Stochastic Simulation
Spring 2025 February 18, 2025

Homework #2: Generating Multivariate RV and Stochastic Processes
Due: March 6, noon

Part I: Theoretic Questions
#1 **(Embedded Chain)** Consider a CTMC with 3 states and transition rate matrix

$$Q = \begin{pmatrix} -10 & 5 & 5 \\ 2 & -5 & 3 \\ 4 & 1 & -5 \end{pmatrix}.$$

Specify the transition probability matrix of its embedded chain and the exponential clocks corresponding to each state.

Part II: Numerical Experiments: For the following problems, report your numerical results with properly designed tables or graphs and clearly explain the results with a few sentences. Put your .py file (or Jupyter notebook files) in a single zip file and submit it via Blackboard. In case you use any data input, you should also include your .csv files or other type of files in your submission. Please name your file `Lastname_StudentID.zip`. Also, give meaningful names to your decision variables and constraints, and add comments to your code liberally.

#2 **(Copulas)** Recall that a bivariate copula $C(x, y)$ is the joint distribution function of two uniform random numbers. In the numerical experiments, you will implement several copulas and compare the dependence structures generated by these copulas.

(a) Implement a function that can generate i.i.d. samples of $(U_1, U_2) \sim C(x, y)$ where $C(x, y)$ is a Gaussian copula induced by two standard normal random variables W_1 and W_2 with correlation coefficient ρ. The parameter ρ should be part of inputs of your function.

(b) Estimated the covariance $Cov(U_1, U_2)$ using your simulation function with 10,000 repetitions for $\rho \in \{\pm 0.8, \pm 0.6, \pm 0.4, \pm 0.2\}$. Produce scatter plots for U_1 and U_2 with $\rho \in \{-0.6, 0, 0.6\}$ and 100 sample points.

(c) Use the Copula to generated copies of two dependent exponential random variables X_1 and X_2, both of which have rate 1.

(d) Estimated the covariance $Cov(X_1, X_2)$ using your simulation function with 10,000 repetitions for $\rho \in \{\pm 0.8, \pm 0.6, \pm 0.4, \pm 0.2\}$.

#3 **(Poisson Processes)** In the lecture, we have discussed how to generate homogeneous Poisson process via inter-arrival times (method 1). We also mentioned that there is an alternative way to generate Poisson process by first generating the total number of arrivals and then distributing each arrival uniformly on the time interval (method 2).

(i) Implement method 1 and generate a Poisson process with rate 5 on time interval $[0, 2]$. Verifying your implementation by showing that the simulated total number of arrivals follows a Poisson distribution with mean 5.

(ii) Implement method 1 and generate a Poisson process with rate 5 on time interval $[0, 2]$. Verifying your implementation by showing that the simulated inter-arrivals follows exponential distributions with rate 5.

(iii) Do you think if method 2 can be extended to non-homogeneous Poisson process? If yes, please write down the pseudo code. If no, please explain your reason.

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