ASSIGNMENT #2

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. All questions except the true and false will be graded for completion. **If you would like feedback on a problem, please indicate it somehow.** You must make an honest attempt on each problem for full points on the completion aspect of your grade.

(1) For the following vector equations, write a system of equations that is equivalent to it.

(a) $x_1 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ -3 \end{bmatrix}$

(b) $x_1 \begin{bmatrix} 2 \\ 0 \\ 0 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ -2 \\ 9 \end{bmatrix} = \begin{bmatrix} 23 \\ 3 \\ 9 \\ 10 \end{bmatrix}$

(2) For the following systems of equations, write the vector equation that is equivalent to it.

(a) $\begin{cases} 2x_1 - 7x_2 + = 9 \\ 6x_2 + x_3 = 2 \\ -2x_1 + 7x_2 + 3x_3 = 1 \end{cases}$

(b) $\begin{cases} -x_1 + x_2 = 0 \\ x_2 = 8 \\ 2x_1 - 3x_2 = 2 \end{cases}$

(3) For the following lists of vectors, determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

(a) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

(b) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

(4) Let $\mathbf{a}_1 = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$, and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$.

(a) List three vectors in $\text{Span}(\mathbf{a}_1, \mathbf{a}_2)$, along with their corresponding weights.

(b) Without drawing, determine if the vector $\begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$ is in $\text{Span}(\mathbf{a}_1, \mathbf{a}_2)$.

(c) Without drawing, determine if the vector $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ is in $\text{Span}(\mathbf{a}_1, \mathbf{a}_2)$.

(5) Let $\mathbf{a}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Draw the points in the Cartesian plane corresponding to the following vectors. After drawing them, do you think every vector in $\mathbb{R}^2$ can be written as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$?

(a) $\mathbf{a}_1, \quad 2\mathbf{a}_1,$

(b) $\mathbf{a}_2, \quad 2\mathbf{a}_2,$

(c) $-\mathbf{a}_1, \quad -2\mathbf{a}_1$

(d) $-\mathbf{a}_2, \quad -2\mathbf{a}_2$

(e) $\mathbf{a}_1 + \mathbf{a}_2, \quad \mathbf{a}_1 + 2\mathbf{a}_2$

(f) $\mathbf{a}_1 - \mathbf{a}_2, \quad \mathbf{a}_2 - \mathbf{a}_1$

(6) Write each matrix equations as a vector equation and vice versa.

(a) $$\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$$

(b) $$x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ 7 \end{bmatrix}$$

(7) Answer the following true and false questions. **No justification needed.**

(a) If $A\mathbf{x} = \mathbf{b}$ is not consistent, then $\mathbf{b}$ is not in the set spanned by the columns of A.

(b) A vector $\mathbf{b}$ is in the space spanned by the columns of A if and only if the solution set of $A\mathbf{x} = \mathbf{b}$ is nonempty.

(c) The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $\begin{bmatrix} A & | & \mathbf{b} \end{bmatrix}$ has a pivot column in every row.

(d) If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then there is a vector $\mathbf{b} \in \mathbb{R}^m$ such that $A\mathbf{x} = \mathbf{b}$ is inconsistent.

(e) Any linear combination can be written as $A\mathbf{x}$ for a suitable matrix A and vector $\mathbf{x}$.