SUMMARY OF IMPORTANT CONCEPTS AND FORMULAE

1. **Waves on parallel-conductor transmission lines:** A pair of parallel conductors serves to guide voltage and current waves from one end to the other. The waves are traveling in the medium surrounding the conductors as electric and magnetic fields which constitute plane waves. The line can be visualized as a distributed-parameter circuit where per-unit-length inductances and capacitances form cells of infinitesimal length distributed along the line. A finite time is required to charge and discharge these elements as the wave propagates along the line, resulting in a time delay of propagation on the line of $T = \mathcal{L}/v$.

2. **The transmission-line equations:** The equations governing the voltage and current on the line are

$$\frac{\partial V(z,t)}{\partial z} = -l\frac{\partial I(z,t)}{\partial t}$$
$$\frac{\partial I(z,t)}{\partial z} = -c\frac{\partial V(z,t)}{\partial t}$$

whose solution is

$$V(z,t) = \underbrace{V^+\left(t - \frac{z}{v}\right)}_{\substack{\text{forward-}\\ \text{traveling } (+z) \text{ wave}}} + \underbrace{V^-\left(t + \frac{z}{v}\right)}_{\substack{\text{backward-}\\ \text{traveling } (-z) \text{ wave}}}$$

$$I(z,t) = \underbrace{\frac{V^+\left(t - \frac{z}{v}\right)}{Z_C}}_{\substack{\text{forward-}\\ \text{traveling } (+z) \text{ wave}}} - \underbrace{\frac{V^-\left(t + \frac{z}{v}\right)}{Z_C}}_{\substack{\text{backward-}\\ \text{traveling } (-z) \text{ wave}}}$$

where the characteristic impedance is $Z_C = \sqrt{l/c}$, and the velocity of propagation is $v = 1/\sqrt{lc}$. The time required for a wave to propagate from one end of the line to the other is $T = \mathcal{L}/v$ and $\mathcal{L}$ is the total line length.

3. **Reflection coefficients:** source end: $\Gamma_S = (R_S - Z_C)/(R_S + Z_C)$, load end $\Gamma_L = (R_L - Z_C)/(R_L + Z_C)$.

4. **The transmission-line equations for sinusoidal excitation:**

$$\frac{d\hat{V}(z)}{dz} = -j\omega l\,\hat{I}(z)$$
$$\frac{d\hat{I}(z)}{dz} = -j\omega c\,\hat{V}(z)$$

whose frequency-domain solution is

$$\hat{V}(z) = \hat{V}^+ e^{-j\beta z} + \hat{V}^- e^{j\beta z}$$
$$\hat{I}(z) = \frac{\hat{V}^+}{Z_C} e^{-j\beta z} - \frac{\hat{V}^-}{Z_C} e^{j\beta z}$$

where the characteristic impedance is, as before, $Z_C = \sqrt{l/c}$ and the phase constant is

$$\beta = \omega\sqrt{lc} = \frac{\omega}{v} \quad \text{radians/m}$$

The corresponding time-domain solution is

$$V(z,t) = V^+\cos(\omega t - \beta z + \theta^+) + V^-\cos(\omega t + \beta z + \theta^-)$$
$$I(z,t) = \frac{V^+}{Z_C}\cos(\omega t - \beta z + \theta^+) - \frac{V^-}{Z_C}\cos(\omega t + \beta z + \theta^-)$$

5. **Reflection coefficient and input impedance:**

$$\hat{\Gamma}_L = \frac{\hat{Z}_L - Z_C}{\hat{Z}_L + Z_C}$$
$$\hat{\Gamma}(z) = \hat{\Gamma}_L e^{j2\beta(z-\mathcal{L})}$$
$$\hat{V}(z) = \hat{V}^+ e^{-j\beta z}[1 + \hat{\Gamma}_L e^{j2\beta(z-\mathcal{L})}]$$
$$\hat{I}(z) = \frac{\hat{V}^+}{Z_C} e^{-j\beta z}[1 - \hat{\Gamma}_L e^{j2\beta(z-\mathcal{L})}]$$
$$\hat{Z}_{in} = Z_C\frac{[1 + \hat{\Gamma}(0)]}{[1 - \hat{\Gamma}(0)]} = Z_C\frac{[1 + \hat{\Gamma}_L e^{-j2\beta\mathcal{L}}]}{[1 - \hat{\Gamma}_L e^{-j2\beta\mathcal{L}}]}$$

6. **Properties of the voltage and current on the line:** Corresponding points on the magnitude of the line voltage (current) are separated by one-half wavelength in distance. The input impedance to the line replicates for multiples of a half wavelength. A maximum and the adjacent minimum are separated by one-quarter wavelength.

7. **Voltage standing wave ratio:** $\text{VSWR} = \dfrac{1 + |\hat{\Gamma}_L|}{1 - |\hat{\Gamma}_L|}$.

8. **Power flow on the line:** $P_{AV} = \dfrac{|\hat{V}^+|^2}{2Z_C}[1 - |\hat{\Gamma}_L|^2]$.