ASSIGNMENT #7

Please do not write your answers on a copy of this assignment, use blank paper. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) Find the eigenvalues of the following matrices. After you find their eigenvalues, describe each eigenvalue's eigenspace using set-builder notation.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -4 & 0 & 2 \\ 0 & -3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(2) Find a basis for the eigenspace corresponding the each listed eigenvalue below.

(a) $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$; $\lambda = 1, 5$

(b) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 3 & 3 \\ 6 & 6 & 2 \end{bmatrix}$; $\lambda = -4$

(3) Is $\lambda = 4$ an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If it is find one corresponding eigenvector.

(4) Is $\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{bmatrix}$? If so, find its eigenvalue.

(5) For each of the following matrices, determine if the matrix is diagonalizable, and if so find P and D such that $A = PDP^{-1}$, where D is a diagonal matrix.

(a) $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

(6) Answer the following true and false questions. No justification is required.

(a) If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A

(b) The eigenvalues of a matrix are the entries of its diagonal.

(c) A square matrix A is invertible if and only if 0 is not an eigenvalue of A.

(d) A square matrix A and it's transpose have the same eigenvalues.

(e) A square matrix A of size n can have more than n eigenvalues.

(f) If two matrices have the same eigenvalues, then they are similar.

(g) A square matrix A of size n is diagonalizable if and only if it has n linearly independent eigenvectors.

(h) $A = PBP^{-1}$ and $A = QBQ^{-1}$, then $P = Q$.