

Capita Selecta AI - Probabilistic Programming Inference for SRL

Thierry Deruyttere (r0660485) & Armin Halilovic (r0679689)

April 1, 2018

Probabilistic Inference Using Weighted Model Counting

1.1 PGM to CNF

1.1.1 ENC 1

Our ENC1 encoding for the Cancer Bayesian network can be found in appendix 4.1.

1.1.2 ENC 2

Our ENC2 encoding for the Cancer Bayesian network can be found in appendix 4.2.

1.2 SRL to CNF

1.2.1 Encoding of Monty Hall as CNF

An encoding of problog programs can be generated by our program as follows:

```
python3 scripts/inference.py --problog files/problog/monty_hall.pl
```

The CNF will be shown using the program's predicates. A version of the CNF in dimacs format will be shown as well. See `README.MD` for more information.

Our CNF encoding for the given Monty Hall ProbLog program is:

$\wedge(\text{open_door}(2) \vee \text{prize}(2) \vee \text{prize}(3) \vee \neg \text{p_open_door}(2)_0)$
 $\wedge(\text{open_door}(2) \vee \text{prize}(2) \vee \neg \text{prize}(3))$
 $\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(2) \vee \neg \text{prize}(2))$
 $\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(2) \vee \text{prize}(3))$
 $\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(3) \vee \neg \text{prize}(2))$
 $\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(3) \vee \text{prize}(3))$
 $\wedge(\neg \text{open_door}(2) \vee \text{p_open_door}(2)_0 \vee \neg \text{prize}(2))$
 $\wedge(\neg \text{open_door}(2) \vee \text{p_open_door}(2)_0 \vee \text{prize}(3))$
 $\wedge(\text{open_door}(3) \vee \text{prize}(2) \vee \text{prize}(3) \vee \neg \text{p_open_door}(3)_0)$
 $\wedge(\text{open_door}(3) \vee \text{prize}(3) \vee \neg \text{prize}(2))$
 $\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(2) \vee \neg \text{prize}(3))$
 $\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(2) \vee \text{prize}(2))$
 $\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(3) \vee \neg \text{prize}(3))$
 $\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(3) \vee \text{prize}(2))$
 $\wedge(\neg \text{open_door}(3) \vee \text{p_open_door}(3)_0 \vee \neg \text{prize}(3))$
 $\wedge(\neg \text{open_door}(3) \vee \text{p_open_door}(3)_0 \vee \text{prize}(2))$
 $\wedge(\text{win_keep} \vee \neg \text{prize}(1))$
 $\wedge(\neg \text{win_keep} \vee \text{prize}(1))$
 $\wedge(\text{win_switch} \vee \neg \text{prize}(2) \vee \text{open_door}(2))$
 $\wedge(\text{win_switch} \vee \neg \text{prize}(3) \vee \text{open_door}(3))$
 $\wedge(\neg \text{win_switch} \vee \text{prize}(2) \vee \text{prize}(3))$
 $\wedge(\neg \text{win_switch} \vee \text{prize}(2) \vee \neg \text{open_door}(3))$
 $\wedge(\neg \text{win_switch} \vee \neg \text{open_door}(2) \vee \text{prize}(3))$
 $\wedge(\neg \text{win_switch} \vee \neg \text{open_door}(2) \vee \neg \text{open_door}(3))$
 $\wedge(\neg \text{prize}(1) \vee \neg \text{prize}(2))$
 $\wedge(\neg \text{prize}(1) \vee \neg \text{prize}(3))$
 $\wedge(\neg \text{prize}(2) \vee \neg \text{prize}(3))$
 $\wedge(\text{prize}(1) \vee \text{prize}(2) \vee \text{prize}(3))$

Weights:

$W(\text{p_open_door}(2)_0) = 0.5$
 $W(\text{p_open_door}(3)_0) = 0.5$
 $W(\text{select_door}(1)) = 1.00$
 $W(\text{prize}(1)) = 0.33$
 $W(\text{prize}(2)) = 0.33$
 $W(\text{prize}(3)) = 0.33$
 $W(\text{open_door}(2)) = 1.00$
 $W(\text{open_door}(3)) = 1.00$
 $W(\text{win_keep}) = 1.00$
 $W(\text{win_switch}) = 1.00$

$W(\neg \text{p_open_door}(2)_0) = 0.5$
 $W(\neg \text{p_open_door}(3)_0) = 0.5$
 $W(\neg \text{select_door}(1)) = 0.00$
 $W(\neg \text{prize}(1)) = 1.00$
 $W(\neg \text{prize}(2)) = 1.00$
 $W(\neg \text{prize}(3)) = 1.00$
 $W(\neg \text{open_door}(2)) = 1.00$
 $W(\neg \text{open_door}(3)) = 1.00$
 $W(\neg \text{win_keep}) = 1.00$
 $W(\neg \text{win_switch}) = 1.00$

1.3 Weighted Model Counting

1.3.1 Weighted model counters on above CNFs

We have selected MiniC2D and Cachet as weighted model counters. We have executed the model counters on DIMACS versions of the CNFs. The DIMACS files can be found under report/encodings. The output of the model counters can be found in the listings below.

MiniC2D

MiniC2D needs to be executed with the $-W$ flag in order for it to do weighted model counting. The resulting probability can be read next to “Count”.

Listing 1.1: MiniC2D on ENC1 encoding of Cancer network

```
Constructing CNF... DONE
CNF stats:
  Vars=30 / Clauses=74
  CNF Time      0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=5, c_con=48 v_con=5
  Vtree Time    0.001s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      75.0%
  lookups      16
  ent count     4
  ent memory    0.2 KB
  ht memory    152.6 MB
  clists       1.0 ave, 1 max
  keys         3.0b ave, 3.0b max, 3.0b min
Count stats:
  Count Time     0.000s
  Count         0.9999999999999999
Total Time: 0.012s
```

Listing 1.2: MiniC2D on ENC2 encoding of Cancer network

```
Constructing CNF... DONE
CNF stats:
  Vars=20 / Clauses=30
  CNF Time      0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=6, c_con=16 v_con=6
  Vtree Time    0.000s
Counting... DONE
  Learned clauses      0
Cache stats:
```

```

hit rate      23.1%
lookups       26
ent count     20
ent memory    1.0 KB
ht memory     152.6 MB
clists        1.0 ave, 1 max
keys          1.8b ave, 3.0b max, 1.0b min
Count stats:
  Count Time  0.000s
  Count       1.0000000000000000
Total Time: 0.012s

```

Listing 1.3: MiniC2D on CNF encoding of Monty Hall

```

Constructing CNF... DONE
CNF stats:
  Vars=10 / Clauses=26
  CNF Time    0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=4, c_con=22 v_con=4
  Vtree Time   0.000s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      20.0%
  lookups       5
  ent count     4
  ent memory    0.2 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          3.2b ave, 4.0b max, 3.0b min
Count stats:
  Count Time    0.000s
  Count         1.0000000000000000
Total Time: 0.011s

```

Cachet

For Cachet, there is no need to use extra parameters to get a probability. It is reported next to “Satisfying probability”.

Listing 1.4: Cachet on ENC1 encoding of Cancer network

```

Number of total components      11
Number of split components      2
Number of non-split components  5
Number of SAT residual formula  12
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0

```

| | |
|-----------------------------|-------------|
| First component split level | 1 |
| Number of Decisions | 11 |
| Max Decision Level | 5 |
| Number of Variables | 30 |
| Original Num Clauses | 74 |
| Original Num Literals | 172 |
| Added Conflict Clauses | 0 |
| Added Conflict Literals | 0 |
| Deleted Unrelevant clauses | 0 |
| Deleted Unrelevant literals | 0 |
| Number of Implications | 124 |
| Total Run Time | 0.0163 |
| Satisfying probability | 8.72319e-08 |
| Number of solutions | 93.6645 |

Listing 1.5: Cachet on ENC2 encoding of Cancer network

| | |
|--------------------------------|-------------|
| Number of total components | 11 |
| Number of split components | 2 |
| Number of non-split components | 5 |
| Number of SAT residual formula | 12 |
| Number of trivial components | 0 |
| Number of changed components | 0 |
| Number of adjusted components | 0 |
| First component split level | 1 |
| Number of Decisions | 11 |
| Max Decision Level | 5 |
| Number of Variables | 20 |
| Original Num Clauses | 30 |
| Original Num Literals | 84 |
| Added Conflict Clauses | 0 |
| Added Conflict Literals | 0 |
| Deleted Unrelevant clauses | 0 |
| Deleted Unrelevant literals | 0 |
| Number of Implications | 72 |
| Total Run Time | 0.017372 |
| Satisfying probability | 1 |
| Number of solutions | 1.04858e+06 |

Listing 1.6: Cachet on WCNF encoding of Monty Hall

| | |
|--------------------------------|---|
| Number of total components | 4 |
| Number of split components | 1 |
| Number of non-split components | 2 |
| Number of SAT residual formula | 5 |
| Number of trivial components | 0 |
| Number of changed components | 0 |

| | |
|-------------------------------|----------|
| Number of adjusted components | 0 |
| First component split level | 2 |
| Number of Decisions | 4 |
| Max Decision Level | 4 |
| Number of Variables | 10 |
| Original Num Clauses | 26 |
| Original Num Literals | 73 |
| Added Conflict Clauses | 0 |
| Added Conflict Literals | 0 |
| Deleted Unrelevant clauses | 0 |
| Deleted Unrelevant literals | 0 |
| Number of Implications | 26 |
| Total Run Time | 0.016062 |
| Satisfying probability | 0.444444 |
| Number of solutions | 455.111 |

For ENC1, we see that with Cachet reports a satisfying probability of almost 0. Similarly, for Monty Hall, we see that we get a probability of 0.44. This is due to the fact that with ENC1, the weights of negated literals are 1, but Cachet expects that $weight(x) + weight(-x) = 1$. In the Monty Hall encoding, we also have weights of negated literals equalling 1, which gives the same problem as with ENC1.

1.3.2 Difference between the selected WMCs

MiniC2D Vs Cachet

MiniC2D and Cachet are weighted model counters that work in different ways. In short, MiniC2D is a top down compiler that compiles CNFs into SDDs, while Cachet uses formula caching combined clause learning and component analysis [[1], [2]]. MiniC2D's compilation to SDDs is faster and uses less memory.

Both weighted model counters use concepts from the SAT literature. They both use clause learning and component caching in order to reuse components that later appear again during search.

Cachet also uses other methods from SAT literature, like an explicit on the fly calculation of connected components. This is different in MiniC2D, as relies on vtrees to identify disconnected CNF components. MiniC2d creates vtrees for CNFs and then creates SDDs based on the created vtrees.

1.3.3 Overview of computational requirements

All the tests can be found in the test folder. We used our scripts to create the dimac files. The input files for our enc1 and enc2 converter are “.dsc” files which can be found at <http://www.bnlearn.com/bnrepository/discrete-small.html#cancer>.

Test 1: Cancer network

Table 1.1: My caption

| | ENC1 | | | ENC2 | | |
|----------------|------|--------|---------|------|--------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 1.0 | 0.2 KB | 0.155s | 1.0 | 1.0 KB | 0.000s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

Test 2: asia network

Table 1.2: My caption

| | ENC1 | | | ENC2 | | |
|----------------|------|--------|---------|------|--------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 1.0 | 0.9 KB | 0.145s | 1.0 | 2.0 KB | 0.139s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

Test 3: sachs network

Table 1.3: My caption

| | ENC1 | | | ENC2 | | |
|----------------|---------|---------|---------|------|---------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 0.99707 | 14.3 KB | 0.184s | 1.0 | 14.5 KB | 0.154s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

Test 4: earthquake network

Table 1.4: My caption

| | ENC1 | | | ENC2 | | |
|----------------|------|--------|---------|------|--------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 1.0 | 0.6 KB | 0.137s | 1.0 | 1.0 KB | 0.153s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

Test 5: survey network

Table 1.5: My caption

| | ENC1 | | | ENC2 | | |
|----------------|------|--------|---------|------|--------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 1.0 | 1.6 KB | 0.125s | 1.0 | 2.0 KB | 0.125s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

Test 6: alarm network

Table 1.6: My caption

| | ENC1 | | | ENC2 | | |
|---------|------|------------|---------|------|--------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 1.0 | 959.7KB KB | 0.268s | 1.0 | 139KB | 0.095s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

Test 6: andes network

Table 1.7: My caption

| | ENC1 | | | ENC2 | | |
|---------|------|--------|---------|------|---------|---------|
| | Prob | Memory | Runtime | Prob | Memory | Runtime |
| Minic2d | 1.0 | 2.7GB | 122.78s | 1.0 | 139.8MB | 6.086s |
| Cachet | val1 | val2 | a | b | val3 | val4 |

1.4 Knowledge compilation

Vtree with the most compact circuit

During our tests

Pattern for a good vtree

As a vtree is a binary tree, which means that a good vtree is compact. We want thus a vtree that is shallow.

Build an Inference Engine

2.0.1 Implementation

We have implemented the pipeline using python. Information about it can be found in `README.MD`.

2.0.2 Pipeline with previous tasks

Cancer Bayesian network

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

Monty Hall

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

2.0.3 Pipeline on Bayesian learning example

DAS NEN DIKKE VETTE TODO

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

2.0.4 Pipeline on alarm Bayesian network

GOD DAMN IT STOM VAK HOE MOETEN WIJ DIT IN 50 UUR DOEN?
PROCESS WORDT GEWOON GEKILLED OMDAT DIE CNF GIGANTISCH
GROOT WORDT.

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

Parameter Learning

learning

Appendix

4.1 ENC1

Indicator clauses:

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse}) \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\
 &\quad \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\
 &\quad \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse})
 \end{aligned}$$

Parameter clauses:

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \theta_{PollutionLow}) \wedge (\lambda_{PollutionLow} \vee \neg \theta_{PollutionLow}) \wedge (\neg \\
 &\lambda_{PollutionHigh} \vee \theta_{PollutionHigh}) \wedge (\lambda_{PollutionHigh} \vee \neg \theta_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \theta_{SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \theta_{SmokerTrue}) \wedge (\neg \\
 &\quad \lambda_{SmokerFalse} \vee \theta_{SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \theta_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \lambda_{CancerTrue} \vee \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg
 \end{aligned}$$

$$\begin{aligned}
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{XrayPositive} \vee \\
& \theta_{XrayPositive|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge \\
& (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\
& \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerTrue}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayPositive} \vee \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerFalse}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \\
& \neg \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerFalse}) \wedge \\
& (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaTrue} \vee \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{CancerTrue} \\
& \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \\
& \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerTrue}) \wedge \\
& (\lambda_{CancerTrue} \vee \neg \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \lambda_{DyspnoeaTrue} \vee \\
& \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge \\
& (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\
& \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \theta_{DyspnoeaFalse|CancerFalse})
\end{aligned}$$

Weights:

$$\begin{aligned}
W(\lambda_{PollutionLow}) &= 1.00 \\
W(\neg \lambda_{PollutionLow}) &= 1.00 \\
W(\lambda_{PollutionHigh}) &= 1.00 \\
W(\neg \lambda_{PollutionHigh}) &= 1.00 \\
W(\lambda_{SmokerTrue}) &= 1.00 \\
W(\neg \lambda_{SmokerTrue}) &= 1.00 \\
W(\lambda_{SmokerFalse}) &= 1.00 \\
W(\neg \lambda_{SmokerFalse}) &= 1.00 \\
W(\lambda_{CancerTrue}) &= 1.00 \\
W(\neg \lambda_{CancerTrue}) &= 1.00 \\
W(\lambda_{CancerFalse}) &= 1.00 \\
W(\neg \lambda_{CancerFalse}) &= 1.00 \\
W(\lambda_{XrayPositive}) &= 1.00 \\
W(\neg \lambda_{XrayPositive}) &= 1.00 \\
W(\lambda_{XrayNegative}) &= 1.00 \\
W(\neg \lambda_{XrayNegative}) &= 1.00 \\
W(\lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\lambda_{DyspnoeaFalse}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaFalse}) &= 1.00
\end{aligned}$$

$$\begin{aligned}
W(\theta_{PollutionLow}) &= 0.90 \\
W(\neg\theta_{PollutionLow}) &= 1.00 \\
W(\theta_{PollutionHigh}) &= 0.10 \\
W(\neg\theta_{PollutionHigh}) &= 1.00 \\
W(\theta_{SmokerTrue}) &= 0.30 \\
W(\neg\theta_{SmokerTrue}) &= 1.00 \\
W(\theta_{SmokerFalse}) &= 0.70 \\
W(\neg\theta_{SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 0.03 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 0.97 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 0.00 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 0.05 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 0.95 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 0.02 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 0.98 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{XrayPositive|CancerTrue}) &= 0.90 \\
W(\neg\theta_{XrayPositive|CancerTrue}) &= 1.00 \\
W(\theta_{XrayNegative|CancerTrue}) &= 0.10 \\
W(\neg\theta_{XrayNegative|CancerTrue}) &= 1.00 \\
W(\theta_{XrayPositive|CancerFalse}) &= 0.20 \\
W(\neg\theta_{XrayPositive|CancerFalse}) &= 1.00 \\
W(\theta_{XrayNegative|CancerFalse}) &= 0.80 \\
W(\neg\theta_{XrayNegative|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerTrue}) &= 0.65 \\
W(\neg\theta_{DyspnoeaTrue|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerTrue}) &= 0.35 \\
W(\neg\theta_{DyspnoeaFalse|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerFalse}) &= 0.30 \\
W(\neg\theta_{DyspnoeaTrue|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerFalse}) &= 0.70 \\
W(\neg\theta_{DyspnoeaFalse|CancerFalse}) &= 1.00
\end{aligned}$$

4.2 ENC2

Indicator clauses

$$\begin{aligned}
&(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
&\lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg
\end{aligned}$$

$$\begin{aligned} & \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse} \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\ & \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

Parameter clauses

$$\begin{aligned} & (\neg \rho_{PollutionLow} \vee \lambda_{PollutionLow}) \wedge (\rho_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\ & \rho_{SmokerTrue} \vee \lambda_{SmokerTrue}) \wedge (\rho_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \\ & \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \\ & \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \lambda_{CancerTrue}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \\ & \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\ & \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \\ & \lambda_{SmokerTrue} \vee \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\ & \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

Weights

$$\begin{aligned} W(\lambda_{PollutionLow}) &= 1.00 \\ W(\neg \lambda_{PollutionLow}) &= 1.00 \\ W(\lambda_{PollutionHigh}) &= 1.00 \\ W(\neg \lambda_{PollutionHigh}) &= 1.00 \\ W(\lambda_{SmokerTrue}) &= 1.00 \\ W(\neg \lambda_{SmokerTrue}) &= 1.00 \\ W(\lambda_{SmokerFalse}) &= 1.00 \\ W(\neg \lambda_{SmokerFalse}) &= 1.00 \\ W(\lambda_{CancerTrue}) &= 1.00 \\ W(\neg \lambda_{CancerTrue}) &= 1.00 \\ W(\lambda_{CancerFalse}) &= 1.00 \\ W(\neg \lambda_{CancerFalse}) &= 1.00 \\ W(\lambda_{XrayPositive}) &= 1.00 \\ W(\neg \lambda_{XrayPositive}) &= 1.00 \\ W(\lambda_{XrayNegative}) &= 1.00 \\ W(\neg \lambda_{XrayNegative}) &= 1.00 \\ W(\lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\rho_{PollutionLow}) &= 0.90 \\ W(\neg \rho_{PollutionLow}) &= 0.10 \end{aligned}$$

$$\begin{aligned}
W(\rho_{SmokerTrue}) &= 0.30 \\
W(\neg \rho_{SmokerTrue}) &= 0.70 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.03 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.97 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 0.00 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 1.00 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.05 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.95 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.02 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.98 \\
W(\rho_{XrayPositive}|CancerTrue) &= 0.90 \\
W(\neg \rho_{XrayPositive}|CancerTrue) &= 0.10 \\
W(\rho_{XrayPositive}|CancerFalse) &= 0.20 \\
W(\neg \rho_{XrayPositive}|CancerFalse) &= 0.80 \\
W(\rho_{DyspnoeaTrue}|CancerTrue) &= 0.65 \\
W(\neg \rho_{DyspnoeaTrue}|CancerTrue) &= 0.35 \\
W(\rho_{DyspnoeaTrue}|CancerFalse) &= 0.30 \\
W(\neg \rho_{DyspnoeaTrue}|CancerFalse) &= 0.70
\end{aligned}$$

Bibliography

- [1] Umut Oztok and Adnan Darwiche. A top-down compiler for sentential decision diagrams. *In Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence (IJCAI)*, pages 3141–3148, 2015.
- [2] Paul Beame Tian Sang and Henry Kautz. Heuristics for fast exact model counting. *Eighth International Conference on Theory and Applications of Satisfiability Testing*, 2005.