ASSIGNMENT #3

As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. I will grade a subset of these problems and will take completion of the ungraded problems into account for the final grade of this assignment. Completion is worth 20% of the final grade of this assignment. To emphasize: **you must make an honest attempt on each problem for full points on the completion aspect of your grade.**

(1) Determine if the columns of each matrix form a linearly independent set.

(a) $\begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 7 & 1 \\ 2 & 3 & 8 & 0 \\ 7 & 4 & 9 & 1 \end{bmatrix}$

(2) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 10 \\ 6 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$

(a) Find all values of h so that $\mathbf{v}_3 \in \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

(b) Find all values of h so that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

(3) For which real values of λ are the vectors $\mathbf{v}_1 = (\lambda, \frac{-1}{2}, \frac{-1}{2})$, $\mathbf{v}_2 = (\frac{-1}{2}, \lambda, \frac{-1}{2})$, and $\mathbf{v}_3 = (\frac{-1}{2}, \frac{-1}{2}, \lambda)$ linearly dependent?

(4) For each of the following, determine which sets of vectors are linearly independent. If they are linearly dependent, give a dependence relations among the vectors.

(a) $\left\{ \begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix} \right\}$.

(b) $\left\{ \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix} \right\}$.

(5) Let $A = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 0 & 7 \\ 0 & 1 & 9 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Calculate the following:

(a) $2A + 7C$

(b) BC

(c) $BA + 2B$

(d) $B(A - 3C)$

(6) Let $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$. Verify that $AB = AC$, yet $B \neq C$. **Fun fact:** if a, b, c are real numbers and $ab = ac$, then $b = c$. So matrices don't have this nice property (called left cancellation) that real numbers have! Isn't math interesting?

(7) Answer the following True/False questions. You do not need to provide justification.

(a) The set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} \right\}$ is linearly independent.

(b) If two column vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are linearly dependent, then $\mathbf{a}_1$ is a scalar multiple of $\mathbf{a}_2$.

(c) If three column vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are linearly dependent, then $\mathbf{v}_1$ is a scalar multiple of $\mathbf{v}_2$ and $\mathbf{v}_3$.

(d) The columns of any 4×5 matrix are linearly dependent.

(e) If S is a linearly dependent set of vectors, then each vector in S is a linearly combination of the other vectors in S.

(f) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has no non-trivial solution.

(g) If the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in $\mathbb{R}^4$ are linearly independent, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly independent.