Lecture 11/6/23

What is factoring? writing down a ~~of~~ number or expression as a product of other numbers/expressions

Eg: ~~9~~ $9 = 3 \cdot 3$

Look for largest # that divides each term

~~Look for things in each term that is divisible by same number.~~

Ex: i) $42 - 6x = 6(7-x)$

ii) ~~16~~ $t^8 - t^5 = t^5(t^3 - 1)$

Take the smallest power that occurs out of all terms.

iii) $16x^2 + 40x^4 = 8x^2(2 + 5x^2)$

largest # dividing each term

smallest power of x occuring in original expression.

To sum up the ideas used in the above examples:

~~Define the GCF greatest common factor~~

Recall: ~~A~~ An expression that is a constant multiplied by variables is called a monomial

Ex: $2xyz^2w$

Non Ex: $x + y$

Defn: The GCF (greatest common factor of two monomial is the ~~smallest~~ largest monomial appearing in each.

▲▲ These can be used to factor expressions ▲▲

Ex: Find the GCF of $2a(x-2y)$ and $9b(x-2y)$

Use this to factor ~~2a~~ $2a(x-2y) + 9b(x-2y)$

Solution: $2a(x-2y)$ $9b(x-2y)$

We can't find a monomial that divides each larger than $(x-2y)$. So,

$$GCF = x-2y$$

We can factor out GCF of $2a(x-2y) + 9b(x-2y)$

$$(x-2y)(2a + 9b)$$

Algorithm: Given an expression to factor:

① Identify largest monomial in each term that divides each term. This is the GCF
② Factor GCF out.

We can extend this by factoring through grouping. This is better seen through examples. This is a divide and conquer method!

Ex: 1) $\underbrace{1-x}_{I} + \underbrace{xy-y}_{II}$ $1-x$ $xy-y$

$GCF(I) = 1-x$ $GCF(II) = y$

So $1-x + xy-y = 1-x + y(x-1)$

common factor so pull out!

$$(1-x)(1+y)$$

ċi) $4xy - zx + 24y - 6z$

       I   II

$= x(4y - z) + 6(4y - z)$

$= (x + 6)(4y - z).$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Ex i) $x^2 - 4 = \cancel{(x+2)(x-2)}$ $x^2 - 2^2 = (x + 2)(x - 2)$

ii) $9m^2 - 25 = 3^2 m^2 - 5^2 = (3m)^2 - 5^2$

$$= (3m + 5)(3m - 5)$$

iii) $18p^2 - 200 p^2 q^2 = p^2(18 - 200q^2)$

$$= \cancel{4}\ \cancel{2p}\ 2p^2(9 - 100q^2)$$

$$= 2p^2(3^2 - 10^2 q^2)$$

$$= 2p^2(3^2 - (10q)^2)$$

$$= 2p^2(3 - 10q)(3 + 10q)$$

Factoring Polynomials Ctd:

We will focus on factoring Quadratics (i.e Trinomials)
These are polynomials of the form

$$ax^2 + bx + c$$

The ac -method: To factor a quadratic ax^2+bx+c

① Find the # ac

② List all ~~factorizations~~ ~~factorization~~ M ways to write $a\cdot c$ as a product of two ~~whole~~ numbers.

③ Find a pair of numbers found in ② that add to b.

④ If there is a pair of #s (m,n) from step ③ write (i.e $m+n=$ ~~b~~ b and $m\cdot n=ac$)

(since $m+n=b$) $\quad ax^2+bx+c = ax^2+mx+nx+c$

⑤ Factor by grouping

☞ we can't always do this.

Ex: i) $y^2+2y+9y+18 = y^2+11y+18 \qquad a\cdot c = 18$

$1\cdot18$	$18+1=19$
$9\cdot2$	$9\cdot2=11=b$

$\rightarrow$

$= y^2 + 9y + 2y + 18$

$= y(y+9) + 2(y+9)$

$= (y+9)(y+2)$

ii) $m^2p^2-7mp+12$. Set $mp=x$ (dummy variable)

Then ~~we get~~ we get

careful!
$\downarrow$ here!

$1\cdot12$	$1+12=13$
$6\cdot2$	$6+2=8$
$3\cdot4$	$3+4=7$

$x^2 - 7x + 12 = x^2 - 3x - 4x + 12 = x(x-3) - 4(x-3)$

$= x(x-3) - 4(x-3)$

$= (x - 4)(x-3)$