Math 101C Benchmark 2

October 20, 2022

Name: _____ Student ID: _____

Indicate your section/instructor.

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101C

Answer the questions in the spaces provided on the question sheets. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but also how you obtained it. Include units in your answer when possible. You may receive 0 points for a problem where you show no work.

Instructions:

1. Do not open this exam until you are told to do so.
2. Write your initials on every page!
3. No books or notes may be used on the exam.
4. You may only use an *approved* calculator on the exam. If you have a problem with your calculator, raise your hand.
5. Read and follow directions carefully.
6. All cell phones must be turned off and put away during the exam. Any device that connects to a phone or the web must be removed and put away.
7. Do not separate the pages of this exam. If they do become separated, point this out to your instructor when you hand in the exam.
8. Make sure your answer is clearly marked.
9. Credit or partial credit will be given only when the appropriate explanation and/or work is shown.
10. This exam has 7 questions, for a total of 60 points. There are 7 pages besides this one.
11. You will have 90 minutes to complete the exam.
12. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

You can use this page for scratch work.

Initials: _____

You might find the following expressions for exponential functions useful on the exam

$$f(t) = a(b)^t \qquad P(t) = a\left(1 + \frac{r}{n}\right)^{nt} \qquad h(t) = ae^{kt}$$

1. Consider the following table for a function $h(x)$:

x	-1	0	1	2	3
$h(x)$	81	135	225	375	625

   (a) (5 points) Does the table represent an exponential function? If so, give a formula for the function. If not, **explain** your answer in a sentence or two.

2. Suppose that $\$5,000.00$ is invested in an account and after 1 year there is $\$5,635.00$ in the account. For all parts of this problem, be sure to **show all work** to receive credit.

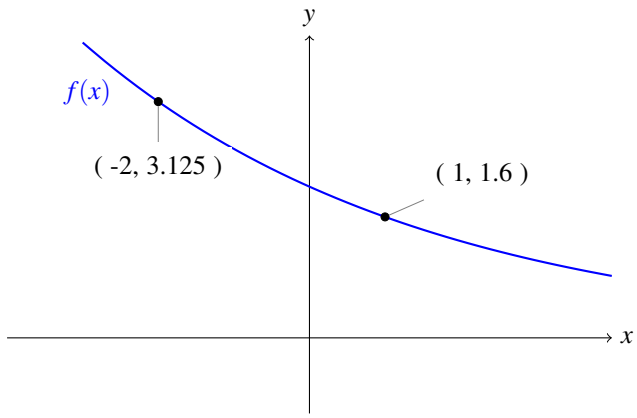
   (a) (3 points) By what percentage is the investment growing every year? That is, find the annual growth rate as percentage.

   (b) (3 points) Write an equation $Q(t)$ for the amount of money in the account after t years.

   (c) (3 points) How much money will be in the account after 5 years?

Initials: _____

3. Below is a graph of an exponential function $f(x)$. For all parts of this problem, be sure to **show all work** to receive credit.



(a) (5 points) Find an equation for $f(x)$ in the form $f(x) = ab^x$.

(b) (4 points) Express $f(x)$ as an equation in the form $f(x) = ae^{kx}$.

Initials: _____

4. A cup of coffee is left sitting on a table and the temperature (in degrees Fahrenheit) of the coffee is given by

$$F(t) = 90(0.95)^t + 65,$$

where t is the number of minutes since the coffee was left on the table. For all parts of this problem, be sure to **show all work** to receive credit.

(a) (3 points) What temperature was the cup of coffee when it was left on the table?

(b) (3 points) What temperature will the coffee be in 10 minutes?

(c) (3 points) What is the average rate that the coffee is cooling between $t = 0$ minutes and $t = 10$ minutes? Be sure to **show all work** to receive credit, and be sure to **include units** on your **final answer**.

(d) (2 points) How many minutes are needed before the coffee reaches 136 degrees?

Initials: _____

5. A certificate of deposit (CD) is a type of investment. Suppose that a particular CD offers a nominal rate of 4% per year, compounded monthly, and suppose that $\$15,000$ is invested into the CD account. For all parts of this problem, be sure to **show all work** to receive credit.

   (a) (3 points) Give an equation, $P(t)$ for the balance of the account after t years.

   (b) (3 points) What is the effective annual rate for this account? Give your **final answer** as a percent, accurate to **at least two decimal places** if you round.

   (c) (3 points) CDs require you to hold your money in the account for a set amount of time until it "matures" and you can take your balance out. If the CD in our example matures in 24 months, how much money will be in the account when the CD matures?

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6. Each part of this problem is unrelated. For all parts of this problem, be sure to **show all work** to receive credit.

(a) (2 points) Let $y = f(x) = 4x^3 - 12$. Find $f^{-1}(y)$.

(b) (1 point) Rewrite $\log_4(1024) = 5$ as an exponential equation.

(c) (1 point) Rewrite $e^{1.25276} = 3.5$ as a logarithmic equation.

(d) (1 point) Evaluate $\log_3(81)$.

(e) (3 points) Rewrite $\ln(17e^2)$ as the sum of two logarithms, then simplify your answer using logarithm rules. Be sure to **show all work** so that your simplification steps are clear.

Initials: _____

7. Technetium-99m (Tc-99 here) is a radioactive isotope that is a byproduct of some fission reactors and is commonly used in medical imaging. For a certain piece of Tc-99 (initially weighing 5 grams), the remaining amount (in grams) of Tc-99 after t hours is given by

$$M(t) = 5e^{-0.1155\,t}$$

For all parts of this problem, be sure to **show all work** to receive credit.

(a) (3 points) How much Tc-99 is left after 8 hours?

(b) (3 points) What is the half-life of Tc-99?

(c) (3 points) Using logarithms, determine how many hours it will take for there to be only one gram of Tc-99 left.