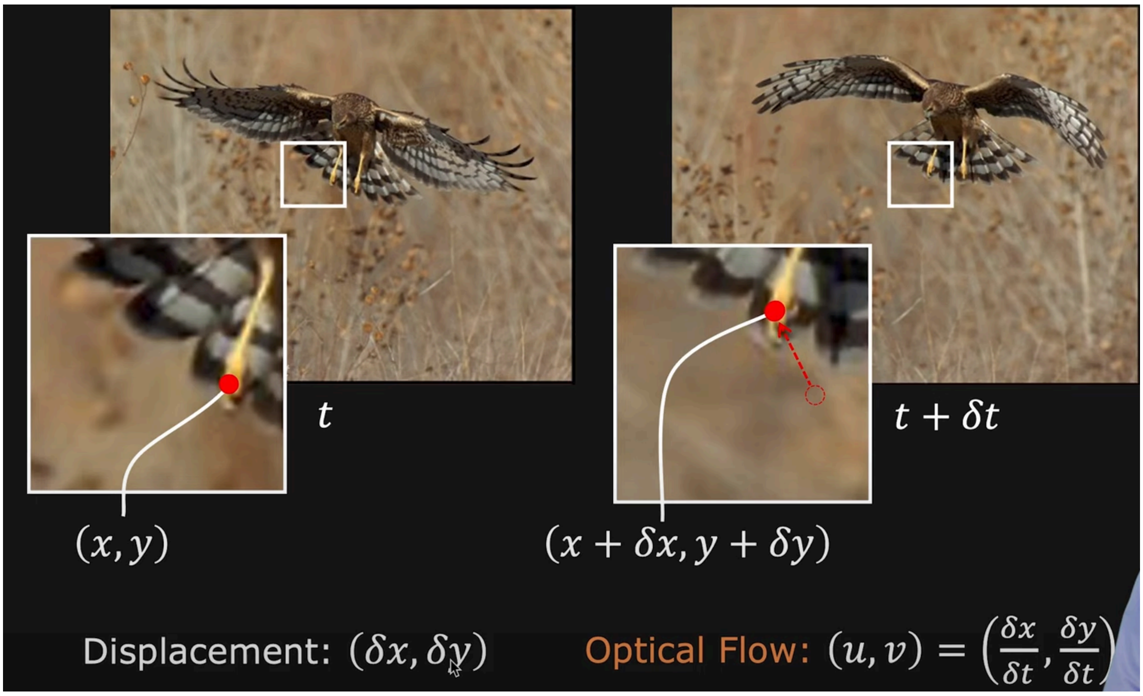
Optical Flow Constraint Equation

Consider a small window (that's the same in both images). Focus on a point in this window. Say we are focused on the foot. Then at time t + delta t, this point has moved to a new location which is x + dx, y + dy.



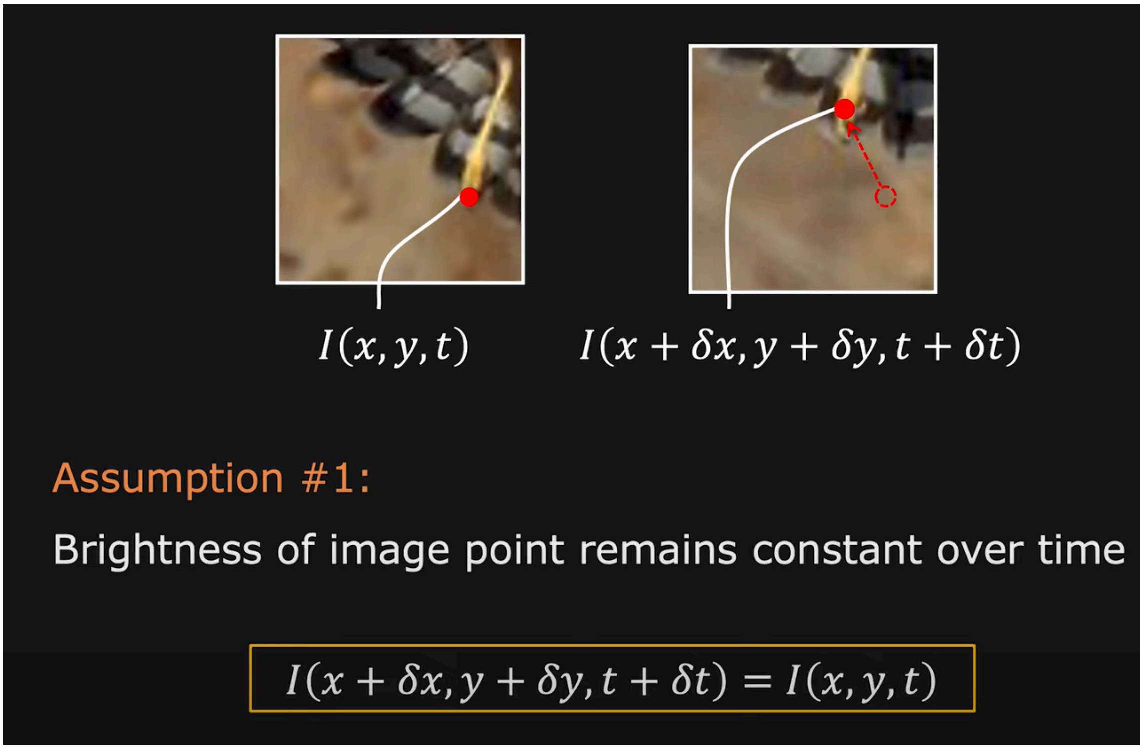
So, the displacement is (dx, dy)

So speed in x direction is dx/dt and speed in y direction is dy/dt. And this is the optical flow.

This is what we want to measure, (u, v)

Assumptions

Assumption #1: That brightness of image points remain the same, at least between consecutive images taken in quick succession. This is a reasonable assumption since dt is small.



Assumption #3: That the displacements of (dx, dy), and dt are small.

Assumption #2:

Displacement $(\delta x, \delta y)$ and time step δt are small

This has to be true for us to derive a constraint equation.

What this allows us to do is to come up with an approximation for I(x + dx, y + dy, t + dt). This approximation is based on the Taylor series expansion

Math Primer

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

Alternatively,

$$\boxed{I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + I_x\delta x + I_y\delta y + I_t\delta t}$$

Results of assumptions

The two assumptions give rise to:

Optical Flow Constraint Equation

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t) \quad \text{-----------} \ (1)$$

$$I(x+\delta x, y+\delta y, t+\delta t) = I(x,y,t) + I_x \delta x + I_y \delta y + I_t \delta t \quad \text{-----------} \ (2)$$

Then simply subtract:

Subtract (1) from (2): $I_x \delta x + I_y \delta y + I_t \delta t = 0$

Then divide by delta_t and take lim to 0

Divide by δt and take limit as $\delta t \to 0$: $I_x \dfrac{\partial x}{\partial t} + I_y \dfrac{\partial y}{\partial t} + I_t = 0$

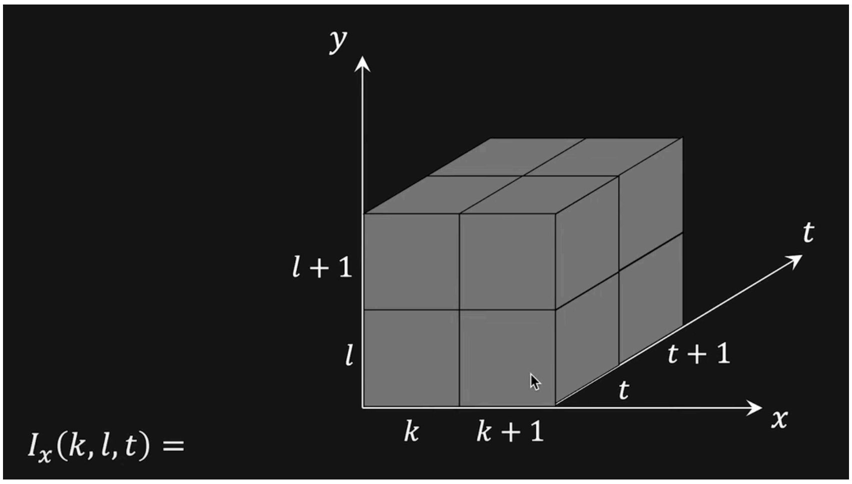
Then deltas turn into derivatives.

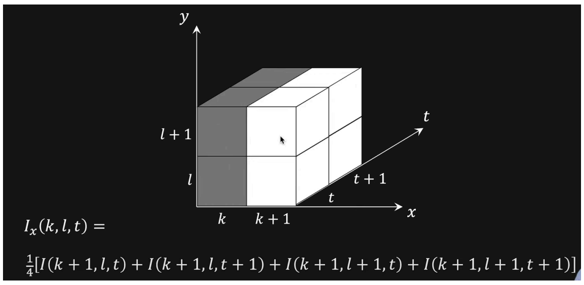
Then delta_x/delta_t becomes dx/dt = u, delta_y/delta_t becomes dy/dt.

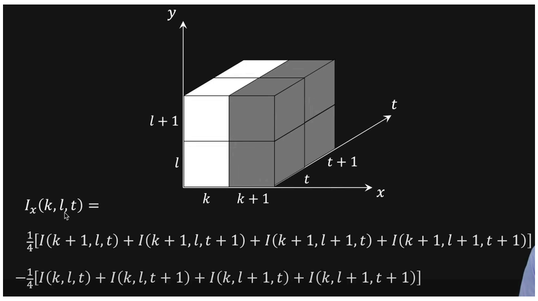
Constraint Equation: $\boxed{I_x u + I_y v + I_t = 0}$ (u, v): Optical Flow

Then! Given two equations taken in quick succession (at t and t + delta t), we can find these derivatives Ix, Iy and It. Done using finite differences using edge detection.

Computing Ix, Iy and It



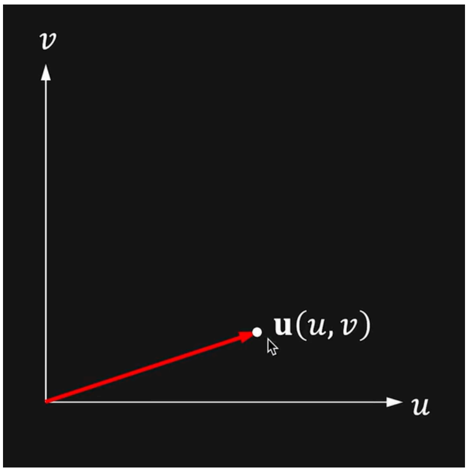




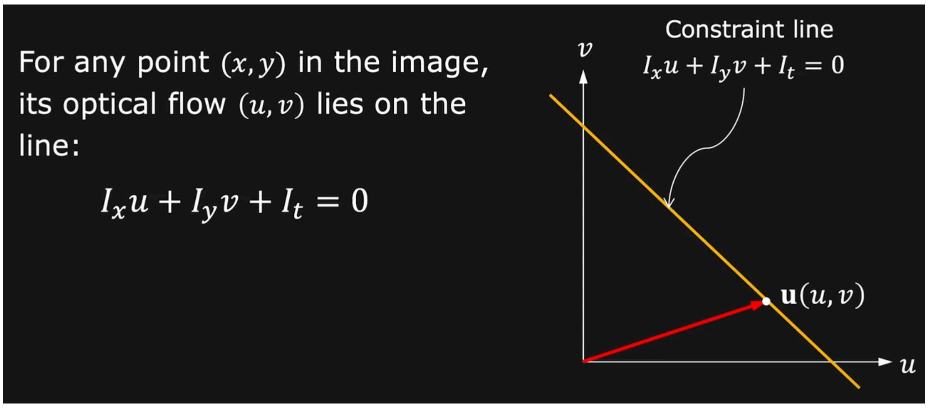
This will give you the derivative of the BRIGHTNESS in the x direction. Similarly you can do so for the y direction and t direction.

Geometrical Interpretation of Optical Flow (to show that u and v are really indeterminable, by showing under-constrainedness)

Let the true flow for a particular point in the image be (u, v)

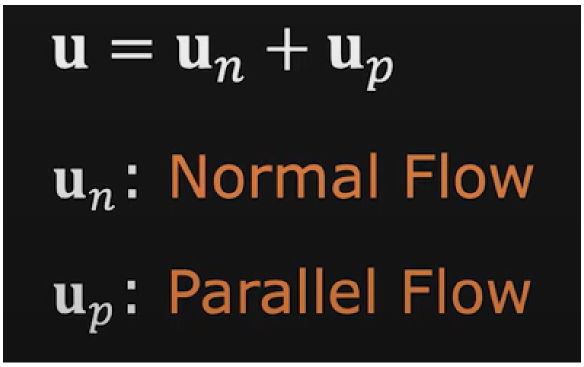


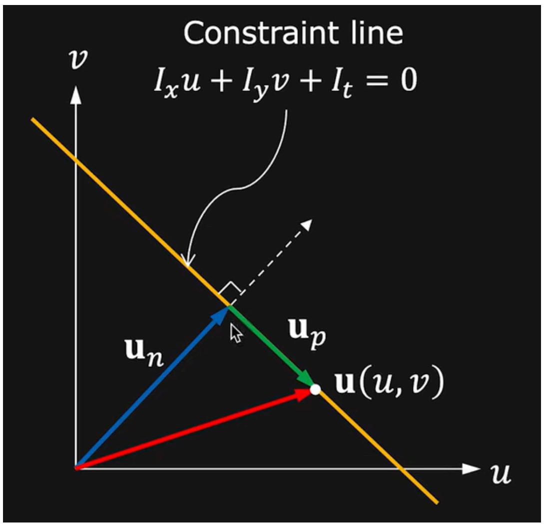
(Unfortunately) the only thing that we know about (u, v) is that it must lie on the constraint line.



We know that (u, v) lies on the line. But don't exactly know where. This is what makes optical flow an under-constrained problem.

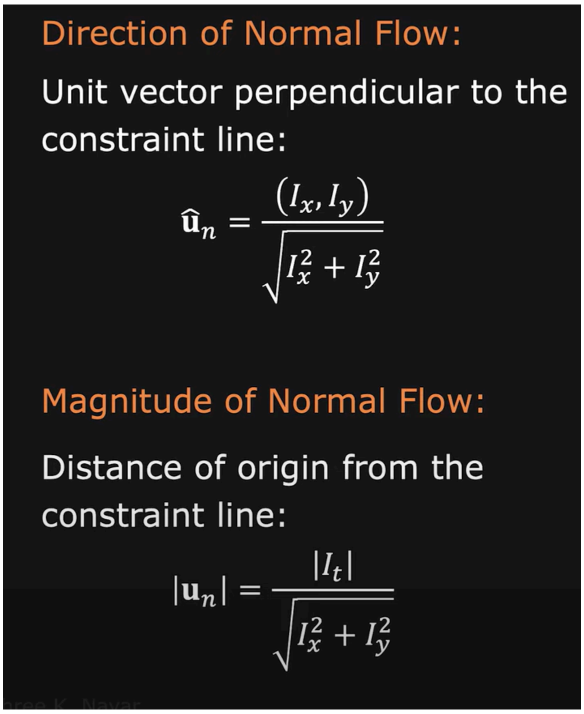
First, note that the vector can be split into two components.

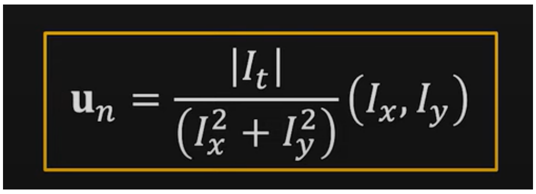




U_n is the normal flow and U_p is the parallel flow

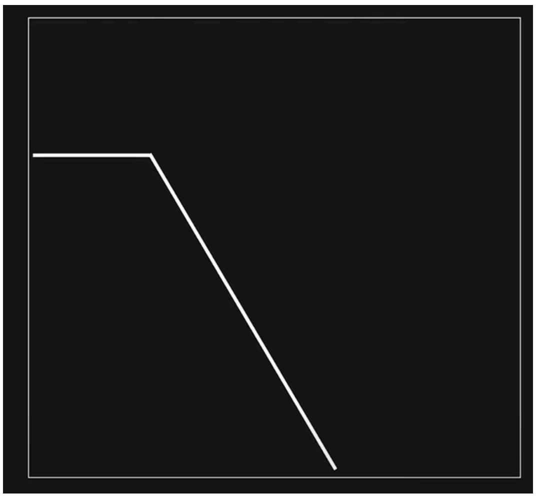
U_n is obviously easy to calculate.

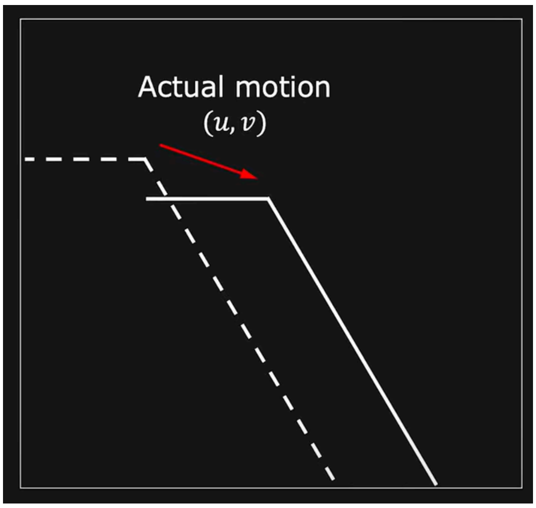




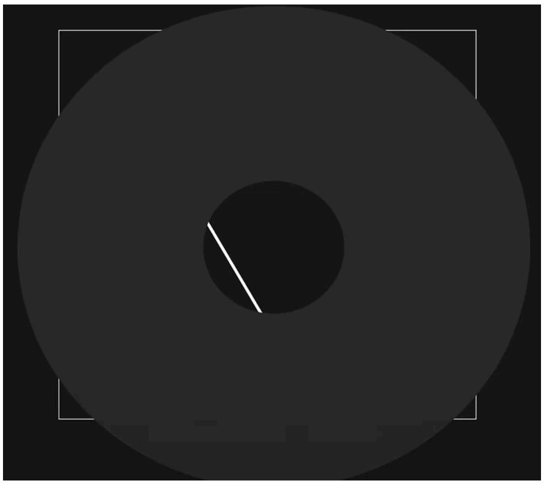
But we cannot compute U_p!

Example of how the problem of inability to calculate U_p manifest in a real scenario?

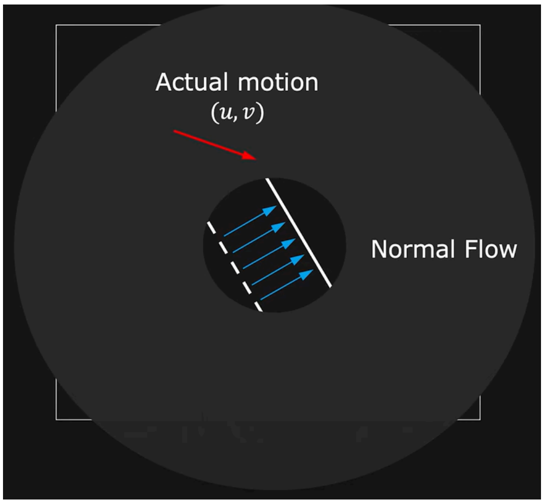




Our image is not one object, we have potentially different flows for every local region. So it necessitates to look at small local patches, called our aperture.



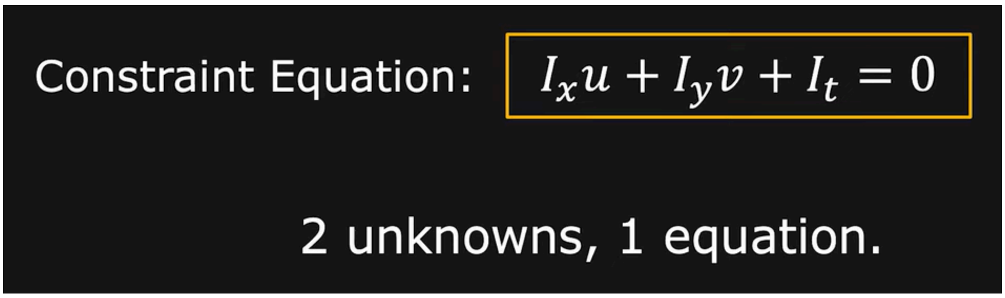
But if you look at the same motion through this aperture, you end up seeing ONLY the normal flow:



Unable to measure the parallel flow, and thus unable to measure the actual flow.

So locally, we are only able to determine the normal flow.

Optical Flow is under constrained!



2 unknowns are u and v.

To solve this, we introduce additional constraints.