

Capita Selecta AI - Probabilistic Programming Inference for SRL

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Probabilistic Inference Using Weighted Model Counting

1.1 PGM to CNF

1.1.1 ENC 1

Our ENC1 encoding for the Cancer Bayesian network can be found in appendix 4.1.

1.1.2 ENC 2

Our ENC2 encoding for the Cancer Bayesian network can be found in appendix 4.2.

1.2 SRL to CNF

1.2.1 Encoding of Monty Hall as CNF

An encoding of problog programs can be generated by our program as follows:

```
python3 scripts/inference.py --problog files/problog/monty_hall.pl
```

The CNF will be shown using the program's predicates. A version of the CNF in dimacs format will be shown as well. See `README.MD` for more information.

Our CNF encoding for the given Monty Hall ProbLog program is:

$$\begin{aligned}
&\wedge(\text{open_door}(2) \vee \text{prize}(2) \vee \text{prize}(3) \vee \neg \text{p_open_door}(2)_0) \\
&\wedge(\text{open_door}(2) \vee \text{prize}(2) \vee \neg \text{prize}(3)) \\
&\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(2) \vee \neg \text{prize}(2)) \\
&\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(2) \vee \text{prize}(3)) \\
&\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(3) \vee \neg \text{prize}(2)) \\
&\wedge(\neg \text{open_door}(2) \vee \neg \text{prize}(3) \vee \text{prize}(3)) \\
&\wedge(\neg \text{open_door}(2) \vee \text{p_open_door}(2)_0 \vee \neg \text{prize}(2)) \\
&\wedge(\neg \text{open_door}(2) \vee \text{p_open_door}(2)_0 \vee \text{prize}(3)) \\
&\wedge(\text{open_door}(3) \vee \text{prize}(2) \vee \text{prize}(3) \vee \neg \text{p_open_door}(3)_0) \\
&\wedge(\text{open_door}(3) \vee \text{prize}(3) \vee \neg \text{prize}(2)) \\
&\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(2) \vee \neg \text{prize}(3)) \\
&\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(2) \vee \text{prize}(2)) \\
&\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(3) \vee \neg \text{prize}(3)) \\
&\wedge(\neg \text{open_door}(3) \vee \neg \text{prize}(3) \vee \text{prize}(2)) \\
&\wedge(\neg \text{open_door}(3) \vee \text{p_open_door}(3)_0 \vee \neg \text{prize}(3)) \\
&\wedge(\neg \text{open_door}(3) \vee \text{p_open_door}(3)_0 \vee \text{prize}(2)) \\
&\wedge(\text{win_keep} \vee \neg \text{prize}(1)) \\
&\wedge(\neg \text{win_keep} \vee \text{prize}(1)) \\
&\wedge(\text{win_switch} \vee \neg \text{prize}(2) \vee \text{open_door}(2)) \\
&\wedge(\text{win_switch} \vee \neg \text{prize}(3) \vee \text{open_door}(3)) \\
&\wedge(\neg \text{win_switch} \vee \text{prize}(2) \vee \text{prize}(3)) \\
&\wedge(\neg \text{win_switch} \vee \text{prize}(2) \vee \neg \text{open_door}(3)) \\
&\wedge(\neg \text{win_switch} \vee \neg \text{open_door}(2) \vee \text{prize}(3)) \\
&\wedge(\neg \text{win_switch} \vee \neg \text{open_door}(2) \vee \neg \text{open_door}(3)) \\
&\wedge(\neg \text{prize}(1) \vee \neg \text{prize}(2)) \\
&\wedge(\neg \text{prize}(1) \vee \neg \text{prize}(3)) \\
&\wedge(\neg \text{prize}(2) \vee \neg \text{prize}(3)) \\
&\wedge(\text{prize}(1) \vee \text{prize}(2) \vee \text{prize}(3))
\end{aligned}$$

Weights:

$$\begin{aligned}
W(\text{p_open_door}(2)_0) &= 0.5 \\
W(\text{p_open_door}(3)_0) &= 0.5 \\
W(\text{select_door}(1)) &= 1.00 \\
W(\text{prize}(1)) &= 0.33 \\
W(\text{prize}(2)) &= 0.33 \\
W(\text{prize}(3)) &= 0.33 \\
W(\text{open_door}(2)) &= 1.00 \\
W(\text{open_door}(3)) &= 1.00 \\
W(\text{win_keep}) &= 1.00 \\
W(\text{win_switch}) &= 1.00
\end{aligned}$$

1.3 Weighted Model Counting

1.3.1 Weighted model counters on above CNFs

We have selected MiniC2D and Cachet as weighted model counters. MiniC2D needs to be executed with the $-W$ flag in order for it to do weighted model counting. The resulting probability can be read next to “Count”.

The output of the model counters can be found in the following listings.

MiniC2D

Listing 1.1: MiniC2D on ENC1

```
Constructing CNF\ldots DONE
CNF stats:
  Vars=30 / Clauses=74
  CNF Time      0.000s
Constructing vtree (from primal graph)\ldots DONE
Vtree stats:
  Vtree widths: con<=5, c_con=48 v_con=5
  Vtree Time    0.001s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      75.0%
  lookups       16
  ent count     4
  ent memory    0.2 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          3.0b ave, 3.0b max, 3.0b min
Count stats:
  Count Time     0.000s
  Count          0.9999999999999999
Total Time: 0.012s
```

Listing 1.2: MiniC2D on ENC2

```
Constructing CNF... DONE
CNF stats:
  Vars=20 / Clauses=30
  CNF Time      0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=6, c_con=16 v_con=6
  Vtree Time    0.000s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      23.1%
  lookups       26
```

```

ent count      20
ent memory     1.0 KB
ht memory      152.6 MB
clists         1.0 ave, 1 max
keys          1.8b ave, 3.0b max, 1.0b min
Count stats:
  Count Time   0.000s
  Count        1.0000000000000000
Total Time: 0.012s

```

- ENC1:

```

Constructing CNF... DONE
CNF stats:
  Vars=30 / Clauses=74
  CNF Time   0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=5, c_con=40 v_con=5
  Vtree Time   0.003s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      16.7%
  lookups       12
  ent count     10
  ent memory    0.5 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          4.2b ave, 6.0b max, 3.0b min
Count stats:
  Count Time   0.000s
  Count/Probability  1.00000
Total Time: 0.128s

```

Figure 1.1: Grounded problog cnf

- ENC2:

```

Constructing CNF... DONE
CNF stats:
  Vars=20 / Clauses=30
  CNF Time   0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=6, c_con=16 v_con=6
  Vtree Time   0.002s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      23.1%
  lookups       26
  ent count     20
  ent memory    1.0 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          1.8b ave, 3.0b max, 1.0b min
Count stats:
  Count Time   0.000s
  Count/Probability  1.00000
Total Time: 0.164s

```

Figure 1.2: Grounded problog cnf

- Monty hall:

```

Constructing CNF... DONE
CNF stats:
  Vars=10 / Clauses=26
  CNF Time    0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=4, c_con=22 v_con=4
  Vtree Time   0.002s
Counting... DONE
  Learned clauses    0
Cache stats:
  hit rate:    20.0%
  lookups     5
  ent count   4
  ent memory   0.2 KB
  ht_memory   152.6 MB
  clists      1.0 ave, 1 max
  keys        3.2b ave, 4.0b max, 3.0b min
Count stats:
  Count Time   0.000s
  Count        1.00000000
Total Time: 0.116s

```

Figure 1.3: Grounded problog cnf

-

Cachet

For Cachet, no need to use extra parameters to get the probability. It is reported next to “Satisfying probability”.

- ENC1:

```

Number of total components      11
Number of split components      2
Number of non-split components  5
Number of SAT residual formula  12
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0
First component split level     1

Number of Decisions             11
Max Decision Level              5
Number of Variables             30
Original Num Clauses            74
Original Num Literals           172
Added Conflict Clauses          0
Added Conflict Literals         0
Deleted Unrelevant clauses      0
Deleted Unrelevant literals     0
Number of Implications          124
Total Run Time                  0.018895

Satisfying probability          8.72319e-08
Number of solutions             93.6645

```

Figure 1.4: Grounded problog cnf

- ENC2:

```

Number of total components      11
Number of split components      2
Number of non-split components  5
Number of SAT residual formula 12
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0
First component split level    1

Number of Decisions            11
Max Decision Level             5
Number of Variables            20
Original Num Clauses           30
Original Num Literals           84
Added Conflict Clauses         0
Added Conflict Literals         0
Deleted Unrelevant clauses     0
Deleted Unrelevant literals     0
Number of Implications         72
Total Run Time                  0.017282

Satisfying probability         1
Number of solutions             1.04858e+06

```

Figure 1.5: Grounded problog cnf

- Monty Hall:

```

Number of total components      4
Number of split components      1
Number of non-split components  2
Number of SAT residual formula  5
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0
First component split level    2

Number of Decisions            4
Max Decision Level             4
Number of Variables            10
Original Num Clauses           26
Original Num Literals           73
Added Conflict Clauses         0
Added Conflict Literals         0
Deleted Unrelevant clauses     0
Deleted Unrelevant literals     0
Number of Implications         26
Total Run Time                  0.016062

Satisfying probability         0.444444
Number of solutions             455.111

```

Figure 1.6: Grounded problog cnf

For ENC1 we see that with Cachet we get a satisfying probability of almost 0. With Monty hall we can also see that we get a probability of 0.4. This is due to the fact that with ENC1 all our negative literals have a weight of 1, while Cachet expects that a literal + its negation = 1. For Monty hall we also have negative literals with weight 1 which gives the same problem as with ENC1.

1.3.2 Difference between the selected WMCs

MiniC2D Vs Cachet

MiniC2D and Cachet are both weighted model counters but how they do this is quite different. MiniC2D is a top down compiler that compiles CNFs into a SDD which results in a faster system but it also uses less space while Cachet is an algorithm that uses formula caching together with clause learning and component analysis. MiniC2D needs vtrees to be able to compile the CNFs into an SDD. They, however, both use things from the SAT literature. They both use clause learning and component caching as to be able to reuse components

that later appear again in the search. Cachet on the other hand also uses some other things from SAT literature like an explicit on the fly calculation of connected components. This is different in MiniC2D as it uses a vtree to identify disconnected CNF components. [?] [?]

1.3.3 Overview of computational requirements

All the tests can be found in the test folder. We used our scripts to create the dimac files. The input files for our enc1 and enc2 converter are “.dsc” files which can be found at <http://www.bnlearn.com/bnrepository/discrete-small.html#cancer>.

Test 1: Cancer network

Table 1.1: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	0.2 KB	0.155s	1.0	1.0 KB	0.000s
Cachet	val1	val2	a	b	val3	val4

Test 2: asia network

Table 1.2: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	0.9 KB	0.145s	1.0	2.0 KB	0.139s
Cachet	val1	val2	a	b	val3	val4

Test 3: sachs network

Table 1.3: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	0.99707	14.3 KB	0.184s	1.0	14.5 KB	0.154s
Cachet	val1	val2	a	b	val3	val4

Test 4: earthquake network

Table 1.4: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	0.6 KB	0.137s	1.0	1.0 KB	0.153s
Cachet	val1	val2	a	b	val3	val4

Test 5: survey network

Table 1.5: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	1.6 KB	0.125s	1.0	2.0 KB	0.125s
Cachet	val1	val2	a	b	val3	val4

Test 6: alarm network

Table 1.6: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	959.7KB KB	0.268s	1.0	139KB	0.095s
Cachet	val1	val2	a	b	val3	val4

Test 6: andes network

Table 1.7: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	2.7GB	122.78s	1.0	139.8MB	6.086s
Cachet	val1	val2	a	b	val3	val4

1.4 Knowledge compilation

Vtree with the most compact circuit

During our tests

Pattern for a good vtree

As a vtree is a binary tree, which means that a good vtree is compact. We want thus a vtree that is shallow.

Build an Inference Engine

2.0.1 Implementation

We have implemented the pipeline using python. Information about it can be found in `README.MD`.

2.0.2 Pipeline with previous tasks

Cancer Bayesian network

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

Monty Hall

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

2.0.3 Pipeline on Bayesian learning example

DAS NEN DIKKE VETTE TODO

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

2.0.4 Pipeline on alarm Bayesian network

GOD DAMN IT STOM VAK HOE MOETEN WIJ DIT IN 50 UUR DOEN?
PROCESS WORDT GEWOON GEKILLED OMDAT DIE CNF GIGANTISCH
GROOT WORDT.

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

Parameter Learning

learning

Appendix

4.1 ENC1

Indicator clauses:

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse}) \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\
 &\quad \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\
 &\quad \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse})
 \end{aligned}$$

Parameter clauses:

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \theta_{PollutionLow}) \wedge (\lambda_{PollutionLow} \vee \neg \theta_{PollutionLow}) \wedge (\neg \\
 &\lambda_{PollutionHigh} \vee \theta_{PollutionHigh}) \wedge (\lambda_{PollutionHigh} \vee \neg \theta_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \theta_{SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \theta_{SmokerTrue}) \wedge (\neg \\
 &\quad \lambda_{SmokerFalse} \vee \theta_{SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \theta_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \lambda_{CancerTrue} \vee \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg
 \end{aligned}$$

$$\begin{aligned}
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{XrayPositive} \vee \\
& \theta_{XrayPositive|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge \\
& (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\
& \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerTrue}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayPositive} \vee \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerFalse}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \\
& \neg \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerFalse}) \wedge \\
& (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaTrue} \vee \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{CancerTrue} \\
& \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \\
& \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerTrue}) \wedge \\
& (\lambda_{CancerTrue} \vee \neg \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \lambda_{DyspnoeaTrue} \vee \\
& \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge \\
& (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\
& \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \theta_{DyspnoeaFalse|CancerFalse})
\end{aligned}$$

Weights:

$$\begin{aligned}
W(\lambda_{PollutionLow}) &= 1.00 \\
W(\neg \lambda_{PollutionLow}) &= 1.00 \\
W(\lambda_{PollutionHigh}) &= 1.00 \\
W(\neg \lambda_{PollutionHigh}) &= 1.00 \\
W(\lambda_{SmokerTrue}) &= 1.00 \\
W(\neg \lambda_{SmokerTrue}) &= 1.00 \\
W(\lambda_{SmokerFalse}) &= 1.00 \\
W(\neg \lambda_{SmokerFalse}) &= 1.00 \\
W(\lambda_{CancerTrue}) &= 1.00 \\
W(\neg \lambda_{CancerTrue}) &= 1.00 \\
W(\lambda_{CancerFalse}) &= 1.00 \\
W(\neg \lambda_{CancerFalse}) &= 1.00 \\
W(\lambda_{XrayPositive}) &= 1.00 \\
W(\neg \lambda_{XrayPositive}) &= 1.00 \\
W(\lambda_{XrayNegative}) &= 1.00 \\
W(\neg \lambda_{XrayNegative}) &= 1.00 \\
W(\lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\lambda_{DyspnoeaFalse}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaFalse}) &= 1.00
\end{aligned}$$

$$\begin{aligned}
W(\theta_{PollutionLow}) &= 0.90 \\
W(\neg\theta_{PollutionLow}) &= 1.00 \\
W(\theta_{PollutionHigh}) &= 0.10 \\
W(\neg\theta_{PollutionHigh}) &= 1.00 \\
W(\theta_{SmokerTrue}) &= 0.30 \\
W(\neg\theta_{SmokerTrue}) &= 1.00 \\
W(\theta_{SmokerFalse}) &= 0.70 \\
W(\neg\theta_{SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 0.03 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 0.97 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 0.00 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 0.05 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 0.95 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 0.02 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 0.98 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{XrayPositive|CancerTrue}) &= 0.90 \\
W(\neg\theta_{XrayPositive|CancerTrue}) &= 1.00 \\
W(\theta_{XrayNegative|CancerTrue}) &= 0.10 \\
W(\neg\theta_{XrayNegative|CancerTrue}) &= 1.00 \\
W(\theta_{XrayPositive|CancerFalse}) &= 0.20 \\
W(\neg\theta_{XrayPositive|CancerFalse}) &= 1.00 \\
W(\theta_{XrayNegative|CancerFalse}) &= 0.80 \\
W(\neg\theta_{XrayNegative|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerTrue}) &= 0.65 \\
W(\neg\theta_{DyspnoeaTrue|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerTrue}) &= 0.35 \\
W(\neg\theta_{DyspnoeaFalse|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerFalse}) &= 0.30 \\
W(\neg\theta_{DyspnoeaTrue|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerFalse}) &= 0.70 \\
W(\neg\theta_{DyspnoeaFalse|CancerFalse}) &= 1.00
\end{aligned}$$

4.2 ENC2

Indicator clauses

$$\begin{aligned}
&(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
&\lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg
\end{aligned}$$

$$\begin{aligned} & \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse} \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\ & \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

Parameter clauses

$$\begin{aligned} & (\neg \rho_{PollutionLow} \vee \lambda_{PollutionLow}) \wedge (\rho_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\ & \rho_{SmokerTrue} \vee \lambda_{SmokerTrue}) \wedge (\rho_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \\ & \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \\ & \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \lambda_{CancerTrue}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \\ & \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\ & \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \\ & \lambda_{SmokerTrue} \vee \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\ & \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

Weights

$$\begin{aligned} W(\lambda_{PollutionLow}) &= 1.00 \\ W(\neg \lambda_{PollutionLow}) &= 1.00 \\ W(\lambda_{PollutionHigh}) &= 1.00 \\ W(\neg \lambda_{PollutionHigh}) &= 1.00 \\ W(\lambda_{SmokerTrue}) &= 1.00 \\ W(\neg \lambda_{SmokerTrue}) &= 1.00 \\ W(\lambda_{SmokerFalse}) &= 1.00 \\ W(\neg \lambda_{SmokerFalse}) &= 1.00 \\ W(\lambda_{CancerTrue}) &= 1.00 \\ W(\neg \lambda_{CancerTrue}) &= 1.00 \\ W(\lambda_{CancerFalse}) &= 1.00 \\ W(\neg \lambda_{CancerFalse}) &= 1.00 \\ W(\lambda_{XrayPositive}) &= 1.00 \\ W(\neg \lambda_{XrayPositive}) &= 1.00 \\ W(\lambda_{XrayNegative}) &= 1.00 \\ W(\neg \lambda_{XrayNegative}) &= 1.00 \\ W(\lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\rho_{PollutionLow}) &= 0.90 \\ W(\neg \rho_{PollutionLow}) &= 0.10 \end{aligned}$$

$$\begin{aligned}
W(\rho_{SmokerTrue}) &= 0.30 \\
W(\neg \rho_{SmokerTrue}) &= 0.70 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.03 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.97 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 0.00 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 1.00 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.05 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.95 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.02 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.98 \\
W(\rho_{XrayPositive}|CancerTrue) &= 0.90 \\
W(\neg \rho_{XrayPositive}|CancerTrue) &= 0.10 \\
W(\rho_{XrayPositive}|CancerFalse) &= 0.20 \\
W(\neg \rho_{XrayPositive}|CancerFalse) &= 0.80 \\
W(\rho_{DyspnoeaTrue}|CancerTrue) &= 0.65 \\
W(\neg \rho_{DyspnoeaTrue}|CancerTrue) &= 0.35 \\
W(\rho_{DyspnoeaTrue}|CancerFalse) &= 0.30 \\
W(\neg \rho_{DyspnoeaTrue}|CancerFalse) &= 0.70
\end{aligned}$$