NUMBER THEORY handout 1

$\log_A(Rhythm)$

August 22, 2023

§1 Problems

Problem 1.1 (2020 AIME II P.1)**.** Find the number of ordered pairs of positive integers (m, n) such that $m^2n = 20^{20}$.

Problem 1.2 (2019 AIME I P.1)**.** Consider the integer

$$N = 9 + 99 + 999 + 9999 + \cdots + \underbrace{99\ldots99}_{\text{321 digits}}.$$

Find the sum of the digits of N.

Problem 1.3 (1995 AIME P.6)**.** Let $n = 2^{31}3^{19}$. How many positive integer divisors of n^2 are less than n but do not divide n?

Problem 1.4 (2021 DIME P.1)**.** Find the remainder when the number of positive divisors of the value

$$(3^{2020} + 3^{2021})(3^{2021} + 3^{2022})(3^{2022} + 3^{2023})(3^{2023} + 3^{2024})$$

is divided by 1000.

Problem 1.5 (2023 MBMT P.7)**.** What is the largest integer n such that 3^n is a factor of $18! + 19! + 20!$?

Problem 1.6 (2022 BMT P.2)**.** Compute the number of positive integer divisors of 100000 which do not contain the digit 0.

Problem 1.7 (2022 IOQM P.6)**.** Let a, b be positive integers satisfying $a^3 - b^3 - ab = 25$. Find the largest possible value of $a^2 + b^3$.

Problem 1.8 (2007 PAMO P.5)**.** For which positive integers n is $231^n - 222^n - 8^n - 1$ divisible by 2007?

Problem 1.9 (1983 AIME P.8)**.** What is the largest 2-digit prime factor of the integer $n = \binom{200}{100}$?

Problem 1.10 (1984 AIME P.2)**.** The integer n is the smallest positive multiple of 15 such that every digit of n is either 8 or 0. Compute $\frac{n}{15}$.