

# Capita Selecta AI - Probabilistic Programming Inference for SRL

Thierry Deruyttere (r0660485) & Armin Halilovic (r0679689)

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# Probabilistic Inference Using Weighted Model Counting

## 1.1 PGM to CNF

### 1.1.1 ENC 1

Our ENC1 encoding for the Cancer Bayesian network can be found in appendix 4.1. The CNF in dimacs format can be found under `report/encodings/cancer/`.

### 1.1.2 ENC 2

Our ENC2 encoding for the Cancer Bayesian network can be found in appendix 4.2. The CNF in dimacs format can be found under `report/encodings/cancer/`.

## 1.2 SRL to CNF

### 1.2.1 Encoding of Monty Hall as CNF

An encoding of problog programs can be generated by our program as follows:

```
python3 scripts/inference.py --problog files/problog/
monty_hall.pl
```

The CNF will be shown using the program's predicates. A version of the CNF in dimacs format will be shown as well. See `README.MD` for more information.

Our CNF encoding for the given Monty Hall ProbLog program is:

$\wedge(\text{open\_door}(2) \vee \text{prize}(2) \vee \text{prize}(3) \vee \neg \text{p\_open\_door}(2)\_0)$   
 $\wedge(\text{open\_door}(2) \vee \text{prize}(2) \vee \neg \text{prize}(3))$   
 $\wedge(\neg \text{open\_door}(2) \vee \neg \text{prize}(2) \vee \neg \text{prize}(2))$   
 $\wedge(\neg \text{open\_door}(2) \vee \neg \text{prize}(2) \vee \text{prize}(3))$   
 $\wedge(\neg \text{open\_door}(2) \vee \neg \text{prize}(3) \vee \neg \text{prize}(2))$   
 $\wedge(\neg \text{open\_door}(2) \vee \neg \text{prize}(3) \vee \text{prize}(3))$   
 $\wedge(\neg \text{open\_door}(2) \vee \text{p\_open\_door}(2)\_0 \vee \neg \text{prize}(2))$   
 $\wedge(\neg \text{open\_door}(2) \vee \text{p\_open\_door}(2)\_0 \vee \text{prize}(3))$   
 $\wedge(\text{open\_door}(3) \vee \text{prize}(2) \vee \text{prize}(3) \vee \neg \text{p\_open\_door}(3)\_0)$   
 $\wedge(\text{open\_door}(3) \vee \text{prize}(3) \vee \neg \text{prize}(2))$   
 $\wedge(\neg \text{open\_door}(3) \vee \neg \text{prize}(2) \vee \neg \text{prize}(3))$   
 $\wedge(\neg \text{open\_door}(3) \vee \neg \text{prize}(2) \vee \text{prize}(2))$   
 $\wedge(\neg \text{open\_door}(3) \vee \neg \text{prize}(3) \vee \neg \text{prize}(3))$   
 $\wedge(\neg \text{open\_door}(3) \vee \neg \text{prize}(3) \vee \text{prize}(2))$   
 $\wedge(\neg \text{open\_door}(3) \vee \text{p\_open\_door}(3)\_0 \vee \neg \text{prize}(3))$   
 $\wedge(\neg \text{open\_door}(3) \vee \text{p\_open\_door}(3)\_0 \vee \text{prize}(2))$   
 $\wedge(\text{win\_keep} \vee \neg \text{prize}(1))$   
 $\wedge(\neg \text{win\_keep} \vee \text{prize}(1))$   
 $\wedge(\text{win\_switch} \vee \neg \text{prize}(2) \vee \text{open\_door}(2))$   
 $\wedge(\text{win\_switch} \vee \neg \text{prize}(3) \vee \text{open\_door}(3))$   
 $\wedge(\neg \text{win\_switch} \vee \text{prize}(2) \vee \text{prize}(3))$   
 $\wedge(\neg \text{win\_switch} \vee \text{prize}(2) \vee \neg \text{open\_door}(3))$   
 $\wedge(\neg \text{win\_switch} \vee \neg \text{open\_door}(2) \vee \text{prize}(3))$   
 $\wedge(\neg \text{win\_switch} \vee \neg \text{open\_door}(2) \vee \neg \text{open\_door}(3))$   
 $\wedge(\neg \text{prize}(1) \vee \neg \text{prize}(2))$   
 $\wedge(\neg \text{prize}(1) \vee \neg \text{prize}(3))$   
 $\wedge(\neg \text{prize}(2) \vee \neg \text{prize}(3))$   
 $\wedge(\text{prize}(1) \vee \text{prize}(2) \vee \text{prize}(3))$

**Weights:**

$W(\text{p\_open\_door}(2)\_0) = 0.5$   
 $W(\text{p\_open\_door}(3)\_0) = 0.5$   
 $W(\text{select\_door}(1)) = 1.00$   
 $W(\text{prize}(1)) = 0.33$   
 $W(\text{prize}(2)) = 0.33$   
 $W(\text{prize}(3)) = 0.33$   
 $W(\text{open\_door}(2)) = 1.00$   
 $W(\text{open\_door}(3)) = 1.00$   
 $W(\text{win\_keep}) = 1.00$   
 $W(\text{win\_switch}) = 1.00$

$W(\neg \text{p\_open\_door}(2)\_0) = 0.5$   
 $W(\neg \text{p\_open\_door}(3)\_0) = 0.5$   
 $W(\neg \text{select\_door}(1)) = 0.00$   
 $W(\neg \text{prize}(1)) = 1.00$   
 $W(\neg \text{prize}(2)) = 1.00$   
 $W(\neg \text{prize}(3)) = 1.00$   
 $W(\neg \text{open\_door}(2)) = 1.00$   
 $W(\neg \text{open\_door}(3)) = 1.00$   
 $W(\neg \text{win\_keep}) = 1.00$   
 $W(\neg \text{win\_switch}) = 1.00$

## 1.3 Weighted Model Counting

### 1.3.1 Weighted model counters on above CNFs

We have selected MiniC2D and Cachet as weighted model counters and have executed them on DIMACS versions of the CNFs of the previous tasks. The DIMACS files can be found under report/encodings. The output of the model counters is listed below.

#### MiniC2D

MiniC2D needs to be executed with the  $-W$  flag in order for it to do weighted model counting. The resulting probability can be read next to “Count”.

Listing 1.1: MiniC2D on ENC1 encoding of Cancer network

```
Constructing CNF... DONE
CNF stats:
  Vars=30 / Clauses=74
  CNF Time      0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=5, c_con=48 v_con=5
  Vtree Time    0.001s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      75.0%
  lookups       16
  ent count      4
  ent memory    0.2 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          3.0b ave, 3.0b max, 3.0b min
Count stats:
  Count Time     0.000s
  Count         0.9999999999999999
Total Time: 0.012s
```

Listing 1.2: MiniC2D on ENC2 encoding of Cancer network

```
Constructing CNF... DONE
CNF stats:
  Vars=20 / Clauses=30
  CNF Time      0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=6, c_con=16 v_con=6
  Vtree Time    0.000s
Counting... DONE
  Learned clauses      0
```

```

Cache stats:
  hit rate      23.1%
  lookups       26
  ent count     20
  ent memory    1.0 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          1.8b ave, 3.0b max, 1.0b min
Count stats:
  Count Time    0.000s
  Count         1.0000000000000000
Total Time: 0.012s

```

Listing 1.3: MiniC2D on CNF encoding of Monty Hall

```

Constructing CNF... DONE
CNF stats:
  Vars=10 / Clauses=26
  CNF Time      0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=4, c_con=22 v_con=4
  Vtree Time    0.000s
Counting... DONE
  Learned clauses      0
Cache stats:
  hit rate      20.0%
  lookups       5
  ent count     4
  ent memory    0.2 KB
  ht memory     152.6 MB
  clists        1.0 ave, 1 max
  keys          3.2b ave, 4.0b max, 3.0b min
Count stats:
  Count Time    0.000s
  Count         1.0000000000000000
Total Time: 0.011s

```

## Cachet

For Cachet, there is no need to use extra parameters to get a probability. It is reported next to “Satisfying probability”.

Listing 1.4: Cachet on ENC1 encoding of Cancer network

Number of total components	11
Number of split components	2
Number of non-split components	5
Number of SAT residual formula	12
Number of trivial components	0

Number of changed components	0
Number of adjusted components	0
First component split level	1
Number of Decisions	11
Max Decision Level	5
Number of Variables	30
Original Num Clauses	74
Original Num Literals	172
Added Conflict Clauses	0
Added Conflict Literals	0
Deleted Unrelevant clauses	0
Deleted Unrelevant literals	0
Number of Implications	124
Total Run Time	0.0163
Satisfying probability	8.72319e−08
Number of solutions	93.6645

Listing 1.5: Cachet on ENC2 encoding of Cancer network

Number of total components	11
Number of split components	2
Number of non-split components	5
Number of SAT residual formula	12
Number of trivial components	0
Number of changed components	0
Number of adjusted components	0
First component split level	1
Number of Decisions	11
Max Decision Level	5
Number of Variables	20
Original Num Clauses	30
Original Num Literals	84
Added Conflict Clauses	0
Added Conflict Literals	0
Deleted Unrelevant clauses	0
Deleted Unrelevant literals	0
Number of Implications	72
Total Run Time	0.017372
Satisfying probability	1
Number of solutions	1.04858e+06

Listing 1.6: Cachet on WCNF encoding of Monty Hall

Number of total components	4
Number of split components	1
Number of non-split components	2

Number of SAT residual formula	5
Number of trivial components	0
Number of changed components	0
Number of adjusted components	0
First component split level	2
Number of Decisions	4
Max Decision Level	4
Number of Variables	10
Original Num Clauses	26
Original Num Literals	73
Added Conflict Clauses	0
Added Conflict Literals	0
Deleted Unrelevant clauses	0
Deleted Unrelevant literals	0
Number of Implications	26
Total Run Time	0.016062
Satisfying probability	0.444444
Number of solutions	455.111

For ENC1, we see that with Cachet reports a satisfying probability of almost 0. Similarly, for Monty Hall, we see that we get a probability of 0.44. This is due to the fact that with ENC1, the weights of negated literals are 1, but Cachet expects that  $weight(x) + weight(-x) = 1$ . In the Monty Hall encoding, we also have weights of negated literals equalling 1, which gives the same problem as with ENC1.

### 1.3.2 Difference between the selected WMCs

#### MiniC2D Vs Cachet

MiniC2D and Cachet are weighted model counters that work in different ways. In short, MiniC2D is a top down compiler that compiles CNFs into SDDs, while Cachet uses formula caching combined with clause learning and component analysis [1], [2].

Both weighted model counters use concepts from the SAT literature. They both use clause learning and component caching in order to reuse components that later appear again during search.

Cachet also uses other methods from SAT literature, like an explicit on the fly calculation of connected components. This is different in MiniC2D, as it relies on vtrees to identify disconnected CNF components. MiniC2d creates vtrees for CNFs and then creates SDDs based on the created vtrees.

### 1.3.3 Overview of computational requirements

We have executed the model counters with various CNFs to build an overview of computational requirements. The files we used for testing can be found under report/encodings. We have used scripts to convert the “.dsc” files to ENC1 and ENC2 encodings in DIMACS format. We downloaded the “.dsc” files from <http://www.bnlearn.com/bnrepository/>.

**Cancer network (small)**

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	0.3 KB	0.053s	1.0	1.0 KB	0.050s
<b>Cachet</b>	0.0	?	0.016s	1.0	?	0.016s

**Asia network (small)**

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	1.2 KB	0.049s	1.0	1.9 KB	0.05s
<b>Cachet</b>	0.0	?	0.018s	1.0	?	0.017s

**Sachs network (small)**

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	0.99707	16.8 KB	0.075s	1.0	13.4 KB	0.07s
<b>Cachet</b>	0.0	?	0.019s	1.0	?	0.017s

**Earthquake network (small)**

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	0.6 KB	0.051s	1.0	1.0 KB	0.05s
<b>Cachet</b>	0.0	?	0.016s	1.0	?	0.017s

**Survey network (small)**

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	0.5 KB	0.035s	1.0	2.0 KB	0.052s
<b>Cachet</b>	0.0	?	0.016s	1.0	?	0.016s

**Alarm network (medium)**

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	0.999	451.1 KB	0.217s	0.999	143.4 KB	0.093s
<b>Cachet</b>	0.0	?	0.176s	1.0	?	0.222s



### Child network (medium)

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	45.8KB	0.076s	1.0	30.8 KB	0.059s
<b>Cachet</b>	0.0	?	0.03s	1.0	?	0.03s

### Hailfinder network (large)

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	0.999	591.8 MB	46.065s	1.0	25.1MB	2.73s
<b>Cachet</b>	0	?	58.86s	1	?	15.21s

### Andes network (very large)

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	5.5GB	266.605s	1.0	122.8 MB	5.646s
<b>Cachet</b>	?	?	> 4h(killed)	?	?	> 4h(killed)

For Cachet, we didn’t find a way to output memory usage, so we cannot compare that aspect to MiniC2D. From the results listed above, we can conclude that ENC2 is a better encoding than ENC1 when it comes to computational requirements. In most of the cases, it has lower memory usage and a lower runtime. Concerning the weighted model counters, we clearly see that Cachet is faster than MiniC2D on small to medium networks. When we use larger networks, Cachet is a lot slower than MiniC2D, and for the very large network, Cachet couldn’t even finish in 4 hours!

## 1.4 Knowledge compilation

### Vtree with the most compact circuit

For this task, we used both the SDD and MiniC2D packages to generate vtrees and SDDs. The resulting vtrees and their corresponding SDDs can be found under report/1.4/. Subdirectories have been made for each WMC’s output for each of the CNFs. The file name in each of these directories follow the following naming convention: (SDD\_vtree\_type | MiniC2D\_vtree\_method)[-SDD\_minimize\_cardinality]\_(vtree | sdd)

- **ENC1:** SDD built the best vtree for the most compact circuit with the “-t balanced -m” flags. The next best circuit can be created via the vtree created by MiniC2D with “-m 3” (reverse elimination order).
- **ENC2:** For this encoding, we have different options that result in the best circuits. The options “-t balanced” for SDD and “-m 3” for MiniC2D

are again one of the best. Like for ENC1, the vtree created by SDD is better than the one from MiniC2D.

- **Monty Hall:** Here we see that by using the option “-t balanced -m”, we receive the most compact circuit and yet again closely followed by “-m 3” from MiniC2D.

#### **Pattern for a good vtree**

From our tests, we’ve noticed that vtrees that are more shallow result in the best circuits. This seems logical to us, since a vtree is a binary search tree, and with such a tree you also prefer it to be shallow.

# Build an Inference Engine

## 2.1 Implementation

We have implemented the pipeline using python. Information about installation and usage can be found in README.MD.

Reporting of statistics other than the resulting probabilities only happens with our pipeline when MiniC2D is selected as model counter (flag `-model.counter minic2d`).

## 2.2 Pipeline with previous tasks

### 2.2.1 Cancer Bayesian network

- Probabilities:

cancer	: 0.01163
dyspnoea	: 0.3040705
pollution("high")	: 0.1
pollution("low")	: 0.9
smoker	: 0.3
xray("negative")	: 0.7918589999999999
xray("positive")	: 0.208141

- Total runtime: 0.594s
- Runtime of the separate parts:

Time to parse ground problog:	0.011s
Time to convert ground problog to first order logic:	0.000s
Time to convert first order logic to CNF:	0.002s
Time for all model counting:	0.58s

- Number of variables and lines in CNF: 17, 77
- Depth of vtree: 11
- Number of edges and nodes in the circuit: 81, 94

### 2.2.2 Monty Hall

- Probabilities:

prize(1)	:	0.3333333333333333
prize(2)	:	0.3333333333333333
prize(3)	:	0.3333333333333333
select_door(1)	:	1.0
win_keep	:	0.3333333333333333
win_switch	:	0.6666666666666666

- Total runtime: 0.469s
- Runtime of the separate parts:

Time to parse ground problog:	0.011s
Time to convert ground problog to first order logic:	0.0s
Time to convert first order logic to CNF:	0.001s
Time for all model counting:	0.458s

- Number of variables and lines in CNF: 10, 26
- Depth of vtree: 8
- Number of edges and nodes in the circuit: 52, 44

## 2.3 Pipeline on Bayesian learning example

- Probabilities:

weight(c1, 0.1)	:	0.0
weight(c1, 0.3)	:	2.167977370376305e-06
weight(c1, 0.5)	:	0.004149743263716653
weight(c1, 0.7)	:	0.13174850211546205
weight(c1, 0.9)	:	0.8640995866434509
weight(c2, 0.1)	:	3.227661043218289e-06
weight(c2, 0.3)	:	0.02410999702877393
weight(c2, 0.5)	:	0.6672475437022279
weight(c2, 0.7)	:	0.3062862583084689
weight(c2, 0.9)	:	0.0023529732994859343

- Total runtime: 3.633s
- Runtime of the separate parts:

Time to parse ground problog:	0.338s
Time to convert ground problog to first order logic:	0.024s
Time to convert first order logic to CNF:	0.088s
Time for all model counting:	3.183s

- Number of variables and lines in CNF: 192, 1062
- Depth of vtree: 95
- Number of edges and nodes in the circuit: 646, 525

## 2.4 Pipeline on alarm Bayesian network

We had to drop many nodes in order for the pipeline to be able to process the ALARM Bayesian network. This is due to how we implemented the conversion from First Order Logic to CNF. We have come across methods with better performance, but were out of time to try other implementations. Only 21 of the 37 nodes are left in the network.

We used the pipeline to convert files/networks/alarm.net to CNF with the following command.

```
python3 scripts/inference.py --bayesian_network files/
networks/alarm.net
```

We kept dropping leaf nodes until a CNF was created within reasonable time (a couple seconds). Then, we used the ground problog file in the output of the pipeline and stored it in files/problog/alarm.pl. Finally, to this file, we added queries for all the nodes in the network.

The probabilities shown below are only those of the nodes lower in the network. To see all of the probabilities, run:

```
python3 scripts/inference.py --problog files/problog/
alarm.pl
```

- Probabilities:

cVP("HIGH")	: 0.1541757517134
cVP("LOW")	: 0.1129475165156
cVP("NORMAL")	: 0.7160774170523999
IVEDVOLUME("HIGH")	: 0.2095
IVEDVOLUME("LOW")	: 0.0886
IVEDVOLUME("NORMAL")	: 0.7019
pAP("HIGH")	: 0.0575
pAP("LOW")	: 0.0496
pAP("NORMAL")	: 0.8929
pCWP("HIGH")	: 0.2064165610039
pCWP("LOW")	: 0.1129475165156
pCWP("NORMAL")	: 0.6755051316624
sHUNT("HIGH")	: 0.103095
sHUNT("NORMAL")	: 0.8969050000000001
sSTROKEVOLUME("HIGH")	: 0.0404
sSTROKEVOLUME("LOW")	: 0.1808
sSTROKEVOLUME("NORMAL")	: 0.7787999999999999
tPR("HIGH")	: 0.2971
tPR("LOW")	: 0.3068
tPR("NORMAL")	: 0.3961

vENTMACH("HIGH")	:	0.056
vENTMACH("LOW")	:	0.056
vENTMACH("NORMAL")	:	0.838
vENTMACH("ZERO")	:	0.05

- Total runtime: 6.413s
- Runtime of the separate parts:

Time to parse ground problog:	0.037s
Time to convert ground problog to first order logic:	0.003s
Time to convert first order logic to CNF:	0.148s
Time for all model counting:	6.225s

- Number of variables and lines in CNF: 112, 1118
- Depth of vtree: 27
- Number of edges and nodes in the circuit: 974, 711

# Parameter Learning

We have extended the pipeline with support for parameter learning. For this functionality, our program expects a file containing tunable probabilities and another file containing values for all probabilities (the ground truth). The ground truth is necessary for generation of interpretations. The amount of interpretations to be generated can be set as well. More information about this feature can be found in `README.MD`.

## 3.1 Generated interpretations

Four interpretations can be generated with the following command:

```
python3 scripts/inference.py --problog_learn files/
    problog/cancer_learn.pl --problog_learn_truth files/
    problog/cancer.pl --learning_interpretations 4
```

Observations will be dropped with a probability of 30% automatically and the resulting interpretations will be written to `src/files/interpretations.txt`.

Here is an example of generated interpretations with the command listed above:

```
evidence(\+cancer).
evidence(\+xray("positive")).
evidence(\+dyspnoea).
evidence(\+pollution("high")).
evidence(\+smoker).
```

---

```
evidence(smoker).
evidence(dyspnoea).
evidence(\+pollution("high")).
evidence(pollution("low")).
```

---

```
evidence(xray("negative")).
evidence(\+dyspnoea).
evidence(\+pollution("high")).
evidence(\+smoker).
```

---

```
evidence(smoker).
evidence(dyspnoea).
```

```
evidence(\+pollution("high")).  
evidence(pollution("low")).
```

## **3.2 Pipeline with interpretations**

### **3.2.1 Parameters with 10 interpretations**

### **3.2.2 Parameters with 100 interpretations**

### **3.2.3 Parameters with 1000 interpretations**

## **3.3 Observations for different number of interpretations**

We notice that TODO ... Voorspelling: Met minder interpretaties zijn de iteraties sneller (logisch, want minder queries per iteratie) dan met meer interpretaties. Met meer interpretaties gebeurt de convergence wel in minder iteraties omdat de EM dan beter werkt. Wel ook de total runtime er bij zetten.



# Appendix

## 4.1 ENC1

**Indicator clauses:**

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse}) \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\
 &\quad \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\
 &\quad \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse})
 \end{aligned}$$

**Parameter clauses:**

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \theta_{PollutionLow}) \wedge (\lambda_{PollutionLow} \vee \neg \theta_{PollutionLow}) \wedge (\neg \\
 &\lambda_{PollutionHigh} \vee \theta_{PollutionHigh}) \wedge (\lambda_{PollutionHigh} \vee \neg \theta_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \theta_{SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \theta_{SmokerTrue}) \wedge (\neg \\
 &\quad \lambda_{SmokerFalse} \vee \theta_{SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \theta_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \lambda_{CancerTrue} \vee \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg
 \end{aligned}$$

$$\begin{aligned}
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{XrayPositive} \vee \\
& \theta_{XrayPositive|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge \\
& (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\
& \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerTrue}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayPositive} \vee \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerFalse}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \\
& \neg \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerFalse}) \wedge \\
& (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaTrue} \vee \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{CancerTrue} \\
& \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \\
& \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerTrue}) \wedge \\
& (\lambda_{CancerTrue} \vee \neg \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \lambda_{DyspnoeaTrue} \vee \\
& \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge \\
& (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\
& \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \theta_{DyspnoeaFalse|CancerFalse})
\end{aligned}$$

**Weights:**

$$\begin{aligned}
W(\lambda_{PollutionLow}) &= 1.00 \\
W(\neg \lambda_{PollutionLow}) &= 1.00 \\
W(\lambda_{PollutionHigh}) &= 1.00 \\
W(\neg \lambda_{PollutionHigh}) &= 1.00 \\
W(\lambda_{SmokerTrue}) &= 1.00 \\
W(\neg \lambda_{SmokerTrue}) &= 1.00 \\
W(\lambda_{SmokerFalse}) &= 1.00 \\
W(\neg \lambda_{SmokerFalse}) &= 1.00 \\
W(\lambda_{CancerTrue}) &= 1.00 \\
W(\neg \lambda_{CancerTrue}) &= 1.00 \\
W(\lambda_{CancerFalse}) &= 1.00 \\
W(\neg \lambda_{CancerFalse}) &= 1.00 \\
W(\lambda_{XrayPositive}) &= 1.00 \\
W(\neg \lambda_{XrayPositive}) &= 1.00 \\
W(\lambda_{XrayNegative}) &= 1.00 \\
W(\neg \lambda_{XrayNegative}) &= 1.00 \\
W(\lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\lambda_{DyspnoeaFalse}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaFalse}) &= 1.00
\end{aligned}$$

$$\begin{aligned}
W(\theta_{PollutionLow}) &= 0.90 \\
W(\neg\theta_{PollutionLow}) &= 1.00 \\
W(\theta_{PollutionHigh}) &= 0.10 \\
W(\neg\theta_{PollutionHigh}) &= 1.00 \\
W(\theta_{SmokerTrue}) &= 0.30 \\
W(\neg\theta_{SmokerTrue}) &= 1.00 \\
W(\theta_{SmokerFalse}) &= 0.70 \\
W(\neg\theta_{SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 0.03 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 0.97 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 0.00 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 0.05 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 0.95 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 0.02 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 0.98 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{XrayPositive|CancerTrue}) &= 0.90 \\
W(\neg\theta_{XrayPositive|CancerTrue}) &= 1.00 \\
W(\theta_{XrayNegative|CancerTrue}) &= 0.10 \\
W(\neg\theta_{XrayNegative|CancerTrue}) &= 1.00 \\
W(\theta_{XrayPositive|CancerFalse}) &= 0.20 \\
W(\neg\theta_{XrayPositive|CancerFalse}) &= 1.00 \\
W(\theta_{XrayNegative|CancerFalse}) &= 0.80 \\
W(\neg\theta_{XrayNegative|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerTrue}) &= 0.65 \\
W(\neg\theta_{DyspnoeaTrue|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerTrue}) &= 0.35 \\
W(\neg\theta_{DyspnoeaFalse|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerFalse}) &= 0.30 \\
W(\neg\theta_{DyspnoeaTrue|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerFalse}) &= 0.70 \\
W(\neg\theta_{DyspnoeaFalse|CancerFalse}) &= 1.00
\end{aligned}$$

## 4.2 ENC2

### Indicator clauses

$$\begin{aligned}
&(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
&\lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg
\end{aligned}$$

$$\begin{aligned} & \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse}) \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\ & \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

### Parameter clauses

$$\begin{aligned} & (\neg \rho_{PollutionLow} \vee \lambda_{PollutionLow}) \wedge (\rho_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\ & \rho_{SmokerTrue} \vee \lambda_{SmokerTrue}) \wedge (\rho_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \\ & \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \\ & \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \lambda_{CancerTrue}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \\ & \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\ & \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \\ & \lambda_{SmokerTrue} \vee \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\ & \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

### Weights

$$\begin{aligned} W(\lambda_{PollutionLow}) &= 1.00 \\ W(\neg \lambda_{PollutionLow}) &= 1.00 \\ W(\lambda_{PollutionHigh}) &= 1.00 \\ W(\neg \lambda_{PollutionHigh}) &= 1.00 \\ W(\lambda_{SmokerTrue}) &= 1.00 \\ W(\neg \lambda_{SmokerTrue}) &= 1.00 \\ W(\lambda_{SmokerFalse}) &= 1.00 \\ W(\neg \lambda_{SmokerFalse}) &= 1.00 \\ W(\lambda_{CancerTrue}) &= 1.00 \\ W(\neg \lambda_{CancerTrue}) &= 1.00 \\ W(\lambda_{CancerFalse}) &= 1.00 \\ W(\neg \lambda_{CancerFalse}) &= 1.00 \\ W(\lambda_{XrayPositive}) &= 1.00 \\ W(\neg \lambda_{XrayPositive}) &= 1.00 \\ W(\lambda_{XrayNegative}) &= 1.00 \\ W(\neg \lambda_{XrayNegative}) &= 1.00 \\ W(\lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\rho_{PollutionLow}) &= 0.90 \\ W(\neg \rho_{PollutionLow}) &= 0.10 \end{aligned}$$

$$\begin{aligned}
W(\rho_{SmokerTrue}) &= 0.30 \\
W(\neg \rho_{SmokerTrue}) &= 0.70 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.03 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.97 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 0.00 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 1.00 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.05 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.95 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.02 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.98 \\
W(\rho_{XrayPositive}|CancerTrue) &= 0.90 \\
W(\neg \rho_{XrayPositive}|CancerTrue) &= 0.10 \\
W(\rho_{XrayPositive}|CancerFalse) &= 0.20 \\
W(\neg \rho_{XrayPositive}|CancerFalse) &= 0.80 \\
W(\rho_{DyspnoeaTrue}|CancerTrue) &= 0.65 \\
W(\neg \rho_{DyspnoeaTrue}|CancerTrue) &= 0.35 \\
W(\rho_{DyspnoeaTrue}|CancerFalse) &= 0.30 \\
W(\neg \rho_{DyspnoeaTrue}|CancerFalse) &= 0.70
\end{aligned}$$

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