

Contents lists available at ScienceDirect

# **Engineering Applications of Artificial Intelligence**

journal homepage: www.elsevier.com/locate/engappai



# SLAM; definition and evolution

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#### ARTICLE INFO

Keywords:
Intelligent robots
Simultaneous localization and mapping
SLAM algorithms
SLAM history
Intelligent agent

#### ABSTRACT

Simultaneous Localization and Mapping (SLAM) is a key problem in the field of Artificial Intelligence and mobile robotics that addresses the problem of localization and mapping when a prior map of the workspace is not accessible. The determination of the SLAM problem has gained significant research momentum up to recent times. In this paper, firstly the problem of SLAM, its general model, framework, the difficulties, and leading approaches are described. Secondly, the progress of SLAM solving algorithms is surveyed throughout history. Pre-development, early SLAM solving algorithms, recent and present methods are presented and the progression of the state-of-art is reviewed based on the impact of leading approaches. We have selected some of the most important approaches of all time (1986–2019) to understand the research development, current trends, and intellectual structure of SLAM. Furthermore, from the trend of recent studies and the existence of difficult problems, a brief but sufficient review in the visual SLAM with the most outstanding approaches is presented. This paper provides one single sufficient review that allows researchers to understand the trend of SLAM, where it has come from, where it is going to and what needs to be more investigated in the SLAM-related field area. The future, in other words, the potential most important approaches inspiring the future researches in the SLAM problem can be seen. This paper will be an efficient overview and a valuable survey for introducing the SLAM solving approaches in mobile robotics as well as the general application of SLAM.

### 1. Introduction

SLAM or simultaneous localization and mapping is a challenging problem in mobile robotics under the area of artificial intelligence that has been taken widely under study for more than two decades where scientists use different techniques to improve autonomy and self-exploration of robot navigation. An autonomous robot is an intelligent system that can navigate through an environment without any human interference. A robot's successful navigation requires the robot to have a good understanding of its environment and to have a steady and accurate tracking of its location within the environment. The robot's location is often described as a robot state which defines the robot's pose including the robot's position and orientation in the map. The map is a set of features describing the environment e.g. walls, obstacles, landmarks, etc. The map is an essential requirement for a mobile robot to find its location in the environment and to complete its path planning tasks accordingly. The task of a robot estimating its poses in known environments using the sensor's data, in other words within a prior given map is termed as localization (Thrun et al., 2005). There have been many different types of researches conducted on the localization problem regarding different criteria as well as different environments. However, a fully intelligent mobile robot should be also capable of exploring the unknown environments where a prior map is either inaccessible or it is rather limited. The emergence of many

indoor applications where the Global Positioning System (GPS) cannot be accessible for localizing the robot, a well-established system to estimate the robot poses is essential. On the other hand, the appeal of applications to remove the user-built maps, have made the requirement of mapping essential for intelligent mobile robots. SLAM solution is the key problem-solving method for such scenarios where the prior knowledge/map is not either accessible or desirable. In other words, SLAM refers to techniques that solve the problem of building the map of the environment without any prior knowledge and at the same time localizing the robot into this built-map without any human interference. It can be said that it provides the ability of the robot's operation without any ad-hoc localization infrastructure. Although, due to the existence of the uncertainty in robot's movements which is a product of many factors (e.g. drifts, dead-reckoning, etc.), the robot needs a map (e.g. a set of recognizable landmarks) to correct its misalignments in coordination with the map. Many scholars have been conducting different approaches to achieve more reliable and faster techniques to solve the SLAM problem with better performance. More than 25 years of researches and experiments on SLAM have expanded it to be a practical, essential, and crucial technology with many indoor and outdoor applications. Many different types of SLAM solving methods have been developed and researchers continue to find better techniques and work on new challenges day by day. Future SLAM methods must deliver fully

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autonomous navigation in any environment such as indoor, outdoor, underwater, air, etc. with the least estimation errors, long lifelong and the fastest mapping while considering the low-power and low computational cost operations and as well as the simplicity. In this paper, we firstly offer a general understanding of SLAM by introducing different aspects and providing challenges and difficulties. Then we go through the key outstanding studies that have had an impact on different approaches throughout history regarding the key leading studies based on the described aspects and terms. The SLAM solving methods are reviewed based on the filtering point of view, application utilization, sensing-based evolution, and the commonly taken in use methods. There have been several surveys in the SLAM area. Younes et al. (2017) reviewed SLAM focusing on keyframe-based visual SLAM methods, which is more concerned with image processing. Saeedi et al. (2016) reviewed the SLAM solutions for multiple mobile robots that focus on cooperating multiple agents. Lowry et al. (2016) reviewed place recognition methods regarding visual SLAM that concerns with environmental visual features. Cadena et al. (2016a) proposed a comprehensive book for recent SLAM methods until 2016 however the presence of the existed works is quite limited in this paper, Taketomi et al. (2017) reviewed visual SLAM methods from 2010 to 2016 which is restricted to visual SLAM during a limited period. Bresson et al. (2017) reviewed SLAM algorithms concentrating on self-driving cars considering long-term maps in different sessional conditions. However, this review also is limited to a specific application. Sualeh and Kim (2019) conducted a survey on SLAM limited to the last decade with its main focus on visual SLAM methods. Zedadra et al. (2017) studied the swarm intelligent based Foraging Problem regarding the SLAM solving based on the Internet of Things (IoT) for mobile devices that focus on network signals and WIFI-based SLAM problem. In this study, different views of the problem are proposed and the classical definitions, as well as an extension to definition, establishment, and comparison of the different existing models, are presented. However, this study also is concentrated on multi-agent and specific techniques. The existing surveys mostly concentrate on one specific aspect or approach of the term regardless of the evolution of the state of the art taking place in pioneer approaches. In other words, the leading works that have either further expanded an approach or changed the direction of the study based on the time still require to be further investigated. In this paper, we review the general SLAM state of the art algorithms throughout history from the beginning to the current time. We indicate the starting point and the key solutions to the challenges that resulted in the development of the most popularly utilized recent SLAM solving techniques. From the forming point of the problem to the current state of the most commonly used methods are introduced that show what has been done towards the term. We cover the key major approaches rather than focusing only on one aspect to indicate the links between the studies that has let the SLAM methods to reach its current state. Moreover, we review the SLAM evolution from filter-base, application and environment based as well as the sensing-based view that covers the key interesting leading approaches designed throughout the history. We update state-of-the-art based on the timeline evolution of presented aspects of SLAM solving approaches rather than the problem appearance and the extension point of view. The future outlook is provided based on its evolution from the perspective of our review. However, there have been many studies been conducted in this field that we cannot mention all of them, so we selected more signification studies concerning their impact on the different directions regarding artificial intelligence-based approaches taken towards the SLAM solving techniques. The rest of the paper is organized as follows. Section 2 presents the structure of the SLAM problem, features, and process overview. Section 3 discusses the difficulties and challenges. Section 4 talks about SLAM history until the recent state. Section 5 discusses visual SLAM. Section 6 is the paper's conclusion.

#### 2. Simultaneous Localization and Mapping (SLAM)

In the past two decades, SLAM solving techniques have had a fast progression. Various SLAM algorithms are developed that use various sensors such as ultrasonic sensors, laser scanners, Red Green Blue (RGB) cameras, etc. for estimating robot's pose and simultaneously building the two-Dimensional (D) or 3D maps. It is said that the 2D SLAM problem with using range finders is thought out as a solved problem (Li et al., 2016), however, this statement is not correct, since the accuracy, speed performance, and the ability to process big data regarding the 2-D SLAM problem growingly require improvements. On the other hand, high quality, robust and 3D visual slams on unmanned systems such as Unmanned Aerial Vehicles (UAVs), self-driving cars, building inspections, surveillance, underwater SLAM and so many other problems related mostly to outdoors, unstable and dynamic environments are also of the remaining of the most challenging unsolved problems. Moreover, there is always a place for any technique to be improved. Instability of perception or uncertainty is mostly the main reason for SLAM failures. SLAM naturally has two parts, Localization, and Mapping. At the beginning of SLAM technology, the mapping and the localization used to be considered separately. Modern researchers discovered that the localization and the mapping are extremely internally dependent on each other. The map is required for precise localization, meanwhile, the localization is essential for mapping. So that the term is referred to as a "Chicken and egg" question. Classical SLAM algorithms estimate the poses and the map cooperatively, but later more advanced techniques considered the localization and mapping as two parallel tasks and so the Parallel Tracking and Mapping (PTAM) (Klein and Murray, 2007) became well known. Odometry in robotic refers to the result of motion integration provided by the robot's motion sensors e.g. wheel encoders, to estimate the robot's motion over time. Several types of odometry models based on varying types of sensors, for instance, Visual Odometry (VO), i.e. odometry using cameras, have been greatly developed. A considerable amount of uncertainty in robot navigation can be eliminated by enhancing the accuracy of the odometry model that results in an enhancement in the mapping process as well. Mapping plays an important role mainly in three key aspects:

- Maps are needed to enable the path planning, obstacle avoidance, etc.
- The map itself is the goal of many mobile robotics applications.
- The robustness and the accuracy of localization depend significantly on the mapping accuracy.

Loop closure is one of the most essential compounds of mapping that makes the robot be able to recognize a visited place and therefore being capable of optimizing its estimated pose. The loop closure reduces the drifts dramatically and lets the robot to correct its odometry errors. The recent visual odometry techniques usually include the mapping procedure too, but the map may not have any application in the path planning or other tasks in the local task execution. So, SLAM differs from the modern odometry models because of the global map optimization, in other words, due to the loop closure (Cadena et al., 2016b). Positioning is an essential concern of SLAM. The techniques for solving the positioning difficulties are classified into the probabilistic and non-probabilistic approaches. Approaches relying on probabilistic methods are the mainstream classification. The probability methods are based on the Bayesian estimation method where mainly Particle Filters (PF) and Kalman Filters (KF) methods are used. The system inputs and outputs, the motion model's output, and the observed data and the observation outputs and the state noises are Gaussian distributions. The optimal estimation of the robot pose can be obtained by KF (Yavuz et al., 2009; Wang et al., 2013). KF-based SLAM approach has a great advantage in convergence and it maintains a domineering place among many solutions because of the simplicity in implementation, however, it suffers from the absence of a loop closure and the problem within

a data association that will ultimately affect the whole system state to fail. Furthermore, the application of KF is for linear systems but most of the real systems are usually nonlinear. Nonlinear systems can be linearized by Extended Kalman Filter (EKF) using the first-order Taylor expansion (Julier and Uhlmann, 2001). Unscented Kalman Filter (UKF) (Wan and van der Merwe, 2000) is a scheme of estimation for nonlinear distribution utilizing a sampling strategy (Julier, 2003) as well that has a more eminent accuracy in linearization. It does not require to compute the Jacobian. However, the difference between EKF and UKF is somewhat small. It does not differ with EKF in linear systems and it works a little slower than EKF in nonlinear systems. EKF and UKF both are in the same complexity class as  $O(n^2)$ . A new SLAM approach was introduced in 1991 (Leonard and Durrant-Whyte. 1991a) which turned to be the standard implementation of SLAM. The technique used EKF to reduce the impact of the sensory information and the inaccuracy in the building map (Naminski, 2013). However, most of the novel methods, for instance, iSAM2 (Kaess et al., 2012) rely on nonlinear optimization and incremental filters but still, it is important to understand the basic and the primary standard implementation of SLAM techniques. An overview of a standard implementation of SLAM algorithm based on the EKF is as follows:

- Based on initial sensory information, the elementary map is built.
- The starting pose and landmarks around the robot are characterized
- As the robot moves around, the map information and the robot location are updated.
- · New landmarks are extracted from the updated map.
- The robot associates the new landmarks with previous ones and localizes itself according to the new landmarks.
- At each step, new path planning is driven based on the new map until the robot successfully reaches its goal.

Fig. 1shows a common block diagram of a feature-based SLAM problem (Leonard and Durrant-Whyte, 1992). In a feature-based SLAM, a motion model generates a distribution around the robot's measured pose obtained from the robot's odometry or velocity information. An observation model samples the landmark information obtained from the sensors and a fusion block fuses the different estimations. A filtering-based method samples the robot's pose according to the observation model and matches the features to correct the robot's poses as well as the environment's feature poses while updating the map according to the information obtained at each stage. In the update stage, sampling and resampling simultaneously correct the map's features and the robot's pose following the map.

There are two topologies of SLAM related to the filter's state information that can be solved by using a Bayesian approach; online SLAM and Full SLAM depending on the filter's state vector information (Thrun et al., 2005; Durrant-Whyte and Bailey, 2006). In an Online SLAM method e.g. EKF SLAM, filters extract the current features of the map, and accordingly, estimate only the most recent pose of the robot, in other words, it recovers only the most recent pose of the robot. Eq. (1) describes an Online SLAM utilizing a well-known Bayesian rule.

$$bel(x_t, m) = p(x_t, m | z_{1:t}, u_{1:t}) \propto p(z_t | x_t, m)$$

$$\int_{x_{t-1}} p(x_t | x_{t-1}, u_t) bel(x_{t-1}, m) dx_{t-1}$$

$$(1)$$

where  $x_t$  represents the latest pose of the robot, m is a set of all landmarks,  $z_{1:t}$  is the set of landmark observations, and the control unit is included as  $u_{1:t}$ . This equation defines the problem of estimating the robot state x and the map of the environment m based on a series of controls  $u_{1:t}$ , and sensor observations  $z_{1:t}$ , by applying the Bayesian rule with coordinating the prior information with the weighted probability of the posterior. Fig. 2 shows the schematic of an Online SLAM system.

As it is indicated in the figure, at each state the system takes the robot's current pose and corresponds to its current measurements

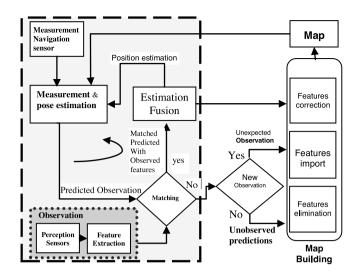


Fig. 1. General schematic of feature-based SLAM approaches.

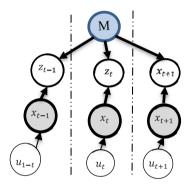


Fig. 2. Online SLAM.

associating with the map; either updating the map or matching the landmarks for localization.

In Full SLAM e.g. GraphSLAM (Thrun and Montemerlo, 2006) the current state and accordingly all the previous states of the robot and the map features are estimated, in other words, it estimates the entire path. Therefore, the computational cost in Full SLAM is relatively high (Buonocore et al., 2016). Eq. (2) represents the Full SLAM problem shown in Fig. 3.

$$bel(x_{0:T}, m) = p(x_{0:T}, m | z_{0:T}, u_{0:T})$$

$$= p(m | x_{0:T}, z_{0:T}) \cdot p(x_{0:T} | z_{0:T}, u_{0:T})$$
(2)

where  $x_{0:T} = \{x_0, x_1, \dots, x_T\}$  represents the trajectory obtained of every pose of the robot,  $m_{0:T} = \{m_0, m_1, \dots, m_T\}$  indicates the collection of all landmarks,  $z_{1:T} = \{z_0, z_1, \dots, z_T\}$  is a set containing all the observations and finally, a set of all control inputs is represented as  $u_{1:T} = \{u_0, u_1, \dots, u_T\}$ .

The equation solves the SLAM problem at each time by estimating the probability of a robot's pose according to the current observation meanwhile updating all previous states and observations according to the new estimated pose, meanwhile, the entire map of the environment is resampled by the probability of the landmarks as the posterior in coordinate to the probability of the robot poses as the prior. These solutions take the advantage of Bayesian rule to correlate between robot's motion model  $p\left(x_t|x_{t-1}\right)$  and the observation model  $p\left(z_t|x_t\right)$  but in contrast with online SLAM that is to re-estimate the whole trajectory which makes this approach relatively slower. However, some techniques e.g. FastSLAM or SEIF (Sparse Extended Information Filters) can also be used for both topologies (Buonocore et al., 2016).

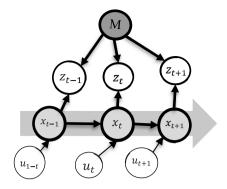


Fig. 3. Full SLAM.

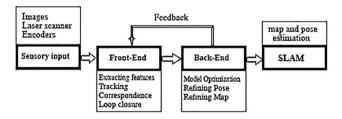


Fig. 4. SLAM architecture.

A SLAM system can be broken down into two partitions, the frontend, and back-end, shown in Fig. 4. The front-end takes the sensory raw data and does some preprocessing on data such as feature extraction, short and long-term data association, i.e. feature tracking and loop closure respectively to be able to transform the geometric information to the mathematical models and send it to the back-end.

In several approaches, the front-end usually uses computer vision and signal processing techniques. For instance, in the feature-based visual SLAM, tracking between frames is operated by the front-end part. The back-end provides feedback to the front end; it is responsible for optimizing the model e.g. in factor graph, refining the pose, and the map.

# 3. Difficulties and challenges

SLAM is a complicated problem in general. The utilization of low-level sensors makes it more difficult. To achieve a reliable SLAM technique, several uncertainty sources and issues need to be coped with. The major issues are uncertainty, the correspondence also called data association, and time complexity.

#### 3.1. Correspondence (data association)

After a long period of exploring an environment, the robot will return to its origin or it will visit previously seen mapped areas multiple times. Here a problem arises where the robot may fail to recognize a visited location. The correspondence which is also known as data association is a difficulty where the SLAM system needs to associate a currently observed landmark with the previously observed landmarks, in other words, estimating the landmark correspondence of the robot in backward to its starting point based on the previously built map is a challenging procedure. Data association errors simply occur when a robot has a wrong perception of the same landmark as the one perceived in another position (Neira and Tardos, 2001). A wrong associated landmark corrupts the mapping. The term also refers to inaccuracy in landmark recognition where an algorithm is not capable of identifying two slightly different features in the environment. For example, differentiating two different landmarks with a similar

shape is difficult, i.e. two different corridors in two different locations within the same environment seem to be the same to the robot that extremely affects the robot's analysis on its environment (Dissanayake et al., 2011; Khairuddin et al., 2015). Besides, due to the imperfect movements and error noises like drifts, etc., the actual motion of a robot may differ from its estimated movement which is another source for data association uncertainty to emerge (Ho et al., 2015). Data association and loop closure are interconnected tasks that are often taken into the same account. But, this statement is not quite correct because in some systems under different criteria, these two need to be handled separately. However, in several visual SLAM algorithms, data association is carried out within the loop closures (Davison et al., 2007b) which have the intention to recognize the previously visited areas during the navigation (Ho and Newman, 2007). When loops are detected the robot can more accurately perform the registration tasks, complete the mapping, and relocalize itself into the map (Williams et al., 2011). Loop closures can highly decrease the optimization errors by putting extra constraints into the pose graph in visual SLAM methods as well (Endres et al., 2014). There have been many descriptors used for loop closure methods such as Fisher Vector (FV) (Perronnin et al., 2012; Perronnin and Dance, 2007), Vector of Locally Aggregated Descriptors (VLAD) (Jegou et al., 2010; Arandjelovic and Zisserman, 2013), and Bag-of-Words (BoW) that is a popular approach which clusters visual features into a set called "dictionary" (Filliat, 2007; Kwon et al., 2013). In the same way, several descriptors are developed to use features of local images (Bay et al., 2008; Lowe, 1999; Rublee et al., 2011). In some approaches such as GIST (Oliva and Torralba, 2001) global descriptors are used. Particle filters and graph-based methods are two common methods that address the local error accumulation (Konolige et al., 2010b; Anon, 2005). Particle filters maintain a representation of a full system-state in every individual particle. Graph-based methods operate on a set of nodes that each node is a representative pose and feature and the edges are constraints created based on observations. Most of the visual-SLAM loop closures are based on the pose graph where a series of adjacent key-frames built the trajectory (Henry et al., 2012; Chen et al., 2007; Strasdat et al., 2012; Tian et al., 2013). The simplest way of detecting loops is wisely paired matching the features of all key-frames, however, this needs a high time complexity. A more effective approach is to randomly check the subset from the previously extracted frames, which has been utilized in several applications (Endres et al., 2014; Stuckler and Behnke, 2014). Scan-to-scan matching approaches (Olson, 2015; Konolige et al., 2010b; Lu and Milios, 1997; Martin et al., 2014) directly calculate comparative pose deviations in laser-based extracted features, Scan-to-map matching uses Gauss-Newton for finding local optima on linear interpolated maps (Kohlbrecher et al., 2011) and Pixel-accurate scan matching e.g. (Olson, 2015) are some the efficient approaches. Many methods extract features from the laser scanner data to reduce the computational cost (Martin et al., 2014). Some loop closure detecting methods use machine learning (Granstrom et al., 2011) and histogram-based matching (Himstedt et al., 2014) for the detected feature. For instance, Bosse and Zlot (2008) proposed a SLAM method for outdoor environments that utilizes a graph-based method, local scan-to-scan matching approach, and overlapping local map matching that works based on histograms of features in the submap. Hess et al. (2016) designed a real-time loop closure method using the branch-andbound approach to build constrains based on scan-to-submap matching. Gao and Zhang (2017) suggested a loop closure solution using a multilayer neural network within a visual SLAM. This system used a Stacked Denoising Auto-encoder (SDA) method to autonomously learn a compressed representation from raw data obtained from a visual sensor. This deep network has the capability of learning a complex inner

structure from images without requiring any visual features.

#### 3.2. Uncertainty

There are two main types of uncertainty in SLAM that affect the robot's capability and performance, location, and hardware uncertainty (Pirahansiah et al., 2013). Hardware uncertainty arises due to hardware errors and noises in the robot's components which causes the wrong and inaccurate information to be extracted and so the inaccurate information results in the inaccurate analysis of the pose, landmarks, and other calculating factors. Location uncertainty arises due to the mobile robot's location and the existence of multiple paths in environments. The movement of a mobile robot from A location to B is simple on a single linear path where the robot can trackback its origin point and it can be easily recognized (Pascal and Kuhn, 2013), although, in the reality, there are multiple pathways for a robot to navigate from the A location to B, So that a high degree of uncertainty happens in mobile robot's location (Khairuddin et al., 2015).

#### 3.3. Sensor's noise/observation error

Sensors as one of the components of the robot's hardware are imperfect and so the sensory information slightly differs from reality. For instance, the changes in climate and physical factors of an environment may vary the output of sensors, for example in terms of using visual sensors such as cameras, the changes in light may affect the perception. A well-detailed study on noise in sensors has been done by Bordoni and Damico (1990). Besides, the computed measurements are outputted from some measurement models which are the mathematical relations to compute the corresponding between the sensory signals and the landmark's states to output the relative information which may not be accurate. The small errors in the perception gradually grow in long-term navigation that will cause the system to fail. Low-level quality sensors make it even worse. So, observation error is an important issue in SLAM that requires efficient solutions.

# 3.4. Time complexity

The number of commands that an algorithm executes in its running time is called time complexity and this number relies on the size of the algorithm's input which in the SLAM system is usually approximately computed based on the number of the landmarks and the computations that it needs for each landmark. The time complexity of techniques is described by the O-notation. For instance, O(N2). The expression O is also called Landau's symbol stands for a set of functions that do not grow faster than the function N<sup>2</sup> in a long time run. In general, the time complexity of the algorithm is important because it makes a huge difference as to if the algorithm whether be practical to be applied in large instances. For as SLAM methods, navigation, mapping, and localization are tasks that need to be operated simultaneously, obviously, any delay in such multiple processing should be avoided, because a delay in one part may result in the other parts failures. For instance, the robot's localization at t=1 needs the mapping at t=1 otherwise wrong states will be outputted to the robot's map as its current state. Moreover, after all, any algorithm must spend reasonable time to finish its task, hence SLAM techniques require to be in fast implemented and calculated methods. Time complexity and computational complexity are internally dependent factors that should be considered in driving a SLAM technique, especially when the landmarks grow in number. Simply because, the more computations of more landmarks the longer an algorithm needs and so the computational complexity results in the time complexity (Ho et al., 2015; Khairuddin et al., 2015). A SLAM technique with unknown correspondence e.g. FastSLAM requires having at maximum a logarithmic time complexity otherwise in terms of quadratic complexity it cannot finish its task in large environments. To name a few, in the Standard EKF-based SLAM (Extended Kalman Filter based) time complexity is quadratic O(N2), some Optimal EKF-based SLAM, for example, the technique by Davison (1998)

obtained a constant time complexity, the Sparse Weight Kalman Filter (SWKF) approach has a linear computational complexity (Julier, 2001), the Computational cost of the Global-Map Postponement approach is O(N) (Nerurkar and Roumeliotis, 2007) and FastSLAM can be executed in O(M log N) (Thrun et al., 2004c).

### 4. Evaluation of SLAM (past 1985-1999)

Throughout the past decades, many scholars have conducted different studies on different aspects of the SLAM problem. Here we take a look at the SLAM evolution history.

The starting point of the SLAM's pre-development occurred in San Francisco 1986 where R.C. Smith with his partner Cheeseman (Smith and Cheeseman, 1986) introduce the scheme of applying the estimated spatial uncertainty in the IEEE ICRA conference. They indicated in this study that estimation on the uncertainty in the relation between a frame to another frame can be obtained and reducing the uncertainty can be mapped into any frame due to sensory information of frames, no matter where the measurement is performed. This makes the algorithm not to be dependent on any specific frame as its fixed reference. Many of the earliest SLAM pioneers such as Peter Cheeseman, Jim Crowley, Hugh Durrant-Whyte, Raja Chatila, Oliver Faugeras, and Randal Smith were introduced. For a few years, most of the progression was conducted for reducing the uncertainty in maps (Smith and Cheeseman, 1986; Durrant-Whyte, 1988; Chatila and Laumond, 1985; Crowley, 1989; Ayache and Faugeras, 1988). In 1988 the most influential paper was conducted by Smith, Self, and Cheeseman (Smith et al., 1988) in which the stochastic map framework was introduced. This study was conducted to estimate the uncertain spatial relationships in robotics. Although some essential properties of this technique towards the SLAM solution were not realized in this paper, the work covered the essence of the SLAM problem using Kalman filter. The technique led to an approach that is referred to as the Absolute Map Filter or AMF (Smith et al., 1987). AMF was a technique that could estimate a state vector containing the robot's pose and the landmark location in a global coordinate frame. In more detail, the AMF identifies a state covariance matrix in which the weight or the importance of robot to landmark, the landmark to landmark, and landmark correlations were carried out. However, The AMF technique suffers from a square computation complexity at its best that grows with the number of features in a mapped area. The complexity is the result of the essential generation of the state cross correlativity carried out by the Kalman filter. However, the AMF algorithm was applied in many studies (Csorba, 1997; Castellanos et al., 1998) and as well as being improved in several studies (Leonard and Durrant-Whyte, 1991b, 1992), but the fact is that the computational complexity and the map complexity prevented it to be used as a standard SLAM solution. In the structure of the AMF method, the state of the robot and all landmark's states are contained in  $X_a$  which is an augmented state vector as follow:

$$X_{a}(t) = \begin{bmatrix} X_{R}(t) \\ L_{1} \\ \vdots \\ L_{N} \end{bmatrix}$$

$$(3)$$

Fig. 5 shows the state vector including the robot's state  $X_R$  and landmark states  $L_i$ , i=1..N. The Robot's state  $X_R$  at time t is driven as follow:

$$X_{R}(t+1) = F_{R}X_{R}(t) + u_{R}(t+1) + e_{R}(t+1)$$
(4

where the  $F_R$  indicates the robot's state transition model,  $u_R$  is the robot's control input and  $e_R$  is a temporally independent noise vector. A noise-free ith landmark's model in a linear discrete-time manner is driven as follow:

$$L_i(t+1) = L_i(t) = L_i, i = 1 \dots N$$
 (5)

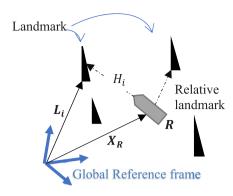


Fig. 5. The AMF state vector.

L indicates a landmark and the relative pose of landmarks concerning the robot's pose is obtained from sensory readings through the observation model which is assumed to be concurrent and linear, as follow:

$$z_i(t) = H_i X_R(t) + e_i(t)$$
(6)

$$H_i = [-H_R \dots 0 \dots H_{Li} \dots 0 \dots]$$
 (7)

where  $X_R$  is the robot's state obtained from robot's kinematics and its relations to the world frame, landmark states  $L_i$ ,  $H_i$  is the observation model and  $e_i(t)$  is the observation error vector. The structure indicates that the observations between the robot and landmarks are relative and it is shown in the form of relative range and bearing in the form of  $H_i$ .

Since the appearance of AMF, many studies had been conducted to deal with the computation complexity and at the same time maintaining the key properties of the theoretic estimation algorithm. Some studies took a 'Divide and Conquer' approach that takes the localization and Mapping as two independent sections of a greater problem that this separating approach results in a remarkable disadvantage towards the SLAM solving. Because, ignoring the cross-correlation between the localization and map building assignments violates the principles of Kalman filter operation and it results in an inconsistency, a vast uncertainty, and an unpredicted amount of errors in the map and the robot's state estimations. As the thesis by Newman (1999) shows, the task-portioning solution ruins the whole framework of the SLAM problem. The importance of cross-correlations evolvement in time was shown in a work by Moutarlier and Chatila (1989) where they expend the concept of the stochastic map presented by Smith et al. (1987) and Smith et al. (1988). They modified the Kalman filter (KF) to be used in an approach assuming that the robot's uncertainty could be affected by the feature uncertainty while the uncertainty in the robot's location is not allowed to affect the feature uncertainty. This assumption avoids the robot state modeling to corrupt the landmark location estimations. Although the computational cost of this technique also is highly dependent on the map size that leads the algorithm to fail in case of large environments. Csorba (1997) developed an algorithm to build a map independent of the robot's location's uncertainty. The technique was based on the works by Jircitano et al. (1990) and Yeh and Kriegman (1995) where the algorithm extracts some independent features and constants of the environment. The relative map presented by Csorb technique is the estimation of the environment's states which is been taken in isolation and it stores only the pose of landmarks in correlation to each other without concern about the robot's pose estimations. Later, this algorithm was called the Relative Map Filter (RMF) that some SLAM solution techniques further studied this technique towards the term. This method uses the estimated uncorrelated maps to estimate the robot's pose using the Kalman filter. The computational complexity of the filter is independent to the map size, however, as the study by Newman (1999) shows, the RMF generally is not representing a consistent landmark location. In 1991 Leonard and Durrant-Whyte developed a SLAM algorithm using the probabilistic approach (Leonard and Durrant-Whyte, 1991a). This technique which was based on the work by Smith and Cheeseman (1986) indicating a technique to implement the Extended Kalman filter that led it to be called EKF-SLAM, the first officially SLAM algorithm. Table 1 presents the most important starting developments of SLAM. Renken (1993) developed an algorithm using ultrasonic sensors wherein the mapping and the localization tasks are executed separately but handled synchronously. This technique assumes that the robot's actual state and its estimations are not correlated. For this reason, the drifts and the uncertainty in robot's motion has a vital impact on the map although the technique tried to counter with this problem by ad hoc tuning on several parameters, it still is not accurate enough. Chong and Kleeman (1997) divided the map into several sub-maps and save them relative to each other in order to have them available for the robot's localization at any time. Although the effect of eliminating the cross-correlation in the filter intersection is unknown because it is not been examined. In a work, by Uhlmann et al. (1997) a technique was presented to provide a recursive sensory data fusing with no need of the dependent correlation information. The method of the data fusion named 'Covariance Intersect' does not rely on the information of state cross-correlations that means the robot can handle fusion robot and feature estimations on time without saving them. Due to the fact that the cross-correlation information does not need to be stored, the computational cost of the algorithm is low but the estimated data are not efficient enough due to the conservative assumptions on the data fusion that consequently it did not make it be a promising standard SLAM solution method. In 1999 Anousaki and Kyriakopoulos (1999) examined an approach towards solving the SLAM problem using ultrasonic sensors by combining model-building modules and localization that makes the robot to be able to map an infinite workspace. This technique uses a combination of the range data from the ultrasonic sensors and the dead-reckoning from the odometry to build an occupancy grid as the map and it updates this map as the robot travels through the environment. The robot uses the information obtained from the sensors in an Extended Kalman filter in its localization module. In this technique also the map is divided into sub-maps as separate cells weighted with probabilistic distributions. The environment's features are extracted using the Hough Transform (HT) and the Clustering algorithm. The localization and the mapping are taken as parallel and separate modules but synchronous that leads the robot to correct its map in its localization module but it still does not offer more accuracy in the built map because the robot suffers from the uncertainties in its motion. The approach requires a longer time for mapping than the localization. It also suffers from the poor measurement of ultrasonic sensors especially in terms of fast navigations and in circumstances where the obstacles are too close to the robot that the sensors are not able to detect. Castellanos et al. (1999) presented a complete framework of a feature-based SLAM solving approach using a 2D laser rangefinder wherein the Symmetries and Perturbations map (SPmap) is produced based on the general representation of the estimated geometric information. This technique which is a combination of probability theory for representing the uncertainty in the feature locations and the theory of symmetries for representing the partiality specified for a different type of geometric elements. It also copes with singularities in the uncertainty representation and it can completely represent the whole environment. The key difference of this technique is the fact that the localization takes the part assuming that the map is fixed and map features are taken static which makes it be an unreliable method for large or dynamic environments, besides of the fact that the optimal representation of the navigation alone has O(N2) complexity time. Another problem with this technique also was that it does not deal with data association as revisiting a place is taken as a new place. There have been many studies attempted to overcome the computational cost problem (Castellanos et al., 1999; Guivant and Nebot, 2001; Thrun et al., 2004b; Paz et al., 2008; Thrun et al., 2004d) and many studies have tried to improve the efficiency of data association in EKF-SLAM (Cox, 1993; Dissanayake et al., 2001; Pedraza et al., 2009).

Table 1 SLAM pre-development.

Year	Title	Reference		
1985	Position referencing and consistent world modeling for mobile robots	Chatila and Laumond (1985)		
1986	On the representation and estimation of spatial uncertainty	Smith and Cheeseman (1986)		
1987	A stochastic map for uncertain spatial relationships (AMF)	Smith et al. (1987)		
1988	Uncertain Geometry in Robotics	Durrant-Whyte (1988)		
1988	Building, Registering, and Fusing Noisy Visual Maps	Ayache and Faugeras (1988)		
1988	Estimating Uncertain Spatial Relationships in Robotics	Smith et al. (1988)		
1989	World Modeling and Position Estimation for a Mobile Robot using Ultra-Sonic Ranging	Crowley (1989)		
1989	Stochastic Multisensory Data Fusion for Mobile Robot Location and Environment Modeling	Moutarlier and Chatila (1989)		
1990	Gravity based navigation of AUV's	Jircitano et al. (1990)		
1991	Mobile robot localization by tracking geometric beacons	Leonard and Durrant-Whyte (1991a		
1991	Simultaneous map building and localization for an autonomous mobile robot.	Leonard and Durrant-Whyte (1991)		
1992	Directed Sonar Sensing for Mobile Robot Navigation. Norwood	Leonard and Durrant-Whyte (1992)		
1993	A review of statistical data association techniques for motion correspondence	Cox (1993)		
1993	Concurrent localization and map building for mobile robots using ultrasonic sensors	Renken (1993)		
1995	Toward selecting and recognizing natural landmarks.	Yeh and Kriegman (1995)		
1997	Large scale sonarray mapping using multiple connected local maps	Chong and Kleeman (1997)		
1997	Nondivergent simultaneous map building and localization using covariance intersection	Uhlmann et al. (1997)		
1997	Simultaneous Localization and Map Building	Csorba (1997)		
1998	Simultaneous map building and localization for mobile robots: A multisensor fusion approach.	Castellanos et al. (1998)		
1999	Simultaneous Localization and Map Building for Mobile Robot Navigation	Anousaki and Kyriakopoulos (1999)		
1999	The SPmap: A Probabilistic Framework for Simultaneous Localization and Map Building	Castellanos et al. (1999)		

# 5. Development of SLAM algorithms (2001-2019)

Ever since the SLAM problem has been termed, many scholars and scientists have conducted different types of researches and projects on it which mostly are based on two approaches, filter-based methods and optimization-based approaches (Sunderhauf, 2012).

# 5.1. SLAM probabilistic approaches

Probabilistic methods which are different mathematical derivations of the Bayesian rule are the leading approaches towards filter based SLAM solutions because the robot maps are often determined by uncertainties and noise, the probabilistic approaches can model various sources of uncertainties (Thrun, 2002). Kalman Filters (KF), Extended KF (EKF), Compressed EKF (CEKF), Unscented KF (UKF), Adaptive Kalman Filter (AKF), Particle Filters (PF), RaoBlackwellized PF, Information Filters (IF), Extended IF (EIF), Sparse EIF (SEIF), Expectation—Maximization (EM) are some of the most common probabilistic approaches used in SLAM solving methods. A review of SLAM methods based on these filters is done by Aulinas et al. (2008).

Optimization or Graph-based approaches often utilize a primary graph scheme for representing the robot's observations. This graph is contained of nodes in which each of them represents a robot's pose as well as the measurements obtained at the pose. The edges in the graph indicate the spatial constraints associated with two robot poses. The

optimization process takes place after the construction of the graph and it is responsible for finding the best configuration among the robot's poses according to the constructed constraints. A constraint is mostly covering a transformation between a robot's pose with its relative pose. Optimization-based methods are mostly obtained of two parts, the graph construction which is in charge of building a graph from sensory readings, and graph optimization which is in charge of estimating the right configuration of the poses based on the graph's edges.

#### 5.1.1. Kalman based approaches

There have been many SLAM methods based on various KF (Davison and Murray, 2002; Newman and Leonard, 2003; Jensfelt et al., 2006; Se et al., 2002) that each attempt to enhance some aspect of the problem. For example, The Unscented KF (Wan and van der Merwe, 2001) improves the approximation problem of the EKF and also it addresses the linearity requirement of the KF.

Traditionally, most SLAM solving techniques use the EKF approach which may introduce contrast and divergences due to the model inaccuracy. Moreover, since every received observation influences all parameters withing the Gaussian, the update state takes a long time to be carried out. This makes the EKF not been able to estimate maps with high dimensions and also it makes EKF based SLAM methods very slow in environments surrounding with many complex features (Thrun et al., 2004a).

In 2016, X. Jiang and T. Li introduced a new method using the Adaptive Kalman Filter (AKF) (Jiang and Li, 2016). This algorithm is capable of adjusting the parameters of the KF in real-time processing and efficiently improves the filtering efficiency and so, the technique can improve the accuracy of localization and overcomes the difficulty of information mismatch that results in an improvement in the mapping. Tian et al. (2020) proposed an Adaptive version of EKF (AEKF) based on AKF approaches but with a recursive statistic noise formation based on building the Maximized Likelihood Estimation (MLE) around Expectation–Maximization (EM) which provide the capability of carrying on a statistic noise approximation. However, the computational cost of this approach is comparatively higher than the EKF and the prior AKF based approaches and it also suffers in data association as well as the landmark matching in the large complex environments.

All various of KF has an advantage of providing an optimal Minimum Mean-Square Error of the state estimations and their covariance matrixes can converge strongly, but the problem with KF based approaches is that the Gaussian noise hypothesis limits the flexibility of the KF regarding a big number of landmarks and the data association within large environments.

# 5.1.2. Submap-based filters

Submap based methods decompose the environment's map into several smaller maps in which one or more of the sub-maps is used for keeping track of the robot's state at each time. This approach makes the computational cost lower, however, the key problem with them is in merging the sub-maps into a global map. For example, The Compressed Filter (CF) also called Compressed EKF (CEKF) was proposed by Guivant and Nebot (2001) works similarly to EKF with a distinction that the local maps are conducted based on a predefined period which every time the algorithm reaches a certain time it incorporates the local map into the global map by a single Kalman update. Even though the computational cost of CF is considerably lower than the EKF based SLAM method, but this approach also similar to the Kalman filter suffers from the quadratic computational cost when the number of submaps growth. Moreover, if the global map did not have a common landmark with the local map, the algorithm will fail. The SubOptimal SLAM Filter (SSF) approach proposed by Shin et al. (2006) employs a limited number of landmarks of a local map and processes them at each performance. The number of local maps is independent of the number of the whole landmarks but since the size of the map at each time step grows according to the new observed landmarks, it makes the convergence considerably slow.

## 5.1.3. Tree-based filters

Assumed Density Filtering (ADF) executes the typical filtering process as long as the state believes is not passed over a determined complexity level, thereafter the ADF projects the belief into another type of distributions so-called Switching Kalman Filter (SKF) method which process the mixing of several Gaussians over time and projects them with a constant, representing the mixed components. Thin Junction Tree Filters (Paskin, 2004) (TJTF) and Treemap Filters (Frese, 2004) (TmF) also are other types of information-based filters that fall into a class of ADF methods with one difference that indicates their structure of the projection not be defined before the execution. Hence the TJTF and TmF have been considered as Adaptive ADF (AADF) methods as well. The online determination of the projection minimizes the estimation error. TJTF and TmF both are of the graph-based estimation for sparsifying a canonical-scheme. These methods which are indeed the graphical model of gaussian both can efficiently represent the features however the trees containing the representations obtained within these modes are not designed for dealing with the revisiting areas within the workspaces, and so in their nature, they have weak constraints. In other words, both of these approaches do not have a data association.

#### 5.1.4. Information Filters (IF)

Information Filters (IF) (Maybeck, 1979) propagates an inverse covariance matrix containing the state error and it filters the data by directly summing vector and matrixes containing the information. IF is also called the Inverse Covariance form of the Kalman Filter (ICKF) (Mutambara, 1998). IF is comparatively more accurate with more stability than KF (Thrun et al., 2004d; Thrun and Liu, 2003). However, IF requires the recovering of the state estimate at each update step which needs an inversion of the matrixes and the vectors that make the computational cost relatively high. Therefore, KF based methods have been more popularly utilized than IF based approaches (Thrun et al., 2006). Extended Information Filters (EIF) (Maybeck, 1979) is an IF but with an extension for nonlinear systems and it is computationally similar to EKF but with a difference that is the EIF carries an inverted covariance matrix. EIF based approaches also have been utilized to solve the SLAM problem (Nettleton et al., 2000a,b), however, the EKF is comparatively more popular than EIF. In 2004 Sparse Extended Information Filter (SEIF) algorithm was introduced by Thrun et al. (2004b) to illustrate the map by graphical grid cells representing the features. SEIF uses the conditional independence approach that allows the system to keep only a small set of landmarks joined with the robot at a time and it eliminates the links between the robot and its associated landmarks by the so-called sparsification that reduces the number of associated links between the landmarks. Moreover, SEIF uses the sparsity of the information matrix in all calculations. The graphical grids are topically unified where limitations represent the relative information between features of two neighbor points and the robot's position correlated to the map. In this technique measurement and motion updates always are carried on in a constant time so that their computational complexity will not be affected by the size of the maps. However, even though the complexity of SEIF is linear but comparing to EKF, it has a lower quality.

Joukhadar et al. (2019) attempted to solve the SLAM problem using the Unscented Kalman Filter (UKF) for landmark tracking and Invariant Feature Transformer (IFT) for landmark detection. In this method, the key idea is that the estimated pose provided by the robot's onboard sensors are not reliable and it results in a low Degree of Belief (DoB) in the robot's localization so that the main focus of the paper is to examine the probabilistic methods to find a more accuracy on the robot's localization part. However, the mapping procedure is not reported and the novelty of the technique is rather limited. A comparison between EKF localization based on a 2D map and a UKF based approach is provided that shows UKF has an advantage in handling the robot's localization with a high level of nonlinearities. Furthermore, the UKF based approach is been done under the assumption that the landmark's poses are prior known that makes the technique out of SLAM usage.

# 5.1.5. Particle filters (PF)

Del Moral (1996) for the first time proposed the term "Particle Filters" also termed as Sequential Monte-Carlo (SMC) by Liu and Rong (1998), that was the latest kind of filters able to efficiently solve the localization problem without considering the system to be linear, nor it needed the restrictions of the Gaussian noise assumption concerning the observations. PF can highly handle the nonlinearity and the non-gaussian models, the computation of PF is simple and it can estimate any likelihood distributions (Arulampalam et al., 2002). Methods based on PF keep several map hypotheses within all particles that each of them contains a robot's sampled stochastic trajectory within the navigation environment. However, the computational cost of PF grows dramatically as the state dimension and the number of the landmarks grows, meaning that this method is not applicable in real-time SLAM within complex environments (Nie et al., 2020). Many studies have been conducted to improve the PF-based SLAM methods to be capable of utilization into environments with similar features. In identical environments, a huge number of particles need to be carried out by the PF based SLAM techniques to reduce perception errors. The data association also is the main problem in similar environments. An improvement in the PF SLAM algorithm is conducted in the work by Zhang et al. (2014) based on optimizing particle swarm. The performance of ordinary particle filter SLAM is improved in this technique wherein the sensory information such as odometry and laser inputs are fused which forms a multimodal proposed distribution of particles with a concentration on the regions with the maximum value of each posterior probability distribution obtained by the particle swarm optimization. Table 2 comparatively shows the different probabilistic methods based on filtering approaches.

### 5.2. Well developed SLAM approaches

One of the most well-known SLAM techniques after the EKF-SLAM named the FastSLAM algorithm was driven in 2002 by Montemerlo et al. (2002).

Murphy (1999) and Murphy and Russell (2001) noticed that when the trajectory of the robot is known, the probability between the landmark locations is conditionally independent. Consequently, Rao-Blackwellized (RB) decomposition was presented and performed in an approach that contributed to a general framework of PF towards solving the SLAM problem. Based on this idea, Montemerlo et al. (2007) proposed the FastSLAM again using the Rao-Blackwellized Particles Filter (RBPF) to estimate the robot path as well as a few low-dimensional EKF to estimate the landmark positions. In other words, FastSLAM uses a hybrid technology that is a combination of EKF and the PF which makes the robot to achieve higher accuracy. In this technique, the algorithm relies on the previous pose estimation of the robot. The technique also assumes that when the robot's pose is known, conditionally the landmarks are not dependent on each other. Moreover, the algorithm breaks down the SLAM into the robot localization problem and the problem of collecting estimated landmarks, both based on the robot's pose estimation. O (M log N) is the complexity of computation in FastSLAM which depends on the number of landmarks (M) and the number of particles (N) which both can be constant numbers. Since each particle prescribes the landmarks differently, the FastSLAM carries out multiple data associations so that the data association in FastSLAM is very robust to errors. FastSLAM has a vital power in data association in comparison with EKF based SLAM approaches and also it is straightforward to implement however under particular circumstances the selected samples are often inefficient. The motion and the measurement models of the robot do not need to be linearized. Its application in Non-Gaussian and Nonlinear systems is better and more accessible. In general, the major advantage of the FastSLAM is that particles perform their independent data associations, while in KF based SLAM method, the system is designed based on one data association assumption for the complete filter. Moreover, the usage of particle filters for sampling the robot trajectories consumes less memory usage computation cost in comparison to KF based methods. However, FastSLAM needs to carry out a data association independently, therefore, the computational cost grows dramatically in noisy environments and due to sparse map and the FastSLAM is sensitive to divergence (Ho et al., 2015). Nevertheless, in FastSLAM instantiations, the dependencies of the feature positions are restricted that results in slow convergence. Furthermore, universal consistency is low that makes the method not to be applicable in long-run navigations in large environments.

Later in the year 2003 Montemerlo et al. introduced an improved version of this algorithm and named it the FastSLAM 2.0 in which, the proposal distribution relies on both, the previously estimated pose and the accrual measurement of the mobile robot (Montemerlo et al., 2003). FastSLAM 2.0, in addition to the advancements of FastSLAM 1.0, also benefits from a more reliable computational cost due to its enhanced proposal distribution.

Carlone et al. (2010) investigated the active SLAM problem regarding navigation using Rao-Blackwellized Particle Filters and proposed a technique for measuring particle-based SLAM posterior estimations that

enhances the robot's awareness of loop closures. They used Kullback-Leibler divergence in their method that employing it under a protocol of recognizing the expected information makes the robot to be capable of self-decision-making to obtain the best motion strategy of navigation, resulting in a reduction of the uncertainty in SLAM posterior, simultaneously. Then the proposed information gain is validated in its probabilistic interpretation. A novel method called Unscented Fast-SLAM was presented in 2008 (Kim et al., 2008). This method provides robustness in the localization and the mapping process based on the usage of Scale Unscented Transformation. In 2015 an improved version of FastSLAM 2.0 named 6-Dof Low Dimensionality SLAM (L-SLAM) was presented by Zikos and Petridis (2015). According to Zikos, L-SLAM is as robust and as accurate as FastSLAM 2.0 but its speed outperforms FastSLAM 2.0 by the factor of 3. L-SLAM also uses the Rao-Blackwellized particle filters for tracking the posteriors and it is a probabilistic and feature-based algorithm. The KF is used for estimating the location of the robot and features, and PF is used to estimate the robot's orientation. Like the FastSLAM algorithm, the pose of the robot and the location of the features are taken as random variables, but the difference is in the scheme wherein L-SLAM, for estimating all the robot poses uses PF but for the pose estimation of each feature uses EKF. Besides, L-SLAM uses the Kalman Smoothing technique to issue the latest measured data as backward to previously estimated robot poses and features. Nie et al. (2020) proposed the LCPF SLAM with an improved loop detection and a new parameter called Usable Ratio for detecting the useful information obtained from laser readings that improve the consistency of the RBPF SLAM to be applicable in relatively larger scenarios. However, the usage of more criteria to decide the reliability of a loop makes the method still suffers from slow performance. A smoothing technique named Square Root Smoothing and Mapping (SAM) was introduced in the year of 2006 by Dellaert and Kaess (2006) to improve the process of mapping in the SLAM and to increase its efficiency. In the year 2009, Moreno et al. (2009) Introduced the Differential Evolution SLAM technique as a new solution towards the SLAM problem. In 2011, Mullane et al. (2011) drove a SLAM algorithm in which a Gaussian mixture filter named Probability Hypothesis Density (PHD) was used in the mapping process to present the environment features as a Random Finite Set (RFS) of landmarks. In this approach, instead of conventional state vectors of error estimation, the statistic finite-set is used for error estimation and a natural finiteset is adopted and applied to represent the environment's map. The technique uses particles for representing the robot's trajectories, PHD filters are utilized on each particle to estimate the robot's location, and the number of features that have gone through the sensor's Field of View (FOV) are estimated.

Bowman et al. (2017) presented an approach based on the iSAM2 method using semantic landmark information along with metric features. This method, contrary to traditional methods, relies on both the semantic information and the low-level geometric features. The data association problem and loop closure are dealt with in an interconnected procedure. This technique can work in indoor and outdoor environments and carries out a full 6-D pose history of sensors for building the map. However, the dynamic obstacles and features can influence the system and the computational cost is slightly higher in comparison to traditional approaches. Besides, the robot's motion is not fully recognized the orientation is not recognized.

Zhang et al. (2017) developed a SLAM solving algorithm utilizing an external storage memory that is used to detached the tasks of storing and computing where the stored data are used as a perceptive representation for the robot's localization and path planning that allows the system to be effective in long-run and in large-scale conditions. The construction inspires the development of SLAM-like-behaviors within a deep differentiable neural network. The mapping, localization, and path planning are taken as individual components that use each other as references for their tasks. Moreover, the generated map is feature-rich distributed. However, utilizing an external

Table 2
Probability approaches

Year	Method	Author	Strength	Problems	
1979	Information Filters (IF) Extended IF (EIF)	P. Maybeck	Simple and direct implementations High accuracy Cope with high dimension maps	Difficulties with Map merging Problem with data association Requires state recovery	
1986 1990 1992	Kalman Filter (KF) Extended KF (EKF)	Smith and Cheeseman; Smith et al; Leonard et al.	Efficient convergence Cope with uncertainties Requires to know only the mean	Gaussian constraints Problem with high dimensional maps High Computational cost Data association in large environments First order Tylor expansion	
1996 1998	Particle filters (PF)	Del Moral; Liu and Chen	Copes with Nonlinearities Copes with non-Gaussian noises	High complexity Data association Higher dimensional map	
2000	Unscented KF (UKF)	E. Wan and R. Merwe	Handling nonlinearities Second order Tylor expansion Accuracy Handling uncertainty	Requires to know the mean and covariance Requires the fast interference	
2000	Rao-Blackwellized PF (RBPF)	Doucet et al.	PF advancements Logarithmic computational cost Multimodal distributions Accuracy Does not need to linearize	Nontrivial particle matching Map merging High cost of data association Limited information of the landmarks Curse of dimensionality	
2001	Compressed EKF (CEKF)	J. Guivant and Nebot	Accuracy Map consistency Low memory cost Cope with larger environments	Problem with data association Relying on robust features Multiple map merging Require common landmarks between maps	
2004 2004	ADF / Switching KF (SKF) Thin Junction Tree Filters (TJTF) Treemap Filters (TmF)	M. Paskin, U. Frese	Simple and direct implementation Tractable approximations Linear computational cost	Lack of Data association Requiring the state recovery Requiring the switching mechanism Unstable in large environment	
2004	Sparse EIF (SEIF)	Thurn et al.	Graphical grids representation Sparsification Constant computational complexity	Lower quality than EKF Data association problem Weak representation Iterative and slow	
2006	SubOptimal SLAM Filter (SSF)	V. Shin et al.	Fast process for small areas Efficient with adequate landmarks	High memory cost Multiple map merging Data association problem Require common landmarks between maps High computational cost	
2016	Adaptive KF (AKF)	X. Jiang and T. Li	Real time gain adjustment Accurate Robust estimation Robust mapping	Gaussian noise Relying on number of landmarks High computational cost Data association in complex environments	
2020	Adaptive Extended Kalman Filter (AEKF)	Y. Tian et al.	Real time gain adjustment Accurate Stability Unbiased estimation	High computational cost Data association problem Problem with high dimensional maps	

memory increases the economic cost of the system and the complicated computations degenerate the applicability of this algorithm on fewer robots. Ji et al., 2018 introduced the Closet Probability and Feature Grid (CPFG) SLAM solution technique that is a novel LIDAR-Based SLAM solution specifically designed in order to be utilized on transports in off-road inactive environments. This approach derives a probability grid in combination with cloud feature extraction within the Expectation-Maximization (EM) scheme. The matching section of the system registers point cloud onto the grid map to be used by the nearest neighbor grid in the pose estimation stage. It additionally extends the point to plane ICP (Low, 2004) algorithm as well as the Normal Distribution Transform (NDT) (Magnusson, 2009) to obtain a more effective matching procedure. To provide the ability of performance in real-time, the pose estimation and feature map updating are considered two parallel tasks. However, the main focus of this research is on the mapping and so it does not enhance the uncertainty in motion and the location. Furthermore, dynamic objects are not recognized that it makes the algorithm not to be effective in real-life experiments. In 2014 Y.J. Shih and et al. introduced a new structure for solving SLAM regarding the data association problem (Shih et al., 2014). The idea of this technique is using a fuzzy filter and the curvature data to choose

the right measurement as the current landmark instead of comparing the current measurement with all previously seen landmarks. Furthermore, it benefits from the triangulation technique for improving robot localization accuracy. However, as the number of landmarks grows the computational complexity remarkably and in the long run, the method fails in finding the right loop closure.

### 5.3. Sense based developments

Different types of sensors can be utilized within SLAM methods to generate 2D and 3D maps. Generally, sonar (Franchi et al., 2020), laser (Dryanovski et al., 2010; Morris et al., 2010), cameras including monocular (Elgayar et al., 2013; Li et al., 2020a; Ye et al., 2020; Lu et al., 2020) stereo (Zeng et al., 2020; Yasuda et al., 2020) and RGB-D sensors (Li et al., 2020b; Zhang et al., 2020; Mota et al., 2020; Ito et al., 2020) are the most well-used sensors that are utilized beside the robot's internal sensors (i.e. odometers and compasses) in SLAM solving techniques. However there have been several different sensors and different configurations (Colosi et al., 2020) as well as multiple-sensors (Chiang et al., 2020) i.e. multiple cameras (Kuo et al., 2020; Yang et al., 2020; Won et al., 2020) and multiple laser scanners (Yadav

and Lohani, 2020), under different data-fusion methods have also been utilized within several SLAM methods. Laser range sensors which are considered as active sensors are accurate however they are expensive. Sonar sensors also are active sensors, relatively fast and cheaper, however, they come with the disadvantage of lacking in perceived data. Visual sensors are passive sensors but with longer range and higher resolution, however, the computational cost of processing the vision-based information is comparatively higher besides of the fact that feature extraction and matching also are difficult tasks.

There have been several approaches designed based on the fusion of Lidar data with other types of sensors (Schenk and Csatho, 2002; Mcgaughey, 2014). Data fusion is useful for producing higher-quality measurements in comparison with using one single fused data stream. There is not a common method for lidar data fusion with other data sources, however, the common technique is to benefit from the object recognition from maps (Sohn and Dowman, 2007; Rottensteiner et al., 2005), and using tree canopy detection (Erdody and Moskal, 2010). Most approaches attempt to find some kind of correspondence between lidar data and other types of data to overcome the fusion. Most of these correspondences are formed on the structure of tensor-based feature points (Li and Olson, 2011), Inertial Measurement Unit (IMU), and Global Positioning System (GPS) (Mcgaughey, 2014), scene edges and patches (Schenk and Csatho, 2002) and matching the detected objects in the maps (Rottensteiner et al., 2005). Gee et al. (2016) addressed the sparse nature of the lidar sensors by merging several lidar scans inside a larger point cloud that results in achieving more accurate measurements and therefore a better SLAM solution. They limited the motion of the lidar to a one-axis translation for interpolating and refining it to achieve a denser form. This novel model is used to drive a basic stereo SLAM algorithm for producing a dense colored point cloud that has more accuracy in comparison with the standard lidar SLAMs. This lidar guided stereo SLAM method achieved a consistent accurate mapping and a more reliable localization. However, multiple scanning the scene makes the computational cost to increase and except the measurement model the method still suffers from the uncertainty in the motion such as drift and etc. Du and Du (2019) used data from a high-precision Laser combining with odometry data from the wheel encoders to decrease the uncertainty in the distribution and established a more accurate map of the environment. This technique is based on the RBPF method that can eliminate the number of samples to almost half. In this work, they examined the same dataset with both the traditional RBPF and their optimized RBPF showing that in their case the number of samples that were needed to construct the map was decreased to 18 samples for the optimized RBPF while the traditional RBPF needed 38 number of samples. However, in the longrun execution of this technique, the uncertainty in the odometry data will be increased significantly that causes the algorithm to again suffer from the map inconsistency. Moreover, the mapping procedure is not reported in this work and the algorithm needs a solution to deal with the map inconsistency.

During the year 2017, Xie, Yu, Lin, and Sun tried to take the calibration of the Sensor Estimation into the SLAM computation (Xie et al., 2017). They improved the EKF technique by enhancing the position estimation accuracy through bias estimation that can be applied to the SLAM problem. This method can ensure the estimation accuracy even when the bias information is unknown. Clemens et al. (2018) proposed a SLAM solving technique based on RBPF SLAM that models the occupancy probabilities of the map with beta distributions in which different types of uncertainties like missing or conflicting information are recognized and represented distinctively in the map. The cell's occupancy probabilities are taken as random variables representing different causes of uncertainty. The RBPF estimates the joint distribution in the map and the path. The additional representation of the uncertainty information contained in the beta distribution enhances the navigation and the path planning that allows the system to provide an active exploration. The map building by this technique is

considerably improved in comparison with the RBPF-SLAM. However, the computational cost regarding the extra distribution is relatively high. Kakoty et al. (2019) proposed a scheme for navigation and map building in an unknown environment based on human behavior. In this technique, the sensor information is used continually to avoid the obstacles assuming that the features are not fixed. A grid occupancy is obtained and updated continually in real-time with the robot scanning 360 degrees around itself and due to the map, and the navigation is done based on a separated set of sensors that are mostly responsible for obstacle avoidance and finding the goal. The purpose of the robot is to reach the goal using online active sensors rather than localizing itself into the map so that the grid map is updated at each time based on the new sensor information. The uncertainty is not dealt with however based on a multiplier as the velocity constraint, maintaining a safe distance from the dynamic obstacles in navigation is obtained. So that, the method is useful when the map of the environment is not important since the technique does not concern about the uncertainty in robot's pose nor about the features possess in the map because the features of the environment are assumed to be dynamic anyway. However, the navigation in the unknown environment is successfully done.

Different types of sensors and mechanisms have been utilized in SLAM solving approaches. Robertson et al. (2009) offered a SLAM solving algorithm for a foot-traveler application that employs foot-mounted inertial sensors to form transition probabilities as the beta distribution in a hexagonal grid rather than the single Bernoulli distribution per grid cell. The technique proposed a new Bayesian estimation structure for covering the noise and drift. However, the method is not appropriate for robots with other classes of locomotion and the created map is only a sparse representation of some selected regions of the environment. In the year 2001, Dissanayake et al. (2001) introduced a new technique for constructing the map of an environment for a mobile robot in which they took the advantage of Millimeter Waves (MMW) to build the map. Evers et al. (2016) developed a SLAM solving technique based on audio sources such as talkers using a microphone array within a surrounded environment. This novel method is a bearing-only acoustic SLAM (a-SLAM) procedure that operates by utilizing a single-Cluster probabilistic representation obtained from a Hypothesis Density Filter (HDF). By letting the array move, the acoustic environment is explored and the map is refined. The source-sensor position is estimated by using spatial diversity in the array's kinematic that results in a fairly accurate map building. However, the map built by this technique is rather sparse and limited to applications of specifically designed scenes. Sensor data fusion has been considered for enhancing observation accuracy. Altan et al. (2016) studied the mine based SLAM and proposed a method for map building and robot positioning based on ultrasonic sensory readings for mine environment exploration. This method works based on processing the ventilation system information using EKF. Several Indoor localization and navigation methods have utilized the magnetic field's pattern and signatures to achieve repeatable and stable pose tracking (Gozick et al., 2011; Haverinen and Kemppainen, 2009; Lu et al., 2013; Sheinker et al., 2013a,b). Fingerprinting is a popular localization method based on the magnetic field ideas using unsteady signal measurements such as WiFi-based and ZigBee-based localization methods. WIFISLAM (Ferris et al., 2007; Vallivaara et al., 2010) similar to Magnetic SLAM (Jung et al., 2015) was implemented based on measuring magnetic fields. In this approach when a location is estimated the estimated value can be updated in a database using interpolation and then the updated database is utilized to estimate the next state. Zedadra et al. (2016) proposed a foraging method for multi-robot cooperation based on the ant colony concept. This technique works based on switching algorithms and provides a quick search for pathfinding as well as localizing the robots into the environment by taking advantage of the C-marking method. This study covered multiple scenarios of the robot's path and locations and improves the speed of the task compellation. Lin et al. (2016) proposed a method for mobile phone localization based on the Internet of Things (IoT) for a location-based service adopted in smart-buildings. The method adopted the Markov-Chain concept for estimating the pedestrian's location with the prior information obtained based on the signal strength and the posterior of the pedestrian's fingerprint to generate a radio map of the environment. Considering the existed electronic devices that generate a considerable amount of noise, this method is claimed to localize a phone with less than 1.5 (m) error in localization. However, the application of this method is rather relying on WIFI signals alone with less concentration on the device's motion. Fortino et al. (2018) studied the IoT concept and offered a method for implementation of the method based on IoT analysis for developing a smart system capable of localizing itself along with utilizing an agent-based cooperative smart object method. Vallivaara et al. (2010, 2011) proposed a method utilizing a Gaussian process for interpolating the magnetic field to be used with Rao-Blackwellized particle filters for determining the robot's pose. Similarly, PF is employed for localization task in SLAM with the extra support from radio signals to solve the robot's pose estimated errors (Jung et al., 2015). Based on the magnetic fields approach Wang et al. (2017) proposed a SLAM-solving method by measuring ambient magnetic fields presented in indoor environments. This technique introduced Kriging interpolating method for map updating and also offered an enhanced exponentially weighted particle filter for estimating the feature's pose distribution. This method achieved an improvement in localization and consequently in the map building, however, in term of a sharp turning in the motion of the robot, the algorithm will face to a shift in the estimated location that affects the performance of the system therefore the navigation of the system is restricted. Liu et al. (2018) proposed a radar-based SLAM using an Adaptive Genetic Algorithm (AGA) that is implemented in an improved closed-loop PF-SLAM to detect the features obtained from microwave radar sensor. This method uses both deterministic and stochastic echo-wave responses to build a scattering map of the surrounding objects and to obtain the position of the radar. The map is also used to improve the accuracy of the radar itself, however the accuracy of the positioning depends on the map while the accuracy of the map depends on the positioning of the radar that makes the uncertainty of the mapping and positioning be ambiguous.

#### 5.4. Application-based recent developments

The main contribution of SLAM is on mobile robot motion and observation where many outstanding methods have been developed. From there, many methods have been reconstructed to be used in other applications. SLAM problem solving has become a technology being utilized in many different applications from wheeled single mobile robots and multi-robot applications (Saeedi et al., 2016) to medical operators, Arm-SLAM for industrial autonomous manipulators (Armslam et al., 2016), flying objects (Shim and Cho, 2017), underwater submarines, sound pitch correcting methods and sound-based applications i.e. A-SLAM (Evers et al., 2016) telecommunication devices i.e. Channel-SLAM (Gentner et al., 2016) for position estimation of mobile receivers, and many other approaches and applications. Different approaches based on different applications and configurations have been widely presented throughout history. In this paper, we briefly review a few interesting configurations based on rather than wheeled single mobile robots.

Vysotska and Stachniss (2017) proposed an algorithm to use available Open-Street-Map (OSM) data inside the SLAM problem to relate the robot's observations with the building's information for generating efficient constraints within pose graph structure. By selecting the robot's goal location from the available map, the method allows the system to accordingly reduce the uncertainty in the robot's pose. Open-Street-Map evaluates the localization and results in an active localization that leads the system to achieve a more accurate map building. However, expanding graph-based SLAM correlating with the OSM increases the computational cost and the method has no application in cases where OSM is not available.

There have been relatively fewer works been done on the underwater SLAM solution (Newman and Leonard, 2003; Eustice, 2005). Most underwater SLMA methods use sonar sensors. However visual sensors also have been utilized in underwater applications (Petillot et al., 2008). A survey on underwater SLAM challenges is conducted by K. Köser (Köser and Frese, 2020).

Turan et al. (2017) proposed a dynamic map fusion-based direct SLAM method for using on endoscopic capsule robots that achieved a relatively accurate pose estimation and map building system on a realistic surgical case. This method uses GPU accelerated none-stationary frame-to-model fusion, the mutual volumetric-photometric position estimating and loop closure based on the full-dense model-to-model approach are used and the system is composed based on RGB Depth SLAM methods (Whelan et al., 2015; Newcombe et al., 2011b). The usage of surfel-based dense data fusion in combination with fame-to-model tracking and non-rigid deformation allows the system to achieve effectiveness among surgical robots.

Several reviews on the extension of SLAM methods to multiple-robot systems are available (Saeedi et al., 2016; Gupta and James, 2019; Howe and Novosad, 2011). However, most of the conservative multirobot SLAM methods suffer from the overdue number of parameters and loop-closure rejections which makes them less accurate. However recently there have been some improvements in multi-robot SLAM methods too. For example, Lajoie et al. (2020) proposed a graph-based method named Door-SLAM designed for multiple-robot systems, that utilizes fewer parameters while it can perform inter-robot loop closure without needing to exchange the row sensory data between the robots.

Another interesting technique among proposed algorithms of this year is the one proposed by Shim and Cho (2017) where they have conducted a technique to process a map through two different robots, UGV (Unmanned Ground Vehicle) and UAV (Unmanned Aerial Vehicle). The idea is that, to make the UGV SLAM capable of sharing a built map between a UAV and a UGV to correct the errors by a robot with more freedom of movement since, for a UGV the location errors occur because of some obstacles in the environment or the drifts in its movement. Regarding the fact that the UGV benefits from longtime battery life, simpler odometry, more accuracy in its movement, and large loading capacity in comparison with a UAV which suffers from these factors but on the other hand it has the advantage of free movement, together can achieve a higher rate of efficiency in solving a problem. ortiz Santos et al. (2018) Proposed Sliding mode three-dimension SLAM (SM-SLAM) to solve the navigation and mapping problem of Unmanned Aerial Vehicles (UAV) systems with 6 Degrees of Freedom (DoF). This method generates a 3D map of the environment and it does not require the uncertainties in motion and observation to be of Gaussian representation with zero-mean. SM-SLAM provides a robust system against restricted perturbations and it achieves a more accurate pose estimation in comparison with EKF-SLAM. However, the computations take a long time because it requires the system to wait for the update at each state.

Dubé et al. (2017) proposed a LIDAR-based SLAM for multi-robots that despite most of the multi-robot SLAM systems (Nagatani et al., 2009; Michael et al., 2012) it has the ability of online performance. This technique is based on a progressive sparse pose-graph optimization approach and it can generate a rich 3D geometric map and localize itself online into the map. Sequential and place recognition constraints are used for loop closure and data association. The robots are employed based on Inter-robot global association to produce a full 3D SLAM soliton with robustness and effectiveness with low-time execution, memory cost, and communication bandwidth required of the system.

Table 3 gives brief information on some of the driven leading algorithms that have had a considerable impact on the direction of the SLAM technology throughout the SLAM mid-time history.

Table 3

Year	Author	Title	Achievement
2001	Dissanayake et al.	A solution to SLAM problem	Utilizing Millimeter Waves
2001	Guivant, Nebot	Optimization of the SLAM for real-time implementation	Reducing computational cost
2002	Montemerlo et al.	FastSLAM	Higher accuracy
2003	Montemerlo et al.	FastSLAM 2.0	Decreasing computational cost
2004	Thrun, et al.	SLAM with Sparse Extended Information Filters	Independent computational cost to size
2006	Dellaert, Kaess	Square Root SAM	Improved mapping process
2007	Montemerlo et al.	FastSLAM	Path and landmark estimation
2008	Kim et al.	Unscented FastSLAM	Robustness
2008	Bossed and Zlot	Map matching and data association for largescale two-dimensional laser scan-based SLAM	Loop closure optimization
2008	Paz et al.	Divide and conquer: EKF slam in o(n)	Improved data association
2009	Pedraza et al.	Extending the limits of f-based slam with B-splines	Improved data association
2009	Moreno et al.	Differential evolution solution to the SLAM problem	Novel Solving approach
2010	Carlone et al.	An application of Kullback–Leibler divergence to active SLAM and exploration with Particle Filters	Enhancement in loop closure
2011	Mullane et al.	A Random- Finite-Set Approach to Bayesian SLAM	Improved mapping process
2014	Zhang et al.	An improved particle filter SLAM algorithm in similar environments	Improved Performanc

# 5.5. Visual SLAM

The present state of the art of SLAM technique is mostly based on two approaches, first, the approaches based on portable laser rangefinders named the Lidar-based SLAM (Hess et al., 2016) and second the approaches based on computer vision known as the vision-based SLAM (Kerl et al., 2013). Lidar SLAM is a comparatively more sophisticated approach towards the SLAM solution that has been favorably implemented in a vast variety of intelligent robots including commercial products. However, the map built by a Lidar-based SLAM method is rather simple that does not allow the robot to achieve well-detailed information of the environment and in many intelligent applications obtaining the environmental information is crucial. Similarly, the visual SLAM has been a fast developing towards solving the SLAM problem that is also capable of reconstructing the 3D maps of environments using cameras instead of laser scanners. Furthermore, images contain a wealth about feature information that allows the robot to be able to accomplish a larger variety of tasks. Moreover, the improvement in the visual sensors and the appearance of the depth cameras as well as different visual sensors has led many of the recent researches and SLAM techniques to be conducted based on the cameras and visual sensory information. Fig. 6 shows the key components of visual SLAM.

Visual SLAM is mostly categorized in Monocular, RGB-D, and Stereo SLAM approaches. Utilizing one camera concerning the SLAM problem is termed as monocular SLAM, combining an infrared sensor and a monocular camera to form the sensor is termed RGB-D SLAM, and employing various cameras is termed stereo vision SLAM. Many visual sensors have visual odometry in themselves and they are robust, accurate, and easy to implement. For instance, a Monocular SLAM method known as MonoSLAM was proposed by Davison et al. (2007a) that is a method of estimating the visual motions using monocular cameras. This method is the earliest real-time high frame-rate Monocular SLAM

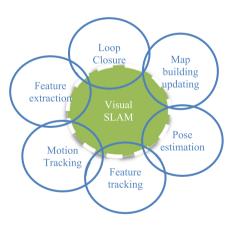


Fig. 6. Contemporary visual SLAM components.

solution. This feature-based technique can estimate the state of the features in the environment as well as the state of the camera at the same time which technically makes it a SLAM solving algorithm (Soto-Alvarez and Honkamaa, 2014; Bailey and Durrant-Whyte, 2006). There are many different monocular SLAM algorithms introduced in the past decade which mainly can be categorized into two classifications, direct methods (Cadena et al., 2016a; Krombach et al., 2017), and feature-based methods (Davison et al., 2007a; Mur-Artal et al., 2015a).

The MonoSLAM uses the EKF algorithm (Bailey et al., 2006) to build a map of landmarks for the environment. The generated map is limited but it is stable that solves the problem of monocular feature initialization. However, the sparse nature of the map also exposes the robot to fail the localization task in situations where more details of the environment are needed. Furthermore, the EKF used in

this approach causes a problem with the linearization that results in emerging drift errors and the complex computational complexity. For this reason, the UKF (Martinez-Cantin and Castellanos, 2005) and the improved UKF (Holmes et al., 2008) were implemented in visual SLAM to deal with the linearization uncertainty. Another approach that could cope with the linearization difficulty was a PF based monocular SLAM proposed by Sim et al. (2006). The method could build a more accurate map, but again the PF approach leads the algorithm to have a high computational complexity that results in the system's failure in large environments. A keyframe-based monocular SLAM approach identified as PTAM (Parallel Tracking And Mapping) was designed by Klein and Murray (2007). PTAM was a method able to map the small areas employing the FAST corners (Rosten and Drummond, 2006) and described one of the earliest marker-less augmented reality applications based on the method. In this method, the localization and mapping are divided into two identical parallel tasks. A keyframe extraction technique was offered by this method and it alternatively used the Nonlinear Optimization method instead of the EKF method to cope with the linearization difficulty that consequently reduces the uncertainty in the robot's localization. However, due to the problem with the PTAM in its global optimization, this technique also is not applicable in large environments. Large-scale direct monocular SLAM (LSD-SLAM) (Engel et al., 2013, 2014), dense tracking and mapping (DTAM) (Newcombe et al., 2011a) are some of the direct methods of visual SLAM that use only RGB images as their input and by using the intensity of all the pixels in the frame it can estimate the camera's trajectory and reconstruct the nature and so the 3D map of the environment. DTAM uses multiple pictures of a static scene available in a video stream to improve the accuracy of a single photometric data term that results in generating an accurate depth map in real-time. LSD-SLAM is capable of building a semi-dense global steady map of the environment, that in comparison with the point-cloud map built by the feather-based methods, is more well-detailed, in other words, it contains a more comprehensive representation of the environment. LSD-SLAM algorithm is also able to be executed in real-time. Furthermore, it offers an innovative direct tracking method that is capable of accurately detecting the scale drift. However, the direct method results in the problem of the Gray-Scale invariant assumption that means the algorithm is applicable in environments where the lighting is not regularly changed. A SLAM solving method based on autonomous object detection without prior information was proposed by Lee and Song (2008). This method generates a grid map of the environment by detecting and characterizing the detected objects as the landmarks of the map. The map built by this technique is a sparse 2D map that in comparison with the prior Visual SLAM methods, enhances the accuracy of the positioning and navigation. However, due to neglecting the 3D map the algorithm cannot model a complete structure of the environment which is the most valuable benefit of visual SLAM. Regarding the navigation especially the localization task of the SLAM problem, an onboard SLAM solving method was presented by Brand et al. (2014). This technique uses local 2.5D sub-maps that can be directly used for local path planning and active obstacle avoidance, when the robot finishes navigation a general grid 2D map will be built by linking the local maps. However, the use of sub-maps makes the algorithm not been applicable in large environments furthermore its computational cost depends on the environment's size. In other words, the application of this method stands only for indoor small environments. RatSLAM (Ball et al., 2013) which is the first biological-inspired visual SLAM, produces a graph map. It can use odometry data along with RGB images to estimate the robot's position and the environment cells. The positioning of the robot using this method is comparatively more accurate however the computational cost grows dramatically in the long time run. Mur-Artal et al. (2015b) presented a new real-time method as a visual-SLAM solution named ORB-SLAM. ORB-SLAM is a feature-based monocular SLAM that can use Monocular, Stereo, or RGB-D cameras for its input to estimate 3D features positions and reconstruct the environment as

a map in a real-time. Furthermore, it supports different optimization mechanisms, and a high accuracy level at positioning is achieved. The majority of the existing monocular SLAM methods as well as ORB-SLAM, use a re-localization function to cope with failures in the tracking procedures which in, the estimation for the failed frame will be suspended until the camera be correctly re-localized. This issue may cause a part of the trajectory to be lost. Moreover, although the ORB-SLAM has the problem with the uncertainty in the monocular scale that leads the system to fail into tracking loss and failing in navigation due to the lack of robustness in changing environments.

Ataer-Cansizoglu et al. (2016) proposed a new approach based on RGB-D SLAM, to overcome registration errors provides a deeper depth range contrary to regular SLAM methods that operate with limitation in the depth range. In this method, both 2D and 3D measurements are used for point feature extractions to generate enhanced interaction among frames. The longer-range constraints are generated by using rays from defined 2D to 3D hyper point correspondences based on 2D point features to "pin-point" matching 3D point features. The technique works with two processing, online and offline. It explores the environment in an online manner and then uses offline processing to improve the results. Pinpoint SLAM achieves more accuracy in generating the map, however, the computational cost especially because of requiring online and postprocessing computations is relatively high. Furthermore, the motion uncertainty is not dealt with in the online exploration but it is dealt with in the postprocessing computations as the trajectory. Yousif et al. (2017) proposed a visual SLAM combining RGB-D camera and monocular camera to construct a larger-scale 3D representation of the environment. The method extracts 3D point features using frames obtained from the RGB-D camera and it also extracts 2D point features obtained from the frames coming from the monocular camera. The different Field-Of-View (FOV) results in generating multiple virtual images for every angle that is used to compute the feature descriptors. A graph is constructed for computing the poses of the frames which its nodes indicate the frames of both camera and the edges represent the storage of pairwise nodes. The global pose of nodes is computed by finding the Minimum Spanning Trees (MSTs) in the graph and eliminating the inconsistent edges. This approach achieved the ability of a larger amount of frames registration and therefore building a larger-scale 3D map. However, the computational cost of the technique is considerably high that in some cases takes hours to finish its computations.

López et al. (2017) introduced a multi-sensorial SLAM solving method with fusing a laser scanner, a monocular visual sensor, and onboard sensors such as IMU and altimeter. This method is designed for a low-cost Micro Aerial Vehicle (MAV) calculating the estimations of the robot's pose and the environment map in a remote-control station. This method works based on a monocular visual SLAM method with the compensation of an integrated EKF-based system for estimating the robot's pose based on the laser scanner data and the onboard sensors. However, the computational cost of this technique is relatively high but because of its execution on a remote-control station, can successfully run on a low-cost robot. The estimated 6 DoF pose, the generated local 2.5D map, trajectory, and the robot's localization are improved in this approach. However, the map is not accessible offline and the requirement of the remote-control base makes this algorithm to limit in its application.

Pumarola et al. (2017) proposed a SLAM solution technique named PL-SLAM (Point and Line SLAM). This approach is based on ORB-SLAM for handling low-textured scenes where most of the points are not detectable. Despite the previous visual SLAM methods which are based on homography (Faugeras and Lustman, 1988) or essential matric estimation (Tan et al., 2013) they operate relying on only point correspondences, this approach also handles line correspondence along with the points. In this method, 5-line correspondences are obtained from three consecutive images to initialize a novel line-based map. Combining points and lines improves the efficiency of ORB SLAM and it also solves its problem with the poor textured images. Re-tracking strategy (RTS) is a new, more efficient, and an active technique to solve the

tracking problem in states where the camera localization has failed (Liu et al., 2017). This technique has locally an auto-functional initialization procedure that will be activated when a tracking procedure has failed. The method launches a new tracking process and when a loop closure happened, the algorithm corrects the trajectory.

A new, more efficient approach that introduced a real-time dense 3D modeling technique based on the ORB-SLAM approach using Kinect 2.0 camera in the real world that was proposed by Lv et al. (2017). In this approach, the ORB-SLAM constructs an Octomap (Hornung et al., 2013) according to the octrees based on probabilistic estimated occupancy. The RTS technique is used to solve the tracking problem in states where the camera localization has failed. However, due to the nature of octomap, the technique does not express the environment intuitively that does not provide direct access to the map which results in less interactivity of the robot. Later in the year 2017, R. Mur-Artal and et al. introduce an improved algorithm based on the ORB-SLAM named ORB-SLAM2 (Mur-Artal and Tardos, 2017) which in addition to the previous method, it supports also the usage of stereo cameras as well as RGB-D cameras outside of the monocular camera.

The ORB-SLAM2 structure mainly relies on three parallel parts: Tracking, responsible for estimating the camera pose based on the feature method at each frame of the input image, Local Map building that receives and processes the new keyframes, and loop cluster that is the loop detection and loop correction. The estimated camera pose is concerned about forming a PnP (Perspective-n-Point) (Hu and Wu, 2006) solving method. Various approaches for the PnP solving have been suggested for example DLT (Direct Linear Transformation) (Abdel-Aziz and Karara, 2015), P3P (Kneip et al., 0000), EPnP (Efficient PnP) (Lepetit et al., 2009), etc., that the EPnP is considered as the most accurate solution to PnP problem that can be used as a pose estimating method within the ORB-SLAM. This technique achieved a good level of accuracy in positioning as well as the robustness against violent movement and changes in the environment and it covered re-usage of the map by the same system, loop cluster, and relocalization ability that makes the method to be useful in different applications such as hand-held cameras, flying drones, and self-driving cars. Mur-Artal and Tardos claim that the ORB-SLAM2 is the most accurate SLAM solution so far and they published the source code of the technique that made it a SLAM solution useful in other fields of robotics. However, the ORB-SLAM2 generates a training vocabulary through large-scale data and store it within a text format that this text file needs to be processed and its procedure is very time-consuming for mobile robots when the size of the vocabulary is large. The application of ORB-SLAM2 on the mobile robot has the difficulty with mobile robots such as slow-startup that is mostly occupied with processing the vocabulary text file. The usage of the large datasets results in having a large amount of invalid data when the robot's working environment is fixed. Furthermore ORB-SLAM2 suffers from lack of the capability of the offline visualization and mapping the trajectories that are essential for many mobile robots.

Since the appearance of the ORB-SLAM2, many researchers have put efforts to examine and to enhance its performance. A tight sensor fusion approach was designed by Caldato et al. (2017) to deal with unexpected situations in featureless environments when tracking is lost. The algorithm enhances the positioning of visual odometry based on integrating the odometry information and imagery data that results in an enhancement in graph constraints among frames. The robustness of the ORB-SLAM2 is improved by this technique but the robot's computational cost is also extended. However, the performance of ORB-SLAM2 in a normal situation is not improved.

A monocular SLAM algorithm based on ORB-SLAM2 pose estimation was introduced by Wang et al. (2018). This technique uses the fusion of IMU (Inertial Measurement Unit) information for determining the scale information and refining the estimated pose when the tracking is lost. This approach improves the robustness of the system but on the other hand, it also increases the computational cost. ORB-SLAM2 is a keyframe-based SLAM that works with keyframes with six degrees of

freedom(6DoF) so that it does not differentiate between 3DoF (Degree of Freedom) and 6 DoF picture frames. To provide the system with the ability to distinguish the differences between 3DoF and 6DoF, Zeng et al. (2018) advanced a scheme to determine 6DoF keyframes to prevent incorrect triangulation to 3DoF keyframes. This technique decreases the number of keyframes in the map because of filtering the 3DoF keyframes out that consequently reduces the level of precision in the loop processing state as well as the tracking so that it degrades the performance of the ORB-SLAM2.

A unified spherical camera model was introduced by Wangl et al. (2018) that extends the structure of the ORB-SLAM2. Using fisheye cameras enables the system to deliver a more extensive perceiving area. For building the semi-dense maps, the technique utilizes high-gradient regions as semi-dense features based on a new semi-dense feature matching method that results in achieving a more comprehensive map than a sparse feature map. The problem with this method is that the uncertainty of mapping in large-scale environments is relatively high and the semi-dense map is not reusable.

Besides of the standard front view vision, the Ceiling vision has an advantage in localization because it concerns only about the rotation and affinal transform with no change in the scale that makes this method to be more beneficial in using ceiling view features in comparison with detected features in 3D environments. There have been several CV-SLAM (Ceiling Vision-based SLAM) methods proposed in mobile robotics (Choi et al., 2012).

Li et al. (2018) introduce a new ceiling vision-based SLAM. This technique employs different feature detection methods for detecting different key points. It uses both corner and circle features of the environment as landmarks and improves the stability of the system by using Saliency measurement. The algorithm generates a hybrid map by utilizing Delaunay triangles among the more stable features that are selected according to their saliency strength. Additionally, an EKF is used to localize the robot into the hybrid map. However, the computational cost is not reported and the uncertainty in the observation is not dealt with.

Hsiao et al. (2018) proposed a new Dense Planar Inertial SLAM (DPI-SLAM) approach that can reconstruct the dense 3D models of large scale indoor environments utilizing a hand-held RGB-D camera along with an Inertial Measurement Unit (IMU). In this full-SLAM framework, a local depth fusion part is defined where the dense Visual Odometry (VO) estimations are fused with the planar measurements in tightlycoupled (Konolige et al., 2010a; Weiss et al., 2012) approach and the fused information loosely-coupled (Keivan et al., 2016; Leutenegger et al., 2015; Mur-Artal and Tardós, 2017) with pre-integrated IMU. The technique similar to iSAM2 forms a Bayes Tree as its global factor graph by optimizing the planar landmarks, velocities, and the poses. The IMU data and the RGB-D frames are used for estimating the odometry that allows the system to keep tracking regardless of sufficient planes or visual features. The uncertainty caused by rotation drift and noises is reduced by forming a fully probabilistic global optimization approach obtained from the IMU states and the modeled planes as the landmarks, However, the translation drift still exists within this algorithm. Furthermore, the keyframes cannot be reused and the loop closure is not considered to be in real-time.

Yang et al. (2019) introduced a fast-real-time positioning approach within the ORB-SLAM2 with using a binary vocabulary saving method and a vocabulary training method based on an enhanced Oriented FAST and Rotated BRIEF (ORB) framework that reduces the dataset size and enhances the startup speed. Furthermore, they offer an offline map building algorithm based on the map component and keyframe database that in comparison with pure ORB-SLAM2 is more suitable for mobile robots. Moreover, a fast relocation method for the offline map is implemented in their technique. This method enhanced the productivity of the ORB-SLAM2.

Mo et al. improved ORB-SLAM with building a 2D dense map that represents the features of environments (Mo et al., 2019). In this

study, an auto-exploring mobile robot is obtained based on Bayesian optimization, and the algorithm benefits from a theoretic information exploration that allows the robot to build a map while using a local map to maintain exploring. The technique uses a selection of a determined amount of information to construct its local occupancy grid map. The local map is used for localization and path planning tasks of the robot. The technique is capable of real-time performance in a complex environment. However, since the map is obtained from the Octomap (Hornung et al., 2013) and it is changed in its resolution some features of the environment are distorted especially when the camera on the robot shakes due to the robot's motion. Table 4 shows the reviewed V-SLAM methods according to their publishing time.

To evaluate the performance of SLAM methods, benchmarking instruments such as TUM RGB-D Benchmark (Sturm et al., 2012) and KITTI Benchmark Suit (Geiger et al., 2012) have been introduced. More Lately, several open-source benchmarking tools have been proposed. Comparative visual-inertial research on several methods was presented by Delmerico and Scaramuzza (2018) and Abouzahir et al. (2018) compared four SLAM algorithms. EVO (Grupp, 2018) is an evaluation structure for visual odometry methods combining several datasets. Tools like DAWNBench (Coleman et al., 2017) were proposed for deep learning AI methods. The first framework for examining the popular SLAM methods was SLAMBench2 (Bodin et al., 2018), a sensoragnostic and dataset-agnostic structure that integrated a variety of data sets, metrics, and methods with plug-and-play algorithm support. The initial report of SLAMBench (Nardi et al., 2015) only covered a single algorithm named KinectFusion (Newcombe et al., 2011b) and it was directly blended with the SLAM method, rather than providing the evaluation as an external module. However, the SLAMBench tools have a lack of coverage of SLAM systems for dynamic environments and the lack of covering the Machin Learning methods such as Chiefly Convolutional Neural Networks (CNNs) (Krizhevsky et al., 2012) that can enhance SLAM algorithms. SLAMBench 3.0 proposed by Bujanca et al. (2019) addresses this issue by combining semantic and dynamic SLAM systems along with depth estimation. It goes beyond classical visual SLAM systems and presents new assistance for scene perception and non-fixed scenes (dynamic SLAM). SLAMBench 3.0 also covers proper metrics, datasets, and sample algorithms. Moreover, it additionally provides the implementation of Dynamic Fusion accompanying with an evaluation framework. However, this tool also does not cover trajectory metrics and the use of multiple cameras.

# 6. Recent and future overview

Most of the recent works regarding SLAM developments have been focused on enhancing the filtering techniques as well as optimization solutions to manage and match the information obtained from landmarks faster and more reliable. Along with traditional filtering methods, fuzzy controllers have been designed to control the parameters of the conventional filters. However, most of the techniques based on fuzzy approaches are obtained from two parallel approaches, the SLAM and the fizzy optimization of the SLAM, which leads to growth in computational cost as well as memory cost. New versions of the all variation of the KF, PF, IF, etc., have been proposed throughout the recent studies. The main focuses of the papers based on the traditional filters are either to develop the more robust adaptive model for them, building faster interferences for the filters or reconfiguration of the parameters within those filters to achieve more robust estimations as well as obtaining more reliable data associations. As the works, we have reviewed in the previous sections a wide range of studies try to deal with enriching the map representations of the well-known SLAM techniques. Utilizing different types of sensors, as well as a multisensory system and various ways of sensor configurations have been taken under study. However, the optimization-based SLAM methods have been proven stronger than the traditional filter-based approaches, but we have presented the recent works, it seems that the probabilistic

approaches still have a great place among researches. Furthermore, the majority of the proposed SLAM techniques are designed to solve the SLAM problem dealing with the single-robot mapping in static environments (Thrun and Leonard, 2008). Although, there are plenty of studies and implementations in modern SLAM to overcome the problem in dynamic environments as well as multiple robot navigation scenarios. A variety of SLAM techniques are developed to improve and tackle different problems of SLAM. Realizing the difficulty with optimum navigation and location recognition. The mapping process also has become the ultimate objective in the SLAM area where the capability of mobile robots towards self-decision-making in dynamic environments without human interference during a limited period is the vital goal. Furthermore, SLAM's solution to a large environment with big data processing where the number of landmarks is extremely big is still of the remaining challenges that also has attracted many studies to be upon this aspect of the area. Table 5 presents some of the interesting works that have been done in recent years.

In recent years, Big data also have come more into the SLAM challenges where the researches try to develop SLAM solutions for processing a huge number of features in very large environments. For instance, Das (2018) refined the FastSLAM and implemented it in a Real-Time Appearance-Based Mapping (RTAB-Map) approach regarding a big dataset application. This study uses a camera as its sensor to build a 2D Occupancy grid map as well as a 3D Octomap. Moreover, a loop closure also is implemented in this approach. The RTAB-Map improved the FastSLAM to be applicable in large-scale scenes with big data by using multiple loop closures. However, the computational cost considering the grid map and the loop closures is not reported, and the uncertainty in the motion is not dealt with.

Bringing up the big data into the account has led many approaches to utilize planners and artificial landmarks to enhance SLAM efficiency. Many recent developed SLAM algorithms use the markers to reduce the computational cost as well as to achieve the robustness in the estimations. Several methods used Squared Planar Markers (SPM) (DeGol et al., 2017; Fiala, 2005, 2010; Garrido-Jurado et al., 2014, 2016; Kato and Billinghurst, 1999; Olson, 2011; Schmalstieg et al., 2002; Wang and Olson, 2016). The markers are usually in the form of some visible black border objects that can be uniquely recognized. The corners of a marker can be used to estimate the camera's pose. Most of the popular methods use a single marker or a limited set of markers that their relation to each other is known which makes the applicability of the systems to be limited to small environments. A few approaches tried to design camera localization based on SPMs for applications in large environments with big data. Lim and Lee (2009) introduced an online visual SLAM solution that uses planar markers and EKF to track a robot's pose. In this technique, new detected markers are added to the map based on their relationship with the robot's current pose. However, this technique does not recognize the uncertainty in robot's motion, neither deals with the loop closure problem. Yamada et al. (2009) presented a similar approach applied to an autonomous blimp that also enhances the SLAM procedure in big data situations. Klopschitz and Schmalstieg (2007) proposed an approach for building a 3D map of fixed markers in an indoor environment. The method estimates the camera position using Structure from Motion (SfM) and the marker locations are obtained by triangulation based on the camera position. However, the SfM method is not always functional, and the technique does not deal with the uncertainty in the coordinates between the marker's and the camera's locations. Shaya et al. (2012) presented an online visual SLAM algorithm using markers based on creating a pose graph. The pose graph is obtained from the markers poses and their associated poses in relation to each other. The key idea of this technique is that when two markers are observed in a frame, their relative positions can be updated. A selected set of visible markers with the condition of having a closer path to an origin are used at each update. However, the uncertainty in the nodes locations is not dealt with in this technique and the application of the technique

Table 4

Year	Method	Author	Category	Input source	Key idea	Output	Problem
2007	MonoSLAM	Davison et al.	Feature- based	RGB images	Feature initialization, feature orientation estimation	Active 3D map, drift free accurate localization	Sparse map
2006	PF Vision	Sim et al.	Feature- based	RGB images	PF	Accurate map	Computational cost
2007	PTAM	Klein and Murray	Feature- based	RGB images	Keyframe extraction, nonlinear optimization	Accurate localization	Not applicable in large environments
2008	Visual SLAM	Lee& song	Direct method	RGB images	Autonomous object detection.	Sparse 2D Grid map, Accurate navigation	Sparse map
2011	DTAM	Rainer et al.	Direct method	RGB images	Using multiple images of the same scene, Using pixel's intensity	Graph-based SLAM, depth 3D map	Computational cost
2012	CV-SLAM	Choi et al.	Feature- based	RGB-images	Ceiling vision	Improved localization,	Poor map
2013	RatSLAM	Ball et al. n.d.	Feature- based	Odometry, RGB images	First biological-inspired method	Graph map, navigation	Limited application
2014	LSD-SLAM	Engel et al.	Direct Method	RGB-images	Multiple pictures of a static scene, innovative direct tracking method	LSD-SLAM, semi-dense map	Computational cost, Changing light violates the system
2014	RTS	Brand et al.	Direct Method	Stereo- images	Using 2D submaps	2.5D Grid map Active obstacle avoidance	Computational cost
2015	ORBSLAM	Mur-Artal et al.	Feature- based	Both	Different optimization methods,	3D ORB features, re-localization	Map is not reusable
2016	Pinpoint SLAM	Cansizoglu et al.	Feature- based	RGB images	Longer-range constraints	Overcome registration errors,	Computational cost, motion uncertainty
2017	RTS	Liu et al.	Direct method	RGB images	Trajectory correction and retracking	Height TCR Low RMSE	Small environments usage only
2017	ORB-SLAM- tracking	Lv et al.	Direct method	Stereo images	Real-time dense 3D modeling, RTS, octomap	Realtime exploration. Dense 3D map	Direct access to the map is not possible
2017	ORBSLAM2	Mur-Artal & Tardos	Feature- based	Stereo & RGB	Forming a PnP, training vocabulary	Robustness against violent movement, accurate reusable map	Time consuming text-file procedure
2017	MonoRGBD	Yousif et al.	Feature- based	RGB images	2D and 3D features, FOV, MST,	Larger-scale 3D map	Computational cost
2017	Tight sensor fusion	Caldato et al.	Both	Stereo or RGB, Odometry	Integrating odometry and visual data	Featureless environments tracking, robustness	Computational cost
2017	Multi- sensorial SLAM	E. López et al.	Feature- based	Monocular & onboard sensors	Data Fusion Remote computations, EKF	2.5D map, improved localization	Offline map not available
2017	PL-SLAM	Pumarola et al.	Feature- based	Stereo or RGB	Point and line correspondence	Handling low textured scenes	Computational cost
2018	Feature Saliency based SLAM	Li et al.	Feature- based	RGB	Different feature detection methods. both corner and circle features. Saliency measurement	Hybrid map Improves the stability,	Uncertainty in observation and motion
2018	DPI-SLAM	Hsiao et al.	Both	RGB and IMU	Local depth fusion, Bayes tree	Large scale 3D map	Lack of loop closure
2018	Sliding mode	Wang et al.	Feature- based	RGB images	The fusion of IMU	Re-tracking, Robustness	Computational cost
2018	ORB-SLAM2 with 6DOF motion	Zeng et al.	Feature- based	Stereo or RGB	Filtering the 3DOF keyframes out	Distinguish the differences between 3DoF and 6DoF frames	Degraded performance
2018	Unified spherical model	Wangl et al.	Feature- based	Stereo	Semi-dense feature matching	More comprehensive map	Uncertainty in large environments, non-reusable map
2019	Rapid Relocation	Yang et al.	Feature- based	Both	Binary vocabulary saving, BRIEF	Enhanced ORBSLAM2	Sparse map
2019	Autonomous SLAM	Mo et al.	Feature- based	Both	Bayesian optimization information exploration	2D dense map, performance in a complex environment	Camera shakes violates the system

in environments with complex features is ambiguous. Neunert et al. (2016) used artificial landmarks to drive a monocular visual-inertial

SLAM based on EKF. An inertial measurement unit is implemented and when the information from markers is received, the EKF does the fusing

Table 5

Year	Author	Technique	Goal	
2014	Shih et al.	Fuzzy filter triangulation	Data association, accuracy	
2015	Zikos & Petridis	RBWPF KF Kalman Smoothing	Enhancing speed	
2016	Jiang & Li	Adaptive KF	Improving real-time SLAM	
2016	Altan et al.	Ultrasonic and mine-based EKF	Optimum map	
2016	Fiala et al.	SPM	Robustness	
2016	Evers et al.	Hypothesis density filter.	Map building with sound	
2016	Gee et al.	Merging multiple scans	Accuracy	
2016	Lin et al.	WIFI Internet of Things (IoT)	Accuracy	
2016	Zedadra et al.	Foraging multi-agent method	Performance speed	
2017	Xie et al.	Bias estimation	Enhancing EKF	
2017	Shim & Cho	Multi robot utilizing map sharing	Higher efficiency in the map	
2017	Turan et al.	Surfer-based dense data fusion	Accurate pose estimation and effectiveness	
2017	Wang et al.	Kriging interpolating, enhanced exponentially weighted PF	Improvement in localization an mapping	
2017	Vysotska and Stachniss	Usage of OSM	Active localization, accuracy in map	
2017	Dube et al.	Progressive sparse pose-graph optimization	Online performance, Rich 3D ma	
2017	Bowman et al.	Fusing semantic information with metric features	Full 6D pose history of measurements	
2017	J. Zhang et al.	Utilizing external storage memory	Long-run effectiveness	
2018	Santos et al.	Sliding mode approach	A robust system against restricted perturbations	
2018	Liu et al.	AGA	Scattering map from radar senso	
2018	Ji et al.	Closet Probability, EM, ICP, NDT	Effective matching, realtime performance	
2018	S. Das	RTAB, multiple loop closure	Large-scale mapping	
2018	Fortino	Cooperative smart object method	Large scale multi agent mapping	
2018	Munoz-Salinas et al.	SPMs, multiple images	Accuracy	
2019	Du and Du	Optimized RBPF multiple sensors	Decreasing computational cost	
2019	Joukhadar et al.	UKF, SIFT	Accuracy in localization	
2019	Kakoty et al.	Online sensory reads, Navigation based on human behavior,	Real time localization	
2019	P. Robertson et al.	Beta distribution, enhanced Bayesian estimation	Map from foot-mounted sensors	
2019	Clements et al.	RBPF, beta distributions	Active exploration, improved mapping	

task. However, the uncertainty in the marker's pose is not recognized in this work and it also suffers from lacking a loop closure structure. Munoz-Salinas et al. (2018) introduced an off-line approach to build a map from SPMs in large indoor environments that is capable of eliminating the position uncertainty in an off-line manner based on using an unlimited number of high-resolution images in order to obtain the high accuracy. The key problem with using SPMs is the ambiguity problem (Oberkampf et al., 1996; Schweighofer and Pinz, 2006; Collins and Bartoli, 2014) that is caused by the uncertainty in localization of the marker's corners that are subjected to noise. Moreover, offline methods have several disadvantages such as requiring the system to wait until the whole process be done in order to detect errors, requiring the repetition of the whole process in case of expansion in the map and finally the high computational demands. To overcome these disadvantages, Munoz-Salinas et al. (2018) developed a novel SLAM algorithm from Squared Planar Markers named (SPM-SLAM) that can efficiently solve the SLAM problem in large-scale environments and

in real-time. The key idea is to place some printed markers in the environment and use a camera to estimate the pose of the markers and the camera's location at the same time. This method primary initials the map from a set of images with ambiguous markers and based on a new method it estimates the pose of markers relying on the observations assuming that the frame poses are priorly known. This technique uses a local optimization method that coordinates the keyframes and marker locations by decreasing the reprojection uncertainty of the markers. The key idea is that new observations constantly improve the poses of the keyframes. Furthermore, the local optimization process involves only the keyframes associated with the observed markers in the associated frame. Therefore, the computational cost is affordable.

## 6.1. Discussion

Reviewing subsystems and different aspects of SLAM frameworks regarding the common ground in the majority of the key SLAM solving methods is essential to understand the evolution of the technology as well as finding out what is needed to be further investigated. Based on the problems and the advantages of each solution, the efficient approaches towards different difficulties can be tracked. A general definition of the key terms within the SLAM problem as well as correcting the wrong statements throughout the existed studies also is beneficial. The difficulties that caused the evolution of the methods to be formed throughout the past helps to determine the state of the art. The SLAM evolution considering the key perspectives; probabilistic approaches from filter-based approaches to information-based methods, the key application-based developments to the major environmentbased method evolution, and the evolution of the technology based on the type of sensors as well as determining the SLAM leading approaches regarding a common ground withing the perspectives is essential to be considered for forming the state of the art. We reviewed the key leading approaches based on SLAM major difficulties and the key aspects. The approaches for dealing with the correspondence and methods based on forming the loop closures that offer useful tools for the matching, as well as the methods designed based on resampling solutions, are reviewed. Throughout history, some methods are proposed based on the map criteria and several approaches are designed based on the different types of sensors. In some points, the different approaches taken on different aspects of SLAM meet at a common spot that leads the SLAM techniques to be developed with more efficiency in solutions

The SLAM development has been carried on in two major types of approaches, filter-based probabilistic approaches, and optimizationbased approaches. The starting point of SLAM can be traced back to the appearance of the spatial uncertainty estimation method which was concerned more with the uncertainties in the map as a subsystem parallel to the localization. The stochastic map framework was the most influential work and based on it, the essence of the SLAM was established based on KF that led to the appearance of the AMF method which was designed to estimate a state vector including the robot and the landmarks poses together. The square computational complexity of AMF resulted in the appearance of many methods, i.e. Divide and Conquer, Relative Map Filters, Parallel taskings, sub-map division, covariant intersect, etc. and their progressions resulted in the appearance of many different SLAM methods based on different variations of KF. Probabilistic methods based on Bayesian rule are the key leading approaches regarding filter-based approaches based on different variations of Kalman Filters such as KF, EKF, AKF, AEKF, CEKF, UKF, SKF, EM, MLE, PF, RBPF, IF or ICKF, EIF, SEIF, SSF, which are the key leading methods in filter-based approaches. Kalman filters-based SLAM methods have a high convergence and can handle the uncertainties successfully but they are slow in higher dimensional environments. Most of the methods based on different variations of KF require robust features, multiple map merging and they mostly have problems withing data association. However, all KF-based methods and its variations have an advantage of providing the optimal Minimum Mean-Square Error of the state estimation and their covariance matrix can converge strongly, but again the problem with KF based approaches is that the Gaussian noise hypothesis limits the flexibility of the KF regarding a big number of landmarks and the data association within large environments. Information filter-based methods are more stable and simpler and they can handle the larger environments faster, but still, they too struggle with data association problems as well as high computational cost in high dimensional maps. Optimization approaches concern about establishing a graph of robot poses along with observations as the edges of the graph nodes and then finding the best configurations based on the edges. Optimization-based methods are relatively more optimal towards the map building and they comparatively handle the data association better; however, they too suffer from the growing computational cost in large environments that makes them inefficient, unstable, and relatively less accurate in estimating the robot pose withing the built map. Treebased filters such as SKF, ADF, AADF, TJTF, TmF methods have been

proposed that work mostly based on sparsification which the sparse nature of the maps is the key weakness of such methods. Particle Filters also traditionally termed as SMC was the latest kind of filters able to solve the localization problem able of handling the non-gaussian models. Particle Filters-based SLAM methods more suitable regarding nonlinearities and facing non-gaussian noises however the complexity in those methods grows dramatically in larger environments as the number of landmarks grows. The main concern of PF-based methods is about data association. In general, the feature-based SLAM methods are highly dependent on the robustness of the landmark detection which in complex environments and noisy situations, it is very challenging. The lack of enough landmarks can fail the SLAM as well. Visual based SLAM methods can comparatively handle the landmark detection better especially in featureless environments i.e. underwater, that can open a wider range of capabilities. Vision-based systems also benefit from a relatively low cost in design in comparison with lidar types of sensors. The SLAM key approaches are conducted based on either Lidar-based SLAM or Vision-based SLAM developments. Lidar SLAM is comparatively more sophisticated and it has been utilized in a wider range of applications. However, the map built by vision SLAM is more detailed and informative in comparison to lidar SLAM that generates sparse maps. Visual-SLAM methods are developed based on homography, essential matrix estimation point correspondences, etc. Visual SLAM is mostly categorized in Monocular, RGB-D, and Stereo SLAM approaches.

Different probabilistic SLAM frameworks based on filtering approaches exist. Fast SLAM utilizing RBPF soon become very popular and took the place of the traditional EKF SLAM. FastSLAM is very robust to errors and it has a vital power in data association in comparison with EKF based SLAM. However, the computational cost in noisy environments grows dramatically, and the FastSLAM is sensitive to divergence. FastSLAM 2.0, overcomes the computational cost and it is less sensitive to the divergence. Unscented FastSLAM improves the ability of the explosion to diverge by utilizing a robust localization. L-SLAM is as robust and as accurate as FastSLAM 2.0 but its faster than FastSLAM 2.0. LCPF SLAM comparatively has a more powerful loop closure and has a better consistency and can be utilized in larger workspaces. However, it has slower performance. Differential Evolution SLAM was another successful SLAM solving method and SAM and PHD methods improved the mapping process of SLAM solving methods. Different methods have been utilized to enhance the methods. Triangulation, Fuzzy filter, and curvature data were utilized in SLAM frameworks to improve the data association. NDT and ICP methods were utilized to improve the matching process. iSAM2 has a powerful data association and it can be utilized in outdoor environments as well and it is able of providing more information on the features, however, it is relatively slower than its pioneer methods and the robot motion is not fully recognized in this method. CPFG SLAM was developed for off-roads inactive environments using the EM scheme.

Different types of sensors, passive i.e. visual sensors and active i.e. laser sensors have been utilized in different SLAM methods as well as different data-fusion methods for utilizing different combinations of several sensors at use. Passive sensors require relatively a higher computational cost. SLAM based on human behaviors has been proposed, foot-traveler applications have been proposed. Sound-based methods using a microphone array, the microwave radar sensor in radar-based SLAM methods have been developed. Wifi-based fingerprinting techniques ZigBee-based methods based on magnetic fields, mine based methods also have been proposed. Foraging and Internet of Things and ant-colony methods, Kriging interpolating, AGA, medical operators, Arm-SLAM in industrial autonomous manipulators, flying objects, underwater submarines, sound pitch correcting methods, and sound-based applications i.e. A-SLAM telecommunication devices i.e. Channel-SLAM for position estimation of mobile receivers, and many other approaches and applications for instance for the endoscopic capsule robots in surgical applications have been taken in SLAM methods. However, there have been relatively fewer works been done on the underwater SLAM solution and sound-based applications.

Many great achievements have been reached to improve SLAM solutions, but among the existing SLAM methods, almost none of them are capable of building complete consistent maps in large environments mostly because of the computation complexity cost, and the growing uncertainty in large environments. Thus, this side of difficulties with SLAM with big data computations has a great room for improvements. Furthermore, since the global maps are mostly obtained from several sub-maps of smaller areas within the environment as for the majority of the methods, the data association still is an issue and it has a place to be further improved. Besides the problem with big data and large environment mapping, SLAM in shapeless environments, i.e. space, air, and underwater, are of the most challenging problems regarding SLAM methods. SLAM approaches considering the service robotics with a concentration on human safety also has been less studied throughout history. Besides, considering the survey, SLAM solving methods can be enhanced by either scheming the system more suitable to larger scenes by improving the filtering methods, or by improving the robustness and efficiency withing the data association aspect.

### 7. Conclusion

SLAM problem is a developing wide area of research and experiments in Artificial Intelligence and Robotics. It has been expanded to other fields. In this paper, we took a tour through this important development in its history by reviewing the works that have had an impact on technology. We focused on the birth year of the algorithms to highlight the way that SLAM has gone through so far and from there we picked up several interesting approaches within different aspects of the SLAM development. However, there has been a huge number of works been conducted in different areas regarding SLAM solution which each of them could be separately reviewed, so that, obviously we cannot present all the works in one single paper, but we attempted to present the most interesting papers according to our opinion that can provide the readers to receive a fairly enough knowledge on almost the all aspects of the area. SLAM is a very wide area that involves many different aspects of different science areas and for a beginner, it usually is very hard to find their way, on the other hand, researches starting to study the SLAM needs to study many papers and many different reviews to get familiar with the major aspects of the technology. We believe this paper is one single efficient paper to understand the important aspects, problems, and directions towards SLAM development that can help researches to jump into the study of the aspect that fits them the best. Moreover, SLAM evolution is covered that allows scientists to have enough understating of the state of the art. Furthermore, the paper shows that the SLAM approaches have been concentrating on filtering measurements, type, and the perspective of the sensors, as well as on the data fusion, uncertainty, mapping, and navigation within different types of environments. However, the strategy of the problem itself has been less studied. Furthermore, although there have been many outstanding accurate algorithms been designed, the solutions still are not perfect that leaves room for improvement. For the future work, we suggest taking a study on SLAM algorithms from the perspective of the applications regarding the SLAM algorithms, and the types of the environments. Last but not the least, a review based on types of maps also may come very useful in the science sharing area.

#### **Funding**

No funding was received.

# Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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