EXTRA CREDIT/ FINAL REVIEW

Please do not write your answers on a copy of this assignment, use blank paper. For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. This extra credit assignment will replace your lowest homework grade with a 25/25. In order to receive full credit, you must make an honest effort on each problem.

(1) Find a basis for the column space of the matrix $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 2 & 3 & 4 & 0 \end{bmatrix}$

(2) Find the rank and nullity of the matrix $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 2 & 3 \end{bmatrix}$

(3) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4\}$ and $\mathcal{C} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ be bases for a vector space V. Suppose that

$$\begin{aligned} \mathbf{b}_1 &= 2\mathbf{v}_1 - \mathbf{v}_3 \\ \mathbf{b}_2 &= \mathbf{v}_2 + \mathbf{v}_4 \\ \mathbf{b}_3 &= 3\mathbf{b}_1 - \mathbf{v}_2 + \mathbf{v}_3 - 2\mathbf{v}_4 \\ \mathbf{b}_4 &= -2\mathbf{v}_4. \end{aligned}$$

Find $P_{\mathcal{B}\to\mathcal{C}}$ and $P_{\mathcal{C}\to\mathcal{B}}$.

(4) Write down what it means for a square matrix A to represent an isomorphism.

(5) Find the determinant of $A = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$. Use the determinant you found to determine whether or not A represents an isomorphism.

(6) Determine if the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is invertible. If so, find its inverse. Otherwise, explain why it is not invertible.

(7) Determine if the matrix $\begin{bmatrix} 10 & 0 & 0 & 0 \\ 1726 & 100000 & 0 & 0 \\ 0 & 14 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ is invertible. If so, find its inverse. Otherwise, explain why it is not invertible.

(8) What does the subspace criteria say?

(9) Does the subset of $\mathbb{R}^2$ consisting of the first and third quadrants constitute a subspace of $\mathbb{R}^2$? Why or why not.

(10) What does it mean for a matrix A to have an eigenvector $\mathbf{v}$ associated to the eigenvalue λ

(11) Find all eigenvalues of $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

(12) Describe, using set-builder notation, all eigenspaces of $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(13) If a $n \times n$ matrix has **distinct** eigenvalues, is it diagonalizable?

(14) Let A be a $n \times n$ matrix with eigenvalue λ. The eigenspace, $E_\lambda(A)$ is the kernel of which matrix?

(15) Determine if the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is diagonalizable. If it is, find a diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$. If it id not diagonalizable, explain why it isn't.

(16) Determine if the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ is diagonalizable. If it is, find a diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$. If it id not diagonalizable, explain why it isn't.

(17) Find an eigenvector for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(18) If an $n \times n$ matrix has a eigenvalue of zero, is it invertible?

(19) What does it mean if $T : V \to W$ is a linear transformation of vector spaces?

(20) Consider the map $T : \mathbb{R}[x, y, z]_{\leqslant 1} \to \mathbb{R}[x]_{\leqslant 1}$ defined by $f(p(x, y, z)) = p(x, 0, 0)$. Show that T is a linear transformation. Additionally, find a 2×4 matrix that represents T.