ASSIGNMENT #3

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) Determine if the columns of each matrix form a linearly independent set.

(a) $\begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 7 & 1 \\ 2 & 3 & 8 & 0 \\ 7 & 4 & 9 & 1 \end{bmatrix}$

(2) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 10 \\ 6 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$

(a) Find all values of h so that $\mathbf{v}_3 \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

(b) Find all values of h so that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

(3) For which real values of λ are the vectors $\mathbf{v}_1 = (\lambda, \frac{-1}{2}, \frac{-1}{2})$, $\mathbf{v}_2 = (\frac{-1}{2}, \lambda, \frac{-1}{2})$, and $\mathbf{v}_3 = (\frac{-1}{2}, \frac{-1}{2}, \lambda)$ linearly dependent?

(4) Answer the following True/False questions. You do not need to provide justification.

(a) If two column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are linearly dependent, then $\mathbf{a}_1$ is a scalar multiple of $\mathbf{a}_2$.

(b) If three column vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are linearly dependent, then $\mathbf{v}_1$ is a scalar multiple of $\mathbf{_2}$ and $\mathbf{_3}$.

(c) The set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} \right\}$ is linearly independent.

(d) The columns of any 4×5 matrix are linearly dependent.

(e) If S is a linearly dependent set of vectors, then each vector in S is a linearly combination of the other vectors in S.

(f) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has no non-trivial solution.

(g) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in $\mathbb{R}^4$ are linearly independent, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly independent.

(5) **Challenge/Extra Credit:** How many pivot columns must a $n \times m$ matrix have if its columns span $\mathbb{R}^n$? You don't need to prove your answer (I encourage you to try, though), but you must give at least some justification for your answer whether it be through examples and/or sentences.