ASSIGNMENT #10

Please do not write your answers on a copy of this assignment, use blank paper. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. All questions except the true and false will be graded for completion. **If you would like feedback on a problem, please indicate it somehow.** You must make an honest attempt on each problem for full points on the completion aspect of your grade.

(1) Find the eigenvalues of the following matrices. After you find their eigenvalues, describe each eigenvalue's eigenspace using set-builder notation.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -4 & 0 & 2 \\ 0 & -3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(2) Find a basis for the eigenspace corresponding the each listed eigenvalue below.

(a) $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$; $\lambda = 1, 5$

(b) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 3 & 3 \\ 6 & 6 & 2 \end{bmatrix}$; $\lambda = -4$

(3) Is $\lambda = 4$ an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If it is find one corresponding eigenvector.

(4) Is $\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{bmatrix}$? If so, find its eigenvalue.

(5) Find an eigenvector for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(6) Find the eigenvalues of the 2×2 matrix that roates points by 45 degrees about the origin. For each eigenvalue you find, find a basis for its eigenspace.

(7) Answer the following true and false questions. No justification is required.

(a) If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A

(b) The eigenvalues of a matrix are the entries of its diagonal.

(c) A square matrix A is invertible if and only if 0 is not an eigenvalue of A.

(d) A square matrix A and it's transpose have the same eigenvalues.

(e) A square matrix A of size n can have more than n eigenvalues.