CS:3330, Algorithms

HW11 - Minimum Spanning Trees

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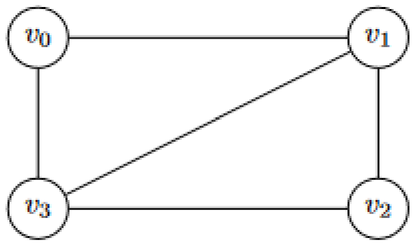
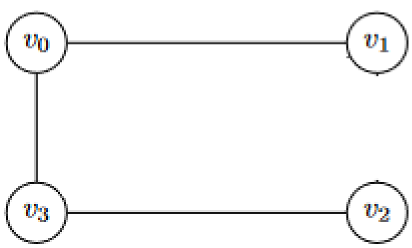
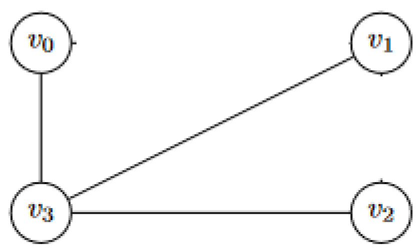
1 All spanning trees

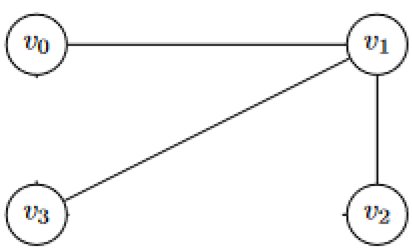
Find all possible spanning trees of the graph shown below.

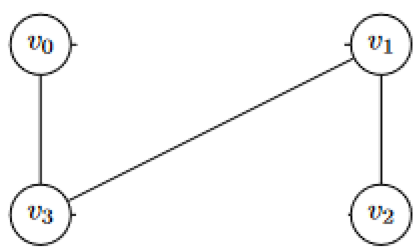


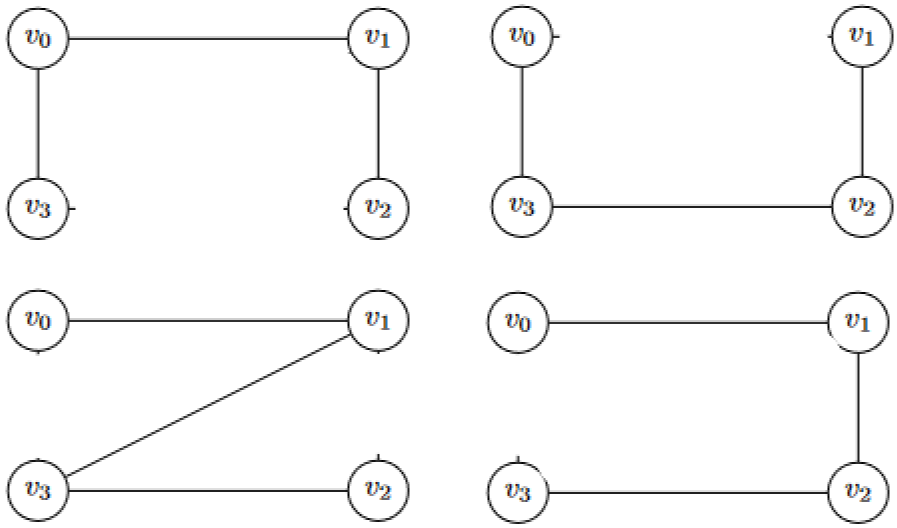
The following are the possible spanning trees for the given graph:





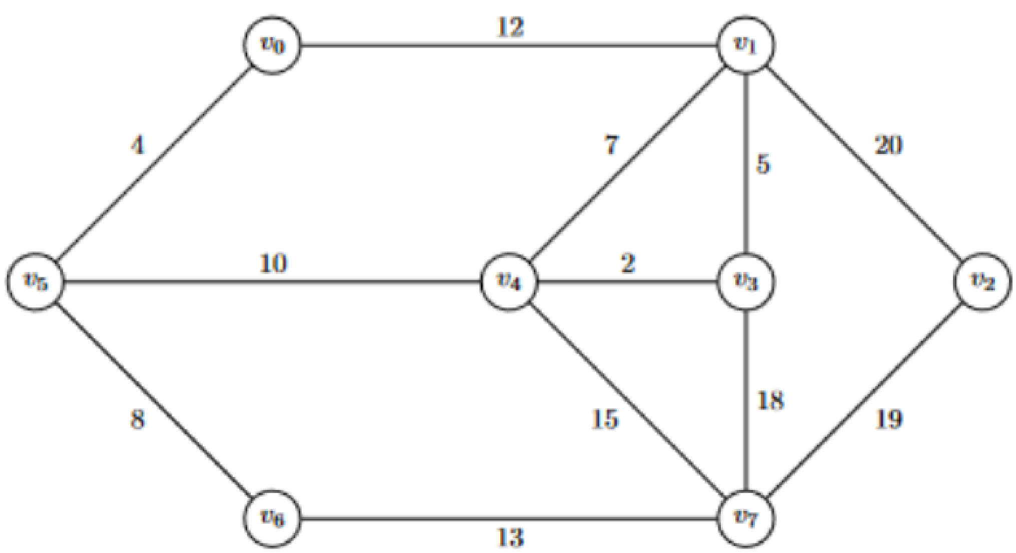






2 Kruskal's

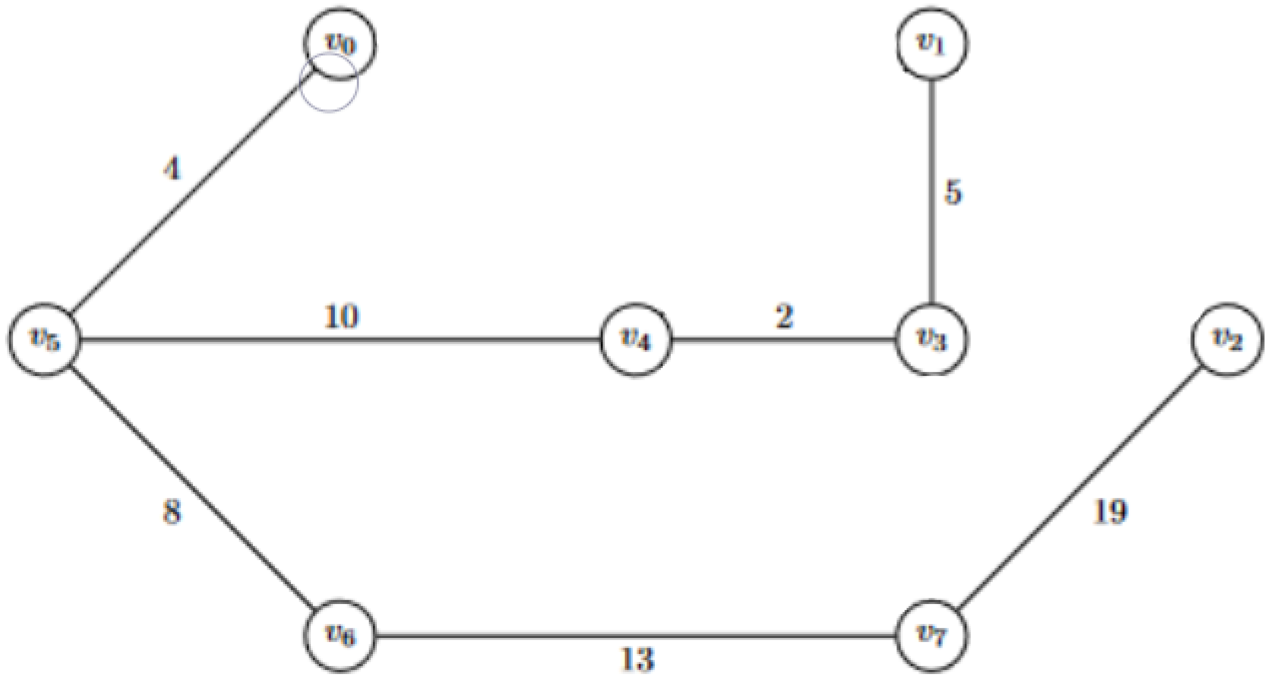
Use Kruskal's algorithm to find a minimum spanning tree for the graph shown below. Indicate the order in which edges are added to form the MST.



The following set of edges, F, represent the order in which edges were added to the MST for this graph.

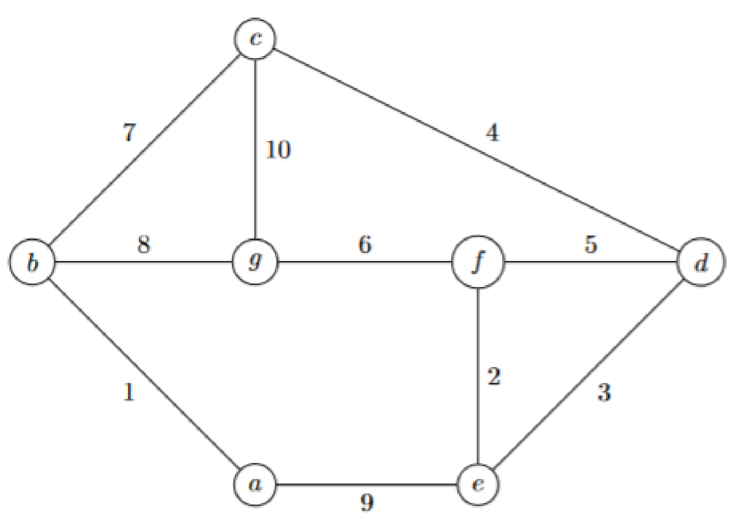
$$F = \{\{v_3, v_4\}, \{v_0, v_5\}, \{v_1, v_3\}, \{v_5, v_6\}, \{v_4, v_5\}, \{v_6, v_7\}, \{v_2, v_7\}\}$$

This set of edges gives the following MST:



3 Prim's

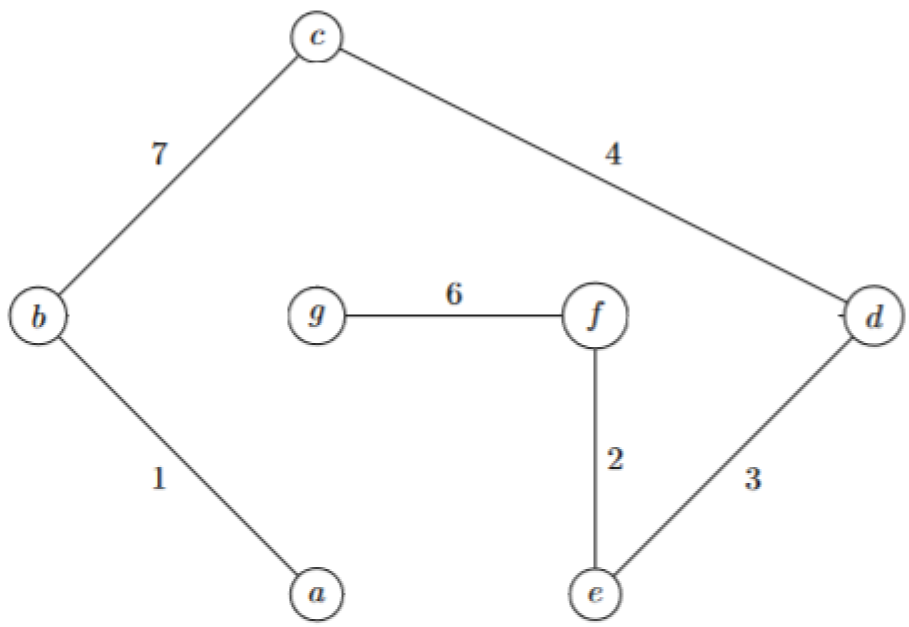
Use Prim's algorithm to find a minimum spanning tree for the graph shown below. Indicate the order in which edges are added to form the MST.



The following set of edges, F, represent the order in which edges were added to the MST for this graph.

$$F = \{\{v_a, v_b\}, \{v_b, v_c\}, \{v_c, v_d\}, \{v_d, v_e\}, \{v_e, v_f\}, \{v_f, v_g\}\}$$

This set of edges give the following MST:



4 G and G_{new}

Let T be a minimum spanning tree of graph G obtained by Prim's algorithm. Let G_{new} be a graph obtained by adding to G a new vertex and some edges, with weights, connecting the new vertex to some vertices in G. Can we construct a minimum spanning tree of G_{new} by adding one of the new edges to T? If you answer yes, expain how; If you answer no, explain why not.

No, you can not get a correct MST for G_{new} by simply adding one of the new edges to the end of the MST for G. This is because you cannot guarantee that the new edges would have been a more locally optimal choice earlier in the algorithm.

For example, take the graph from problem three and a new vertex h to it with edges $\{c, h\}$ and $\{d, h\}$ with weights of 1 and 2 respectively. If you were to add one of these edges to T (the MST for G), edge $\{c, h\}$ would be added. However, if you run all of Prim's algorithm again with the new vertex and edges, the edge $\{c, d\}$ will be replaced by the new edges $\{c, h\}$ and $\{d, h\}$ because there combined weight of 3 is 2 less than the combined weight of $\{c, d\}$ and $\{c, h\}$ which is 5. In another situation, if the weights of both edges were greater than the weight of $\{c, d\}$ (4) the MST for G_{new} would just be an extension of the original T.

This example demonstrates my explanation as to why when adding a vertex to a graph, the MST for the new graph can not be guaranteed to be a simple extension of the original graph and the MST for the new graph must be recalculated start to finish.

5 Disconnected Kruskal's

Suppose Kruskal's algorithm is run on a disconnected graph. What will be the output?

The output of Kruskal's algorithm on a disconnected graph is more than one spanning tree, one for each group of connected vertices.

6 Disconnected Prim's

Suppose Prim's algorithm is run on a disconnected graph. What will be the output.

Prim's algorithm on a disconnected graph will have no output, as the algorithm runs until it connects all vertices to one another and by nature of a disconnected graph, the algorithm will never be able to meet that condition.