

# „Vehicle Dynamics Modelling and Simulation”

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ROZDZIAŁ 7

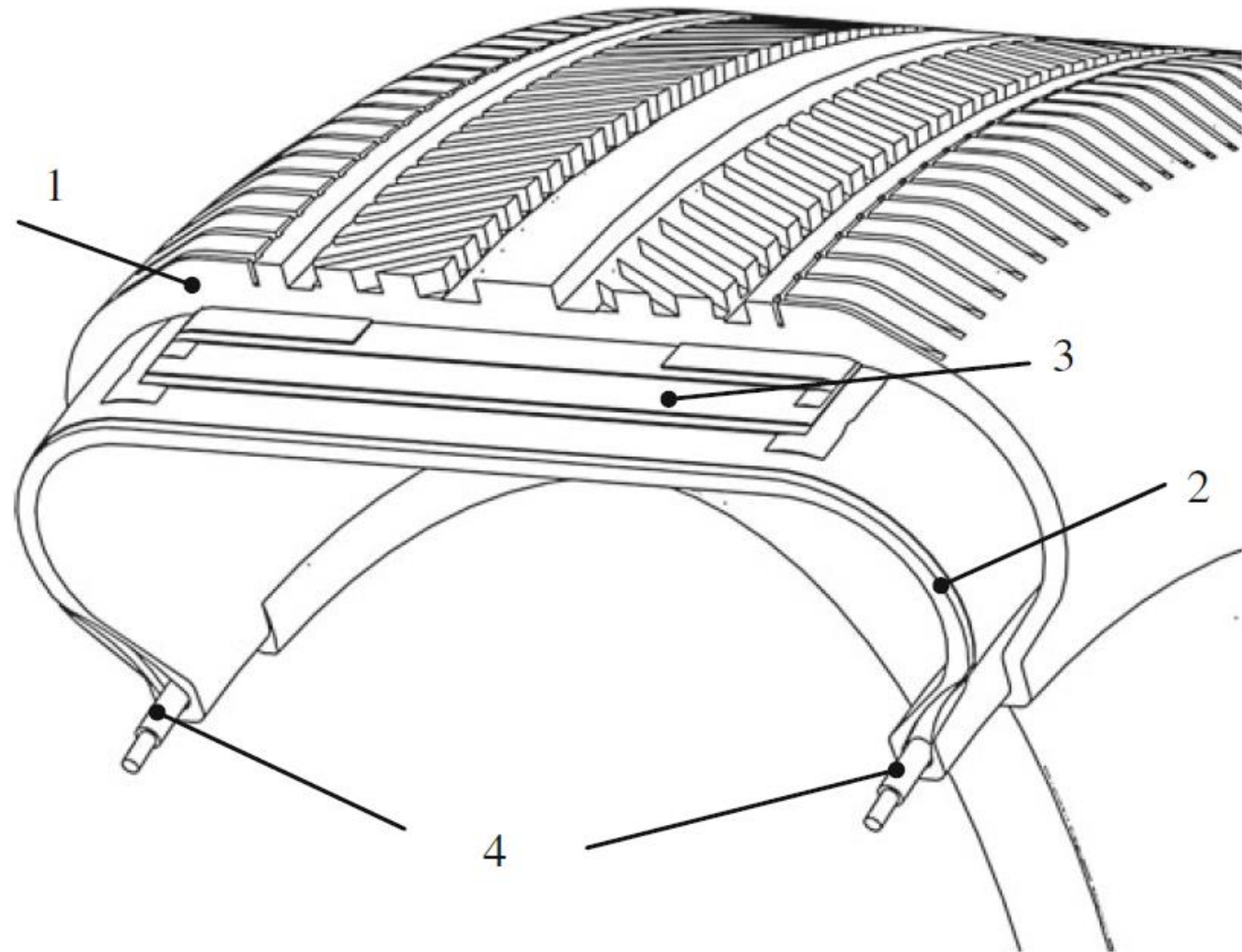
# Budowa opony

1 – bieżnik, powierzchnia toczna

2 – osnowa

3 – opasanie

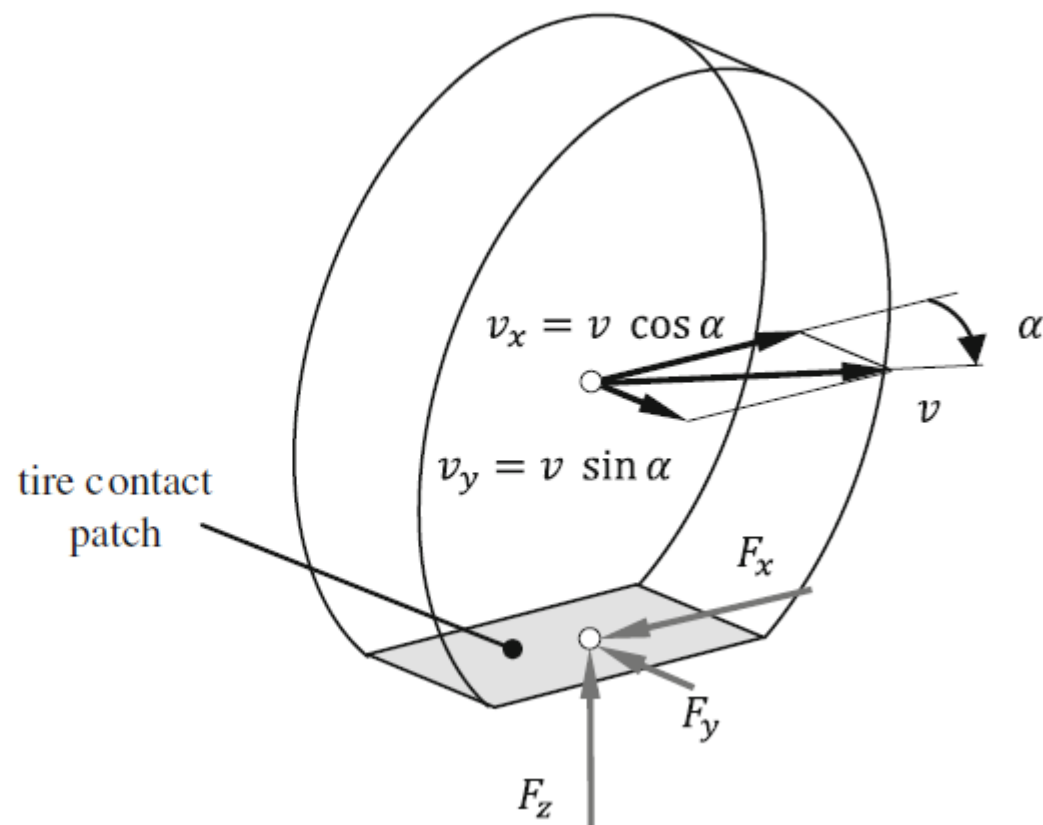
4 – drutówka (bead ring)



**Fig. 7.1** Basic construction of a radial tire

# Rozkład sił

**Fig. 7.2** Contact forces between road surface and tire



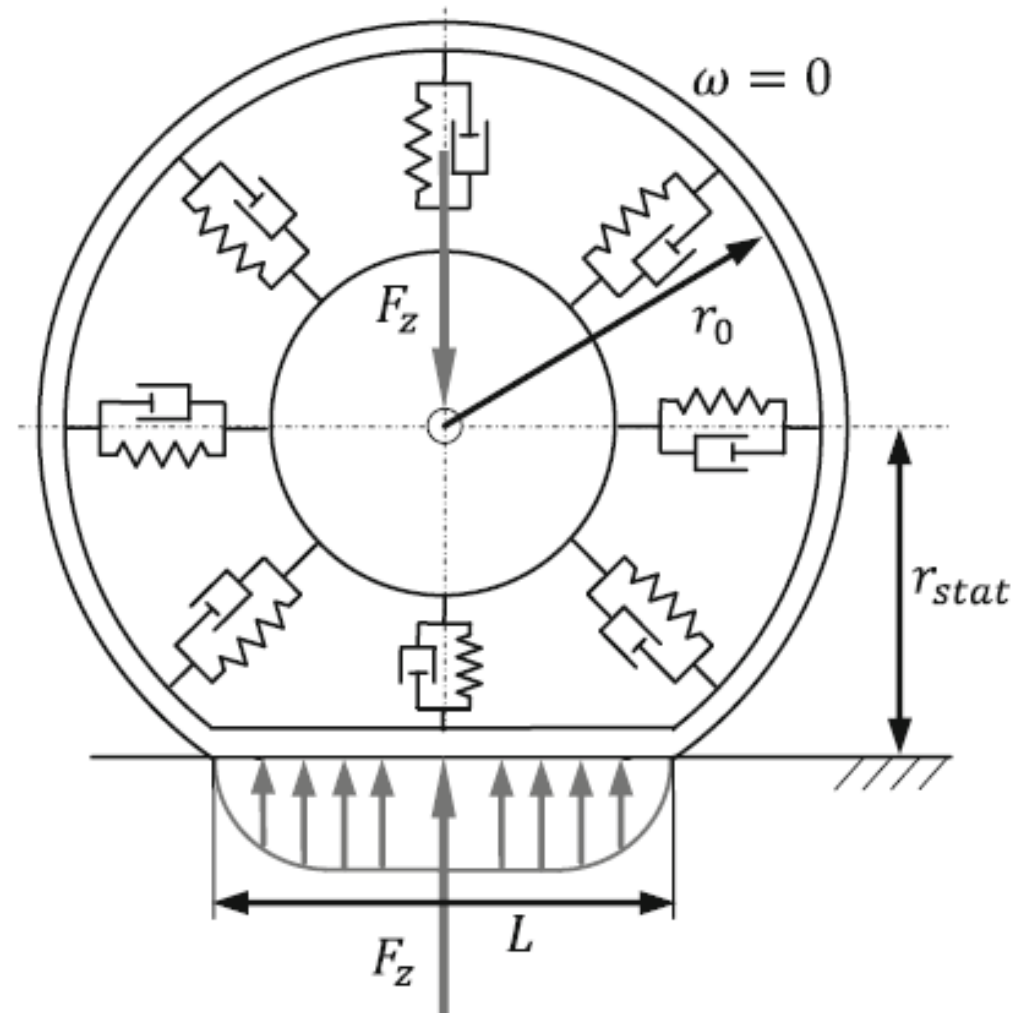
**Table 7.1** Components of the contact forces at the tires

Direction	Force description	Symbol
Longitudinal, circumferential	Longitudinal force, circumferential force	$F_x$
Transversal, lateral	Transversal force, side force, lateral force	$F_y$
Vertical	Vertical force, upward force, tire contact force	$F_z$

# Ugięcie statyczne

$$\Delta r = r_0 - r_{stat}$$

$$L \approx 2\sqrt{2}r_0\Delta r$$

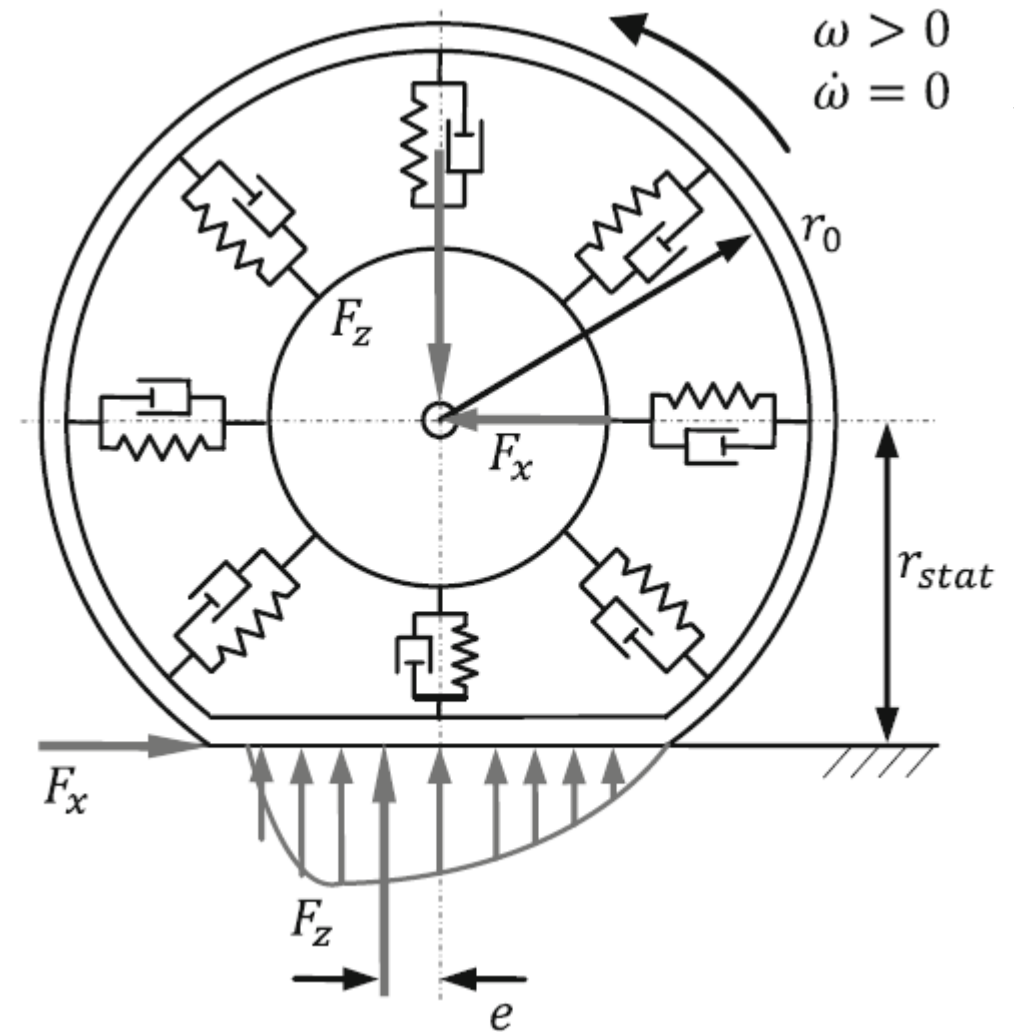


# Opór toczenia

$$F_x r_{stat} - F_z e = 0$$

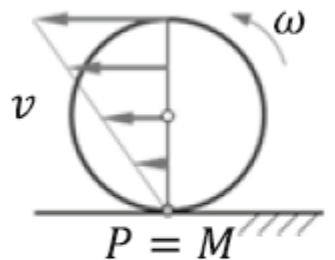
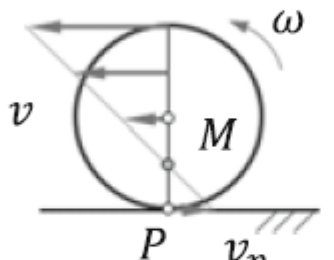
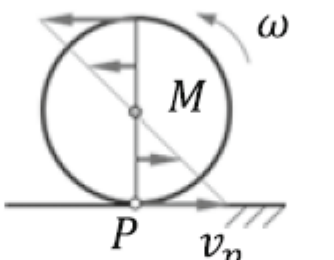
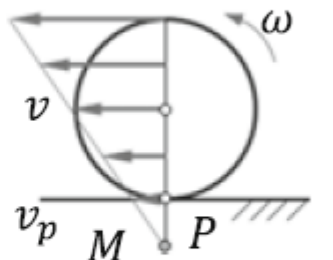
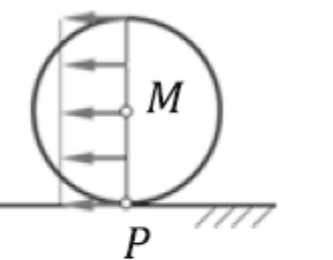
$$F_R = F_x = \frac{e}{r_{stat}} F_z$$

$$\mu = \frac{F_x}{F_z}$$



# Poślizg wzdluzny

**Table 7.2** Rigid body slip

$v = \omega r$	$v < \omega r$		$v > \omega r$	
Rolling wheel	Driven wheel	Spinning wheel	Braked wheel	Blocked wheel
				
No slip	Drive slip		Brake slip	
$s_A = 0$	$s_A = \frac{v_P}{\omega r}$	$s_A = 1$	$s_B = \frac{v_P}{v}$	$s_B = 1$
$s_B = 0$	$= \frac{\omega r - v}{\omega r}$		$= \frac{v - \omega r}{v}$	

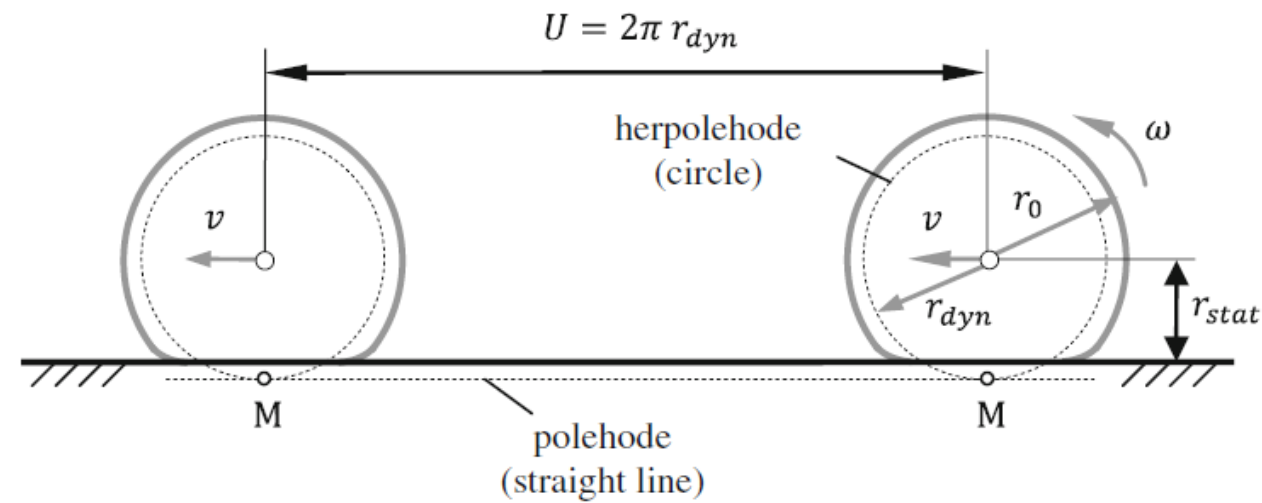
# Obwód toczenia i promień dynamiczny

$$r_{dyn} = \frac{U}{2\pi}$$

$$U = r_{dyn}\alpha = r_0 \sin \alpha \rightarrow r_{dyn} = r_0 \frac{\sin \alpha}{\alpha} \approx r_0 \left(1 - \frac{\alpha^2}{6}\right).$$

$$r_{stat} = r_0 \cos \alpha \approx r_0 \left(1 - \frac{\alpha^2}{2}\right) \rightarrow \frac{\alpha^2}{2} \approx \left(1 - \frac{r_{stat}}{r_0}\right)$$

$$r_{dyn} \approx r_0 \left(1 - \frac{\alpha^2}{6}\right) \approx r_0 \left(1 - \frac{1}{3} + \frac{r_{stat}}{3r_0}\right) = \frac{2}{3}r_0 + \frac{1}{3}r_{stat}$$



$$s_A = \frac{\omega r_{dyn} - v}{\omega r_{dyn}}$$

$$s_B = \frac{v - \omega r_{dyn}}{v}$$

# Prędkość poślizgu

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$$v_P = v - \omega r_{dyn}.$$

- Slip-less motion:  $v = \omega r_{dyn} \rightarrow v_P = 0$
- Driven wheel  $v < \omega r_{dyn} \rightarrow v_P < 0$
- Braked wheel:  $v > \omega r_{dyn} \rightarrow v_P > 0$



# Odkształcenie

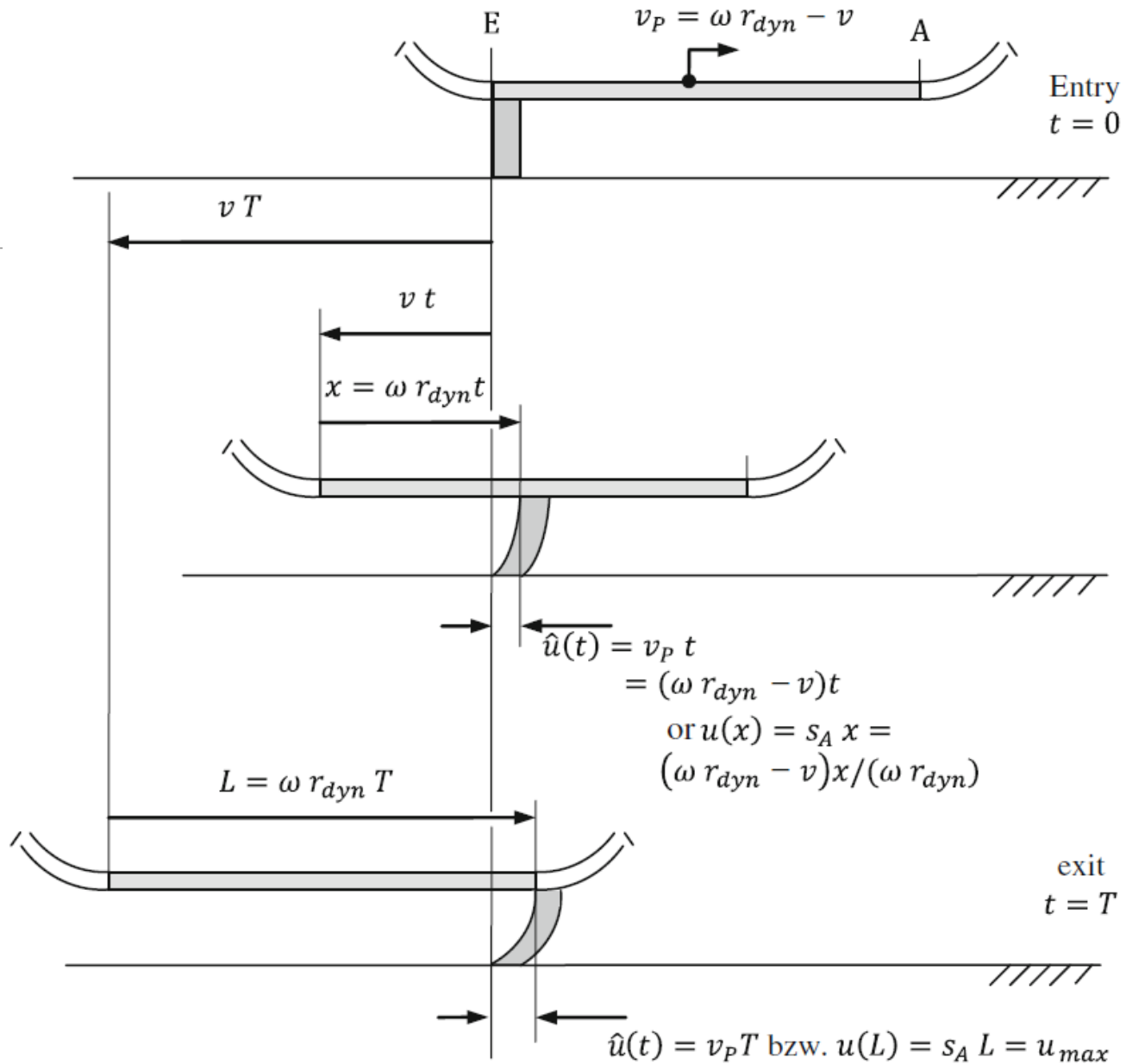
$$x = \omega r_{dyn} t$$

$$\hat{u}(t) = v_P t = (\omega r_{dyn} - v)t$$

$$u(x) = (\omega r_{dyn} - v) \frac{x}{\omega r_{dyn}} = s_A x$$

$$T = \frac{L}{\omega r_{dyn}}$$

$$u_{max} = u(L) = s_A L$$



# Naprężenia styczne

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$$\tau_x(x) = ku(x)$$

- Profil
- Moduł sprężystości
- Wysokość bieżnika

$$F_x = \int_A \tau_x(x) dA = \int_0^L \tau_x(x)b dx = \frac{1}{2}kbL^2s_A$$

# Adhezja

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Siła pionowa

$$F_Z = \int_A p_Z(x) dA = \int_0^L p_Z(x) b dx$$

Warunek dla naprężeń stycznych

$$\tau_x(x) \leq \tau_{xH}(x) \quad \text{with} \quad \tau_{xH}(x) = \mu_H p_Z(x)$$

Naprężenia podczas poślizgu

$$\tau_{xG}(x) = \mu_G p_Z(x)$$

Tarcie opony o asfalt

$$\mu_G < \mu_H \rightarrow \tau_{xG}(x) < \tau_{xH}(x) = \mu_H p_Z(x)$$

# Krzywa siły poślizgu

Siła obwodowa i sztywność

$$F_x = c_s s$$

$$c_s = \frac{1}{2} k b L^2$$

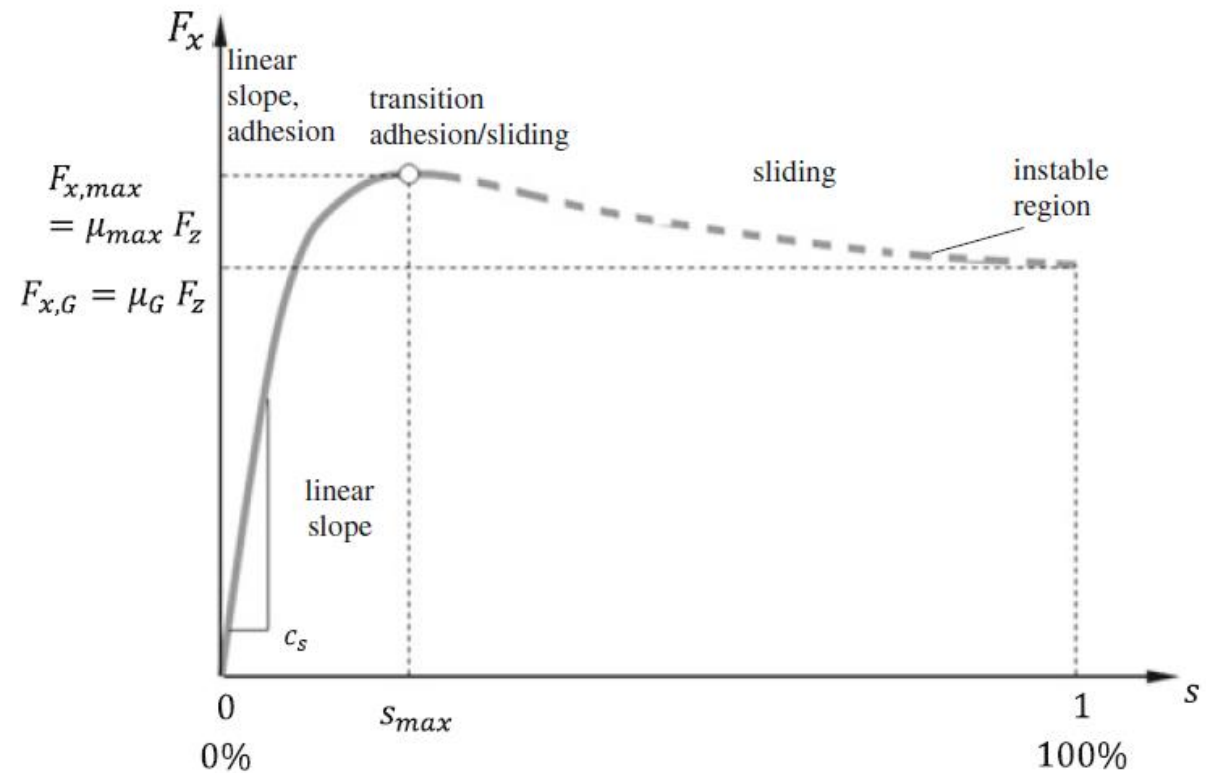
Współczynnik przyczepności max

$$\mu_{max} = \frac{F_{x,max}}{F_Z}$$

$$\mu_{max} < \mu_H$$

Współczynnik tarcia ślizgowego

$$\mu_G = \frac{F_{x,G}}{F_Z}$$

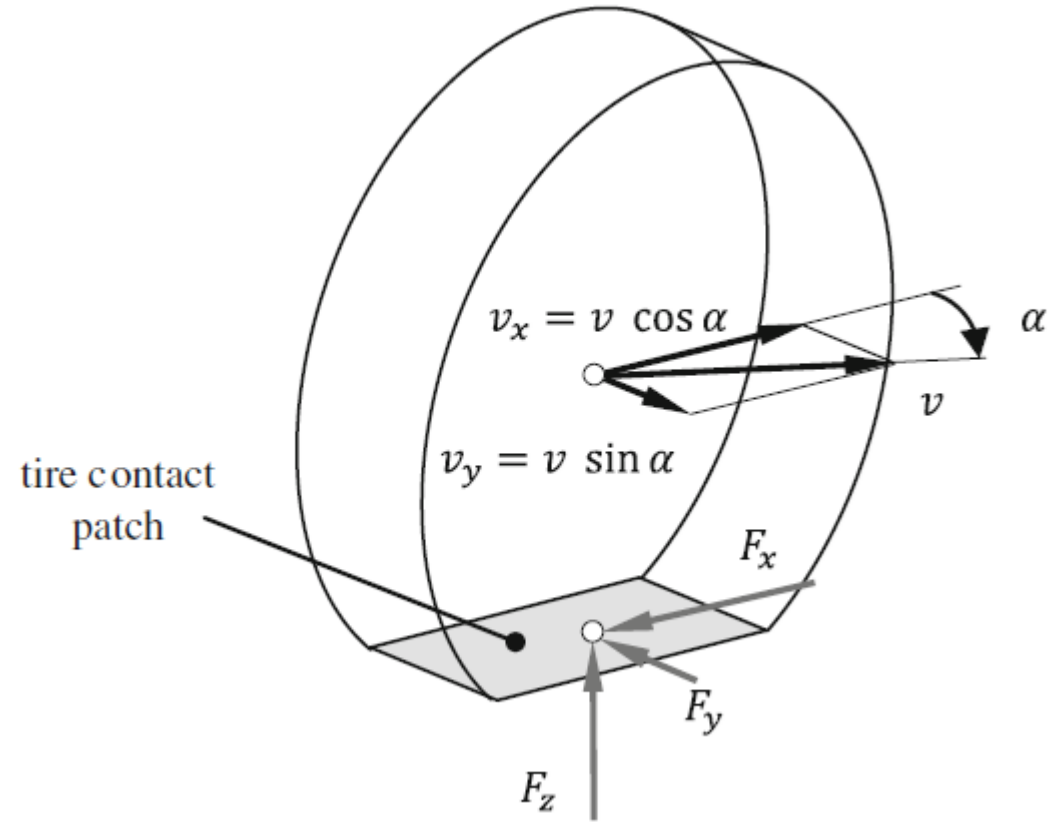


# Kąt poślizgu, siła boczna

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$$\tan \alpha = \frac{v_y}{v_x} \quad \sin \alpha = \frac{v_y}{v}$$

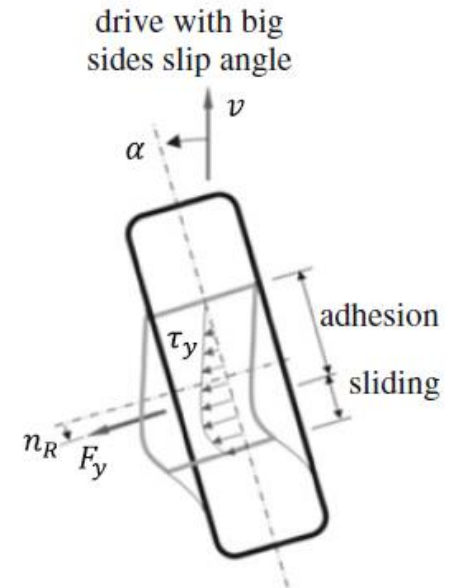
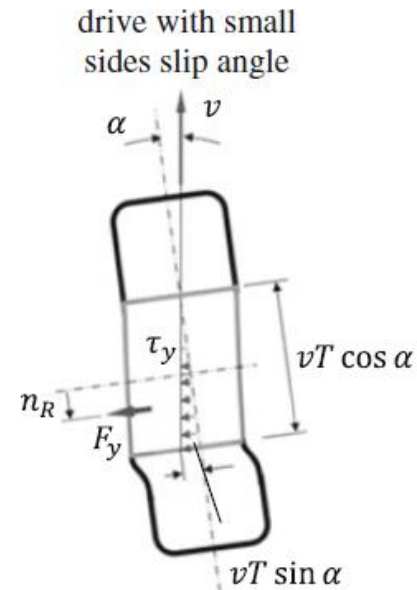
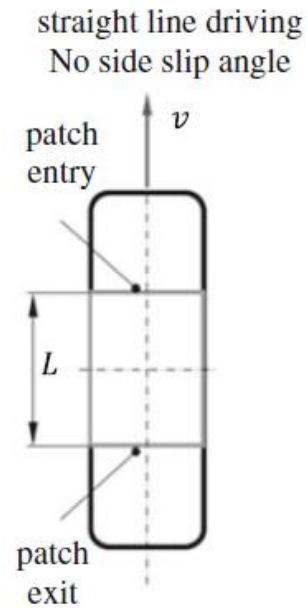
$$F_y = c_\alpha \sin \alpha \cos \alpha$$



# Moment obrotowy

$$M_z = n_R F_y$$

$$n_R \approx \frac{1}{6}L$$



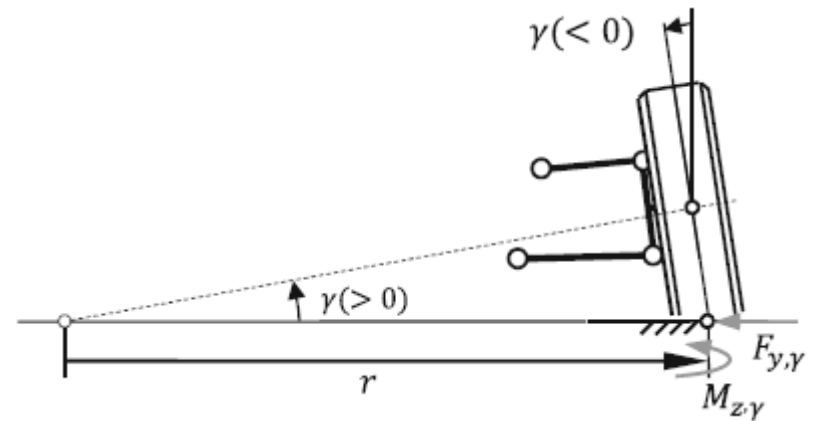
# Wpływ kąta pochylenia koła

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Dla kąta pochylenia koła  $< 5^\circ$

$$F_{y,\gamma} = -c_\gamma \gamma$$

$$M_{z,\gamma} = -c_{M,\gamma} \gamma$$



# Superpozycja

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$$\sqrt{F_x^2 + F_y^2} \leq \mu_{\max} F_z$$

