

ASSIGNMENT #10

Please do not write your answers on a copy of this assignment, use blank paper. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words.** In addition to the true and false section being graded, I will grade one other problem; this will account for 10 points out of 25. The other 15 will be based on completion. **If you would like feedback on a particular problem, please indicate it somehow.** You must make an honest attempt on each problem for full points on the completion aspect of your grade.

- (1) Find the eigenvalues of the following matrices. After you find their eigenvalues, describe each eigenvalue's eigenspace using set-builder notation.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -4 & 0 & 2 \\ 0 & -3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- (2) Find a basis for the eigenspace corresponding the each listed eigenvalue below.

(a) $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}; \lambda = 1, 5$

(b) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 3 & 3 \\ 6 & 6 & 2 \end{bmatrix}; \lambda = -4$

- (3) Is $\lambda = 4$ an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If it is find one corresponding eigenvector.

- (4) Is $\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{bmatrix}$? If so, find its eigenvalue.

- (5) Find an eigenvector for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

- (6) Find the eigenvalues of the 2×2 matrix that rotates points by 45 degrees about the origin. For each eigenvalue you find, find a basis for its eigenspace.

- (7) Answer the following true and false questions. No justification is required.

- (a) If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector \mathbf{x} , then λ is an eigenvalue of A
- (b) The eigenvalues of a matrix are the entries of its diagonal.

- (c) A square matrix A is invertible if and only if 0 is not an eigenvalue of A .
- (d) A square matrix A and its transpose have the same eigenvalues.
- (e) A square matrix A of size n can have more than n eigenvalues.