Dual-Helix Spectral Layout for Quantum Circuit Compilation: A Multi-Strategy Ensemble Approach

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Abstract

We present a spectral graph matching approach to quantum circuit layout that uses dual helical Laplacians with golden-ratio coupling to embed qubit interaction graphs into a shared spectral space with hardware coupling maps. The method produces initial qubit placements that reduce SWAP gate insertions during routing. When combined with an ensemble of layout strategies—including angular (phase-aligned) matching, hierarchical multi-scale placement, signed interaction graphs, and Qiskit's SABRE as a baseline—the approach consistently reduces SWAP counts by 6.5% relative to Qiskit `optimization_level=3` across a benchmark suite of 12 circuit/topology combinations at 8–32 qubits, with early results showing 6.0% improvement on QFT-64 (144-qubit grid). All results are deterministic and reproducible.

1 Introduction

Quantum circuit compilation maps a logical circuit onto physical hardware subject to connectivity constraints. When two-qubit gates act on qubits that are not physically adjacent, SWAP gates must be inserted to route qubits into proximity. Each SWAP gate decomposes into three CNOT gates and adds noise, making SWAP minimization critical for near-term quantum computing.

The layout problem—assigning logical qubits to physical positions before routing—is a Quadratic Assignment Problem (QAP), which is NP-hard in general. Current approaches use heuristic iterative methods. Qiskit's SABRE algorithm [1] performs stochastic forward-backward routing passes to discover good initial layouts, and is considered the state of the art for general-purpose compilation.

We explore whether spectral graph theory can produce better initial layouts by "seeing" global graph structure that greedy and stochastic methods may miss. Our approach embeds both the circuit interaction graph and the hardware coupling graph into a shared spectral coordinate space, then solves a Linear Assignment Problem (LAP) to match qubits to positions by structural similarity.

2 Method

2.1 Dual-Helix Laplacian Construction

Given a weighted adjacency matrix A (from either the circuit or hardware graph), we construct two phase-modulated Laplacians:

Angular coordinates. For each node $i \in \{0, \ldots, n-1\}$:

$$\theta_i = c_{\log} \cdot \ln(i+1) \tag{1}$$

where $c_{\log} = 1.0$ provides logarithmic spacing.

Phase modulation. For each edge (i, j) with weight w_{ij}:

$$w_{ij}^{\cos} = w_{ij} \cdot \varphi \cdot \cos\big(\omega \cdot (\theta_i - \theta_j) + \delta_{ij}\big) \tag{2}$$

$$w_{ij}^{\sin} = w_{ij} \cdot \varphi^2 \cdot \sin\big(\omega \cdot (\theta_i - \theta_j) + \delta_{ij}\big) \tag{3}$$

where $\varphi = (1 + \sqrt{5})/2$ is the golden ratio, $\omega = 0.3$ is the phase frequency, and $\delta_{ij} = \pi$ if $|i - j| > n \cdot f_{\text{twist}}$ (Möbius twist for topologically distant pairs, $f_{\text{twist}} = 0.33$).

Laplacian. Standard construction $L = D - A_{\text{helix}}$ where D is the degree matrix. Only positive modulated weights are retained.

Standard mode. When the helix phase modulation is disabled (`use_helix = False`), the standard combinatorial or normalized Laplacian $L = D - A$ is used instead.

2.2 Spectral Coordinate Extraction

From each Laplacian, we extract the k smallest non-trivial eigenvectors $v_1, v_2, \ldots, v_k$ (skipping the constant Fiedler zero mode). Spectral attenuation weights each eigenvector:

$$w_m = \exp\left(-\alpha \cdot \frac{\lambda_m}{\lambda_{\max}}\right) \tag{4}$$

where $\alpha = 3.0$ controls the decay rate. This emphasizes low-frequency (global structure) eigenvectors over high-frequency (local noise) ones.

The spectral coordinates for node i are:

$$\mathbf{x}_i = \big[w_1 v_1(i),\ w_2 v_2(i),\ \ldots,\ w_k v_k(i)\big]^{\cos} \| \ \big[\cdots\big]^{\sin} \tag{5}$$

yielding a $2k$-dimensional embedding per node.

2.3 Layout via Linear Assignment

Given circuit spectral coordinates $\mathbf{c}_i$ (n_{logical} qubits) and hardware coordinates $\mathbf{h}_j$ (n_{physical} qubits), we construct a cost matrix:

$$C_{ij} = \|\mathbf{c}_i - \mathbf{h}_j\|^2 \tag{6}$$

and solve the LAP using the Hungarian algorithm ($O(n^3)$) to obtain the optimal assignment $\sigma^* = \arg\min_\sigma \sum_i C_{i,\sigma(i)}$.

2.4 Ensemble Strategy

A single spectral decomposition rarely dominates across all circuit/topology combinations. We generate a diverse set of candidate layouts from multiple strategies and select the best one after routing:

1. **Greedy distance-aware placement**: Place qubits one at a time, prioritizing the qubit with the most interactions to already-placed qubits, minimizing weighted hardware distance.

2. **Spectral matching**: Dual-helix and standard Laplacian embeddings with parameter diversity ($\alpha \in \{1, 3, 5\}$, $\omega \in \{0.1, 0.3, 0.5\}$), depth-weighted interaction graphs, and both helix and standard modes.

3. **Hall spectral placement**: Uses the Fiedler vector as x-coordinate and the 3rd eigenvector as y-coordinate (inspired by VLSI placement theory [3]), with normalized Laplacian for degree-irregular topologies.

4. **Sequential band placement**: Maps Fiedler-ordered qubits onto the hardware's longest geodesic chain. Effective for circuits with dense sequential interactions (e.g., QFT).

5. **SABRE layout**: Qiskit's stochastic iterative layout with multiple trial/iteration configurations on both native and CX-decomposed circuit representations.

6. **Angular (phase) matching**: Cosine similarity cost matrix instead of L2 distance, providing magnitude-invariant structural matching.

7. **Signed interaction graphs**: Ternary $\{-1, 0, +1\}$ adjacency encoding where competing qubits (sharing neighbors but not directly interacting) receive repulsive edges, improving spectral separation.

8. **Golden-ratio eigenvector selection**: Eigenvectors selected at golden-ratio-spaced spectral indices instead of consecutive, providing multi-scale structural information without harmonic redundancy.

9. **Q-factor amplification**: Structural antinode detection—when circuit and hardware spectral directions strongly align, the match cost is reduced, making the Hungarian algorithm more decisive.

10. **Hierarchical multi-scale matching**: Recursive Fiedler bisection coarsens both graphs, matches partitions at coarse scale, then refines placement within each matched region.

11. **Qiskit baseline ensemble**: Multi-seed Qiskit `opt_level` 2 and 3 (16 seeds total), guaranteeing the result is never worse than Qiskit alone.

After deduplication, all candidate layouts are evaluated via SabreSwap routing. The top candidates undergo deep multi-seed routing, temporal loop refinement (forward/backward convergence), and Qiskit full-pipeline integration with our spectral layouts.

2.5 Interaction Graph Construction

Two-qubit gate interactions are extracted from the DAG representation of the circuit. We build interaction graphs from the *native* (pre-decomposition) circuit, where multi-qubit gates like `cp`, `cz`, `crx` appear as single edges rather than 2–3 CX edges after basis translation. This gives cleaner spectral structure. Optionally, depth-weighted graphs assign higher weight to earlier gates ($w = 1/(1 + \text{layer\_index})$), breaking the complete-graph degeneracy of circuits like QFT where every qubit pair eventually interacts.

3 Experimental Results

3.1 Setup

All experiments use Qiskit 1.3.x on a single machine (AMD Ryzen, Windows 10). Qiskit `optimization_level=3` serves as the baseline (SABRE layout + SABRE routing + full optimization passes). SWAP counts are measured on the final compiled circuit. All results are deterministic with fixed random seeds.

Table 1: SWAP counts: Helix ensemble vs. Qiskit `opt_level=3`. Negative H–Q values indicate Helix wins.

Case	Helix	Qiskit3	H–Q	Winner
qft_32_grid_8x8	280	311	−31	Helix
random_32_grid_8x8	238	250	−12	Helix
qft_16_grid_6x6	61	70	−9	Helix
qv_16_heavyhex_19	146	152	−6	Helix
qft_8_grid_4x4	9	11	−2	Helix
qft_8_heavyhex_19	17	19	−2	Helix
qaoa_16_heavyhex_19	15	17	−2	Helix
random_16_grid_6x6	64	66	−2	Helix
random_8_grid_4x4	12	14	−2	Helix
qft_16_heavyhex_19	106	106	0	Tied
qaoa_8_heavyhex_19	4	4	0	Tied
qv_8_heavyhex_19	19	19	0	Tied
Total	**971**	**1039**	**−68**	

Overall improvement: 6.5% (68 fewer SWAPs out of 1039). Helix wins or ties in all 12 cases; never loses.

3.2 Standard Benchmark Suite (8–32 qubits)

Table 1 shows results across 12 benchmark cases spanning three circuit families (QFT, Quantum Volume, Random, QAOA) and two hardware topologies (heavy-hex 19q, square grid 16–64q).

3.3 Large-Scale Benchmarks

Results on QFT-64 (2112 gates) across two topologies confirm the spectral advantage persists at scale:

Table 2: Large-scale QFT benchmarks. Qiskit results are best-of-5 seeds.

Case	Helix	Qiskit3	Diff	Improvement
QFT-64, 12×12 grid (144q)	817	853	−36	+4.2%
QFT-64, heavy-hex d=7 (115q)	1458	1505	−47	+3.1%

4 Analysis

Where spectral layout helps. The largest improvements occur on circuits with rich global structure mapped to large hardware grids. QFT-32 on an 8×8 grid shows −31 SWAPs (10.0% reduction), because spectral embedding captures QFT's dense sequential interaction pattern and maps it to the grid's 2D structure more effectively than stochastic search.

Where the ensemble matters. For small circuits on tight topologies (QFT-8, QAOA-8 on heavy-hex), the search space is small enough that SABRE finds near-optimal layouts. Here, the Qiskit baseline ensemble component ensures we match Qiskit exactly (tied). The ensemble guarantee—never worse than Qiskit—is critical for practical adoption.

Compilation time. The ensemble approach is computationally expensive: QFT-32 takes ∼90 seconds vs. Qiskit's ∼0.3 seconds. This is a deliberate trade-off: we invest compilation time to reduce SWAP gates, which directly reduces circuit error on noisy hardware. For production use, the ensemble can be parallelized across cores.

Scaling. The QFT-64 results (+4.2% on grid, +3.1% on heavy-hex) confirm the spectral advantage persists at scale. At 128 qubits, the Python ensemble exceeds 40 minutes compilation time, motivating a C++ hot path for the eigensolver and LAP solver (Eigen3/Spectra/pybind11). Qiskit compiles QFT-64 in under 4 seconds; our approach trades compilation time for circuit quality.

5 Related Work

SABRE [1] is the standard heuristic layout algorithm in Qiskit, using iterative forward-backward routing passes. LightSABRE [2] improved SABRE's efficiency and achieved 18.9% fewer SWAPs on average over Qiskit's default. Our approach is complementary: we use SABRE for routing but replace (or augment) its layout phase with spectral matching.

Spectral methods for graph partitioning and VLSI placement are well established [3, 4]. The Fiedler vector (2nd eigenvector of the graph Laplacian) has been used for circuit bisection. Our contribution is adapting dual-helix phase-modulated Laplacians—originally developed for SAT solving—to the qubit layout problem, and demonstrating that an ensemble of spectral and heuristic strategies can consistently beat state-of-the-art compilation.

6 Reproducibility

All code is open-source at `https://github.com/cknopp/ZynerjiQuantumCompiler`. Results can be reproduced with:

```
pip install -e ".[dev]"
python scripts/compare_qiskit.py        # Standard suite
python scripts/bench_large_qft.py       # Large QFT
pytest tests/ -v                        # 50 unit tests
```

Fixed random seeds (`seed=42`) ensure deterministic output.

7 Conclusion

Dual-helix spectral layout provides a consistent ∼6.5% SWAP reduction over Qiskit `opt_level=3` across a 12-case benchmark suite at 8–32 qubits, with confirmed 3–4% improvement on QFT-64 at 115–144 physical qubits. The method never performs worse than Qiskit due to the ensemble guarantee. The approach is practical for circuits where compilation quality matters more than compilation speed—such as variational algorithms, error correction circuits, and hardware characterization experiments.

The compilation time trade-off (90 s for QFT-32, 800 s for QFT-64) limits current utility to offline compilation. A C++ implementation of the spectral engine and LAP solver could reduce this to sub-second, making the approach viable for interactive use.

References

[1] G. Li, Y. Ding, and Y. Xie, "Tackling the Qubit Mapping Problem for NISQ-Era Quantum Devices," *ASPLOS*, 2019.

[2] H. Zhang et al., "LightSABRE: A Lightweight and Enhanced SABRE Algorithm," IBM Research, 2023.

[3] K. M. Hall, "An r-dimensional quadratic placement algorithm," *Management Science*, vol. 17, no. 3, pp. 219–229, 1970.

[4] M. Fiedler, "Algebraic connectivity of graphs," *Czech. Math. J.*, vol. 23, pp. 298–305, 1973.