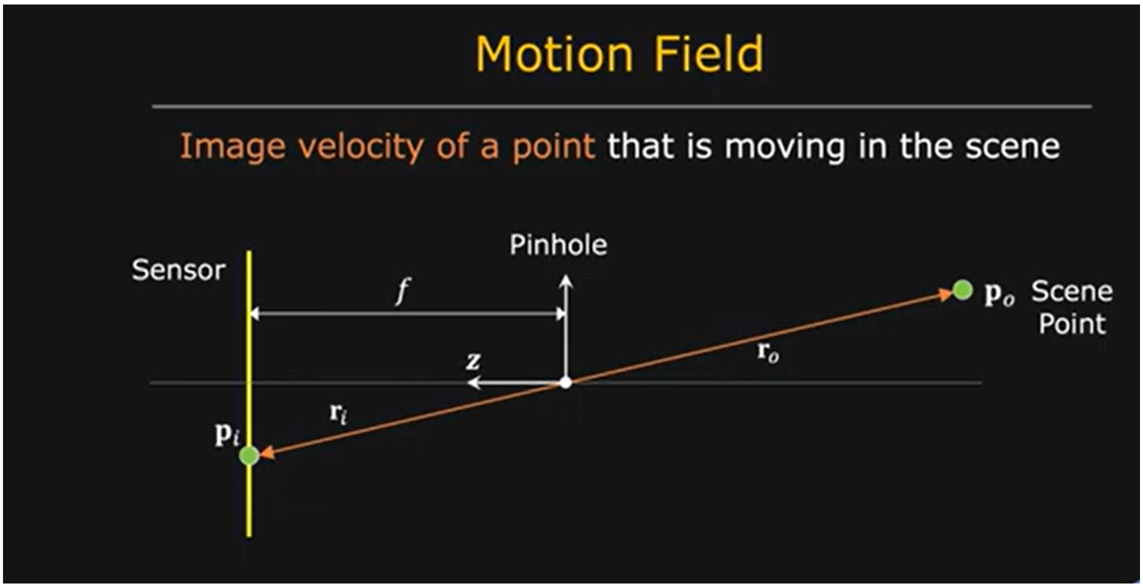
Motion Fields and Optical Flow

Motion Fields



p0 is a point in the 3d scene and it's location is given by the vector r0. p0 projects onto the point pi via perspective projection and it's location on the image plane is described by the vector ri.

If p0 has some velocity in the 3d world, say v0, then it's displacement is given by v0 * dt. So it's new location is r0 + delta r0.

Also by definition, scene velocity v0 is the rate of change of r0, so

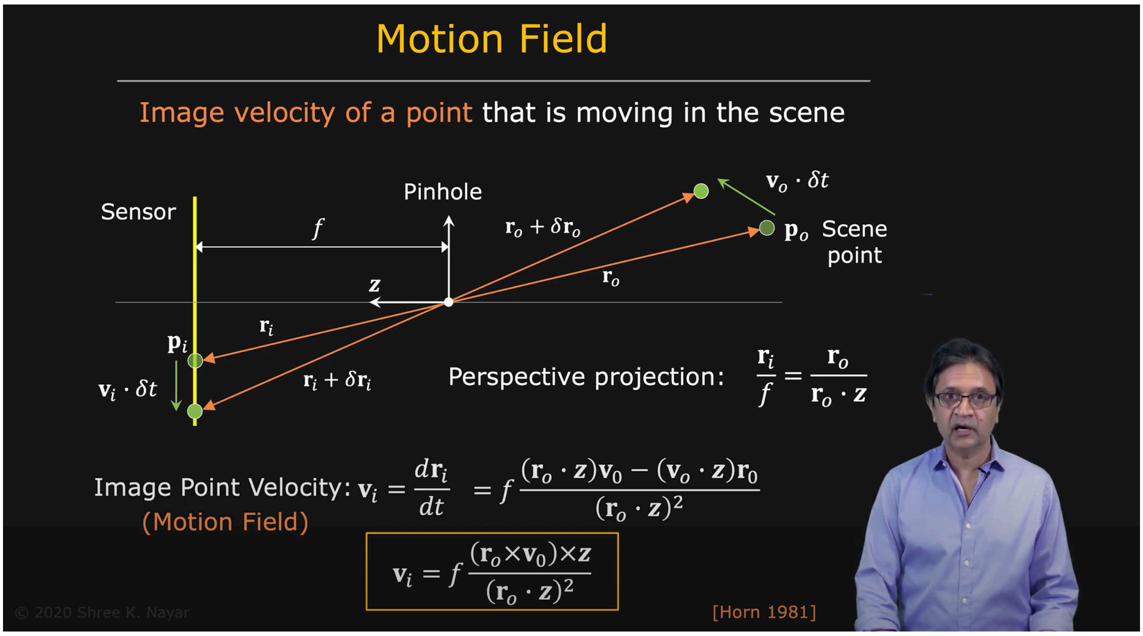
$$\text{Scene Point Velocity:} \quad \mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$$

What we are interested in is the Image point velocity:

$$\text{Image Point Velocity:} \quad \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$$

(Motion Field)

We know v0 but we don't know vi. So try to relate these to each other using perspective projection. Then simply take derivatives (using quotient rule) and then use cross product and get the answer.



This vi is the motion field corresponding to the moving point in the 3D scene.

This is what we hope to measure, but there is no guarantee that we can measure this.

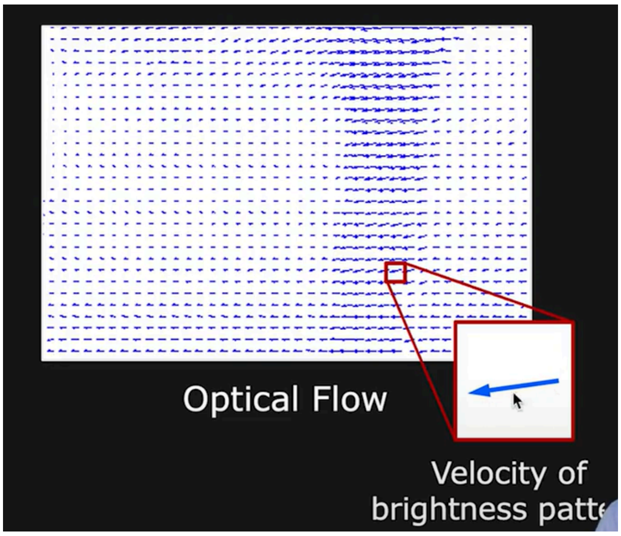
Optical Flow

Instead, what we hope to measure is the motion of brightness patterns in the image.

(Here, if you take each point in the first frame and try to figure out where the point landed in the second image, that would be the motion field)

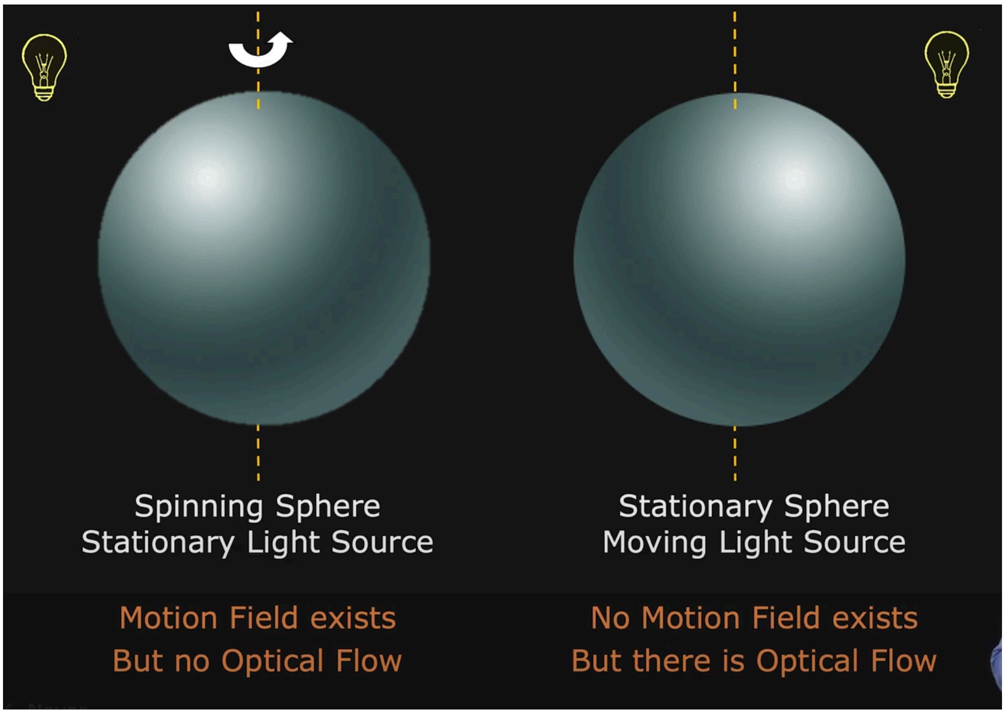
But what we can do is take a brightness pattern in one image and figure out where the brightness pattern is in the second image.

And if you have a successful algorithm that can do that, we can get a result that looks like this:

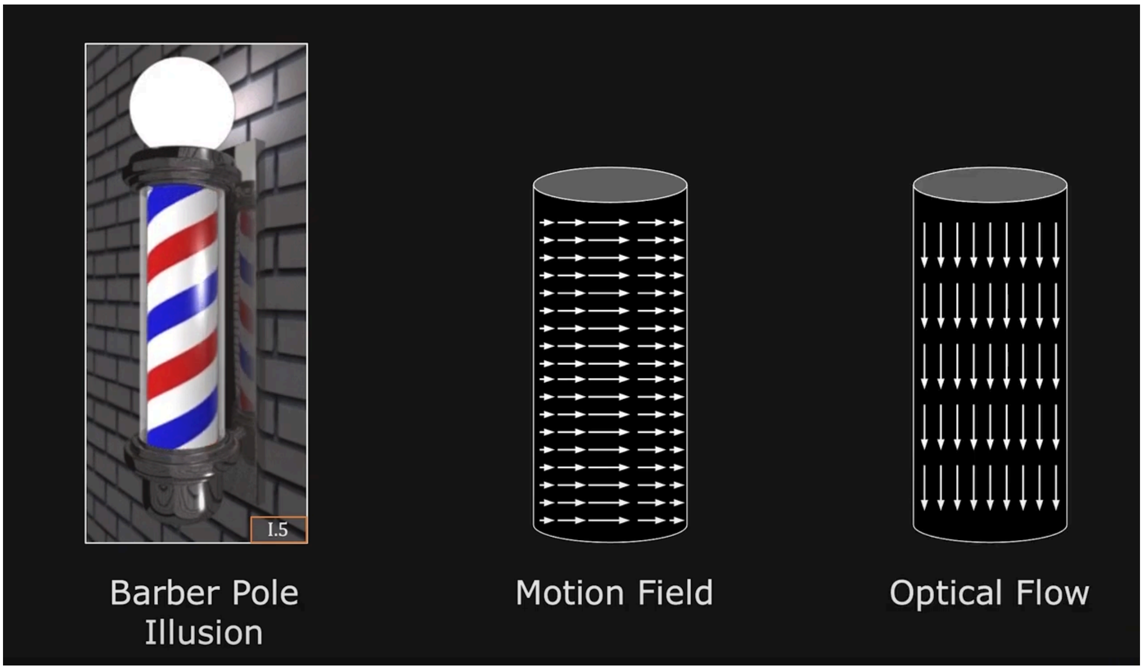


At each pixel, you have a vector that tells you what the optical flow at the point is. So, the motion of brightness patterns is the optical flow. And the optical flow vector's length tells you how fast its moving and direction tells you direction it's moving in the IMAGE PLANE.

Ideally Optical Flow = Motion Field. Unfortunately, not most of the time.



Another case where motion field and optical flow exists, but not the same



I.5
Barber Pole Illusion
Motion Field
Optical Flow