Discrete Structures (CS 335)

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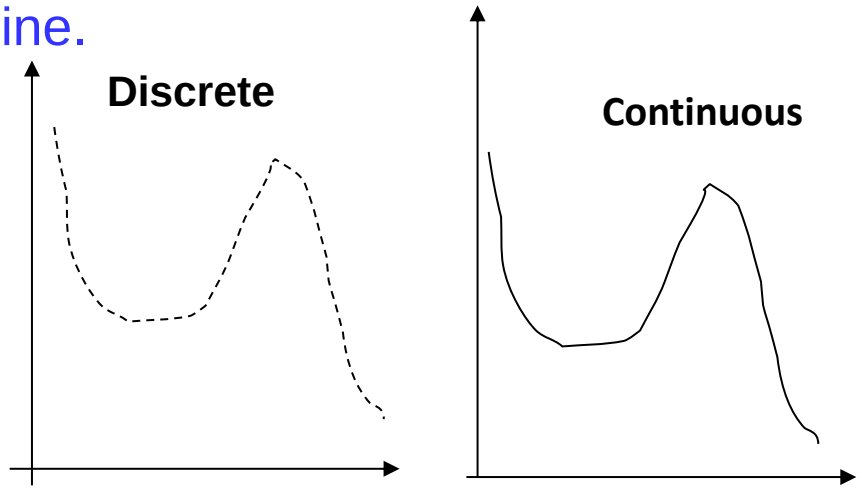
Discrete vs Continuous

- Examples of discrete Data
  - Number of boys in the class.
  - Number of candies in a packet.
  - Number of suitcases lost by an airline.
- Examples of continuous Data
  - Height of a person.
  - Time in a race.
  - Distance traveled by a car.



What is discrete Structures?

- Discrete mathematics is the part of mathematics devoted to the study of discrete objects (Kenneth H. Rosen, 6th edition).

- Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous (wikipedia).

Syllabus (Topics to be covered in this course)

- Logic
- Elementary Number Theory and Methods of Proof
- Set Theory
- Relations
- Sequences and Recursion
- Mathematical Induction
- Counting
- Relations and Equivalence Relations
- Graphs
- Trees

Reference Books

- Discrete Mathematics and its Applications
  (with Combinatorics and Graph Theory)
  5th Edition, The McGraw-Hill Companies, 2007,
  Kenneth H. Rosen.
- Discrete Mathematics with Applications
  4th Edition, Thomson Learning, 1995,
  Susanna S. Epp.
- Discrete Mathematics for Computer Scientists
  2nd Edition, Addison-Wesley, 1999, John
  Truss.

Logic

- Propositional Logic
- Logic of Compound Statements
- Propositional Equivalences
- Conditional Statements
- Logical Equivalences
- Valid and Invalid Arguments
- Applications: Digital Logic Circuits
- Predicates and Quantifiers
- Logic of Quantified Statements

Propositional Logic

Proposition: A proposition (or Statement) is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.

Examples

1. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

   Islamabad is the capital of Pakistan.

   This makes a declarative statement, and hence is a proposition. The proposition is TRUE (T).

Examples (Propositions Cont.)

2. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

Can Ali come with you?.

This is a question not the declarative sentence and hence not a proposition.

Examples (Propositions Cont.)

3. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

Take two aspirins.

This is an imperative sentence not the declarative sentence and therefore not a proposition.

Examples (Propositions Cont.)

4. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

$x + 4 > 9$.

Because this is true for certain values of x (such as $x = 6$) and false for other values of x (such as $x = 5$), it is not a proposition.

Examples (Propositions Cont.)

5. Is the following sentence a proposition? If it is a proposition, determine whether it is true or false.

He is a college student.

Because truth or falsity of this proposition depend on the reference for the pronoun *he*. it is not a proposition.

Notations

- The small letters are commonly used to denote the propositional variables, that is, variables that represent propositions, such as, *p*, *q*, *r*, *s*, ….

- The truth value of a proposition is true, denoted by T or 1, if it is a true proposition and false, denoted by F or 0, if it is a false proposition.

Compound Propositions

Producing new propositions from existing propositions.

Logical Operators or Connectives

1. Not $\neg$
2. And $\wedge$
3. Or $\vee$
4. Exclusive or $\oplus$
5. Implication $\rightarrow$
6. Biconditional $\leftrightarrow$

Compound Propositions

Negation of a proposition

Let *p* be a proposition. The negation of *p*, denoted by ¬ *p (also denoted by ~p)*, is the statement

"It is not the case that p".

The proposition ¬ *p* is read as "not *p*". The truth values of the negation of *p*, ¬ *p,* is the opposite of the truth value of *p*.

Examples

1. Find the negation of the following proposition

p : Today is Friday.

The negation is

$\neg p$: It is not the case that today is Friday.

This negation can be more simply expressed by

$\neg p$: Today is not Friday.

Examples

2. Write the negation of

"6 is negative".

The negation is

"It is not the case that 6 is negative".

or "6 is nonnegative".

Truth Table (NOT)

- Unary Operator, Symbol: ¬

p	¬p
true	false
false	true

Conjunction (AND)

Definition

Let *p* and *q* be propositions. The conjunction of *p* and *q*, denoted by *p*^*q*, is the proposition "*p and q*".

The conjunction *p*^*q* is true when *p and q* are both true and is false otherwise.

Examples

1. Find the conjunction of the propositions p and q, where

p : Today is Friday.

q : It is raining today.

The conjunction is

$p \wedge q$: Today is Friday and it is raining today.

Truth Table (AND)

- Binary Operator, Symbol: $\wedge$

p	q	$p \wedge q$
true	true	true
true	false	false
false	true	false
false	false	false

Disjunction (OR)

Definition

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "*p or q*".

The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Examples

1. Find the disjunction of the propositions p and q, where

p : Today is Friday.

q : It is raining today.

The disjunction is

$p \vee q$: Today is Friday or it is raining today.

Truth Table (OR)

- Binary Operator, Symbol: $\vee$

p	q	$p \vee q$
true	true	true
true	false	true
false	true	true
false	false	false

Exclusive OR (XOR)

Definition

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition "$p \oplus q$".

The *exclusive or*, $p \oplus q$, is true when exactly one of p and q is true and is false otherwise.

Examples

1. Find the *exclusive or* of the propositions p and q, where

   p : Atif will pass the course CSC102.

   q : Atif will fail the course CSC102.

The *exclusive or* is

$p \oplus q$ *:* Atif will pass or fail the course CSC102.

Truth Table (XOR)

- Binary Operator, Symbol: $\oplus$

p	q	$p \oplus q$
true	true	false
true	false	true
false	true	true
false	false	false

Examples (OR vs XOR)

The following proposition uses the (English) connective "or". Determine from the context whether "or" is intended to be used in the inclusive or exclusive sense.

1. "Nabeel has one or two brothers".

A person cannot have both one and two brothers.
Therefore, "or" is used in the exclusive sense.

Examples (OR vs XOR)

2. To register for BSC you must have passed the qualifying exam or be listed as an Math major.

Presumably, if you have passed the qualifying exam and are also listed as an Math major, you can still register for BCS. Therefore, “or” is inclusive.

Composite Statements

Statements and operators can be combined in any way to form new statements.

p	q	$\neg p$	$\neg q$	$(\neg p) \vee (\neg q)$
true	true	false	false	false
true	false	false	true	true
false	true	true	false	true
false	false	true	true	true

Lecture Summery

- Introduction to the Course
- Propositions
- Logical Connectives
- Truth Tables
- Compound propositions