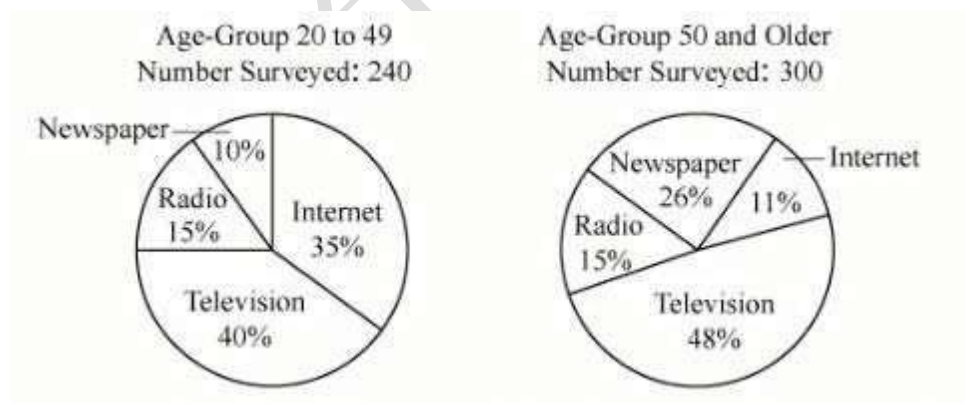
D.  $y = x + \frac{1}{2}$

E.  $y = \frac{1}{2}x + 3$

13. A veterinarian has 70 clients who own cats, dogs, or both. Of these clients, 36 own cats, including 20 clients who own both cats and dogs. Which of the following statements must be true? Indicate all such statements.

- A. There are 54 clients who own dogs.
- B. There are 34 clients who own dogs but not cats.
- C. There are 16 clients who own cats but not dogs.

14-16 are based on the following data.



Survey\* of preferred method to obtain news, by age-group

\*Each person surveyed indicated one of the four methods as his or her preferred method to obtain news.

14. What fraction of the people in the age-group 20 to 49 indicated newspaper or the Internet as their preferred method to obtain news?
15. Which of the following is closest to the percent of all the people survey who indicated the Internet as their preferred method to obtain news?
- A. 18.8%
- B. 21.7%
- C. 23.0%
- D. 33.3%
- E. 46.0%
16. For the age-group 50 and older, the number of people who indicated the Internet as their preferred method to obtain news is approximately what percent less than the number of people who indicated radio?
- A. 12%
- B. 27%
- C. 36%
- D. 45%
- E. 50%
17. When the positive integer  $x$  is divided by 42, the remainder is 19. What is the remainder when  $x$  is divided by 7?

- A. 0
- B. 2
- C. 3
- D. 4
- E. 5

18. If  $x$  is 4 more than half of  $y$  and if  $y$  is 10 more than half of  $x$ , what is the value of  $x$ ?

19. A pianist agreed to perform one concert at a fee 12.5 percent less than her usual fee and a second concert at a fee 20 percent greater than the first fee. The fee for the second concert was what percent greater than her usual fee?

- A. 5%
- B. 7.5%
- C. 15%
- D. 16.25%
- E. 32.5%

20.

Textbook	Numbers of
----------	------------

	Pages
A	510
B	480
C	490
D	520
E	$x$

The table shows the number of pages in each of 5 textbooks. What is the greatest possible value of  $x$  for which the average (arithmetic mean) number of pages of the 5 textbooks is equal to the median number of pages of the 5 textbooks?

## 综合练习 1-Section2

1. For the 500 measurements obtained in experiment X, the average (arithmetic mean) value is 280 and the value  $k$  is at the 75<sup>th</sup> percentile. For the 500 measurements obtained in experiment Y, the average value is 280 and the value  $n$  is at the 75<sup>th</sup> percentile.

Quantity A:  $k$

Quantity B:  $n$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

2. Quantity A: The greater possible value of  $\frac{2}{x-y}$ , where  $9 \leq x \leq 12$  and  $-2 \leq y \leq 8$

Quantity B: 2

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

3.  $x - y = 5$

Quantity A:  $x^2 - y^2$

Quantity B: 5

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

4.  $f(x) = 4x^2 + 28x + 49$  for all  $x$ .

Quantity A: The number  $b$  such that  $f(b)$  is the minimum value of  $f$

Quantity B: -3

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

5. Quantity A:  $(27)^{-8}$

Quantity B:  $(81)^{-6}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

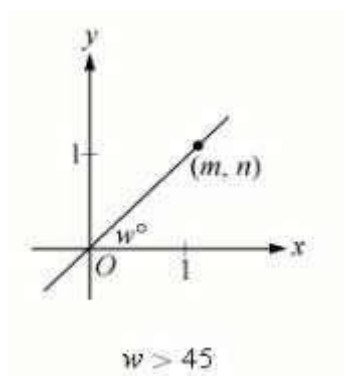
6.  $m$  and  $n$  are integers.

Quantity A:  $(\sqrt{10^{2m}})(\sqrt{10^{2n}})$

Quantity B:  $10^{mn}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

7.



Quantity A:  $m+n$

Quantity B:  $2m$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.
8. Company A has twice as many employees as Company B, and the two companies have no employees in common. If 64 percent of the employees of Company A are women and 52 percent of the employees of Company B are women, what percent of all the employees of the two companies are women?
9. A bookcase has  $s$  shelves with  $n$  books on each shelf, where  $n$  is a multiple of both  $s$  and  $s-1$ . If all of the books on the highest shelf were removed and redistributed equally among the other shelves, which of the following represents the number of books that would be on each of the other shelves?
- A.  $\frac{ns}{s-1}$
- B.  $\frac{n(s+1)}{s}$
- C.  $\frac{(n+1)s}{s-1}$



D.  $\frac{(n-1)s}{s-1}$

E.  $\frac{(n+1)(s-1)}{s}$

10. Which of the following pairs of integers have reciprocals whose sum is either less than  $\frac{1}{3}$  or greater than  $\frac{1}{2}$ ? Indicate all such pairs.

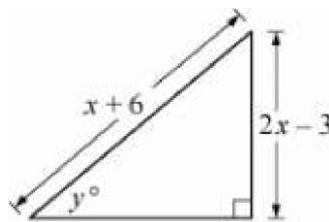
A. 1 and 14

B. 3 and 12

C. 5 and 10

D. 7 and 8

11.



In the triangle, if  $y = 30$ , then  $x =$

A. 3

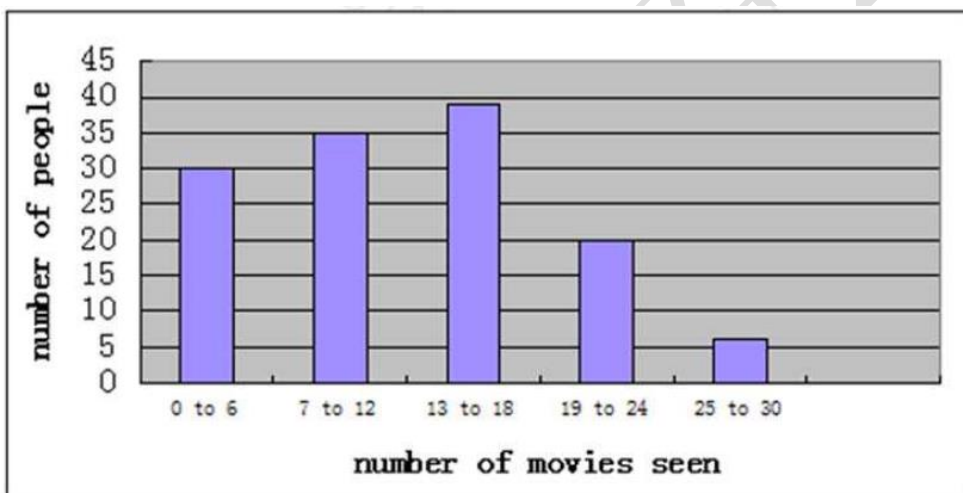
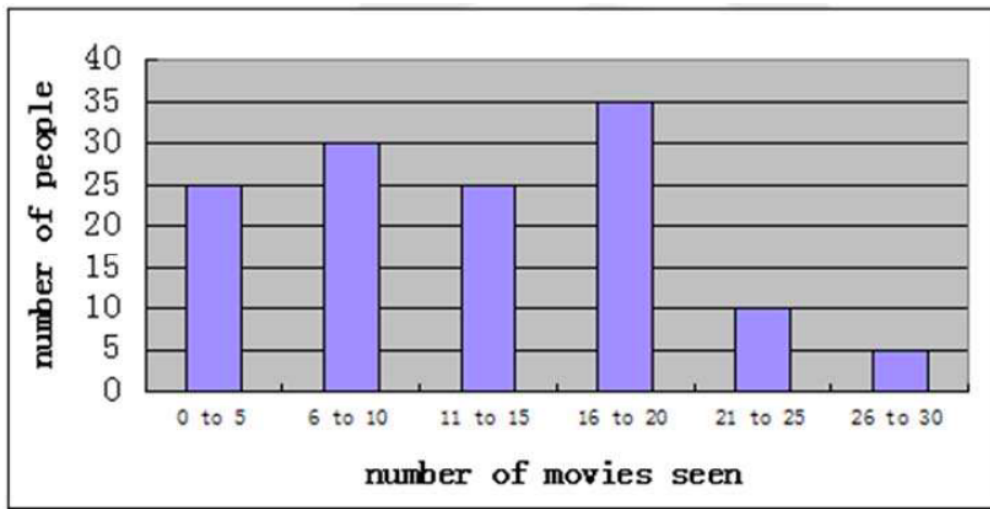
B. 4

C. 5

D. 8

E. 9

12.



In a survey, 130 people were asked how many movies they had seen in the preceding year. Their responses varied from 0 to 30 movies. The graphs above show two different summaries of the same survey results. How many people responded that they had seen 11 or 12 movies?

- A. 10
- B. 12

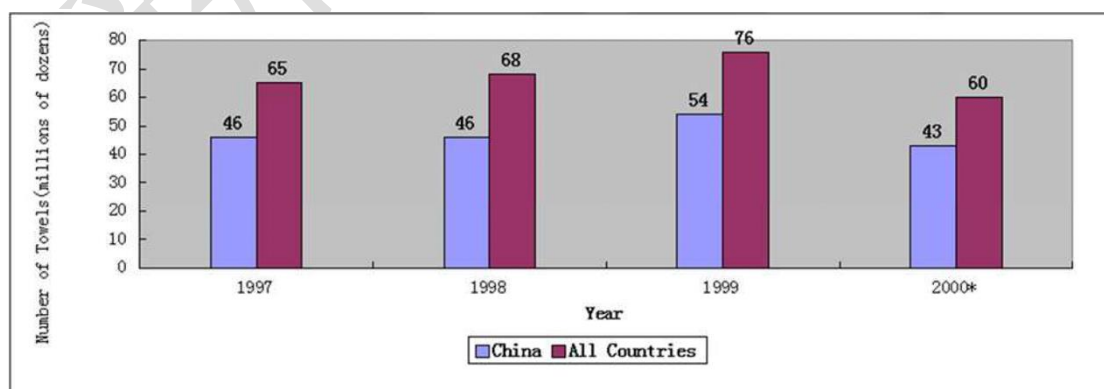
- C. 15
- D. 20
- E. 23

13. The width and the length of a rectangular piece of plywood are 4 feet and 8 feet, respectively. Along one edge of the plywood, a strip  $x$  inches wide and 8 feet long is removed. Then, along an edge perpendicular to the 8-foot edge, a strip  $x$  inches wide is removed. For what value of  $x$  will the remaining rectangular piece have width and length in the ratio of 2 to 5? (1 foot= 12 inches)

14-16

For each of the years 1997 through 2000\*, the graph shows the number of towels imported to Japan from China, and the total number of towels imported to Japan from all countries, including China.

Number of Towels Imported to Japan, 1997-2000\* (in millions of dozens\*\*)



\*For the first nine months of 2000

\*\*1 dozen= 12

14. In 1998, how many of the imported towels were not imported from China?
- A. 260 million  
B. 264 million  
C. 268 million  
D. 272 million  
E. 276 million
15. If the average (arithmetic mean) number of towels imported from China per month was the same for the last 3 months of 2000 as it was for the first 9 months of 2000, approximately how many million dozen towels were imported from China during the 12 months of 2000?
- A. 57  
B. 63  
C. 76  
D. 80  
E. 86
16. In 1999, the ratio of the number of towels imported from China to the total number of towels imported from countries other than China was closest to which of the following?

- A. 7 to 2
- B. 3 to 1
- C. 5 to 2
- D. 2 to 1
- E. 3 to 2

17. If  $x$  is a positive integer such that the units digit of  $x^3$  is 3, what is the units digit of  $x^{15}$ ?

- A. 1
- B. 3
- C. 5
- D. 7
- E. 9

18.  $\frac{60! - 59!}{58!}$

- A.  $(59)(58)$
- B.  $(60)(59)$
- C.  $(58)^2$
- D.  $(59)^2$
- E.  $(60)^2$

19. If a square region with side  $x$  and a circular region with radius  $r$  have the

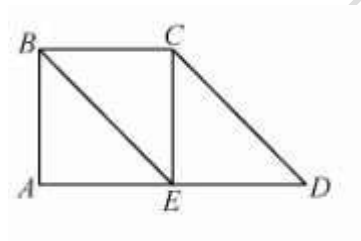
same area, then  $x$  must be how many times as great as  $r$ ?

- A.  $\frac{1}{\pi}$
- B.  $\frac{1}{\sqrt{\pi}}$
- C.  $\sqrt{\pi}$
- D.  $\pi$
- E.  $\pi^2$

20. The sum of  $n$  numbers is greater than 48. If the average (arithmetic mean) of the  $n$  numbers is 1.2, what is the least possible value of  $n$ ?

## 综合练习 2-Section1

1.



ABCE is a square, and BCDE is a parallelogram. Quantity A: The area of square ABCE

Quantity B: The area of parallelogram BCDE

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

2.  $n$  is an integer.

Quantity A:  $(-1)^n(-1)^{n+2}$

Quantity B: 1

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3. The population of Country X for 1980 was  $p$ . The population of Country X increased by 3.8 percent in each of the next two years.

Quantity A: The population of Country X for 1982

Quantity B:  $1.076p$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

4.  $x \neq 0$

Quantity A:  $x$

Quantity B:  $x(x+5)$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

5.  $x=2, y=3, z=5$

Quantity A:  $x^{-1}yz^{-2}$

Quantity B:  $(\frac{xz}{y})^{-2}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

6.  $x < y-2$

Quantity A: The average (arithmetic mean) of  $x$  and  $y$

Quantity B:  $y-1$



- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

7.  $x$  is an integer greater than 3.

Quantity A: The number of even factors of  $2x$

Quantity B: The number of odd factors of  $3x$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

8. If  $(a,b)$  is a point in the  $xy$ -plane, then the distance between  $(a,b)$  and the  $x$ -axis is  $|b|$  and the distance between  $(a,b)$  and the  $y$ -axis is  $|a|$ .

Quantity A: The total number of points  $P$  in the  $xy$ -plane such that the distance between  $P$  and one of the axes is 10 and the distance between  $P$  and the other axis is 8

Quantity B: The total number of points  $Q$  in the  $xy$ -plane such that the distance

between Q and one of the axes is 5 and the distance between Q and the other axis is 4.

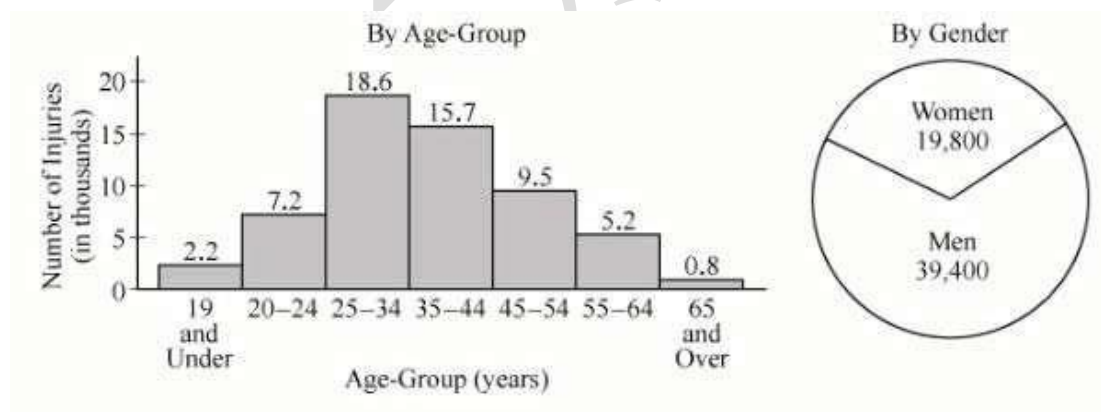
- A. Quantity A is greater.
  - B. Quantity B is greater.
  - C. The two quantities are equal
  - D. The relationship cannot be determined from the information given.
9. For a sample of 210 households, one-third of the households do not have any pets, one-third of the households each have 1 pet, and the rest of the households each have 2 pets. Which of the following statistics for the sample are equal to 1? Indicate all such statistics.
- A. The average (arithmetic mean) number of pets per household.
  - B. The median number of pets per household.
  - C. The range of the numbers of pets per household.
10. According to a tax rate formula for a certain year, the amount of tax owed by an individual whose annual income was between \$31,850 and \$77,100 was equal to a base tax of \$4,386 plus 24 percent of the annual income that exceeded \$31,850. According to this formula, what was the amount of tax owed by an individual whose annual income that year was \$42,000?

11. Each week a salesperson receives a commission that is equal to 12 percent of the first \$500 of sales plus 20 percent of additional sales. If the salesperson received a commission of \$380 last week, what was the total amount of the sales that the salesperson made last week?
- A. \$1,600  
B. \$1,660  
C. \$1,860  
D. \$2,000  
E. \$2,100
12. Last Monday a certain store sold 17 wrenches at  $x$  dollars each. Last Tuesday the store reduced its prices and sold an additional 8 wrenches at  $0.5x$  dollars each. Which of the following is equal to the average (arithmetic mean) price, in dollars, of the 25 wrenches that the store sold last Monday and Tuesday?
- A.  $0.68x$   
B.  $0.73x$   
C.  $0.76x$   
D.  $0.81x$   
E.  $0.84x$

13. In a distribution of 8,500 different measurements of the variable  $x$ , 26.5 is the 56th percentile and 37.1 is the 78<sup>th</sup> percentile. Which of the following is closest to the number of measurements of  $x$  that are in the distribution such that  $26.5 \leq x \leq 37.1$ ?

- A. 1,850
- B. 2,200
- C. 3,500
- D. 4,750
- E. 6,650

Questions 14 and 16 are based on the following data. NUMBER OF OCCUPATIONAL INJURIES IN STATE X, 1998



14. How many of the age-groups each accounted for more than 15 percent of the total number of occupational injuries in State X in 1998?

- A. One
- B. Two
- C. Three

D. Four

E. Five

15. In 1998, if one-half of the occupational injuries in the combine 34-and-under age-groups were incurred by men, what was the number of occupational injuries incurred by men in the combined 35-and-over age-groups?

A. 33,500

B. 31,900

C. 30,500

D. 25,400

E. 21,700

16. For the 55-64 age-group in 1998, the average (arithmetic mean) number of work-hours lost per occupational injury was 48.5. If a workweek is 40 work-hours, which of the following is closets to the total number of workweeks lost due to occupational injuries in the 55-64 age-group in 1998?

A. 4,500

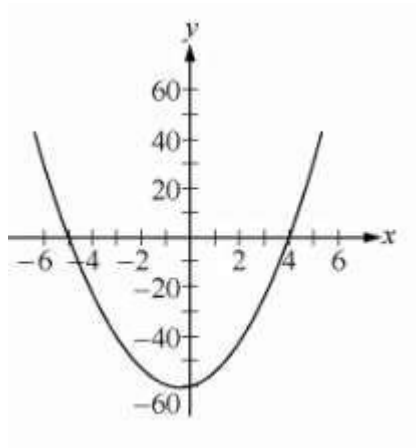
B. 5,200

C. 5,500

D. 5,900

E. 6,300

17.



Which of the following could be the equation of the graph in the  $xy$ -plane shown above?

A.  $y = x^2 + x - 60$

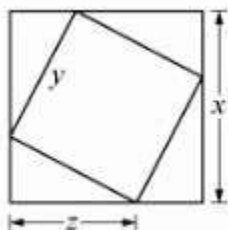
B.  $y = x^2 + x - 20$

C.  $y = x^2 + 3x - 60$

D.  $y = 3x^2 + x - 60$

E.  $y = 3x^2 + 3x - 60$

18.



The figure shows a smaller square with sides of length  $y$  inscribed in a larger square with sides of length  $x$ . Which of the following relationships between  $x$ ,  $y$ , and  $z$  must be true ?

- A.  $x^2 = y^2 + z^2$
- B.  $x^2 = y^2 - z^2$
- C.  $(x-z)^2 = y^2$
- D.  $(x-z)^2 + z^2 = y^2$
- E.  $(x-z)^2 + z^2 = y^2$

19.

X	Frequency
0	6
1	11
2	18
3	23
4	15

The table shows the frequency distribution of the random variable  $X$ . What is the median of the distribution of the values of  $X$ ?

- A. 1.0
- B. 1.8
- C. 2.0

D. 2.5

E. 3.0

20. The functions  $f$  and  $g$  are defined by  $f(x) = |2x + 1|$  and  $g(x) = 3$  for all numbers  $x$ . What is the least value of  $c$  for which  $f(c) = g(c)$ ?

## 综合练习 2-Section2

1. Of the students in a certain group, 22 percent are juniors and 26 percent are seniors.

Quantity A: The ratio of the number of juniors in the group to the number of seniors in the group.

Quantity B:  $\frac{4}{5}$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.



2. The area of a circular region is  $5\pi$

Quantity A: The diameter of the circular region

Quantity B:  $\sqrt{20}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

3. The reciprocal of  $x - 2$  is  $x + 2$

Quantity A:  $x$

Quantity B: 3

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

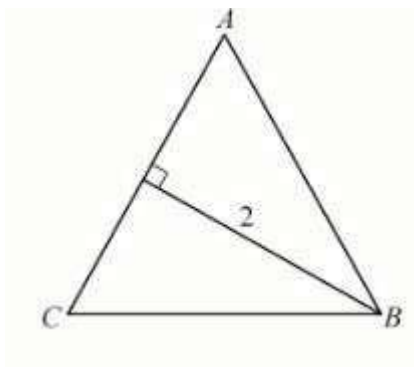
4.  $x > 0$

Quantity A: The area of a square region with diagonal of length  $\sqrt{2}x$

Quantity B: The area of a circular region with diameter of length  $x$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

5.



ABC is an equilateral triangle.

Quantity A: The length of AB

Quantity B:  $2\sqrt{3}$

- A. Quantity A is greater
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

6.  $\frac{1}{2} < r < 1$

Quantity A:  $2r$

Quantity B:  $\frac{1}{r}$

- A. Quantity A is greater.
  - B. Quantity B is greater.
  - C. The two quantities are equal
  - D. The relationship cannot be determined from the information given.
7. In a dataset of 10,000 numbers varying from 20 to 80, the number 62 is the 60<sup>th</sup> percentile and the number 74 is the  $n$ th percentile..

Quantity A:  $n$

Quantity B: 70

- A. Quantity A is greater.
  - B. Quantity B is greater.
  - C. The two quantities are equal
  - D. The relationship cannot be determined from the information given.
8. A historian asserts that at the beginning of 1852, the population of a certain mining town was 16,000. The historian also asserts that for each of

the years from 1849 through 1853, the town's population at the beginning of the year was twice that of the preceding year. According to the historian, what was the range of the town's populations at the beginning of each year from 1848 through 1853?

- A. 14,000
- B. 15,000
- C. 28,000
- D. 30,000
- E. 31,000

9. A box at a yard sale contains 3 different china dinner sets, each consisting of 5 plates. A customer will randomly select 2 plates to check for defects. What is the probability that the 2 plates selected will be from the same dinner set?

- A.  $\frac{2}{7}$
- B.  $\frac{2}{5}$
- C.  $\frac{2}{3}$
- D.  $\frac{5}{6}$
- E.  $\frac{3}{2}$

10. Line  $l$  in the  $xy$ -plane contains points  $A$  and  $B$  with coordinates  $(-4,5)$  and  $(6,-1)$ , respectively. Line  $k$  is perpendicular to  $l$  and contains the midpoint of line segment  $AB$ . Which of the following statements are true? Indicate all such statements.

- A. The slope of line  $l$  is  $\frac{-3}{5}$ .
- B. Line  $k$  has a negative slope.
- C. Line  $k$  contains the point  $(1,2)$ .

11. What is the remainder when  $3^{283}$  is divided by 5?

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

12. If  $x < y$ , which of the following must be true?

- A.  $2x < y$
- B.  $2x > y$
- C.  $x^2 < y^2$
- D.  $2x - y < y$
- E.  $2x - y < 2xy$

13. A rectangular solid P has height  $2c$  and a base of width  $a$  and length  $b$ .

Two other rectangular solids, Q and R, each have height  $c$  and bases of width  $a$  and length  $b$ . Which of the following represents the amount by which the sum of the surface areas of Q and R exceeds the surface areas of P?

- A.  $2ab$
- B.  $4ab$
- C.  $2ab + 2bc$
- D.  $2ab + 4ac$
- E.  $2ab + 4ac + 4bc$

Questions 14 and 16 are based on the following data.

A survey of 550 male managers and 650 female managers was conducted.

All 1,200 managers identified whether, for each of six characteristics, the characteristic is important to consider when hiring a new employee. For each of the six characteristics, the percent of managers surveyed who identified that characteristic as important to consider is given in the following table.

#### SURVEY RESULTS

Characteristic	Percent
Work experience	72%

Proficiency in English	68%
Ability to follow directions	65%
Specific occupational skill	60%
Computer expertise	58%
Appropriate attire and behavior	55%

14. Which of the

following statements about the managers surveyed must be true?

Indicate all such statements.

- A. Less than 55 percent were male managers.
  - B. Of the male managers, more identified work experience as an important characteristic to consider than identified proficiency in English.
  - C. Less than 60 percent of the male managers identified specific occupational skill as important to consider.
15. The number of managers surveyed who identified work experience as an important characteristic to consider was approximately what percent greater than the number who identified appropriate attire and behavior as an important characteristic to consider?
- A. 15%
  - B. 20%

- C. 25%
- D. 30%
- E. 35%

16. If 48 percent of the managers surveyed identified both ability to follow directions and computer expertise as an important characteristics to consider, what percent of the managers surveyed identified neither of these characteristics as important to consider?

- A. 15%
- B. 18%
- C. 23%
- D. 25%
- E. 28%

17. If the product of 7 consecutive integers is equal to the median of the integers, what is the least of the 7 integers?

18. On his trip to the airport, Grant drove a total of 9 miles. His average speed on the trip was  $x$  miles per hour, where  $30 \leq x \leq 35$ . Which of the following could be the total number of minutes that Grant took to make the trip? Indicate all such numbers of minutes.

- A. 15



- B. 16
- C. 17
- D. 18
- E. 19

19. If  $n$  is an integer, what is the least possible value of  $3^n + (3)(3^{-n})$ ?

- A. 1
- B. 2
- C. 3
- D.  $3\frac{1}{3}$
- E. 4

20. At a certain elementary school, 10 percent of the fifth-grade students are members of the school band. If 12 percent of the fifth-grade boys and 8 percent of the fifth-grade girls are members of the band, what percent of the fifth-grade students at the school are boys?

- A. 10%
- B. 12%
- C. 20%
- D. 30%
- E. 50%

## 7. 模考 (Mock Test1-2)

### 模考 1-Section2

1. Each of the 120 people in a group donated one of three different amounts to charity. Of the people in the group,  $\frac{2}{3}$  donated \$10.00 each,  $\frac{1}{4}$  donated \$15.00 each, and the rest donated \$25.00 each.

Quantity A: The average (arithmetic mean) amount donated per person in the group

Quantity B: \$12.5

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.  
D. The relationship cannot be determined from the information given.

2.  $-|x|=|x|$

Quantity A:  $x$

Quantity B: 0

- A. Quantity A is greater.  
B. Quantity B is greater.  
C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

3. Quantity A: The greatest possible value of  $\frac{2}{x-y}$ , where  $6 \leq x \leq 8$  and  $2 \leq y \leq 5$

Quantity B:  $\frac{2}{3}$

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

4. The function  $f$  is defined by  $f(x)=x(x^2-4)$  for all numbers  $x$ .

Quantity A: The number of points at which the graph of  $y=f(x)$  intersects the  $x$ -axis in the  $xy$ -plane

Quantity B: 3

A. Quantity A is greater.

B. Quantity B is greater.

C. The two quantities are equal.

D. The relationship cannot be determined from the information given.

5.  $0 < r < v < x < y < z$

Quantity A: The average (arithmetic mean) of the 4 numbers  $r$ ,  $v$ ,  $y$ , and

$z$

Quantity B: The average (arithmetic mean) of the 5 numbers  $r$ ,  $v$ ,  $x$ ,  $y$ , and  $z$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

6. The volume of a right circular cylinder is  $2,000\pi$ , and its height is 16 times its radius.

Quantity A: The radius of the cylinder

Quantity B: 5

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

7.  $C_1, C_2, C_3, \dots, C_j, \dots$

The sequence shown is defined by  $C_1 = 5$  and  $C_{j+1} = \frac{1}{5}C_j$  for each positive integer  $j$ .

Quantity A:  $C_{10}$

Quantity B:  $(5^{15})C_{25}$

- A. Quantity A is greater.
  - B. Quantity B is greater.
  - C. The two quantities are equal.
  - D. The relationship cannot be determined from the information given
8. An investor placed a total of \$6,400 in two accounts for one year. One of the accounts earned simple annual interest at a rate of 5 percent and the other earned simple annual interest at a rate of 3 percent. The investor made no deposits or withdrawals from the accounts. If each account earned the same amount of interest after one year, what was the total amount of interest earned from both accounts?
- A. \$128
  - B. \$144
  - C. \$240
  - D. \$256
  - E. \$512
9. A flat rectangular tile has a length that is between 4 inches and 6 inches and a width that is between 3 inches and 6 inches. Which of the following

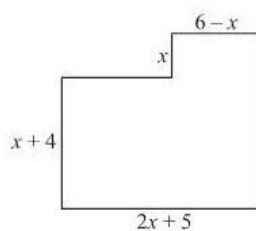
could be the value of the area, in square feet, of the top surface of the tile? (1 foot = 12 inches)

- A.  $\frac{1}{8}$
- B.  $\frac{1}{6}$
- C.  $\frac{1}{2}$
- D.  $\frac{4}{3}$

10. If 2, 4, 6, and 9 are the digits of two 2-digit integers, what is the least possible positive difference between the integers?

- A. 28
- B. 27
- C. 17
- D. 13
- E. 9

11.



In the figure, all intersecting line segments meet at right angles. Which of the following represents the perimeter of the figure in terms of  $x$ ?

A.  $3x + 15$

B.  $4x + 9$

C.  $5x + 19$

D.  $6x + 16$

E.  $8x + 18$

12. A certain experiment has only three possible outcomes. The probabilities of the outcomes are  $p$ ,  $r$ , and  $s$ . If  $r = 1 - 3p$ , what is  $s$  in terms of  $p$ ?

A.  $p$

B.  $2p$

C.  $3p$

D.  $1 - p$

E.  $1 - 2p$

13. When the positive integer  $d$  is divided by 12, the remainder is 5. What is the remainder when  $d^2$  is divided by 8?

A. 1

B. 3

C. 5

D. 6

E. 7

Questions 14 to 16 are based on this passage:

### Percent Distribution of Primary Modes of Transportation

Used by Commuters in Country *S* in June 2016

Mode of Transportation	Percent of Commuters
Drive alone	35%
Bus	25%
Train	20%
Bicycle or motorcycle	10%
Car pool	4%
Other	6%

Total number of commuters in Country *S* in June 2016: 8 million

14. If 75 percent of the commuters were local commuters and if 20 percent of the local commuters used buses as their primary mode of transportation, what percent of all commuters who used buses as their primary mode of transportation were local commuters?

A. 6%



- B. 15%
- C. 30%
- D. 60%
- E. 75%

15. For commuters who used car pools as their primary mode of transportation, the average (arithmetic mean) number of commuters per car pool vehicle was 2.5. Which of the following is closest to the total number of car pool vehicles for these commuters?

- A. 0.02 million
- B. 0.13 million
- C. 0.32 million
- D. 0.80 million
- E. 1.20 million

16. From June 2016 to December 2016, the total number of commuters increased by  $x$  percent while the percent of commuters who used trains as their primary mode of transportation remained the same. If the number of commuters who used trains as their primary mode of transportation increased by 16,000 from June to December, what is the value of  $x$ ?

- A. 1

- B. 2
- C. 3
- D. 4
- E. 5

17. The sale price of a certain radio is 25 percent less than the list price and 40 percent greater than the wholesale price of the radio. If the wholesale price of the radio is \$30, what is the list price of the radio?

- A. \$52
- B. \$53
- C. \$54
- D. \$55
- E. \$56

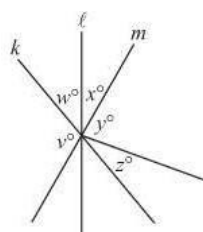
18. A list consists of three different positive integers whose sum is 10.

Which of the following statements individually provide(s) sufficient additional information to determine the value of the greatest integer in the list?

Indicate all such statements.

- A. The median of the integers in the list is 3.
- B. The range of the integers in the list is 5.
- C. The sum of the greatest integer and the least integer in the list is 7.
19. A group of  $n$  college students bought three identical round cakes to share. They divided the first cake into equal-sized pieces, one piece for each of them. They did the same with the second cake. After 3 of the students decided they did not want any more cake, the remaining students divided the third cake into equal sized pieces, one piece for each of them. If Silvia received 1 piece from each of the three cakes, then, in terms of  $n$ , the amount of cake that she received was the same as what fraction of 1 cake?
- A.  $\frac{n+2}{n(n-3)}$
- B.  $\frac{2n-3}{n(n-3)}$
- C.  $\frac{3n-3}{n(n-3)}$
- D.  $\frac{3n-6}{n(n-3)}$
- E.  $\frac{3n-3}{2n(n-3)}$

20.



In the figure, lines  $k$ ,  $l$ , and  $m$  intersect at a single point, which is the vertex of all the angles shown. If  $x = z$ ,  $y = 2w$ , and  $v = 110$ , what is the ratio of  $x$  to  $w$ ?

Give your answer as a fraction.

## 模考 1-Section4

1.  $s = \{2, 7, 9, 10, 11, 15\}$

A number  $r$  is to be selected at random from set  $S$ .

Quantity A: The probability that the value of  $(-1)^{r^2}$  will be 1

Quantity B:  $\frac{1}{2}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2.  $\frac{y}{8} = \frac{z}{16}$

Quantity A:  $\frac{y+1}{4}$

Quantity B:  $\frac{z+1}{8}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3. Quantity A:  $\frac{2}{\sqrt[3]{6}} + \frac{3}{\sqrt[3]{6}}$

Quantity B:  $\frac{\sqrt[3]{6}}{2} + \frac{\sqrt[3]{6}}{3}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

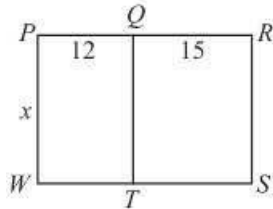
4. The length of each side of square  $PQRS$  is greater than 3. The radius of circle  $C$  is 3.

Quantity A: The area of square  $PQRS$

Quantity B: The area of circle  $C$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

5.



The ratio of  $PW$  to  $PR$  in rectangle  $PRSW$  is equal to the ratio of  $PQ$  to  $QT$  in rectangle  $PQTW$ .

Quantity A:  $x$

Quantity B: 20

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

6. For 1 year, Maria invested \$10,000 at a simple annual interest rate of  $x$  percent and \$8,000 at a simple annual interest rate of  $y$  percent, where  $x = \frac{3y}{4}$  and  $y > 0$ .

Quantity A: The amount of interest earned on the \$10,000 investment

Quantity B: The amount of interest earned on the \$8,000 investment

- A. Quantity A is greater.

- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.
7. The points  $(r, s)$  and  $(t, u)$  are in the  $xy$ -plane, where  $r, s, t$ , and  $u$  are all less than 0 and  $r \neq t$ .

Quantity A: The slope of the line that passes through  $(r, s)$  and  $(t, u)$

Quantity B: 0

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given

8. 14.6, 12.6, 16.3, 16.2, 16.2, 17.8, 17.7, 16.0, 14.6, 12.6

The readings of the electric meter for a certain business for each of 10 months are listed above. What is the median of the readings?

- A. 16.0
- B. 16.1
- C. 16.2
- D. 16.25

E. 16.3

9. When measured from sea level, the height of Mountain  $C$  is 40 percent of the height of Mountain  $D$ . If the height of Mountain  $C$  is greater than 800 meters, which of the following values could be the height, in meters, of Mountain  $D$ ? Indicate all such values. A. 1,200

A. 1,500

B. 1,600

C. 1,800

D. 2,500

E. 3,000

10. An empty box weighs 50 ounces. The box is to be packed with at least 98 units of a certain product but not more than 102 units. If each unit weighs 0.82 ounce and the packed box weighs  $w$  ounces, then  $w$  must satisfy which of the following inequalities?

A.  $80.36 \leq w \leq 83.64$

B.  $88.56 \leq w \leq 112.2$

C.  $119.50 \leq w \leq 132.64$

D.  $130.36 \leq w \leq 133.64$

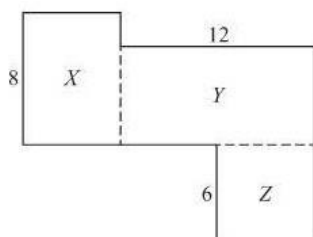
E.  $131.55 \leq w \leq 137.56$



11. The integers  $s$  and  $t$  are divisible by 5. Which of the following is NOT necessarily true?

- A.  $s-t$  is divisible by 5.
- B.  $s+t$  is divisible by 10.
- C.  $st$  is divisible by 25.
- D.  $s^2-t^2$  is divisible by 5.
- E.  $s^2+t^2$  is divisible by 25.

12.



The figure shows some of the dimensions, in meters, of a building with eight sides. The perimeters of rectangular regions  $X$ ,  $Y$ , and  $Z$  are 28, 34, and 22 meters, respectively. What is the perimeter of the building, in meters?

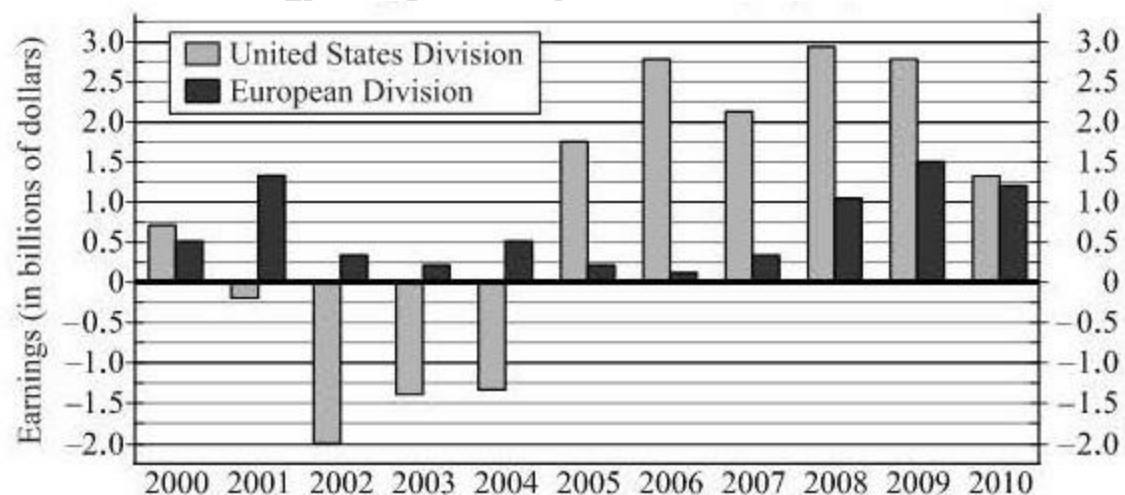
- A. 60
- B. 64
- C. 70
- D. 74
- E. 84

13. If  $x = 3$  is a solution of the equation  $x^2 + bx - 18 = 0$ , where  $b$  is a constant, then the other solution is

- A.  $x = 15$
- B.  $x = 9$
- C.  $x = 6$
- D.  $x = -3$
- E.  $x = -6$

Questions 14 to 16 are based on this passage:

Annual Earnings of Two Divisions of Company X, 2000-2010



Note: Negative earnings are considered as losses.

14. For 2008 the annual earnings of the European division were approximately

what percent of the combined annual earnings of the two divisions?

- A. 25%
- B. 40%
- C. 50%
- D. 60%
- E. 75%

15. If the annual earnings of the European division for 2009 represented 7 percent of the division's annual revenues, approximately how much were its annual revenues?

- A. \$11 billion
- B. \$21 billion
- C. \$37 billion
- D. \$105 billion
- E. \$214 billion

16. For the years shown, the range of the annual earnings of the United States division was approximately how much greater than the range of the annual earnings of the European division?

- A. \$1.5 billion
- B. \$2.7 billion

- C. \$3.6 billion
- D. \$4.3 billion
- E. \$5.0 billion

17. The average (arithmetic mean) of  $w$ ,  $x$ ,  $y$ , and  $z$  is equal to the average of  $x$ ,  $y$ , and  $z$ . Which of the following expressions represents  $w$  in terms of  $x$ ,  $y$ , and  $z$ ?

- A.  $\frac{x+y+z}{4}$
- B.  $\frac{x+y+z}{3}$
- C.  $\frac{3(x+y+z)}{4}$
- D.  $\frac{4(x+y+z)}{3}$
- E.  $12(x+y+z)$

18. How many integers between 115 and 969 are cubes of integers?

19. The width of rectangle  $T$  is 25 percent greater than the width of rectangle  $R$ , and the length of  $T$  is 10 percent greater than the length of  $R$ . The area of  $T$  is what percent greater than the area of  $R$ ?

- A. 2.5%
- B. 15%
- C. 27.3%
- D. 35%
- E. 37.5%

20. If  $-1 < x < 1$  and  $x \neq 0$ , which of the following statements must be true?

Indicate all such statements.

- A.  $x < \frac{1}{x}$
- B.  $x^2 < x$
- C.  $x^3 < x^2$

## 模考 2-Section2

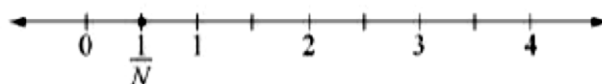
1.  $0 < x < y < 1$ .

Quantity A:  $y-x$

Quantity B:  $(x-y)^2$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2.



Quantity A:  $N$

Quantity B:  $\frac{1}{2}$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3. Last Monday, James paid \$8.00 per share for 180 shares of Stock X, paid \$10.00 per share for 160 shares of Stock Y, paid \$12.00 per share for 200 shares of Stock Z, and bought no other shares of stock.

Quantity A: The average (arithmetic mean) price per share that James paid for all of the shares of stock he bought last Monday

Quantity B: \$10.00

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

4.  $2x + cy = 10$

$$6x + 2cy = 40$$

The system of equations shown has a solution, where  $c$  is a constant.

Quantity A:  $c$

Quantity B: 1

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

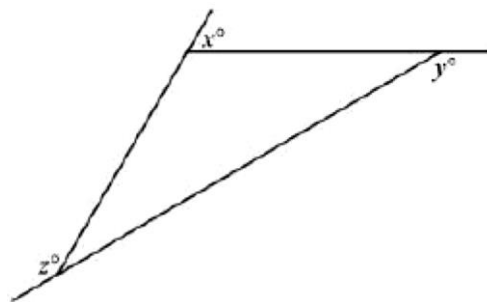
5. The average (arithmetic mean) of the 6 numbers  $q$ ,  $r$ ,  $s$ ,  $t$ ,  $u$ , and  $v$  is 36.

Quantity A:  $\frac{q}{6} + \frac{r}{6} + \frac{s}{6} + \frac{t}{6} + \frac{u}{6} + \frac{v}{6}$

Quantity B: 6

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal
- D. The relationship cannot be determined from the information given.

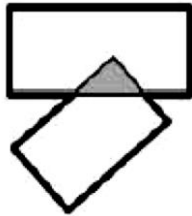
6.



Quantity A:  $x + y + z$

Quantity B: 360

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.



7.

In the figure shown, two rectangular regions overlap to form a triangular region, which is shaded. The perimeters of the rectangles are 15 and 21, and the perimeter of the triangle is 6.

Quantity A: The sum of the lengths of the thick line segments

Quantity B: 30

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given

Answer:



8.  $x$  and  $y$  are integers.

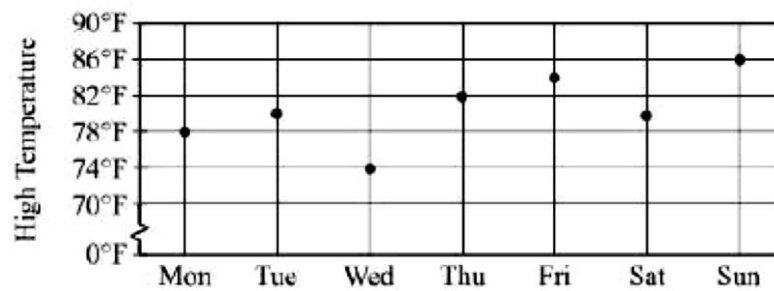
$$2^x - 3y = 20$$

Quantity A:  $y$

Quantity B: 0

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given
9. If  $x$  is 50 percent of  $y$  and if  $z$  is 30 percent of  $y$ , where  $y > 0$ , then  $z$  is what percent of  $x$ ?
10. A total of \$36,000 was invested for one month in a new money market account that paid simple annual interest at the rate of  $r$  percent. If the investment earned \$360 in interest for the month, what is the value of  $r$ ?
- A. 10.0
- B. 10.5
- C. 11.0
- D. 12.0
- E. 12.5

11.



$$C = \frac{F - 32}{1.8}$$

The equation represents the relationship between temperature  $C$  in degrees Celsius and the corresponding temperature  $F$  in degrees Fahrenheit. The graph shows the high temperature, in degrees Fahrenheit, on each day of a certain week. Which of the following is closest to the range of the high temperatures for the week, in degrees Celsius?

- A.  $4^{\circ}\text{C}$
- B.  $7^{\circ}\text{C}$
- C.  $8^{\circ}\text{C}$
- D.  $12^{\circ}\text{C}$
- E.  $14^{\circ}\text{C}$

12. The circular bases of a right circular cylinder are inscribed in two opposite faces of a cube. If the volume of the cube is 64, which of the following is closest to the volume of the cylinder?

- A. 46
- B. 50
- C. 54
- D. 58
- E. 62

13.

Score Interval	Number of Students
90-100	5
80- 89	8
70-79	10
60-69	2

All of the students in an English class took a test, and each student received a test score that was a whole number between 60 and 100, inclusive. The table shows the number of students who received a test score in each of four score intervals. Which of the following numbers could be the median test score for all of the students?

Indicate all such numbers.

- A. 85
- B. 80
- C. 75
- D. 70

E. 65

F. 60

Questions 14 to 16 are based on this passage:

Book-Publishing Industry Sales in Region R

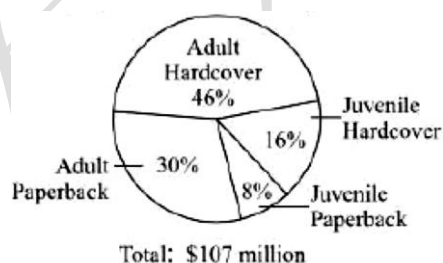
Summary of Book Sales for 2005, 2010, and 2015

(in millions of dollars)

Market Category	2005	2010	2015
Mass-market	\$54	\$93	\$107
Professional	\$41	\$87	\$105
Other	\$146	\$157	\$198
Total	\$241	\$337	\$410

Sales of Mass-Market Books

by Age Level and Cover Type, 2015



14. For mass-market books in 2015, which of the following is closest to the ratio of the dollar amount of sales of adult hardcover books to the dollar amount of sales of juvenile hardcover books?

A. 7 to 2

B. 5 to 2

- C. 4 to 1
- D. 3 to 1
- E. 2 to 1

15. For 2005, if sales of law books accounted for 39 percent of the dollar amount of sales of professional books, then sales of law books accounted for what percent of the dollar amount of all book sales for 2005?

Give your answer to the nearest whole percent.

16. If the sales of mass-market adult hardcover books accounted for 33 percent of mass-market book sales in 2010, by approximately what percent did the dollar amount of sales of this type of book increase from 2010 to 2015?

- A. 20%
- B. 30%
- C. 40%
- D. 50%
- E. 60%

17.

Online News Source	Percent of Those Surveyed
Computer home page headlines	52%
Newspaper website	48%
News service website	35%
Magazine website	24%
Other	20%

In a study of online news sources, computer users were surveyed to determine what computer sources they used for online news. The responses of those surveyed are summarized in the table shown. If 15 percent of those surveyed responded that they used both newspaper and magazine websites, what percent of those surveyed used newspaper websites but not magazine websites?

18. The circumference of circle X is  $24\pi$ . If the radius of circle Y is equal to  $\frac{1}{12}$  of the circumference of circle X, what is the circumference of circle Y?

- A.  $\pi$
- B.  $2\pi$
- C.  $2\pi^2$
- D.  $4\pi^2$
- E.  $2\pi^3$

19. If  $t \neq 0$ , which of the following is equivalent to  $\frac{1}{1 + \frac{1}{t + \frac{1}{t}}}$

- A.  $\frac{1}{t}$

B.  $\frac{1}{t^2+1}$

C.  $\frac{t^2+1}{t}$

D.  $\frac{1}{t^2+t+1}$

E.  $\frac{t^2+1}{t^2+t+1}$

20.  $6 \leq |x| \leq 8$

$1 \leq |y| \leq 2$

$3 \leq |z| \leq 4$

If  $x$ ,  $y$ , and  $z$  satisfy the inequalities shown, what is the least possible value of  $|x + y + z|$ ?

A. 0

B. 1

C. 2

D. 3

E. 4

## 模考 2-Section4

1.  $|x| + |y| = 3$

Quantity A:  $x+y$

Quantity B:  $-2$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

2. The sum of 11 consecutive integers is 22.

Quantity A: The median of the 11 integers

Quantity B: 3

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

3. When the positive integer  $q$  is divided by 2, the remainder is 0.

Quantity A: The remainder when  $q$  is divided by 8

Quantity B: 4

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.



D. The relationship cannot be determined from the information given.

4.

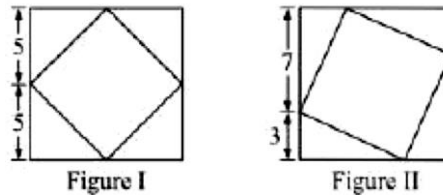


Figure I and Figure II each show a small square inscribed in a large square that has sides of length 10.

Quantity A: The perimeter of the small square in Figure I

Quantity B: The perimeter of the small square in Figure II

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

5.

Quantity A: The area of a circle with circumference  $8\pi x$

Quantity B: The area of a circle with radius  $4x$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal

D. The relationship cannot be determined from the information given.

6. A certain car travels 120 miles in  $t$  hours at a constant rate of  $r$  miles per hour.

Quantity A: The number of hours required for the car to travel 240 miles at a constant rate of  $3r$  miles per hour

Quantity B:  $t$

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given.

7. For all numbers  $x$ , the function  $f$  is defined as follows.

$$f(x) = \begin{cases} 2^{-x} & \text{if } x > 0 \\ 2^x & \text{if } x \leq 0 \end{cases}$$

Quantity A:  $f(-4) + f(0) - f(4)$

Quantity B: 1

- A. Quantity A is greater.
- B. Quantity B is greater.
- C. The two quantities are equal.
- D. The relationship cannot be determined from the information given

8. If  $x = -3$ , which of the following is greatest?
- A.  $1+x$
- B.  $x-5$
- C.  $\frac{1}{x}$
- D.  $3x$
- E.  $x^3$
9. A class contains 120 students, each of whom is either a part-time student or a full-time student. If a student is to be selected at random from the class, then the probability that the student will be a part-time student is  $\frac{2}{5}$ . How many of the students in the class are full-time students?
- A. 24
- B. 48
- C. 60
- D. 72
- E. 96
10. The least positive integer that is divisible by 2, 3, 15, and 28 is
- A. 210

- B. 420
- C. 840
- D. 1,260
- E. 2,520

11. In a certain city, 15 percent of the total population is more than 60 years old. If 60 percent of the people older than 60 voted in a recent election, then these voters comprised what percent of the total population?

- A. 6%
- B. 8%
- C. 9%
- D. 15%
- E. 60%

12. According to the scale given on a certain map, 1 inch is equivalent to 20 miles. Which of the following scales are the same as the scale given on the map? Indicate all such scales.

- A.  $\frac{1}{4}$  inch is equivalent to 5 miles.
- B.  $\frac{1}{2}$  inch is equivalent to 10 miles.
- C.  $\frac{5}{2}$  inches are equivalent to 50 miles.

D. 4 inches are equivalent to 80 miles.

13. If  $x$  is an odd integer, which of the following must be an odd integer?

A.  $x^4 + 1$

B.  $x^4 - 1$

C.  $(x + 1)^5 - 2$

D.  $(x + 1)^5 + 1$

E.  $(x - 1)^6 + 2$

Questions 14 to 16 are based on this passage:

Distribution of Level of Educational Attainment and Unemployment Rates for  
25 to 64 Year Olds in Selected Countries, 1992

Country	Distribution of Level of Educational Attainment			Unemployment Rate
	No High School Diploma	High School Diploma Only	Two-year College Degree or Higher	
Austria	32%	60%	8%	3.6%
Belgium	55%	25%	20%	7.8%
France	48%	36%	16%	8.8%
Germany	18%	60%	22%	6.2%
Italy	72%	22%	6%	7.4%
Netherlands	43%	36%	21%	5.6%
Spain	77%	9%	14%	14.7%
Switzerland	19%	61%	20%	2.5%

14. For the countries shown, what is the range of the percent of 25 to 64 years old with no high school diploma?

A. 52%

- B. 59%
- C. 63%
- D. 68%
- E. 69%

15. For how many of the countries was the percent of 25 to 64 year olds with no high school diploma greater than the percent with a two-year college degree or higher?

- A. Three
- B. Four
- C. Five
- D. Six
- E. Seven

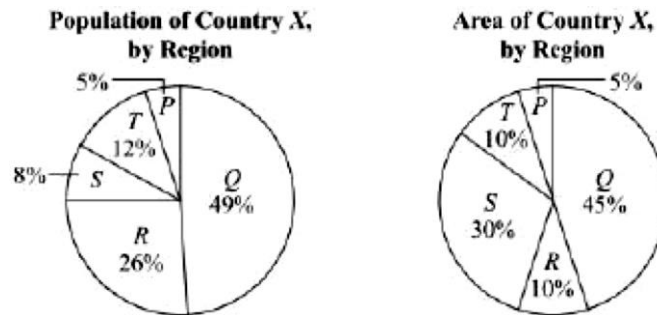
16. What is the median unemployment rate for 25 to 64 year olds in those countries for which more than 15 percent of 25 to 64 year olds had a level of educational attainment of a two- year college degree or higher?

- A. 5.6%
- B. 6.2%
- C. 6.8%

D. 7.8%

E. 8.8%

17.



Country X consists of five regions— $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $T$ . The circle graphs show the percent distributions of the population and the area of Country X, by region. Which region of Country X has the greatest population per unit of area?

A.  $P$

B.  $Q$

C.  $R$

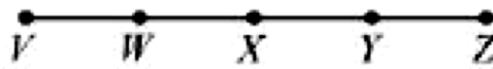
D.  $S$

E.  $T$

18. Which two of the following points in the  $xy$ -plane lie on the same horizontal line? Indicate both of the points.

- A. (3,4)
- B. (6,8)
- C. (4,5)
- D. (5,4)
- E. (6,2)

19.



In the figure shown,  $X$  is the midpoint of line segment  $VZ$ , and  $W$  and  $Y$  are the midpoints of line segments  $VX$  and  $XZ$ , respectively. What is the ratio of the length of  $VY$  to the length of  $WY$ ?

- A. 2 to 1
- B. 2 to 3
- C. 3 to 2
- D. 3 to 4
- E. 4 to 3

20. The sum of the squares of three consecutive positive integers is 149. What is the sum of the three integers?



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