ASSIGNMENT #4

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) Let $A = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 0 & 7 \\ 0 & 1 & 9 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Calculate the following:

(a) $2A + 7C$

(b) BC

(c) $BA + 2B$

(d) $B(A - 3C)$

(2) Find the transpose of each matrix below:

(a) $\begin{bmatrix} 1 & 0 & 8 \\ 2 & 6 & 7 \\ 2 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 3 & 9 & 0 & 1 & 7 \\ 7 & 2 & 1 & 5 & 8 \\ 1 & 2 & 1 & 7 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(3) Let $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$. Verify that $AB = AC$, yet $B \neq C$. **Fun fact:** if a, b, c are real numbers and $ab = ac$, then $b = c$. So matrices don't have this nice property (called left cancellation) that real numbers have! Isn't math interesting?

(4) Find the inverse of each matrix below:

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

(5) List all the statements that are equivalent (i.e the same) as: A is an invertible matrix. **Suggestion: look at the online notes.**

(6) Determine if the following are true or false. No justification necessary.

(a) $(AB)^T = A^T B^T$.

(b) $(AB)^{-1} = B^{-1}A^{-1}$.

(c) $AC = CA$

(d) If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b} \in R^n$.

(e) If AB is invertible, then B is invertible.

(f) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad = bc$, then A is invertible.

(g) $A\mathbf{x} = 0$ has no non-trivial solution, then A is invertible.

(h) If A^T is not invertible, then A is not invertible.