CS:3330, Algorithms

HW04 Order Problems

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1 Big-O #1

Show directly, using the definition of Big-O, that $2n^2 + 9n \in O(n^2)$.

The following is the definition for Big-O which will be used for problems 1-3 on this assignment. For a given complexity function $f(n)$, $O(f(n))$ is the set of complexity functions $g(n)$ for which there exists some positive real constant c and some nonnegative integer N such that for all $n \geq N$,

$$g(n) \leq c * f(n)$$

I show that $2n^2 + 9n \in O(n^2)$. Because, for $n \geq 1$,

$$\begin{aligned} 2n^2 + 9n &\leq 2n^2 + 9n^2 \\ &\leq 11n^2 \end{aligned}$$

Where $c = 11$ and $N = 1$ were used to obtain the result.

2 Big-O #2

Show directly, using the definition of Big-O, that $5n^2 + 10 \in O(n^3)$.

I show that $5n^2 + 10 \in O(n^2)$. Because, for $n \geq 2$,

$$5n^2 + 10 \leq 5n^3$$

Where $c = 5$ and $N = 2$ were used to obtain the result.

3 Big-O #3

Show directly, using the definition of Big-O, that $6n^2 + 12n \in O(n^2)$.

I show that $6n^2 + 12n \in O(n^2)$. Because, for $n \geq 1$,

$$\begin{aligned} 6n^2 + 12n &\leq 6n^2 + 12n^2 \\ &\leq 18n^2 \end{aligned}$$

Where $c = 18$ and $N = 1$ were used to obtain the result.

4 Omega # 1

Show directly, using the definition of Ω, that $6n^3 - 12n \in \Omega(n^3)$.

The following is the definition for Omega and will be used for problems 4-6 on this assignment. For a given complexity function $f(n)$, $\Omega(f(n))$ is the set of complexity functions $g(n)$ for which there exists some positive real constant c and some nonnegative integer N such that for all $n \geq N$,

$$g(n) \geq c * f(n)$$

I show that $6n^3 - 12n \in \Omega(n^3)$. Because, for $n \geq 2$,

$$6n^3 - 12n \geq 1 * n^3$$

Where $c = 1$ and $N = 2$ were used to obtain the result. For this question and the other two Ω questions on this HW, the constant was picked as 1 before the N, because that keeps the right side of the inequality as low as possible. Then, the N value was calculated by doing a simple calculation as to when N is the lowest and the inequality is true.

5 Omega # 2

Show directly, using the definition of Ω, that $4n^3 + 2n^2 \in \Omega(n^2)$.

I show that $4n^3 + 2n^2 \in \Omega(n^2)$. Because, for $n \geq 0$,

$$4n^3 + 2n^2 \geq 1 * n^2$$

Where $c = 1$ and $N = 0$ were used to obtain the result.

6 Omega # 3

Show directly, using the definition of Ω, that $6n^2 + 12n \in \Omega(n^2)$.

I show that $6n^2 + 12n \in \Omega(n^2)$. Because, for $n \geq 0$,

$$6n^2 + 12n \geq 1 * n^2$$

Where $c = 1$ and $N = 0$ were used to obtain the result.