

ASSIGNMENT #3

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words.** I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

- (1) Determine if the columns of each matrix form a linearly independent set.

(a)
$$\begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 1 & 1 & 7 & 1 \\ 2 & 3 & 8 & 0 \\ 7 & 4 & 9 & 1 \end{bmatrix}$$

(2) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 10 \\ 6 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$

- (a) Find all values of h so that $\mathbf{v}_3 \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.
(b) Find all values of h so that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

- (3) For which real values of λ are the vectors $\mathbf{v}_1 = (\lambda, \frac{-1}{2}, \frac{-1}{2})$, $\mathbf{v}_2 = (\frac{-1}{2}, \lambda, \frac{-1}{2})$, and $\mathbf{v}_3 = (\frac{-1}{2}, \frac{-1}{2}, \lambda)$ linearly dependent?

- (4) Answer the following True/False questions. You do not need to provide justification.

- (a) If two column vectors \mathbf{a}_1 and \mathbf{a}_2 are linearly dependent, then \mathbf{a}_1 is a scalar multiple of \mathbf{a}_2 .
(b) If three column vectors \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 are linearly dependent, then \mathbf{a}_1 is a scalar multiple of \mathbf{a}_2 and \mathbf{a}_3 .

- (c) The set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} \right\}$ is linearly independent.

- (d) The columns of any 4×5 matrix are linearly dependent.

- (e) If S is a linearly dependent set of vectors, then each vector in S is a linearly combination of the other vectors in S .
- (f) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has no non-trivial solution.
- (g) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in \mathbb{R}^4 are linearly independent, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are also linearly independent.

- (5) **Challenge/Extra Credit:** How many pivot columns must a $n \times m$ matrix have if its columns span \mathbb{R}^n ? You don't need to prove your answer (I encourage you to try, though), but you must give at least some justification for your answer whether it be through examples and/or sentences.