ASSIGNMENT #8

Please do not write your answers on a copy of this assignment, use blank paper. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) Find bases for the following subspaces. Some of these spaces may look familar...

(a) The subspace $\{(v_1, v_2, 0) \mid v_1, v_2 \in R\}$ of $\mathbb{R}^3$.

(b) The vector space $\mathbb{R}[x]_{\leqslant 4}$.

(c) Let A be any $m \times n$ matrix. Is the image of the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ (recall T_A is multiplication by A on the left) a subspace of $\mathbb{R}^m$?

(d) Is the set of points inside and on the unit circle in $\mathbb{R}^2$ a subspace of $\mathbb{R}^2$? *Hint:* the set of points inside on the unit circle can be described as $H = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leqslant 1\}$?

(e) The set of polynomials of $\mathbb{R}[x]_{\leqslant 21}$ that has 1 and 2 as roots.

(f) The subspace $V = \{ax^2 \in \mathbb{R}[x] \mid a \in \mathbb{R}\}$ of $\mathbb{R}[x]_{\leqslant 2}$.

(2) Using your answers to problem (1), find the dimensions of the following spaces.

(a) The subspace $\{(v_1, v_2, 0) \mid v_1, v_2 \in R\}$ of $\mathbb{R}^3$.

(b) The vector space $\mathbb{R}[x]_{\leqslant 4}$.

(c) Let A be any $m \times n$ matrix. Is the image of the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$ (recall T_A is multiplication by A on the left) a subspace of $\mathbb{R}^m$?

(d) Is the set of points inside and on the unit circle in $\mathbb{R}^2$ a subspace of $\mathbb{R}^2$? *Hint:* the set of points inside on the unit circle can be described as $H = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leqslant 1\}$?

(e) The set of polynomials of $\mathbb{R}[x]_{\leqslant 21}$ that has 1 and 2 as roots.

(f) The subspace $V = \{ax^2 \in \mathbb{R}[x] \mid a \in \mathbb{R}\}$ of $\mathbb{R}[x]_{\leqslant 2}$.

(3) Write down two different bases for $\mathbb{R}^3$ that are not the standard basis (remember, the standard basis for $\mathbb{R}^3$ is $\{(1, 0, 0), (0, 1, 0), (0, 0, 1\})$.

(4) Answer the following true and false questions. You do not need to provide justification.

(a) The vector space $\mathbb{R}[x]_{\leqslant n}$ is isomorphic to $\mathbb{R}^{n+1}$.

(b) The vector space $\mathbb{R}^3$ is isomorphic to $\mathbb{R}^2$.

(c) Let V be a vector space with two subspaces (not necessarily the same) U and W. The subset $U \cup W := \{v \in V \mid v \in U \text{ or } v \in W\}$ is a subspace of V.