ASSIGNMENT #6

Please do not write your answers on a copy of this assignment. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) Find the determinants of the following matrices using cofactor expansion along the row or column of your choice.

(a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$

(b) $\begin{bmatrix} 1 & 0 & 2 & 8000 \\ 11 & 0 & 1 & 121 \\ 0 & 0 & 0 & 21 \\ 12 & 0 & 21 & 90 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 & 0 \\ 7 & 1 & 0 \\ 23 & 9 & 3 \end{bmatrix}$

(d) For each of the matrices above, determine which ones represent linear transformations (via multiplication on the left) that are isomorphisms (i.e bijective funtions).

(2) For each of the following matrices, use row operations to determine their determinants. Use your answers to problem 1 to check your work.

(a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$

(b) $\begin{bmatrix} 1 & 0 & 2 & 8000 \\ 11 & 0 & 1 & 121 \\ 0 & 0 & 0 & 21 \\ 12 & 0 & 21 & 90 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 & 0 \\ 7 & 1 & 0 \\ 23 & 9 & 3 \end{bmatrix}$

(3) What is the determinant of the 2×2 matrix that first rotates vectors 45 degrees about the origin and then scales vectors by a factor of 2.

(4) Consider the vectors $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ in $\mathbb{R}^2$. Answer the following questions.

(a) Find the determinant of $A = \begin{bmatrix} \mathbf{a} & \mathbf{b} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$.

(b) Find the area of the parallelagram in $\mathbb{R}^2$ that is determined by $\mathbf{a}$ and $\mathbf{b}$.

(c) How do your answers in parts (a) and (b) compare?

(5) Answer the following as true or false. No justification is needed.

(a) For $n \times n$ matrices A and B, we have $\det(A + B) = \det(A) + \det(B)$.

(b) Let A be an $n \times n$ matrix. If $\det(A) = 0$, then two rows or two columns are the same, or a row or a column is zero. *Hint*: Consider some cocrete examples of 2×2 matrices.

(c) Let A and P be $n \times n$ matrices. Assume that P is invertible. Then, $\det(PAP^{-1}) = \det(A)$.

(d) If $\det(A) = 0$, then A is invertible.

(e) If A is not invertible, then $\det(A) = 0$.