

# Capita Selecta AI - Probabilistic Programming Inference for SRL

Thierry Deruyttere (r0660485) & Armin Halilovic (r0679689)

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# Probabilistic Inference Using Weighted Model Counting

## 1.1 PGM to CNF

### 1.1.1 ENC 1

Our ENC1 encoding for the Cancer Bayesian network can be found in appendix 4.1.

### 1.1.2 ENC 2

Our ENC2 encoding for the Cancer Bayesian network can be found in appendix 4.2.

## 1.2 SRL to CNF

### 1.2.1 Encoding of Monty Hall as CNF

An encoding of problog programs can be generated by our program as follows:

```
python3 scripts/inference.py --problog files/problog/monty_hall.pl
```

The CNF will be shown using the program's predicates. A version of the CNF in dimacs format will be shown as well. See `README.MD` for more information.

Our CNF encoding for the given Monty Hall ProbLog program is:

$$\begin{aligned}
&\wedge(\textit{open\_door}(2) \vee \textit{prize}(2) \vee \textit{prize}(3) \vee \neg \textit{p\_open\_door}(2)\_0) \\
&\wedge(\textit{open\_door}(2) \vee \textit{prize}(2) \vee \neg \textit{prize}(3)) \\
&\wedge(\neg \textit{open\_door}(2) \vee \neg \textit{prize}(2) \vee \neg \textit{prize}(2)) \\
&\wedge(\neg \textit{open\_door}(2) \vee \neg \textit{prize}(2) \vee \textit{prize}(3)) \\
&\wedge(\neg \textit{open\_door}(2) \vee \neg \textit{prize}(3) \vee \neg \textit{prize}(2)) \\
&\wedge(\neg \textit{open\_door}(2) \vee \neg \textit{prize}(3) \vee \textit{prize}(3)) \\
&\wedge(\neg \textit{open\_door}(2) \vee \textit{p\_open\_door}(2)\_0 \vee \neg \textit{prize}(2)) \\
&\wedge(\neg \textit{open\_door}(2) \vee \textit{p\_open\_door}(2)\_0 \vee \textit{prize}(3)) \\
&\wedge(\textit{open\_door}(3) \vee \textit{prize}(2) \vee \textit{prize}(3) \vee \neg \textit{p\_open\_door}(3)\_0) \\
&\wedge(\textit{open\_door}(3) \vee \textit{prize}(3) \vee \neg \textit{prize}(2)) \\
&\wedge(\neg \textit{open\_door}(3) \vee \neg \textit{prize}(2) \vee \neg \textit{prize}(3)) \\
&\wedge(\neg \textit{open\_door}(3) \vee \neg \textit{prize}(2) \vee \textit{prize}(2)) \\
&\wedge(\neg \textit{open\_door}(3) \vee \neg \textit{prize}(3) \vee \neg \textit{prize}(3)) \\
&\wedge(\neg \textit{open\_door}(3) \vee \neg \textit{prize}(3) \vee \textit{prize}(2)) \\
&\wedge(\neg \textit{open\_door}(3) \vee \textit{p\_open\_door}(3)\_0 \vee \neg \textit{prize}(3)) \\
&\wedge(\neg \textit{open\_door}(3) \vee \textit{p\_open\_door}(3)\_0 \vee \textit{prize}(2)) \\
&\wedge(\textit{win\_keep} \vee \neg \textit{prize}(1)) \\
&\wedge(\neg \textit{win\_keep} \vee \textit{prize}(1)) \\
&\wedge(\textit{win\_switch} \vee \neg \textit{prize}(2) \vee \textit{open\_door}(2)) \\
&\wedge(\textit{win\_switch} \vee \neg \textit{prize}(3) \vee \textit{open\_door}(3)) \\
&\wedge(\neg \textit{win\_switch} \vee \textit{prize}(2) \vee \textit{prize}(3)) \\
&\wedge(\neg \textit{win\_switch} \vee \textit{prize}(2) \vee \neg \textit{open\_door}(3)) \\
&\wedge(\neg \textit{win\_switch} \vee \neg \textit{open\_door}(2) \vee \textit{prize}(3)) \\
&\wedge(\neg \textit{win\_switch} \vee \neg \textit{open\_door}(2) \vee \neg \textit{open\_door}(3)) \\
&\wedge(\neg \textit{prize}(1) \vee \neg \textit{prize}(2)) \\
&\wedge(\neg \textit{prize}(1) \vee \neg \textit{prize}(3)) \\
&\wedge(\neg \textit{prize}(2) \vee \neg \textit{prize}(3)) \\
&\wedge(\textit{prize}(1) \vee \textit{prize}(2) \vee \textit{prize}(3))
\end{aligned}$$

**Weights:**

$$\begin{aligned}
W(\textit{p\_open\_door}(2)\_0) &= 0.5 \\
W(\textit{p\_open\_door}(3)\_0) &= 0.5 \\
W(\textit{select\_door}(1)) &= 1.00 \\
W(\textit{prize}(1)) &= 0.33 \\
W(\textit{prize}(2)) &= 0.33 \\
W(\textit{prize}(3)) &= 0.33 \\
W(\textit{open\_door}(2)) &= 1.00 \\
W(\textit{open\_door}(3)) &= 1.00 \\
W(\textit{win\_keep}) &= 1.00 \\
W(\textit{win\_switch}) &= 1.00
\end{aligned}$$

## 1.3 Weighted Model Counting

### 1.3.1 Weighted model counters on above CNFs

We will use MiniC2D and Cachet as WMC counters.

#### MiniC2D

MiniC2D needs to use the  $-W$  option to do weighted model counting. The probability can be read next to “Count”.

- ENC1:

```
Constructing CNF... DONE
CNF stats:
  Vars=30 / Clauses=74
  CNF Time 0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=5, c_con=40 v_con=5
  Vtree Time 0.003s
Counting... DONE
  Learned clauses 0
Cache stats:
  hit rate 16.7%
  lookups 12
  ent count 10
  ent memory 0.5 KB
  ht memory 152.6 MB
  clists 1.0 ave, 1 max
  keys 4.2b ave, 6.0b max, 3.0b min
Count stats:
  Count Time 0.000s
  Count/Probability 1.00000
Total Time: 0.128s
```

Figure 1.1: Grounded problog cnf

- ENC2:

```
Constructing CNF... DONE
CNF stats:
  Vars=20 / Clauses=30
  CNF Time 0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=6, c_con=16 v_con=6
  Vtree Time 0.002s
Counting... DONE
  Learned clauses 0
Cache stats:
  hit rate 23.1%
  lookups 26
  ent count 20
  ent memory 1.0 KB
  ht memory 152.6 MB
  clists 1.0 ave, 1 max
  keys 1.8b ave, 3.0b max, 1.0b min
Count stats:
  Count Time 0.000s
  Count/Probability 1.00000
Total Time: 0.164s
```

Figure 1.2: Grounded problog cnf

- Monty hall:

```

Constructing CNF... DONE
CNF stats:
  Vars=10 / Clauses=26
  CNF Time    0.000s
Constructing vtree (from primal graph)... DONE
Vtree stats:
  Vtree widths: con<=4, c_con=22 v_con=4
  Vtree Time   0.002s
Counting... DONE
  Learned clauses    0
Cache stats:
  hit rate:    20.0%
  lookups     5
  ent count   4
  ent memory   0.2 KB
  ht_memory   152.6 MB
  clists      1.0 ave, 1 max
  keys        3.2b ave, 4.0b max, 3.0b min
Count stats:
  Count Time    0.000s
  Count        1.00000000
Total Time: 0.116s

```

Figure 1.3: Grounded problog cnf

- 

## Cachet

For Cachet, no need to use extra parameters to get the probability. It is reported next to “Satisfying probability”.

- ENC1:

```

Number of total components      11
Number of split components      2
Number of non-split components  5
Number of SAT residual formula  12
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0
First component split level     1

Number of Decisions             11
Max Decision Level              5
Number of Variables             30
Original Num Clauses            74
Original Num Literals           172
Added Conflict Clauses          0
Added Conflict Literals         0
Deleted Unrelevant clauses      0
Deleted Unrelevant literals     0
Number of Implications          124
Total Run Time                  0.018895

Satisfying probability          8.72319e-08
Number of solutions             93.6645

```

Figure 1.4: Grounded problog cnf

- ENC2:

```

Number of total components      11
Number of split components      2
Number of non-split components  5
Number of SAT residual formula 12
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0
First component split level    1

Number of Decisions             11
Max Decision Level              5
Number of Variables             20
Original Num Clauses            30
Original Num Literals           84
Added Conflict Clauses          0
Added Conflict Literals         0
Deleted Unrelevant clauses      0
Deleted Unrelevant literals     0
Number of Implications          72
Total Run Time                  0.017282

Satisfying probability          1
Number of solutions             1.04858e+06

```

Figure 1.5: Grounded problog cnf

- Monty Hall:

```

Number of total components      4
Number of split components      1
Number of non-split components  2
Number of SAT residual formula  5
Number of trivial components    0
Number of changed components    0
Number of adjusted components   0
First component split level    2

Number of Decisions             4
Max Decision Level              4
Number of Variables             10
Original Num Clauses            26
Original Num Literals           73
Added Conflict Clauses          0
Added Conflict Literals         0
Deleted Unrelevant clauses      0
Deleted Unrelevant literals     0
Number of Implications          26
Total Run Time                  0.016062

Satisfying probability          0.444444
Number of solutions             455.111

```

Figure 1.6: Grounded problog cnf

For ENC1 we see that with Cachet we get a satisfying probability of almost 0. With Monty hall we can also see that we get a probability of 0.4. This is due to the fact that with ENC1 all our negative literals have a weight of 1, while Cachet expects that a literal + its negation = 1. For Monty hall we also have negative literals with weight 1 which gives the same problem as with ENC1.

### 1.3.2 Difference between the selected WMCs

#### MiniC2D Vs Cachet

MiniC2D and Cachet are both weighted model counters but how they do this is quite different. MiniC2D is a top down compiler that compiles CNFs into a SDD which results in a faster system but it also uses less space while Cachet is an algorithm that uses formula caching together with clause learning and component analysis. MiniC2D needs vtrees to be able to compile the CNFs into an SDD. They, however, both use things from the SAT literature. They both use clause learning and component caching as to be able to reuse components

that later appear again in the search. Cachet on the other hand also uses some other things from SAT literature like an explicit on the fly calculation of connected components. This is different in MiniC2D as it uses a vtree to identify disconnected CNF components. [1] [2]

### 1.3.3 Overview of computational requirements

All the tests can be found in the test folder. We used our scripts to create the dimac files. The input files for our enc1 and enc2 converter are “.dsc” files which can be found at <http://www.bnlearn.com/bnrepository/discrete-small.html#cancer>.

#### Test 1: Cancer network

Table 1.1: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	0.2 KB	0.155s	1.0	1.0 KB	0.000s
<b>Cachet</b>	val1	val2	a	b	val3	val4

#### Test 2: asia network

Table 1.2: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	0.9 KB	0.145s	1.0	2.0 KB	0.139s
<b>Cachet</b>	val1	val2	a	b	val3	val4

#### Test 3: sachs network

Table 1.3: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	0.99707	14.3 KB	0.184s	1.0	14.5 KB	0.154s
<b>Cachet</b>	val1	val2	a	b	val3	val4

#### Test 4: earthquake network

Table 1.4: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
<b>Minic2d</b>	1.0	0.6 KB	0.137s	1.0	1.0 KB	0.153s
<b>Cachet</b>	val1	val2	a	b	val3	val4

#### Test 5: survey network

Table 1.5: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	1.6 KB	0.125s	1.0	2.0 KB	0.125s
Cachet	val1	val2	a	b	val3	val4

#### Test 6: alarm network

Table 1.6: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	959.7KB KB	0.268s	1.0	139KB	0.095s
Cachet	val1	val2	a	b	val3	val4

#### Test 6: andes network

Table 1.7: My caption

	ENC1			ENC2		
	Prob	Memory	Runtime	Prob	Memory	Runtime
Minic2d	1.0	2.7GB	122.78s	1.0	139.8MB	6.086s
Cachet	val1	val2	a	b	val3	val4

## 1.4 Knowledge compilation

### Vtree with the most compact circuit

During our tests

### Pattern for a good vtree

As a vtree is a binary tree, which means that a good vtree is compact. We want thus a vtree that is shallow.



# Build an Inference Engine

## 2.0.1 Implementation

We have implemented the pipeline using python. Information about it can be found in `README.MD`.

## 2.0.2 Pipeline with previous tasks

### Cancer Bayesian network

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

### Monty Hall

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

### 2.0.3 Pipeline on Bayesian learning example

DAS NEN DIKKE VETTE TODO

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

### 2.0.4 Pipeline on alarm Bayesian network

GOD DAMN IT STOM VAK HOE MOETEN WIJ DIT IN 50 UUR DOEN?  
PROCESS WORDT GEWOON GEKILLED OMDAT DIE CNF GIGANTISCH  
GROOT WORDT.

- Probability:
- Total runtime:
- Runtime of the separate parts:
- Number of variables in CNF:
- Number of lines in CNF:
- Depth of vtree:
- Number of edges and nodes in the circuit:

# Parameter Learning

learning

# Appendix

## 4.1 ENC1

Indicator clauses:

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse}) \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\
 &\quad \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\
 &\quad \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse})
 \end{aligned}$$

Parameter clauses:

$$\begin{aligned}
 &(\neg \lambda_{PollutionLow} \vee \theta_{PollutionLow}) \wedge (\lambda_{PollutionLow} \vee \neg \theta_{PollutionLow}) \wedge (\neg \\
 &\lambda_{PollutionHigh} \vee \theta_{PollutionHigh}) \wedge (\lambda_{PollutionHigh} \vee \neg \theta_{PollutionHigh}) \wedge (\neg \\
 &\quad \lambda_{SmokerTrue} \vee \theta_{SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \theta_{SmokerTrue}) \wedge (\neg \\
 &\quad \lambda_{SmokerFalse} \vee \theta_{SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \theta_{SmokerFalse}) \wedge (\neg \\
 &\quad \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \lambda_{CancerTrue} \vee \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{PollutionLow} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
 &\quad \theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
 &\theta_{CancerFalse|PollutionLow,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
 &\quad \theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
 &\theta_{CancerTrue|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\
 &\quad \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{PollutionHigh} \vee \neg
 \end{aligned}$$

$$\begin{aligned}
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{SmokerTrue} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerTrue} \vee \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{CancerTrue|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\
& \neg \lambda_{CancerFalse} \vee \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{PollutionHigh} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{SmokerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{CancerFalse|PollutionHigh,SmokerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{XrayPositive} \vee \\
& \theta_{XrayPositive|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge \\
& (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\
& \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{CancerTrue} \vee \neg \\
& \theta_{XrayNegative|CancerTrue}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerTrue}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayPositive} \vee \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{XrayPositive|CancerFalse}) \wedge (\lambda_{XrayPositive} \vee \neg \theta_{XrayPositive|CancerFalse}) \wedge (\neg \\
& \lambda_{CancerFalse} \vee \neg \lambda_{XrayNegative} \vee \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \\
& \neg \theta_{XrayNegative|CancerFalse}) \wedge (\lambda_{XrayNegative} \vee \neg \theta_{XrayNegative|CancerFalse}) \wedge \\
& (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaTrue} \vee \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{CancerTrue} \\
& \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \wedge (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerTrue}) \\
& \wedge (\neg \lambda_{CancerTrue} \vee \neg \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerTrue}) \wedge \\
& (\lambda_{CancerTrue} \vee \neg \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \lambda_{DyspnoeaTrue} \vee \\
& \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge \\
& (\lambda_{DyspnoeaTrue} \vee \neg \theta_{DyspnoeaTrue|CancerFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\
& \lambda_{DyspnoeaFalse} \vee \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{CancerFalse} \vee \neg \\
& \theta_{DyspnoeaFalse|CancerFalse}) \wedge (\lambda_{DyspnoeaFalse} \vee \neg \theta_{DyspnoeaFalse|CancerFalse})
\end{aligned}$$

**Weights:**

$$\begin{aligned}
W(\lambda_{PollutionLow}) &= 1.00 \\
W(\neg \lambda_{PollutionLow}) &= 1.00 \\
W(\lambda_{PollutionHigh}) &= 1.00 \\
W(\neg \lambda_{PollutionHigh}) &= 1.00 \\
W(\lambda_{SmokerTrue}) &= 1.00 \\
W(\neg \lambda_{SmokerTrue}) &= 1.00 \\
W(\lambda_{SmokerFalse}) &= 1.00 \\
W(\neg \lambda_{SmokerFalse}) &= 1.00 \\
W(\lambda_{CancerTrue}) &= 1.00 \\
W(\neg \lambda_{CancerTrue}) &= 1.00 \\
W(\lambda_{CancerFalse}) &= 1.00 \\
W(\neg \lambda_{CancerFalse}) &= 1.00 \\
W(\lambda_{XrayPositive}) &= 1.00 \\
W(\neg \lambda_{XrayPositive}) &= 1.00 \\
W(\lambda_{XrayNegative}) &= 1.00 \\
W(\neg \lambda_{XrayNegative}) &= 1.00 \\
W(\lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\
W(\lambda_{DyspnoeaFalse}) &= 1.00 \\
W(\neg \lambda_{DyspnoeaFalse}) &= 1.00
\end{aligned}$$

$$\begin{aligned}
W(\theta_{PollutionLow}) &= 0.90 \\
W(\neg\theta_{PollutionLow}) &= 1.00 \\
W(\theta_{PollutionHigh}) &= 0.10 \\
W(\neg\theta_{PollutionHigh}) &= 1.00 \\
W(\theta_{SmokerTrue}) &= 0.30 \\
W(\neg\theta_{SmokerTrue}) &= 1.00 \\
W(\theta_{SmokerFalse}) &= 0.70 \\
W(\neg\theta_{SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 0.03 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 0.97 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 0.00 \\
W(\neg\theta_{CancerTrue|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\neg\theta_{CancerFalse|PollutionLow,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 0.05 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 0.95 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerTrue}) &= 1.00 \\
W(\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 0.02 \\
W(\neg\theta_{CancerTrue|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 0.98 \\
W(\neg\theta_{CancerFalse|PollutionHigh,SmokerFalse}) &= 1.00 \\
W(\theta_{XrayPositive|CancerTrue}) &= 0.90 \\
W(\neg\theta_{XrayPositive|CancerTrue}) &= 1.00 \\
W(\theta_{XrayNegative|CancerTrue}) &= 0.10 \\
W(\neg\theta_{XrayNegative|CancerTrue}) &= 1.00 \\
W(\theta_{XrayPositive|CancerFalse}) &= 0.20 \\
W(\neg\theta_{XrayPositive|CancerFalse}) &= 1.00 \\
W(\theta_{XrayNegative|CancerFalse}) &= 0.80 \\
W(\neg\theta_{XrayNegative|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerTrue}) &= 0.65 \\
W(\neg\theta_{DyspnoeaTrue|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerTrue}) &= 0.35 \\
W(\neg\theta_{DyspnoeaFalse|CancerTrue}) &= 1.00 \\
W(\theta_{DyspnoeaTrue|CancerFalse}) &= 0.30 \\
W(\neg\theta_{DyspnoeaTrue|CancerFalse}) &= 1.00 \\
W(\theta_{DyspnoeaFalse|CancerFalse}) &= 0.70 \\
W(\neg\theta_{DyspnoeaFalse|CancerFalse}) &= 1.00
\end{aligned}$$

## 4.2 ENC2

### Indicator clauses

$$(\neg \lambda_{PollutionLow} \vee \neg \lambda_{PollutionHigh}) \wedge (\lambda_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \lambda_{SmokerTrue} \vee \neg \lambda_{SmokerFalse}) \wedge (\lambda_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg$$

$$\begin{aligned} & \lambda_{CancerTrue} \vee \neg \lambda_{CancerFalse} \wedge (\lambda_{CancerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{XrayPositive} \vee \neg \lambda_{XrayNegative}) \wedge (\lambda_{XrayPositive} \vee \lambda_{XrayNegative}) \wedge (\neg \\ & \lambda_{DyspnoeaTrue} \vee \neg \lambda_{DyspnoeaFalse}) \wedge (\lambda_{DyspnoeaTrue} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

### Parameter clauses

$$\begin{aligned} & (\neg \rho_{PollutionLow} \vee \lambda_{PollutionLow}) \wedge (\rho_{PollutionLow} \vee \lambda_{PollutionHigh}) \wedge (\neg \\ & \rho_{SmokerTrue} \vee \lambda_{SmokerTrue}) \wedge (\rho_{SmokerTrue} \vee \lambda_{SmokerFalse}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \lambda_{SmokerTrue} \vee \\ & \rho_{CancerTrue|PollutionLow,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionLow} \vee \neg \\ & \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \lambda_{CancerTrue}) \wedge (\neg \\ & \lambda_{PollutionLow} \vee \neg \lambda_{SmokerFalse} \vee \rho_{CancerTrue|PollutionLow,SmokerFalse} \vee \\ & \lambda_{CancerFalse}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerTrue} \vee \neg \\ & \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \\ & \lambda_{SmokerTrue} \vee \rho_{CancerTrue|PollutionHigh,SmokerTrue} \vee \lambda_{CancerFalse}) \wedge (\neg \\ & \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \neg \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \\ & \lambda_{CancerTrue}) \wedge (\neg \lambda_{PollutionHigh} \vee \neg \lambda_{SmokerFalse} \vee \\ & \rho_{CancerTrue|PollutionHigh,SmokerFalse} \vee \lambda_{CancerFalse}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{XrayPositive|CancerTrue} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayPositive}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{XrayPositive|CancerFalse} \vee \lambda_{XrayNegative}) \wedge (\neg \lambda_{CancerTrue} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerTrue} \vee \\ & \rho_{DyspnoeaTrue|CancerTrue} \vee \lambda_{DyspnoeaFalse}) \wedge (\neg \lambda_{CancerFalse} \vee \neg \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaTrue}) \wedge (\neg \lambda_{CancerFalse} \vee \\ & \rho_{DyspnoeaTrue|CancerFalse} \vee \lambda_{DyspnoeaFalse}) \end{aligned}$$

### Weights

$$\begin{aligned} W(\lambda_{PollutionLow}) &= 1.00 \\ W(\neg \lambda_{PollutionLow}) &= 1.00 \\ W(\lambda_{PollutionHigh}) &= 1.00 \\ W(\neg \lambda_{PollutionHigh}) &= 1.00 \\ W(\lambda_{SmokerTrue}) &= 1.00 \\ W(\neg \lambda_{SmokerTrue}) &= 1.00 \\ W(\lambda_{SmokerFalse}) &= 1.00 \\ W(\neg \lambda_{SmokerFalse}) &= 1.00 \\ W(\lambda_{CancerTrue}) &= 1.00 \\ W(\neg \lambda_{CancerTrue}) &= 1.00 \\ W(\lambda_{CancerFalse}) &= 1.00 \\ W(\neg \lambda_{CancerFalse}) &= 1.00 \\ W(\lambda_{XrayPositive}) &= 1.00 \\ W(\neg \lambda_{XrayPositive}) &= 1.00 \\ W(\lambda_{XrayNegative}) &= 1.00 \\ W(\neg \lambda_{XrayNegative}) &= 1.00 \\ W(\lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaTrue}) &= 1.00 \\ W(\lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\neg \lambda_{DyspnoeaFalse}) &= 1.00 \\ W(\rho_{PollutionLow}) &= 0.90 \\ W(\neg \rho_{PollutionLow}) &= 0.10 \end{aligned}$$

$$\begin{aligned}
W(\rho_{SmokerTrue}) &= 0.30 \\
W(\neg \rho_{SmokerTrue}) &= 0.70 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.03 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerTrue) &= 0.97 \\
W(\rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 0.00 \\
W(\neg \rho_{CancerTrue}|PollutionLow,SmokerFalse) &= 1.00 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.05 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerTrue) &= 0.95 \\
W(\rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.02 \\
W(\neg \rho_{CancerTrue}|PollutionHigh,SmokerFalse) &= 0.98 \\
W(\rho_{XrayPositive}|CancerTrue) &= 0.90 \\
W(\neg \rho_{XrayPositive}|CancerTrue) &= 0.10 \\
W(\rho_{XrayPositive}|CancerFalse) &= 0.20 \\
W(\neg \rho_{XrayPositive}|CancerFalse) &= 0.80 \\
W(\rho_{DyspnoeaTrue}|CancerTrue) &= 0.65 \\
W(\neg \rho_{DyspnoeaTrue}|CancerTrue) &= 0.35 \\
W(\rho_{DyspnoeaTrue}|CancerFalse) &= 0.30 \\
W(\neg \rho_{DyspnoeaTrue}|CancerFalse) &= 0.70
\end{aligned}$$



# Bibliography

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