Markov's Property on Geometric Distribution Proof

Zyon Rams

March 2024

Given a probability of success p, s.t. $p \in [0, 1]$, since within our program we are currently using the geometric distribution, let us use the given geometric distribution equation which is $P(X = x) = q^{x-1}p$. To prove Markov's Property we need to show that:

$$P(X = x + n|X > x) = P(X = n)$$

Let us use the definition of Conditional Probabilities:

$$P(X = x + n|X > x) = \frac{P(X = x + n \cap X > x)}{P(X > x)}$$

We know that the intersection of $P(X = x+n\cap X > x)$ is simply $P(X = x+n)$, therefore we can say that:

$$P(X = x + n|X > x) = \frac{P(X = x + n)}{P(X > x)}$$

Now let us use the definition of the Geometric Distribution where the Probability density function is denoted as $P(X = x) = q^{x-1}p$ whilst the cumulative probability distribution is denoted as $P(X <= x) = 1 - (1 - p)^x$, with this definition we can say that $P(X > x) = 1 - (1 - (1 - p)^x) = q^x$, therefor our equation is:

$$P(X = x + n|X > x) = \frac{q^{x+n-1}p}{q^x}$$

$$P(X = x + n|X > x) = q^{n-1}p$$

Since we know that $P(X = n) = q^{n-1}p$ by the definition of the Geometric Distribution Probability Density Function we have shown that:

$$P(X = x + n|X > x) = P(X = n)$$