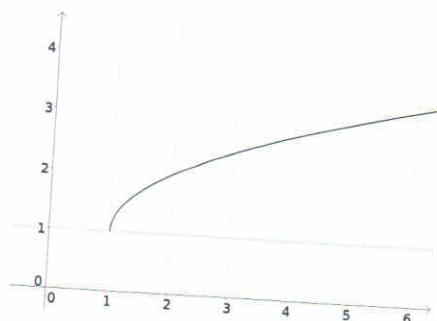


Worksheet 5.6: Combining Transformations

Problem 1 (Warm-Up). The graph of $f(x)$ appears below.



Do the Simpler things first!
Test points if you are unsure!

Match each of the following functions with the correct graph:

(a) $f(x) + 0.5$ i)

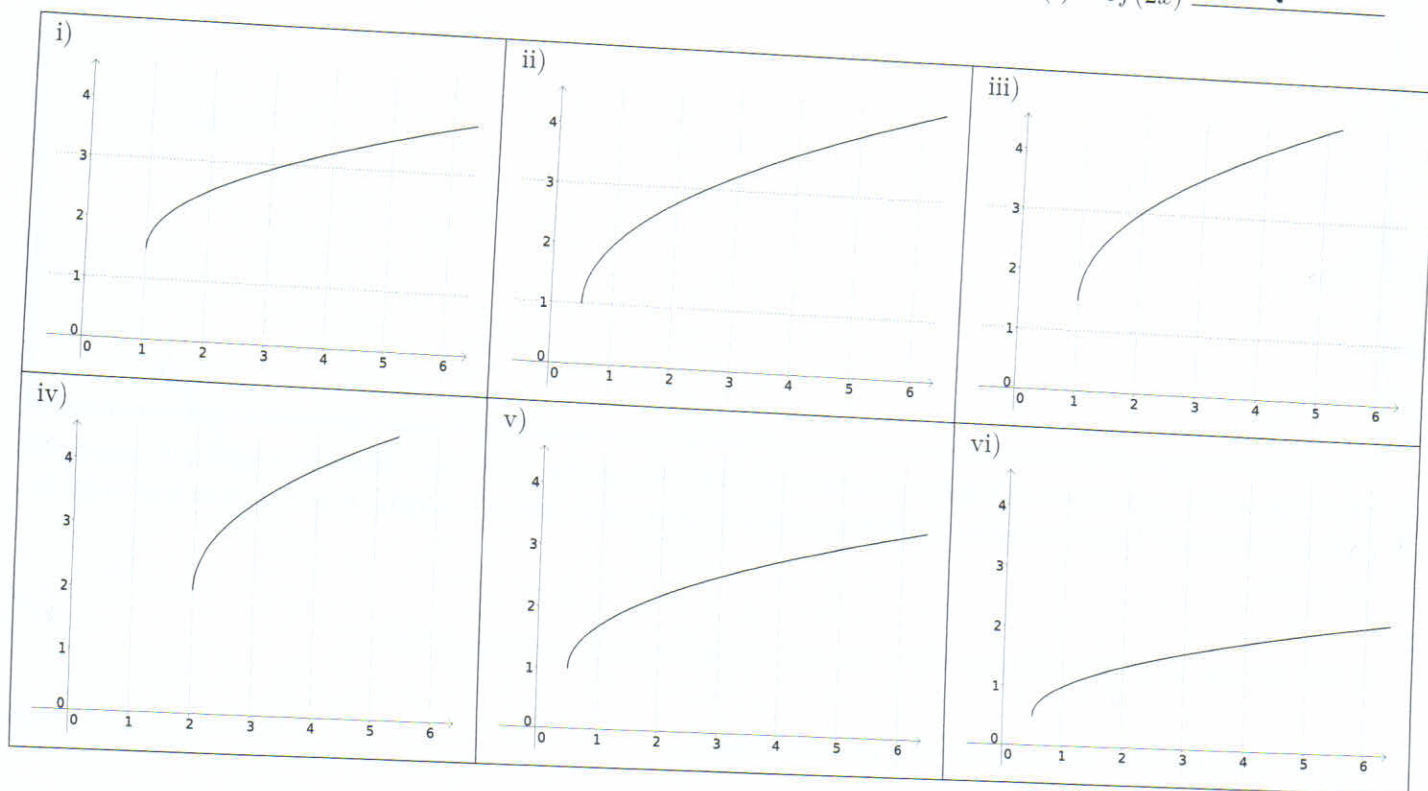
(b) $1.5f(x)$ ~~iii)~~ ii)

(c) $f(2x)$ ~~ii)~~ i)

(d) $2f(0.5x)$ iv)

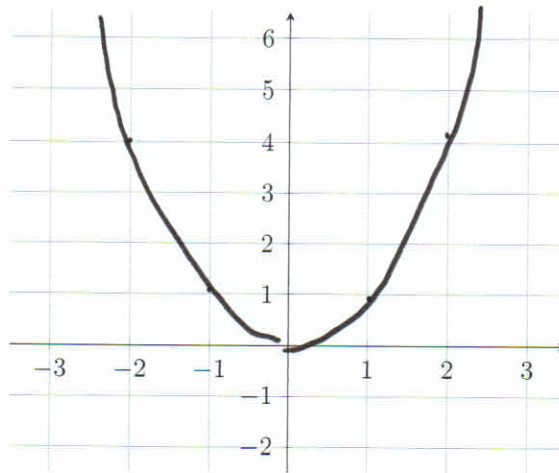
(e) $f(x + 0.5)$ ~~i)~~ v)

(f) $0.5f(2x)$ vi)



Worksheet 5.6: Combining Transformations

Problem 2. Let $f(x) = x^2$. Graph $f(x)$ on the axes below:



(a) Consider the point $(-1, 1)$ on the graph of $f(x)$.

- What point would be on the graph $g(x)$ given by first shifting $f(x)$ up by 1 unit, then stretching it vertically by a factor of 3?

$$(-1, 6)$$

- What point would be on the graph $h(x)$ given by first stretching $f(x)$ vertically by a factor of 3, then shifting it up by 1 unit?

$$(-1, 4)$$

(b) Did changing the order of the transformations change the point?

Yes!

(c) There are two formulas indicating shifting $f(x)$ up by 1 and stretching vertically by a factor of 3: $3f(x) + 1$ and $3(f(x) + 1)$. Try to determine which of these is $g(x)$ and which is $h(x)$. *Hint: Think about order of operations.*

- $g(x)$:

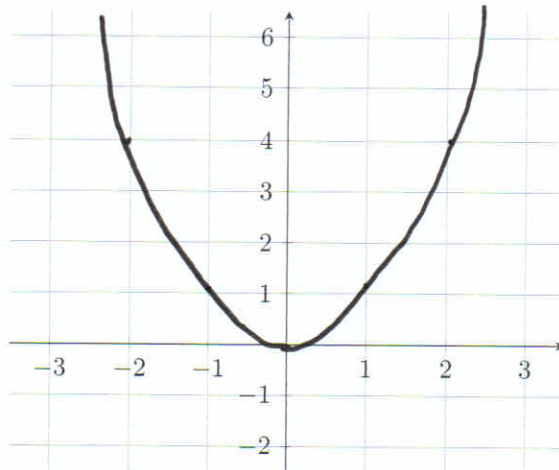
$$3(f(x) + 1)$$

- $h(x)$:

$$3f(x) + 1$$

Worksheet 5.6: Combining Transformations

Problem 3. Let $f(x) = x^2$. Graph $f(x)$ on the axes below:



(a) Consider the point $(-1, 1)$ on the graph of $f(x)$.

- What point would be on the graph $g(x)$ given by first shifting $f(x)$ left by 1 unit, then compressing it horizontally by a factor of 3?

$$\left(-\frac{2}{3}, 1\right)$$

- What point would be on the graph $h(x)$ given by first compressing $f(x)$ horizontally by a factor of 3, then shifting it left by 1 unit?

$$\left(-\frac{4}{3}, 1\right)$$

(b) Did changing the order of the transformations change the point?

Yes

(c) There are two formulas which indicate that we are shifting left 1 and compressing horizontally by a factor of 3 (much like on the front side of this worksheet). What do you think these two formulas are? *Hint: Think about order of operations.*

$$g(x) = f(3x+1)$$
$$h(x) = f(3(x+1))$$

Worksheet 5.6: Combining Transformations

Problem 4. If $(-1, 3)$ is on the graph of $f(x)$, what point is on the graph of

Trick: Find right x value to get -1 inside function.

a) $f(-x) + 1$?

$$\begin{aligned} \text{If } x &= 1 \\ f(-1) + 1 \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

$(1, 4)$

b) $2f(x)$?

$$\begin{aligned} \text{If } x &= -1 \\ 2f(x) &= 2 \cdot (3) \\ &= 6 \end{aligned}$$

$(-1, 6)$

c) $f(0.5x)$?

$$\begin{aligned} \text{If } x &= 2 \\ f(0.5 \cdot 2) \\ &= f(1) = 3 \end{aligned}$$

$(2, 3)$

d) $0.5f(2x)$?

$$\begin{aligned} \text{If } x &= \frac{1}{2} \\ 0.5f(2 \cdot \frac{1}{2}) \\ &= 0.5f(1) \\ &= 0.5 \cdot 3 \\ &= 1.5 \end{aligned}$$

$(\frac{1}{2}, 1.5)$

Problem 5. Describe the graphs of the following as the result of applying shifts, reflections, stretches, and/or compressions to the graph of $y = m(x)$. Note that, in some cases, order matters.

(a) $y = m(\frac{1}{5}x) - 3$

- ① Stretch Horiz. by a factor of 5
- ② Shift down by 3

(b) $y = 3m(x) + 14$

- ① Stretch vert. by a factor of 3
- ② Shift up by 14.

(c) $y = -(m(\frac{1}{4}(x+3))) - 20$ *← Not in standard form! Put into standard form first!*

$$= -m(\frac{1}{4}(x+3)) - 20$$

- ① Stretch Horizontally by factor of 4
- ② Shift left by 3
- ③ Reflect across x-axis
- ④ Shift up 20.

(d) $y = 3m(-9x - 18) + 2$ *← Not in standard form! Put in standard form first!*

$$= 3m(-9(x+2)) + 2$$

You can switch these

- ① Reflect across y-axis
- ② Compress horiz. by factor of 9
- ③ Shift right by 2
- ④ Stretch vertically by a factor of 3
- ⑤ Shift up by 2

Worksheet 5.6: Combining Transformations

Plot a couple of points!

Problem 6. The graph of $f(x)$ appears in the upper left square. Sketch the graph of the remaining functions.

