ASSIGNMENT #2

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) For the following vector equations, write a system of equations that is equivalent to it.

(a) $x_1 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ -3 \end{bmatrix}$

(b) $x_1 \begin{bmatrix} 2 \\ 0 \\ 0 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 9 \end{bmatrix} = \begin{bmatrix} 23 \\ 3 \\ 9 \\ 10 \end{bmatrix}$

(2) For the following systems of equations, write the vector equation that is equivalent to it.

(a) $\begin{cases} 2x_1 - 7x_2 + = 9 \\ 6x_2 + x_3 = 2 \\ -2x_1 + 7x_2 + 3x_3 = 1 \end{cases}$

(b) $\begin{cases} -x_1 + x_2 = 0 \\ x_2 = 8 \\ 2x_1 - 3x_2 = 2 \end{cases}$

(3) For the following lists of vectors, determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

(a) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

(b) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

(4) Let $\mathbf{a}_1 = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$, and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

(a) List three vectors in $\mathrm{Span}(\mathbf{a}_1, \mathbf{a}_2)$, along with their corresponding weights.

(b) Without drawing, determine if the vector $\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$ is in $\text{Span}(\mathbf{a}_1, \mathbf{a}_2)$.

(c) Without drawing, determine if the vector $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ is in $\text{Span}(\mathbf{a}_1, \mathbf{a}_2)$.

(5) Let $\mathbf{a}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Draw the points in the Cartesian plane corresponding to the following vectors. After drawing them, do you think every vector in $\mathbb{R}^2$ can be written as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$?

(a) $\mathbf{a}_1, \quad 2\mathbf{a}_1,$

(b) $\mathbf{a}_2, \quad 2\mathbf{a}_2,$

(c) $-\mathbf{a}_1, \quad -2\mathbf{a}_1$

(d) $-\mathbf{a}_2, \quad -2\mathbf{a}_2$

(e) $\mathbf{a}_1 + \mathbf{a}_2, \quad \mathbf{a}_1 + 2\mathbf{a}_2$

(f) $\mathbf{a}_1 - \mathbf{a}_2, \quad \mathbf{a}_2 - \mathbf{a}_1$

(6) Do the following problems in 1.4 of our textbook: 5, 7, 14, 25, 26, 27, 28, 30, 31. **No justification needed in 25-31.**