mathhw5

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CS 28150 Homework 5

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We wish to devise a class for edge-tridiagonal matrices. It is represented by four arrays, easily enough. Multiplication of a ETM A with a vector B is also very easy to compute, as many of the entries in the matrix are zero. We use a looping function that does at most 4 multiplications per row. The test of the multiplication method checks out.

Solving a solution requires a bit more work, but still only four multiplications and four subtractions per row. This allows for the solving to take significantly less time than a brute-force solving, where each cell would have to be checked. The test for solving also matches our expectations, even at larger n (although with worse quality).

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import math
import time

class Edge_Tridiag:
    size = 0
    d = []
    a = []
    b = []
    e = []
    def __init__(self, *args):
        if len(args) == 1:
            self.size = args[0]
            self.a = [1] * (self.size - 1)
            self.d = [2] * self.size
            self.b = [3] * (self.size - 1)
            self.e = [4] * (self.size - 2)
        else :
            self.a = args[1]
            self.d = args[2]
            self.b = args[3]
            self.e = args[4]
            self.size = args[0]
```

```python
def mult(self, v):
    #multiplies the edge tridiagonal with a given vector
    n = self.size
    sol = [0] * n
    #sol[0] = self.d[0] * v[0] + self.a[0] * v[1] + self.e[0] * v[n-1]
    for i in range(n):
        b_term = 0
        a_term = 0
        e_term = 0
        d_term = self.d[i] * v[i]
        if (i - 1 >= 0):
            b_term = self.b[i-1] * v[i-1]
        if (i < n - 1):
            a_term = self.a[i] * v[i+1]
        if (i < n-2):
            e_term =  self.e[i] * v[n-1]
        sol[i] = d_term + b_term + a_term + e_term
    return sol

def solve(self, v):
    n = self.size
    sol = [0] * n
    #forward solving part
    for i in range(n - 2):
        #scale current row
        scale = self.d[i]
        self.d[i] /= scale
        v[i] /= scale
        if (i < n - 1):
            #print("Orig a is", self.a[i])
            self.a[i] /= scale
            #print("New a is", self.a[i])
        if (i < n - 2):
            self.e[i] /= scale

        self.d[i+1] -= (self.a[i] * self.b[i])
        v[i+1] -= (v[i] * self.b[i])
        if (i < n - 3):
            self.e[i+1] -= (self.e[i] * self.b[i])

    #second-to-last loop special
    scale = self.d[n-2]
    self.a[n-2] -= (self.e[n-3] * self.b[n-3])
    self.a[n-2] /= scale
    self.d[n-2] /= scale
    v[n-2] /= scale
    v[n-1] -= (v[n-2] * self.b[n-3])
```

```
        #last loop special
        self.d[n-1] -= (self.a[n-2] * self.b[n-2])
        scale = self.d[n-1]
        self.d[n-1] /= scale
        v[n-1] /= scale
        #backward solution
        for j in range(n): #check this
            a_term = 0
            e_term = 0
            if ((n - j) < n):
                a_term = self.a[n - j - 1] * sol[n - j]
            if (n - j) < n - 1:
                e_term = self.e[n - j - 1] * sol[n-1]
            sol[n- j - 1] = v[n- j - 1] - a_term - e_term
        return sol
```

[ ]:
```
#Test One: Multiplication
C = Edge_Tridiag(5)
C.d = list(map(lambda x: 4 + .1 * x, range(5)))
C.a = list(map(lambda x: 1 + .01 * (x ** 2) , range(4)))
C.b = list(map(lambda x: 1 - .01 - (x * .03) , range(4)))
C.e = list(map(lambda x: 1 - (x * .05) , range(3)))
```

[ ]:
```
#C-Matrix
#4      1      0       0      1

#.99  4.1    1.01     0     .95

#0    .96   4.2    1.04     .9

#0      0     .93     4.3    1.09

#0      0      0       .9    4.4
```

[ ]:
```
prod = C.mult([1, 2, 3, 4, 5])
prod
```

[ ]:
```
[11.0, 16.97, 23.18, 25.439999999999998, 25.6]
```

[ ]:
```
C.solve(prod)
```

[ ]:
```
[1.0078005466331463,
 2.005500540531604,
 3.0045473597669186,
 4.008320215794588,
 4.963297272935812]
```

[ ]:
```
big = Edge_Tridiag(40000)
big.d = list(map(lambda x: 4 + .1 * (math.sin(x/1000)), range(40000)))
big.a = list(map(lambda x: .2 * (math.sin(x/2000)) , range(39999)))
big.b = list(map(lambda x: .3 * (math.sin(x/3000)) , range(39999)))
big.e = list(map(lambda x: .4 * (math.sin(x/4000)) , range(39998)))
x = list(range(1,40001))
prod2 = big.mult(x)
```

[ ]:
```
begin = time.time()
big.solve(prod2)
print(time.time() - begin, 's')
```

```
0.06947779655456543 s
```

[ ]:
```
big.solve(prod2)
```

[ ]:
```
[1.0,
 2.1738990471396487,
 3.34755898222339,
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Here we see that the solving method, even for a matrix that is 4k x 4k, takes less than 0.1 seconds to evaluate. The solution provided is a bit off, due to the rounding errors of each row, but it maintains a roughly similar shape to the expected solution.